Effect of magnetic-vortex interaction on the kinetics of magnetic-flux creep in type II superconductors

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Magnetic vortices localized by the lattice's pinning potential in type II superconductors are used to study the repulsive interaction of quasiparticles on the kinetics of magnetic vortex creep. A microscopic model is developed in which the interaction of the quasiparticles (vortices) is allowed for in addition to their thermal activation through the pinning-potential barriers. When the localization of magnetic vortices is strong and their number density low, allowing for the repulsive interaction between vortices leads to a considerable increase in magnetic vortex diffusion, a situation ignored in previous models of magnetic flux creep. A kinetic equation describing the dynamics of magnetic flux creep under these conditions is derived. Finally, numerical and analytic solutions of the equation that describes the kinetics of magnetic flux creep are given. These solutions illustrate the variation of creep kinetics due to vortex interaction. © *1995 American Institute of Physics.*

1. INTRODUCTION

The kinetics of quasiparticles localized in a spatially inhomogeneous crystal lattice potential is an interesting and important problem in solid state physics. Its practical importance becomes obvious when the properties of polycrystalline bulk samples, films, and any other disordered systems are studied. The quasiparticles may be quite different: for example, they may be electrons if we are interested in hopping conductivity, or localized spins if we are studying spin tunneling. In hard type II superconductors, the quasiparticles may be magnetic vortices of the Abrikosov or Josephson type. Under strong localization, the quasiparticles are in wells of the effective potential produced by crystal lattice inhomogeneities, and may hop from one local minimum of this potential to another. Note that in addition to the height of the barriers and their separation, the interaction between quasiparticles may also be important.

In this paper we discuss this problem as it applies to the interaction of magnetic (e.g., Abrikosov) vortices hopping between wells of the pinning potential. We show that allowing for the repulsive interaction between vortices in the event of strong localization of magnetic vortices in pinning-potential wells may lead to a considerable increase in magnetic flux diffusion; this was not previously taken into account in studies of one-particle models of hopping creep in superconductors.[1–4] We examine a microscopic model of vortex hopping that allows for vortex interaction and obtain a kinetic equation for describing variations in magnetic vortex number density. Since in this approach magnetic vortices are considered to be interacting quasiparticles, the results may have consequences for other types of quasiparticles in solids.

A highly important problem of superconductivity theory involves the conditions for the emergence of resistive states in superconductors, and their properties, which arise due to various external perturbations such as optical and microwave radiation, magnetic and electric fields, or a transport current. At present there are several approaches to the problem of nonequilibrium superconductivity. The first among these is the study of quasi-one-dimensional situations, in which resistive states result from heating of the system of quasiparticles.[5,6] Another mechanism in which a resistive state emerges as a result of a transport current flowing through a quasi-one-dimensional structure is the mechanism of phase-slip centers.[7] But when resistive states are studied in two-dimensional superconducting films and bulk superconductor samples (in particular, in spatially inhomogeneous high-T_c superconductivity films), dissipative mechanisms related to the motion of magnetic vortices begin to dominate.[3,4,8,9] Hence a detailed study of the kinetics of the hopping creep of vortices, including the interaction of magnetic vortices, is essential for a description of the properties of nonequilibrium resistive states in superconductors (see also Ref. 9), and is highly important both theoretically and in interpreting the experimental data.

There are various approaches to describing magnetic flux creep in superconductors (see, e.g., Ref. 10). A popular model is that of one-particle magnetic flux creep.[1–4,11] This examines the one-particle independent motion of each vortex due to transport current forces, and ignores vortex interaction. A vortex is identified with a quasiparticle localized in a pinning-potential well. Here a mean magnetic vortex flux arises, for instance, when there is a transport current. Because of the Lorentz force on a vortex,[1] the probability of a vortex hopping over the pinning-potential barrier depends on direction. This is a tight-binding approximation: the vortices are localized in the pinning-potential wells. No allowance is made for the interaction between vortices, however.

Another limiting case is represented by models that assume strong interaction in the system of vortices, as a result of which magnetic vortices form a regular lattice[12] and exhibit collective creep behavior.[13,14] Here the pinning potential is usually considered a perturbation (see also Ref. 15). Related papers examine type II superconductors with weak inhomogeneities, and the interaction between vortices is assumed stronger than the interaction with pinning centers.

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Note that there can be no vortex lattice in disordered superconductors (e.g., high-T_c ceramic and polycrystalline samples) in moderate magnetic fields H ($H>H_{c1}$, where H_{c1} is the lower critical field) because the pinning potential in such high-T_c superconductors is extremely deep and aperiodic (see also Ref. 9). Under such conditions, the location of vortices is determined by their interaction with the spatially inhomogeneous pinning potential.

The models cited here thus serve as approximations that can be used to describe magnetic flux kinetics in superconductors. The first model assumes essentially no interaction among vortices, and studies the interaction of vortices with the pinning potential and transport current. The second, in contrast, assumes a strong interaction between vortices, while the pinning potential is considered a perturbation.

Generally, a situation may arise—the one studied here—in which, on the one hand, the pinning potential is not small and cannot be considered a perturbation, and on the other, the interaction between vortices cannot be ignored. We set up a microscopic model to describe magnetic flux creep in type II superconductors. The pinning potential is assumed to be fairly strong, so that hopping creep remains a valid notion.[1] But in contrast to Ref. 1 and similar studies, we allow for the fact that there can be more than one vortex in a pinning-potential well. The interaction of vortices changes their mean energy and hence the probability of hopping to neighboring wells. We show that the interaction of vortices pinned in the potential wells can increase the diffusion of magnetic vortices considerably. Even at low vortex number densities, the interaction between vortices may have a strong influence on magnetic flux kinetics.

2. MICROSCOPIC MODEL

We consider magnetic fields $H>H_{c1}$, in which magnetic vortices have already penetrated the superconductor. Under these conditions, Bean–Livingston surface barriers[16,17] play no role in magnetic flux creep kinetics, which is governed principally by the interaction between vortices and the interaction of vortices with the transport current and the pinning potential.

We have the following equation for the variation in the vortex number density n:

$$\frac{\partial n}{\partial t}=-\operatorname{div}\langle \mathbf{q}\rangle, \tag{1}$$

where $\langle \mathbf{q}\rangle$ is the mean vortex flux.

To calculate $\langle \mathbf{q}\rangle$ we study the following simple model. We assume a pinning potential generated by the inhomogeneities of the crystal lattice, with G_0 the mean depth of the potential wells. Suppose that in the (x,y) plane there exists a vortex number density distribution $n(x,y,t)$ whose self-consistent variation must be found. Self-consistent means the goal is to obtain a kinetic equation for n, in whose derivation we allow for the interaction between vortices, the interaction of vortices with the pinning potential, and the variation of the pinning potential caused by variation of the vortex number density. We assume this to be a "long-wave" theory, i.e., we examine the characteristic variations in the vortex number density and the pinning-potential envelope on a scale much longer than the mean hopping length l of a pinned vortex. Here it is necessary to examine a single hop only to calculate the mean vortex flux.

We assume a certain vortex number density gradient ∇n. Since repulsion increases the mean vortex energy (see also Ref. 1), the average energy level of a vortex in a potential well rises with the vortex number density in that well. Hence the pinning potential F becomes smaller than the lattice potential F_0 by the value of the interaction energy:

$$F(n)=F_0-\langle \varepsilon_{\text{int}}(n)\rangle, \tag{2}$$

where $\langle \varepsilon_{\text{int}}(n)\rangle$ is the mean interaction energy per vortex.

We further assume that the vortex number density is low, and that the most important interaction is the one between vortices within each pinning-potential well. Here we assume the vortex interaction range to be small compared to the scale l; below we discuss this point in greater detail. We take two neighboring pinning-potential wells, 1 and 2. A vortex in a well has a uniform probability of hopping in any direction in the (x,y) plane at a frequency:[1]

$$\nu=\nu_0\, e^{-F/T}, \tag{3}$$

where ν_0 is the characteristic frequency of oscillations of the pinned vortex. On average, the height of the barrier produced by lattice inhomogeneities is the same in all directions. But if there is a number density gradient ∇n, an average flux $\langle \mathbf{q}\rangle \neq 0$ associated with this gradient should obviously arise. Let us assume that $n_1>n_2$, where n_1 and n_2 give the number density of vortices in the first and second wells, respectively. Then in accordance with Eq. (2), $F_1<F_2$, where F_1 and F_2 are the height of the energy barrier for particles in the first and second wells, respectively, and $\nu_1>\nu_2$, where ν_1 and ν_2 are the hopping frequencies defined according to Eq. (3), with the result that there is an average vortex flux.

To calculate the average vortex flux in some direction (e.g., along the x axis), we must consider two neighboring wells (1 and 2) and calculate the average flux that crosses a surface between them.

We first consider a vortex in well 1, and associate the origin of the xy coordinate system with that well. Suppose that the vortex hops to a certain point A on the circumference at radius $\sim l/2$. That hop is associated with a flux whose components are

$$q_x(\varphi)=nv_x^{\text{hop}}=n\nu l_x=\tfrac{1}{2}\, n\nu l\, \cos\varphi,$$

$$q_y(\varphi)=nv_y^{\text{hop}}=n\nu l_y=\tfrac{1}{2}\, n\nu l\, \sin\varphi, \tag{4}$$

where $v_x^{\text{hop}}=\nu l_x$ and $v_y^{\text{hop}}=\nu l_y$ are the x- and y-components of the hop velocity. The angle φ between the hop direction and the x axis can be entirely arbitrary. Hence, to calculate the vortex flux associated with vortices hopping in the positive x direction, we must integrate Eqs. (4) over φ from $-\pi/2$ and $\pi/2$:

$$q_x^+=\int_{-\pi/2}^{\pi/2} q_x(\varphi)\, d\varphi=n\nu l, \quad q_y^+=\int_0^{\pi} q_y(\varphi)\, d\varphi=n\nu l. \tag{5}$$

Here the average flux associated with hopping out of well 1 in the negative x direction is obviously of the same magnitude, $q_x^- = q_x^+$, but in the opposite direction. The average flux of vortices hopping out of one well (say, well 1) is therefore

$$\langle q_1\rangle = q_{x1}^+ - q_{x1}^- = n_1\nu_1 l - n_1\nu_1 l = 0.$$

The situation is different if we consider the flux through a specified surface between wells 1 and 2. To calculate the average flux in the x direction, we must subtract the "negative" flux q_{x2}^- from the second well from the "positive" flux q_{x1}^+ from the first well:

$$\langle q_x\rangle = q_{x1}^+(n_1,F_1) - q_{x2}^-(n_2,F_2) = n_1\nu_1 l - n_2\nu_2 l. \quad (6)$$

Hence, if there is no vortex gradient and $n_1 = n_2$, $\nu_1 = \nu_2$ and $\langle q_x\rangle = 0$.

Now let us assume that there is a certain number density gradient ∇n in the (x,y) plane. We denote the number density for well 1 by $n_1 = n$ and the height of the energy barrier by $F_1 = F$. Here we consider the "long-wave" theory, assuming that the characteristic scales of number density variation in the system of all other quantities are much greater than l. Under these conditions, the expressions for the number density and the energy-barrier height in the second well can be written as Taylor series expansions:

$$n_2 \simeq n_1 + \left.\frac{\partial n}{\partial x}\right|_{x=x_1} \Delta x = n + l\frac{\partial n}{\partial x}, \quad (7)$$

$$F_2 \simeq F_1 + \left.\frac{\partial F}{\partial n}\frac{\partial n}{\partial x}\right|_{x=x_1} \Delta x = F + l\frac{\partial n}{\partial x}\left(\frac{\partial F}{\partial n}\right), \quad (8)$$

where F is the average height of the energy barrier.

We now insert Eqs. (7) and (8) into Eq. (6) and calculate the average vortex flux in the x direction to first order in the number density gradient (in our case $\partial n/\partial x$):

$$\begin{aligned}\langle q_x\rangle &= l(n_1\nu_1 - n_2\nu_2)\\ &= l\left\{n_1\nu_0\exp\left(-\frac{F_1}{T}\right) - n_2\nu_0\exp\left(-\frac{F_2}{T}\right)\right\}\\ &\simeq -l^2\nu_0\exp\left(-\frac{F}{T}\right)\left\{1-\frac{n}{T}\frac{\partial F}{\partial n}\right\}\frac{\partial n}{\partial x}.\end{aligned} \quad (9)$$

Since all directions in the plane are equivalent, we can use the same reasoning as in (9) to obtain

$$\langle q_u\rangle \simeq -l^2\nu_0\exp\left(-\frac{F}{T}\right)\left\{1-\frac{n}{T}\frac{\partial F}{\partial n}\right\}\frac{\partial n}{\partial y}. \quad (10)$$

As a result we arrive at a formula for the average vortex flux to first order in ∇n:

$$\begin{aligned}\langle \mathbf{q}\rangle &= \langle q_x\rangle \mathbf{e}_x + \langle q_y\rangle \mathbf{e}_y\\ &\approx -l^2\nu_0\exp\left(-\frac{F}{T}\right)\left(1-\frac{n}{T}\frac{\partial F}{\partial n}\right)\nabla n,\end{aligned} \quad (11)$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors. Note that $\partial F/\partial n < 0$, as follows from Eq. (2) for the repulsive interaction of vortices.

We compare the derived expression (11) for the vortex flux with the formula for the vortex flux $\mathbf{q_j}$ due to interactions with the transport current and obtainable within the framework of common models of one-particle creep (see, e.g., Refs. 1 and 11).

The drift vortex velocity due to interactions with the current is given by[1,11]

$$v = v_0\exp\left(-\frac{F}{T}\right)\sinh\left(\frac{\beta j}{T}\right), \quad (12)$$

where $v_0 = l\nu_0$, j is the local current density, and β is a constant. Vortex drift is perpendicular to the current. Let us now assume that the current density has components j_x and j_y. Then if α is the angle between current motion and the x axis, $\sin\alpha = j_y/j$ and $\cos\alpha = j_x/j$.

Let us find the components $\langle q_j\rangle_x$ and $\langle q_j\rangle_y$ of the average vortex flux induced by the Lorentz force:

$$\langle q_j\rangle_x = nv_x = nv\sin\alpha = nvj_y/j,$$

$$\langle q_j\rangle_y = nv_y = -nv\cos\alpha = -nvj_x/j.$$

Then

$$\mathbf{q_j} = \frac{nv}{j}j_y\mathbf{e}_x - \frac{nv}{j}j_x\mathbf{e}_y \equiv \frac{v}{\phi_0 j}[\mathbf{jB}],$$

where $\mathbf{B}$ is the induction vector, ϕ_0 is the quantum of flux. Substituting Eq. (12) for v, the average vortex flux becomes

$$\mathbf{q_j} = l\nu_0\exp\left(-\frac{F}{T}\right)\sinh\left(\frac{\beta j}{T}\right)\frac{[\mathbf{jB}]}{j\phi_0}. \quad (13)$$

We transform this formula bearing in mind the Maxwell equation

$$\mathrm{curl}\mathbf{B} = \frac{4\pi}{c}\mathbf{j} \quad (14)$$

and allowing for the fact that in the chosen geometry the induction vector $\mathbf{B}$ is simply $(0,0,B_z)$. Then $\mathbf{B} = n\phi_0\mathbf{e}_z$.

$$\mathrm{curl}\mathbf{B} = \phi_0\frac{\partial n}{\partial y}\mathbf{e}_x - \phi_0\frac{\partial n}{\partial x}\mathbf{e}_y,$$

where n is the vortex number density.

The final result is

$$[\mathbf{jB}] = -\frac{c\phi_0^2 n}{4\pi}\left\{\frac{\partial n}{\partial x}\mathbf{e}_x + \frac{\partial n}{\partial y}\mathbf{e}_y\right\} \equiv -\frac{c\phi_0^2}{4\pi}n\nabla n. \quad (15)$$

Hence, we can rewrite Eq. (13) as

$$\mathbf{q_j} = -\frac{c\phi_0 n}{4\pi j l}\sinh\left(\frac{\beta j}{T}\right)l^2\nu_0\exp\left(-\frac{F}{T}\right)\nabla n. \quad (16)$$

Since Eq. (11) was derived to first order in ∇n, we must let j tend to zero if we want to compare $\langle\mathbf{q}\rangle$ and $\mathbf{q_j}$. We thus obtain

$$\beta = \frac{4\pi l T}{c\phi_0 n}\left(1-\frac{n}{T}\frac{\partial F}{\partial n}\right). \quad (17)$$

Combining (1) and (11), we obtain an equation for the vortex number density:

$$\frac{\partial n}{\partial t} = l^2\nu_0\nabla\left[\left(1-\frac{n}{T}\frac{\partial F}{\partial n}\right)\exp\left(-\frac{F}{T}\right)\nabla n\right]. \quad (18)$$

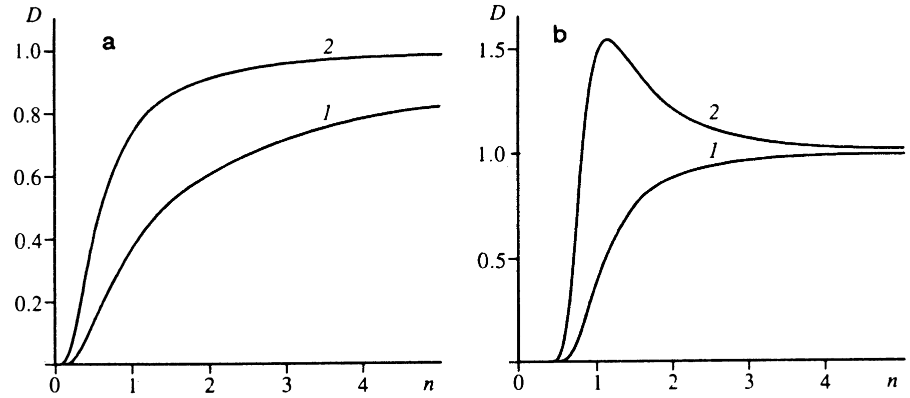

FIG. 1. Diffusion coefficients as a function of vortex number density for two values of α [see Eqs. (19) and (21)]: (a) $\alpha=1$, and (b) $\alpha=3$. Curves marked *1* represent the diffusion coefficient $D_1=\exp(-1/n^\alpha)$, which does not allow for vortex interaction, and curves marked *2* represent $D(n)$ defined by Eq. (21).

Note that in deriving this equation, we did not assume that the term $(n/T)(\partial F/\partial n)$ is small compared to unity. Also, the additional terms obtained in the process of deriving Eq. (18) do not originate in the expansion of $\exp(-F/T)$ in the interaction energy. The equation obtained for magnetic flux creep is nonlinear, and the nonlinearity results from the dependence of F on n and the fact that we allowed for the interaction of vortices in the pinning-potential wells (the term containing $\partial F/\partial n$). Equation (19) describes the kinetics of magnetic flux creep.

The higher the value of $|\partial F/\partial n|$, the greater the contribution of the vortex interaction mechanism under discussion. The greatest effect must occur for superconductors with inhomogeneity scales $L_p \sim \lambda$, where λ is the size of a vortex. Here the conditions for strong localization of vortices in pinning-potential wells are satisfied (if $L_p \ll \lambda$, the vortex does not fit into the well). The spatial structure of high-T_c superconductors can be very small-scale (see, e.g., Ref. 18, where the inhomogeneity scales L_p do not exceed 1 μm). Also $L_p \sim \lambda_L$, where λ_L is the characteristic size of an Abrikosov vortex, which implies that vortex interaction is significant.

Current experimental studies of high-T_c superconductivity (see, e.g., Ref. 19) show that sometimes vortex pinning on large-scale inhomogeneities may play an important role. For instance, Xenikos *et al.*[19] experimentally studied the transport properties of high-T_c films whose thickness was $d \sim 1000$ Å. They discovered that the characteristic scale L_p of the inhomogeneities that significantly affect the pinning of vortices is of order d. This coincides in order of magnitude with the characteristic scale λ_L. Note that in polycrystalline high-T_c samples the characteristic inhomogeneity scales may be even greater. For instance, in their experiments Wang *et al.*[20] and Routbort *et al.*[21] showed that the microstructure of polycrystalline Y–Ba–Cu–O samples and the like contains a broad spectrum of characteristic inhomogeneity scales out to $L_p \sim 1-10$ μm.

The changes obtained in the kinetic equation lead to a change in the kinetics of magnetic flux; in the presence of transport current, they lead to a change in the current–voltage characteristic. To illustrate this we examine the penetration of the superconductor by the magnetic field in the one-dimensional case. For the F vs n dependence, we use the phenomenological formula[1)] (see, e.g., Ref. 22)

$$F=F(n)=\frac{G(T)}{n^\alpha} \propto \frac{(1-T/T_c)^\gamma}{n^\alpha}, \tag{19}$$

where γ and α are constants ($\alpha \approx 1-3$), and $G(T)$ is the "strength" of the pinning potential.

Although the F vs n dependence can be different, the changes obtained in the kinetic equation (18) are important even within the framework of simple phenomenological ideas about the dependence $F=F(n) \propto 1/n^\alpha$. It is important to note that using a specific form of the $F=F(n)$ dependence has no effect on the basic result of our work—the kinetic equation (18) for magnetic vortices, including their interaction—and we employ it here only to illustrate the importance of allowing for the interaction of magnetic vortices in localized states. Establishing the $F=F(n,T)$ dependence that allows for the interaction and the strong localization of vortices constitutes an interesting problem both theoretically and as it applies to the interpretation of the experimental data. Here, obviously, the result depends on the type of lat-

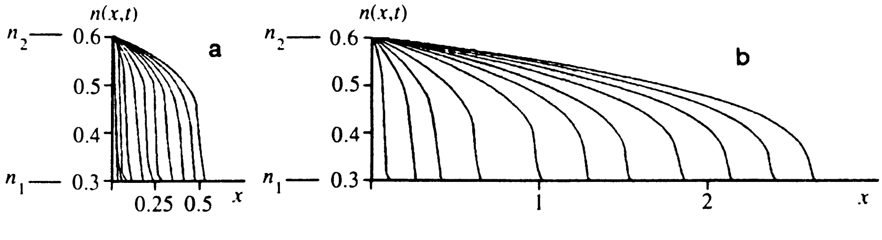

FIG. 2. Plots of $n(x,t)$ as a function of x constructed at different times: $t=0.1$, 0.5, 1, 2, 4, 6, 8, 11, 14, 17, 20 (left to right); $n(x \geq 0,t=0)=n_1$; $n(x=0,t>0)=n_2$; $\alpha=2$. (a) Numerical solutions of Eq. (20) in the absence of vortex interaction (the term with $\partial F/\partial n$ in the factor preceding the exponential is ignored). (b) Solutions of the nonlinear equation (20) with vortex interaction taken into account; the solutions are for the same times.

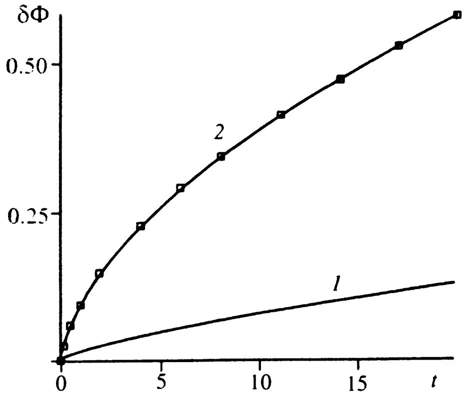

FIG. 3. Time dependence of dimensionless magnetic flux, Eq. (24), when the magnetic field penetrates the sample, for the nonlinear flux diffusion depicted in Fig. 2: (a) without vortex interaction [as in Fig. 2(a)], and (b) with allowance for vortex interaction [as in Fig. 2(b)].

tice defects generating the pinning potential for magnetic vortices (what is important is the shape and size of the pinning-potential wells), and in that sense cannot be universal. The decreasing F vs n power-law dependence nevertheless seems quite reasonable. For instance, the qualitative reasoning behind the vortex lattice case treated by Yeshurun and Malozemoff[23] leads precisely to such a power-law dependence, $F=F(n) \propto 1/n^{\alpha}$, where $\alpha=1$ or $\alpha=\frac{3}{2}$ (see Ref. 23).

3. ONE-DIMENSIONAL MODEL

Transforming to dimensionless variables, $x/l \to x$, $\nu_0 t \to t$, and $n[T/G(T)]^{1/\alpha} \to n$, we obtain

$$\frac{\partial n}{\partial t}=\frac{\partial}{\partial x}D(n)\frac{\partial n}{\partial x}. \qquad (20)$$

Here the nonlinear diffusion coefficient is given by

$$D(n)=\left(1+\frac{\alpha}{n^{\alpha}}\right)\exp\left(-\frac{1}{n^{\alpha}}\right) \qquad (21)$$

and consists of two terms. The first, $D_1=\exp(-1/n^{\alpha})$, corresponds to diffusion with no vortex interaction. The second term preceding the exponential in (21), $\alpha/n^{\alpha} \propto (n/T) \times |\partial F/\partial n|$, corresponds to a situation in which vortex interaction is taken into account. For $\alpha>1$, the nonlinear diffusion coefficient D is a nonmonotonic function of n. $D_1(n)$ and $D(n)$ are plotted for $\alpha=1$ and $\alpha=3$ in Fig. 1. From Eq. (21) and Fig. 1 it is clear that over the full range of number density for which hopping magnetic flux creep is significant ($0<n\leqslant 1$),[2)] because for $n \gg 1$ we have $\exp(-F/T)\sim 1$, and there is no localization of vortices at pinning centers), vortex interaction substantially increases vortex diffusion. At high vortex number densities, $(n/T)|\partial F/\partial n| \to 0$ (for $\alpha>0$) and $D(n)\approx D_1(n)$. Physically this means that potential wells become shallow in strong magnetic fields, and vortices thus easily hop out of the wells, with the result that their interaction has little effect on diffusion.

The strong effect of vortex interaction on the kinetics of magnetic flux creep in the event of strong vortex localization can be understood on the basis of the following qualitative arguments, adopting Eq. (3) for the hop frequency. Allowing for vortex interaction in the event of strong vortex localization ($F/T \gg 1$) leads to a change in the height of the energy barrier F. Since F exhibits exponential behavior, this leads to a marked change in ν. In calculating the average vortex flux, this leads to an additional term in the kinetic equation that turns out not to be small. Thus, vortex interaction, which was previously ignored in discussing magnetic flux creep, is is extremely important in fields $H \gg H_{c1}$.

To illustrate the increase in vortex diffusion due to vortex interaction, we also present numerical solutions of Eq. (20).[3)] Let us examine Eq. (20) on the ray $x>0$:

$$n=n(x,t), \quad n(x,0)=n_1, \quad n(0,t)=n_2. \qquad (22)$$

Hence, the time-independent state for the solutions $n(x,t)$ of Eq. (20) is in this case

$$n=n(x\geqslant 0, t\to +\infty)=n_2=\text{const}. \qquad (23)$$

Figure 2 depicts the numerical solutions of the nonlinear equation (20): plots of $n(x,t)$ at different times. Clearly, vortex interaction increases magnetic vortex diffusion considerably. Another illustration of the sharp increase in the rate at which magnetic flux penetrates the sample because of vortex interaction in this case (see Fig. 2) is Fig. 3, which depicts the time dependence of the dimensionless magnetic flux $\delta\Phi(t)$,

$$\delta\Phi(t)=\int_0^{\infty}[n(x,t)-n_1]\,dx, \qquad (24)$$

in the following cases: (*1*) without magnetic vortex interaction, and (*2*) with allowance for such interaction.

It is interesting to compare the solutions of the nonlinear equation (20) with an analytic solution, which can be obtained in the limit of small values of the gradient ∇n. Let us examine the case in which $n(x,t)\approx n_1 \approx n_2$ and linearize Eq. (20). For small ∇n, Eq. (20) leads to the linear equation

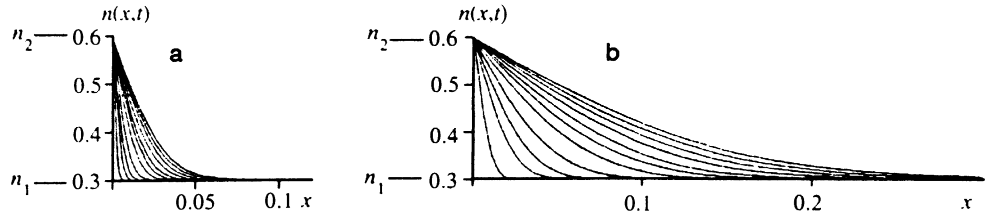

FIG. 4. Solutions (26) of the linear diffusion equation (25) without vortex interaction (a) and with vortex interaction (b). The parameters of the problem are the same as in the solution of the nonlinear problem, as depicted in Fig. 2 for $\alpha=2$. The curves are for the same times as in Fig. 2.

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}, \quad (25)$$

with the diffusion coefficient $D \approx D(n_1) = (1 + \alpha/n_1^\alpha) \times \exp(-1/n_1^\alpha)$. The solution of Eq. (25) consistent with (22) is

$$n(x,t) = n_2 + (n_1 - n_2)\operatorname{erf}\frac{x}{\sqrt{4Dt}}, \quad (26)$$

where

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-y^2)\, dy \quad (27)$$

is the error function. Figure 4 depicts the solution (26) of the linear equation (25). Both without vortex interaction [case (a)] and with such interaction [case (b)] the pattern is exactly the same as for the solutions of nonlinear equations, as depicted in Fig. 2 (the curves were constructed for the same times as in Fig. 2). In contrast to the solutions of the nonlinear equation, $n(x)$ contains no points of inflection in the solutions of the linear problem. Note that the scale in the x direction has been expanded by a factor of ten compared to that of Fig. 2, which depicts the solutions of the nonlinear equation. As a result, magnetic flux diffusion in the given case of a linear equation is much weaker, i.e., the nonlinear equation (18) plays a significant role in the diffusion of magnetic flux in superconductors. The nonlinear nature of diffusion leads to a change in the spatial profile of magnetic induction.

4. CONCLUSION

When there is strong localization of magnetic vortices on pinning centers, the interaction of these vortices is manifested in the kinetics of magnetic flux. Vortex repulsion increases the average energy of the vortices and lowers the potential barrier for the activation of vortices from pinning-potential wells: these changes lead to a considerable modification of creep kinetics. Note that in high-T_c superconductors, magnetic flux creep cannot be described solely by the one-particle model, since collective effects come into play (see, e.g., Refs. 3, 4, 10, 13–15, and 24). Hence, it is generally important to allow for vortex interaction.

In the presence of fairly large-scale inhomogeneities in the pinning potential[19–21] and in the event of strong vortex localization, the collective effects can be described by models like those suggested in this paper. Interpretation of experimental data is in many cases made more difficult by the presence of different inhomogeneity scales, whose spatial spectrum can be extremely broad. Establishing the role of vortex interaction requires experiments with a well-understood pinning structure (for example, films with artificially induced inhomogeneities).

[1)] The singularity at $n=0$ is related only to the choice of the approximation (19). Actually at $n=0$ the value of F is large but finite.

[2)] In dimensional variables this condition corresponds to $n < [G(T)/T]^{1/\alpha}$, where $G(T)$ is the strength of the pinning potential from Eq. (19).

[3)] The characteristic diffusion time for the nonlinear equation (20) can be estimated only very approximately because of the sharp exponential dependence of D on n.

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