Resonance phenomena in the reflection of an elastic wave at the boundary between a hexagonal crystal and an anisotropic film

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We investigate the properties of resonant reflection of a shear acoustic wave in a hexagonal crystal whose surface is coated by an elastically anisotropic film that is thin compared to a wavelength and acts to transform the "supersonic" Rayleigh wave into a leaky wave. We show that within a narrow interval of angles of incidence corresponding to excitation of this leaky wave, the magnitudes of the excitation coefficients for near-surface vibrations increase markedly, and the phase of the reflection coefficient changes abruptly. The value of the resonant angle of incidence itself, like the width of the resonance interval, is regulated by the frequency of the incident wave.

1. INTRODUCTION

The problem of how thin layers (films) affect the propagation of surface acoustic waves (Rayleigh waves) has always attracted the attention of investigators. However, to our knowledge none of the many publications on this topic have treated the situation where a thin coating at the crystal surface mediates the conversion of a Rayleigh wave into a pseudosurface (leaky) wave, although a number of papers discuss the feasibility of such a conversion (see, e.g., Refs. 1–3). In this paper we will show that the appearance of the leaky (pseudosurface) wave is accompanied by an interesting physical effect-resonant reflection of acoustic waves at the crystal-film boundary.

The leaky wave evolves from a "supersonic" Rayleigh wave modified by the deposited film. Such a "supersonic" Rayleigh wave can occur only in anisotropic media; it differs from an ordinary "subsonic" surface wave in that the velocity v_R of the "supersonic" wave lies within a range of velocities where the wave equation in the crystal has both nonuniform (near-boundary) solutions and a solution in the form of a uniform bulk wave. The additional elastically anisotropic layer at the surface of the medium can change the boundary conditions in such a way that the "supersonic" Rayleigh wave ceases to exist—the wave field in the crystal, which is made up of nonuniform modes alone, cannot satisfy the boundary conditions in the vicinity of v_{R}. However, it then turns out that these new boundary conditions can be satisfied for velocities close to v_{R} if a bulk mode is added to the nonuniform modes with an energy flux directed into the bulk of the crystal. The amplitude of this bulk wave is small as long as the thickness of the film is small compared to a wavelength. A solution of the boundary value problem of this type is referred to as a leaky wave.

On the one hand, the coupling of surface modes to bulk modes via the boundary conditions causes attenuation of the leaky wave due to radiation. On the other hand, this coupling allows effective excitation of the leaky wave by waves incident from within the crystal at angles of incidence such that the velocity v of the incident wave along the surface, i.e., $v=\omega/k$, is close to the real part of the velocity of the leaky wave (where ω is the frequency and k is the tangential projection of the wave vector). In this case the reflection becomes resonant and, as we show below, is accompanied by an anomalously large increase in the magnitudes of the excitation coefficients for surface modes, and also an extremely abrupt change in the phase of the reflection coefficient over a narrow interval of angles of incidence.

In this paper we develop a theory of resonant reflection of elastic waves at a boundary between a crystal and a film for the simplest situation: a crystal with hexagonal symmetry.

2. GENERAL RELATIONS

The propagation of elastic waves in crystals is described by the equation

$$\rho(\partial^2 u_q/\partial t^2)=C_{qjkl}(\partial^2 u_k/\partial x_j\partial x_l), \quad q,j,k,l=1,2,3, \tag{1}$$

where C_{qjkl} are the elastic moduli of the crystal, ρ is its density, u_q are the components of the mechanical displacement vector $\mathbf{u}(\mathbf{r},t)$, and $\mathbf{r}\equiv(x_1,x_2,x_3)$ is the radius vector.

In a medium bounded by a planar surface whose orientation is given by the unit normal vector $\mathbf{n}$, we will seek a solution $\mathbf{u}(\mathbf{r},t)$ to the boundary value problem in terms of plane waves. Such a solution consists of a linear combination of partial solutions $\mathbf{u}_\alpha(\mathbf{r},t)$ to the wave equation (1):

$$\mathbf{u}(\mathbf{r},t)=\sum_\alpha b_\alpha \mathbf{u}_\alpha(\mathbf{r},t), \tag{2}$$

$$\mathbf{u}_\alpha(\mathbf{r},t)=\mathbf{A}_\alpha \exp(ik[(\mathbf{m}+p_\alpha\mathbf{n})\mathbf{r}-vt]), \tag{3}$$

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where the b_α are constants determined from the boundary conditions, $\mathbf{m}$ indicates the direction of propagation along the surface ($|\mathbf{m}|=1$), and $\mathbf{A}_\alpha$ is the polarization vector of the partial mode identified by the label α.

After substituting (3) into Eq. (1) we have

$$\hat{M}^{(\alpha)}\mathbf{A}_\alpha=0, \quad (4)$$

where $\hat{M}^{(\alpha)}$ is a matrix with elements

$$(\hat{M}^{(\alpha)})_{ql}=(m_k+p_\alpha n_k)C_{qklj}(m_j+p_\alpha n_j)-\rho v^2\delta_{ql}. \quad (5)$$

From the condition for existence of a nontrivial solution to the system (4), which consists of the requirement that the determinant $\|\hat{M}^{(\alpha)}\|$ of the matrix $\hat{M}^{(\alpha)}$ equal zero:

$$\|\hat{M}^{(\alpha)}\|=0, \quad (6)$$

we find the $p_\alpha(v)$. It is not difficult to verify that six functions $p_\alpha(v)$ are obtained in all, which can take on both real and complex values depending on the value of the velocity v. The modes with real p_α correspond to uniform (bulk) waves, while the partial solutions to (3) with complex p_α are customarily referred to as nonuniform modes. In the case where the crystal is treated as a semi-infinite medium, we must include in the solution only those nonuniform modes for which the sign of the imaginary part of p_α ensures that $\mathbf{u}_\alpha(\mathbf{r},t)$ decreases as the value of $|\mathbf{nr}|$ increases, so that the amplitude of the total field $\mathbf{u}(\mathbf{r},t)$ remains bounded at an infinite distance from the surface. If, however, we assume that the crystal has finite thickness, then there is no longer any need to select modes based on the sign of the imaginary parts of the p_α, and we may include all six partial solutions to the wave equation in the total solution.

Each of the partial modes (3) acts on a unit area of the surface $\mathbf{nr}=\text{const}$ with an elastic force $\mathbf{f}_\alpha$:

$$f_{\alpha q}=C_{qjkl}n_j(\partial u_{\alpha k}/\partial x_l)=-ikL_{\alpha q}\times\exp(ik[(\mathbf{m}+p_\alpha\mathbf{n})\mathbf{r}-vt]),$$

$$L_{\alpha q}=-n_jC_{qjkl}(m_l+p_\alpha n_l)A_{\alpha k}. \quad (7)$$

In Ref. 4 it was shown that for fixed values of v the vectors $\mathbf{A}_\alpha,\mathbf{L}_\alpha$, and $\mathbf{A}_\beta,\mathbf{L}_\beta$ that correspond to modes α and β with $p_\alpha\neq p_\beta$ satisfy the relation

$$\mathbf{A}_\alpha\mathbf{L}_\beta+\mathbf{A}_\alpha\mathbf{L}_\beta=0, \quad \alpha\neq\beta. \quad (8)$$

Furthermore, according to Ref. 4, with the normalization

$$2\mathbf{A}_\alpha\mathbf{L}_\alpha=1, \quad (9)$$

the following identities hold:

$$\sum_{\alpha=1}^{6}\mathbf{L}_\alpha\otimes\mathbf{L}_\alpha=\hat{0}, \quad \sum_{\alpha=1}^{6}\mathbf{A}_\alpha\otimes\mathbf{A}_\alpha=\hat{0}, \quad \sum_{\alpha=1}^{6}\mathbf{A}_\alpha\otimes\mathbf{L}_\alpha=\hat{I}, \quad (10)$$

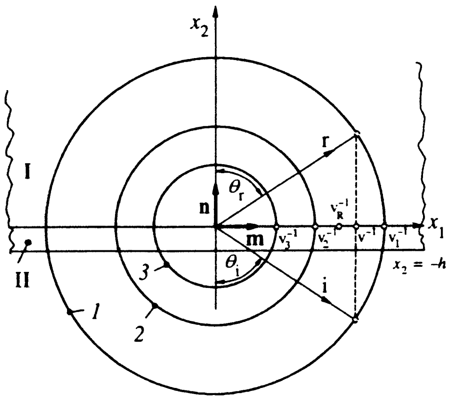

FIG. 1. Geometry of the problem. The axis x_3 is perpendicular to the plane of the figure; I—crystal, II—film. The circles are cross sections in the plane x_1x_2 of cavities cut in the slowness surface for bulk waves in the crystal: *1*—shear-wave branch ($v_1=\sqrt{[c_{44}/\rho)}$; *2*—the branch of transverse waves polarized in the x_1x_2 plane ($v_2=\sqrt{c_{66}/\rho}$); *3*—the longitudinal wave branch ($v_3=\sqrt{c_{11}/\rho}$). Here v_R is the velocity of the "supersonic" Rayleigh wave, v is the velocity of the incident wave along the surface, and θ_i, θ_r are the angles of incidence and reflection ($\theta_i=\theta_r$).

where $\hat{O},\hat{I}$ are the zero and unit matrices respectively; the symbol "$\otimes$" denotes the dyadic product of the vectors.

3. A CRYSTAL WITH A FREE SURFACE

Assume that the hexagonal crystal occupies the half-space $x_2>0$ relative to the system of coordinates $x_1x_2x_3$ (Fig. 1). The sixfold axis of symmetry is directed along the x_3 axis. The crystal is characterized by a density ρ and the elastic moduli

$$C_{1111}=C_{2222}=c_{11}, \quad C_{3333}=c_{33}, \quad C_{1122}=c_{12},$$
$$C_{1133}=C_{2233}=c_{13}, \quad C_{2323}=C_{1313}=c_{44},$$
$$C_{1212}=(C_{1111}-C_{1122})/2=c_{66} \quad (11)$$

(see, e.g., Ref. 5). We also assume that

$$c_{44}<c_{66}, \quad (2-c_{44}/c_{66})^2<4\sqrt{1-c_{44}/c_{66}}\ \sqrt{1-c_{44}/c_{11}}; \quad (12)$$

the condition $c_{44}<c_{66}$ implies the inequality $c_{44}<c_{11}$, since in hexagonal media we always have $c_{66}<c_{11}$ for thermodynamic reasons. Note that there is a large group of crystals for which relations (12) actually hold.

We are interested in solutions to the wave equation in the crystal for the x_1x_2 plane. When $\mathbf{m}$ and $\mathbf{n}$ are directed along the x_1 and x_2 axes respectively, the matrix $\hat{M}^{(\alpha)}$ has the form

$$\hat{M}^{(\alpha)}=\begin{pmatrix} c_{11}+p_\alpha^2c_{66}-\rho v^2 & p_\alpha(c_{11}-c_{66}) & 0 \\ p_\alpha(c_{11}-c_{66}) & c_{66}+p_\alpha^2c_{11}-\rho v^2 & 0 \\ 0 & 0 & c_{44}(1+p_\alpha^2)-\rho v^2 \end{pmatrix}. \quad (13)$$

Expanding the determinant of $\hat{M}^{(\alpha)}$ and setting it equal to zero, we obtain

$$(p_\alpha^2+1-v^2/v_1^2)(p_\alpha^2+1-v^2/v_2^2)(p_\alpha^2+1-v^2/v_3^2)=0, \tag{14}$$

where

$$v_1=\sqrt{c_{44}/\rho}, \quad v_2=\sqrt{c_{66}/\rho}, \quad v_3=\sqrt{c_{11}/\rho}$$

are the phase velocities of bulk waves in the plane x_1x_2, $v_1<v_2<v_3$.

For the interval of velocities we will discuss below,

$$v_1<v<v_2, \tag{15}$$

we find from Eq. (14) the following p_α, α=i,r,2,3,5,6:

$$p_{\rm i}=-p_{\rm r}=-\sqrt{v^2/v_1^2-1}, \quad p_2=p_5^*=i\sqrt{1-v^2/v_2^2}$$

$$p_2=p_6^*=i\sqrt{1-v^2/v_3^2}. \tag{16}$$

Then using (4), (7), (9) we compute the normalized vectors $\mathbf{A}_\alpha$, $\mathbf{L}_\alpha$:

$$\mathbf{A}_{\rm i}=\frac{1}{\sqrt{2c_{44}p_{\rm r}}}\begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \mathbf{L}_{\rm i}=\frac{1}{\sqrt{2c_{44}p_{\rm r}}}\begin{pmatrix}0\\0\\c_{44}p_{\rm r}\end{pmatrix}, \tag{17}$$

$$\mathbf{A}_{\rm r}=\frac{i}{\sqrt{2c_{44}p_{\rm r}}}\begin{pmatrix}0\\0\\-1\end{pmatrix}, \quad \mathbf{L}_{\rm r}=\frac{i}{\sqrt{2c_{44}p_{\rm r}}}\begin{pmatrix}0\\0\\c_{44}p_{\rm r}\end{pmatrix}, \tag{18}$$

$$\mathbf{A}_2=\frac{i}{\sqrt{2c_{66}p_2}}\frac{v_2}{v}\begin{pmatrix}-p_2\\1\\0\end{pmatrix},$$

$$\mathbf{L}_2=\frac{i}{\sqrt{2c_{66}p_2}}\frac{v_2}{v}\begin{pmatrix}c_{66}(p_2^2-1)\\-2c_{66}p_2\\0\end{pmatrix}, \tag{19}$$

$$\mathbf{A}_3=\frac{i}{\sqrt{2c_{66}p_3}}\frac{v_2}{v}\begin{pmatrix}1\\p_3\\0\end{pmatrix},$$

$$\mathbf{L}_3=\frac{i}{\sqrt{2c_{66}p_3}}\frac{v_2}{v}\begin{pmatrix}-2c_{66}p_3\\c_{66}(1-p_3^2)\\0\end{pmatrix}, \tag{20}$$

$$\mathbf{A}_{\alpha+3}=\mathbf{A}_\alpha^*, \quad \mathbf{L}_{\alpha+3}=\mathbf{L}_\alpha^*, \quad \alpha=2,3. \tag{21}$$

Labels α=i and α=r are used to identify the uniform shear waves that propagate normal to the crystal boundary (incident wave) and from the boundary into the interior of the medium (reflected wave) respectively.

Let us discuss the situation where there is no additional layer at the crystal surface. We will seek the wave field in the crystal in the form of a linear combination (2), which can contain the modes α=i,r,2,3. At the free boundary of the medium, the elastic forces reduce to zero; therefore, the coefficients b_α in (2) should be chosen so that

$$\sum_\alpha b_\alpha \mathbf{L}_\alpha=0. \tag{22}$$

Condition (22) is always satisfied by a superposition of modes α=i,r, since for any v on the interval (15) we have

$$\mathbf{L}_{\rm i}+i\mathbf{L}_{\rm r}=0. \tag{23}$$

This solution describes the reflection problem for shear waves. Furthermore, when $v=v_{\rm R}$, where $v_{\rm R}$ is the real nonzero root of the equation

$$f_{\rm R}(v)=(2-v^2/v_2^2)^2-4\sqrt{1-v^2/v_2^2}\ \sqrt{1-v^2/v_3^2}=0, \tag{24}$$

yet another solution appears: the surface Rayleigh wave, which is formed from the nonuniform modes α=2,3, because in this case

$$\mathbf{L}_2+i\mathbf{L}_3=0. \tag{25}$$

According to Ref. 6, it follows from the inequality (12) that

$$v_1<v_{\rm R}<v_2, \tag{26}$$

i.e., in a hexagonal medium with elastic constants that are subject to conditions (12), the Rayleigh wave for this geometry is "supersonic" with respect to the shear bulk waves (Fig. 1).

4. THE PROBLEM OF REFLECTION FOR SHEAR WAVES AT THE BOUNDARY BETWEEN CRYSTAL AND FILM

We now assume that an elastically anisotropic film of thickness h is deposited on the crystal surface, and that rigid acoustic contact is maintained between the film and the crystal. The film is oriented so that the even-order axis of symmetry and the plane of symmetry of the film, if there is such a plane, do not coincide with the x_3 axis and the plane x_1x_2, respectively.

We consider the reflection problem for shear waves. We look for the wave field in the crystal in the form

$$\mathbf{u}(\mathbf{r},t)=\sum_{\alpha={\rm i,r},2,3} b_\alpha \mathbf{A}_\alpha \exp(ik[x_1+p_\alpha x_2-vt]), \tag{27}$$

and in the layer in the form

$$\mathbf{u}^{\rm f}(\mathbf{r},t)=\sum_{\beta=1}^{6} d_\beta \mathbf{A}_\beta^{\rm f} \exp(ik[x_1+p_\beta^{\rm f}x_2-vt]), \tag{28}$$

where $v=v_1/\sin\theta_{\rm i}$, and $\theta_{\rm i}$ is the angle of incidence; β enumerates the partial solutions to the wave equation in the layer, while $p_\beta^{\rm f}$, $\mathbf{A}_\beta^{\rm f}$, and $\mathbf{L}_\beta^{\rm f}$ can be found by following the procedure (4)–(7). We assume that $2\mathbf{A}_\beta^{\rm f}\ \mathbf{L}\beta{\rm f}=1$.

The boundary conditions in this case are continuity of the displacement and elastic forces at the crystal boundaries, i.e., the layer ($x_2=0$), and also the vanishing of elastic forces at the free surface of the layer ($x_2=-h$). This gives nine equations for determining b_α, d_β:

$$\sum_{\beta=1}^{6} d_\beta \mathbf{A}_\beta^{\rm f}=\sum_{\alpha={\rm i,r},2,3} b_\alpha \mathbf{A}_\alpha,$$

$$\sum_{\beta=1}^{6} d_\beta \mathbf{L}_\beta^{\rm f}=\sum_{\alpha={\rm i,r},2,3} b_\alpha \mathbf{L}_\alpha,$$

$$\sum_{\beta=1}^{6} d_\beta \mathbf{L}_\beta^f \exp(-ikp_\beta^f h)=0. \tag{29}$$

For the specified orientation of the layer, the partial modes $\beta=1,...,6$ do not split up into purely shear waves with $\mathbf{A}_\beta^f$, $\mathbf{L}_\beta^f$ directed along the axis x_3, and modes for which $\mathbf{A}_\beta^f$, $\mathbf{L}_\beta^f$ lie in the plane x_1x_2 (see, e.g., Ref. 7). Therefore, the modes $\alpha=$i,r and $\alpha=2,3$ cannot independently satisfy the boundary conditions (29).

In order to calculate the reflection coefficient $R(v,h)=b_r(v,h)/b_i(v,h)$ and the coefficients for excitation of the nonuniform modes $T_\alpha(v,h)=b_\alpha(v,h)/b_i(v,h)$, $\alpha=2,3$, we proceed as follows. We will successively dot the first vector equation in (29) with the vector $\mathbf{L}_\beta^f$ and the second with $\mathbf{A}_\beta^f$, with $\beta=1,...,6$. Each time we add right side to right side, left side to left side. Taking (8), (9) into account, we find

$$d_\beta=\sum_{\alpha=\mathrm{i,r},2,3} b_\alpha(\mathbf{A}_\alpha\mathbf{L}_\beta^f+\mathbf{L}_\alpha\mathbf{A}_\beta^f), \quad \beta=1,...,6. \tag{30}$$

Substituting (30) into the last vector equation in (29), we obtain an equation that contains only the b_α:

$$\sum_{\alpha=\mathrm{i,r},2,3} b_\alpha \mathbf{L}'_\alpha=0, \tag{31}$$

where

$$\mathbf{L}'_\alpha=\sum_{\beta=1}^{6}(\mathbf{A}_\alpha\mathbf{L}_\beta^f+\mathbf{L}_\alpha\mathbf{A}_\beta^f)\mathbf{L}_\beta^f\exp(-ikp_\beta^f h)$$
$$=\left(\sum_{\beta=1}^{6}\mathbf{L}_\beta^f\otimes\mathbf{L}_\beta^f\exp(-ikp_\beta^f h)\right)\mathbf{A}_\alpha+\left(\sum_{\beta=1}^{6}\mathbf{L}_\beta^f\otimes\mathbf{A}_\beta^f\exp(-ikp_\beta^f h)\right)\mathbf{L}_\alpha$$
$$=\mathbf{L}_\alpha+\Delta\mathbf{L}'_\alpha; \tag{32}$$

$$\Delta\mathbf{L}'_\alpha=\left(\sum_{\beta=1}^{6}\mathbf{L}_\beta^f\otimes\mathbf{L}_\beta^f\exp(-ikp_\beta^f h)\right)\mathbf{A}_\alpha+\left(\sum_{\beta=1}^{6}\mathbf{L}_\beta^f\otimes\mathbf{A}_\beta^f[\exp(-ikp_\beta^f h)-1]\right)\mathbf{L}_\alpha. \tag{33}$$

From (31) it follows that

$$R(v,h)=-\frac{[\mathbf{L}'_i\mathbf{L}'_2]\mathbf{L}'_3}{[\mathbf{L}'_r\mathbf{L}'_2]\mathbf{L}'_3}, \tag{34}$$

$$T_2(v,h)=-\frac{[\mathbf{L}'_r\mathbf{L}'_i]\mathbf{L}'_3}{[\mathbf{L}'_r\mathbf{L}'_2]\mathbf{L}'_3}, \quad T_3(v,h)=-\frac{[\mathbf{L}'_r\mathbf{L}'_2]\mathbf{L}'_i}{[\mathbf{L}'_r\mathbf{L}'_2]\mathbf{L}'_3}. \tag{35}$$

5. RESONANCE BEHAVIOR IN THE REFLECTION NEAR A LEAKY WAVE

Let us consider the behavior of the conversion coefficients (34), (35), assuming that the thickness of the layer is much smaller than the wavelength ($kh\ll1$). By separating the mixed products that include only the $\mathbf{L}_\alpha$ out of the numerators and denominators of (34), (35), and taking into account that $[\mathbf{L}_r\mathbf{L}_i]\mathbf{L}_2$, $[\mathbf{L}_r\mathbf{L}_i]\mathbf{L}_3$ are identically equal to zero, we obtain

$$R(v,h)=i\,\frac{f_R(v)+\Delta b_i^*(v,kh)}{f_R(v)+\Delta b_i(v,kh)}, \tag{36}$$

$$T_\alpha(v,h)=\frac{\Delta b_\alpha(v,kh)}{f_R(v)+\Delta b_i(v,kh)}, \quad \alpha=2,3, \tag{37}$$

where the functions $\Delta b_i(v,kh)$ and $\Delta b_\alpha(v,kh)$ describe the contribution of the film and have small values when kh is small; $\Delta b_i^*(v,kh)$ is the complex conjugate of the function $\Delta b_i(v,kh)$, and $f_R(v)$ is defined according to (24). From this it is clear that when there is no Rayleigh wave on the interval $v_1<v<v_2$, for $kh\ll1$ the coefficients $R(v,h), T_\alpha(v,h)$ will differ only slightly from their values in the absence of the coating. From (24) it follows that for $v_1<v<v_2$ the values of $f_R(v)$ will be of order unity; therefore, the presence of the layer causes the phase of the reflection coefficient to depend only weakly on the angle of incidence, and the coefficient for excitation of the nonuniform modes will be small: $T_\alpha\approx\Delta b_\alpha/f_R\ll1$.

The situation is different when the Rayleigh wave is "supersonic." Consider the behavior of the denominator $b_i(v,kh)=f_R(v)+\Delta b_i(v,kh)$ of the coefficients (36), (37). When $v=v_R$, $kh=0$, we have $b_i(v_R,0)=f_R(v_R)=0$. Then for small kh the quantity $b_i(v,kh)$ will go to zero, but only for a certain complex value $v = v_1 - iv'_1$ ($\Delta b_i(v,kh)$ is a complex quantity). This assertion follows because the derivative $\partial f_R(v_R)/\partial v$ does not equal zero when $v=v_R$. Accordingly, we assume that for v near v_R,

$$f_R(v)+\Delta b_i(v,kh)\propto v-v_1+iv'_1; \tag{38}$$

the computations show that

$$v_1-v_R\propto v_R(k_Rh)+..., \quad v'_1\propto v_R(k_Rh)^2+..., \quad v'_1>0, \tag{39}$$

where $k_R=\omega/v_R$. Likewise, the numerators of the coefficients (37) $\Delta b_\alpha(v,kh)$, $\alpha=2,3$, have the following forms when $kh\ll1$:

$$\Delta b_\alpha(v,kh)\approx b_\alpha(v)kh+..., \quad \alpha=2,3, \tag{40}$$

where $b_\alpha(v_R)\neq0$.

To summarize, it turns out that for v close to v_R and $k_Rh\ll1$ we have

$$R(v,h)\approx i\,\frac{v-v_1-iv'_1}{v-v_1+iv'_1}=\exp\left\{i\left[\frac{\pi}{2}-\psi(v)\right]\right\},$$

$$\psi(v)=2\,\mathrm{arcctg}\left(\frac{v-v_1}{v'_1}\right), \tag{41}$$

$$T_\alpha(v,h)\approx T_\alpha^{(0)}\frac{\sqrt{v_1'v_R}}{v-v_1+iv_1'}$$

$$=T_\alpha^{(0)}\sqrt{\frac{v_1'v_R}{(v-v_1)^2+v_1'^2}}\exp\left[-i\frac{\psi(v)}{2}\right],$$

$$\alpha=2,3, \tag{42}$$

and $T_\alpha^{(0)}$ does not depend on v or k_Rh.

From Eqs. (41), (42) it follows that in a neighborhood of $v=v_1$ with width of order v_1', $v_1'\ll v_1$, the phase $\psi(v)$ of the reflection coefficient changes by 2π. In this same region, the phases of the coefficients for excitation of nonuniform modes $T_\alpha(v,h)$ change by π, and their magnitudes increase greatly:

$$|T_\alpha(v,h)|\sim|T_\alpha^{(0)}|\sqrt{v_R/v_1'}\sim|T_\alpha^{(0)}|(k_Rh)^{-1}\gg 1 \tag{43}$$

for $|v-v_1|\leqslant v_1'$, while

$$|T_\alpha(v,h)|\sim|T_\alpha^{(0)}|\sim 1 \tag{44}$$

for $|v-v_1|\gtrsim\sqrt{v_Rv_1'}\sim v_R(k_Rh)$.

Thus, we see that when the Rayleigh-wave velocity is larger than the phase velocity of the shear wave, for angles of incidence such that the velocity v of the shear wave along the surface is close to the value v_1, the presence of a layer on the surface causes the conversion coefficients $R(v,h),T_\alpha(v,h)$ to behave in a way utterly unlike their behavior in its absence. This difference is enhanced as the parameter k_Rh decreases. However, according to (41) and (42) we find that $R(v,h),T_\alpha(v,h)$ do not have a limit as $v\to v_R$, $h\to 0$: their values at the point $v=v_R$, $h=0$ depend on the way the limit is taken. This indicates that Eqs. (41), (42) cannot correctly describe the behavior of the conversion coefficients near v_1 for values of k_Rh that are too small. We can establish the limits of applicability of (41), (42) from the following considerations.

Note first that $v=v_1-iv_1'$ the wave field in the crystal that satisfies the boundary conditions (29) does not contain the mode α=i: in the linear combination (27) we have $b_i=0$, but $b_r\neq 0$, $b_2\neq 0$, $b_3\neq 0$. The amplitude b_r is small compared to b_2, b_3, i.e.,

$$\left|\frac{b_r}{b_2}\right|\approx\left|\frac{b_r}{b_3}\right|\approx\frac{2}{|T_\alpha^{(0)}|}\sqrt{\frac{v_1'}{v_R}}\sim k_Rh, \tag{45}$$

while the ratio of amplitudes for the nonuniform modes themselves (α=2,3) turns out to be practically the same as for the Rayleigh wave: $b_2/b_3\approx -i$. Such solutions are usually called leaky waves, and it is convenient to treat them as a near-surface perturbation that decays as it propagates along the boundary due to radiation into the interior of the crystal.[8] The imaginary correction $-iv_1'$ characterizes the attenuation due to radiation; the leakage of energy from the boundary is provided by the mode α=r.

In view of this assertion, we see that in the reflection problem the condition $v=v_1$ corresponds to resonant excitation of the leaky wave by the incident wave; accordingly, the behavior near v_1 can be interpreted as a consequence of this resonance. In particular, from this point of view the value of $|T_\alpha(v,h)|$ for $|v-v_1|\leqslant v_1'$ is large because "almost" characteristic surface oscillations are excited. The abrupt change in the phase of the reflected wave is caused by interference between the wave that is reflected nonresonantly and the bulk component of the leaky wave. The sharpness of the resonance will be determined by the attenuation of the leaky wave, which in reality is due not only to radiation but also to absorption. We did not take into account the presence of dissipation in deriving Eqs. (41), (42), which is valid provided that the losses due to radiation in the leaky wave exceed losses due to absorption. If we describe the absorption of the leaky wave by adding an additional imaginary component $-iv_d'$ to the velocity v_1, we find that Eqs. (41), (42) are valid for $v_1'\gg v_d'$. Note that absorption of the leaky wave will be determined primarily by absorption of surface vibrations; therefore, using the quantity v_R' that characterizes the attenuation of Rayleigh waves as our estimate of v_d', we have a bound on the value of the parameter k_Rh in (41), (42):

$$(k_Rh)^2\gg v_R'/v_R. \tag{46}$$

Of course, if the absorption of sound in the film is anomalously large compared to that in the crystal, then v_d' can differ significantly from v_R'.

In order to represent the behavior of the coefficients of reflection and excitation of nonuniform modes for $v_1'\gtrsim v_d'$, we replace v_1 by v_1-iv_d' in Eqs. (41), (42). Then in place of v_1' the sum $v_1'+v_d'$ appears in the denominators of $R(v,h),T_\alpha(v,h)$, and the difference $v_1'-v_d'$ appears in the numerator of $R(v,h)$. From the expressions obtained in this way, it is clear in particular that as long as $v_1'>v_d'$ holds, the magnitude of the coefficient of excitation of nonuniform modes at $v=v_1$ will increase as the parameter k_Rh decreases, pass through a maximum $|T_\alpha(v,h)|\sim\sqrt{v_R/v_d'}$, and then decrease, falling to zero when $k_Rh=0$. Furthermore, it is now apparent that for $v_1'\approx v_d'$, within the resonance interval the magnitude of the reflection coefficient should increase abruptly. The explanation for this is that when the amplitude of the surface vibrations are large, even their weak absorption leads to the loss of a considerable fraction of the energy fed to the surface (see, e.g., Ref. 9). In the case of strong absorption ($v_d'\gg v_1'$), all the resonant features disappear.

6. COMPUTING THE PARAMETERS IN THE CONVERSION COEFFICIENTS OF A SHEAR WAVE AT THE CRYSTAL-FILM BOUNDARY

Let us find approximate expressions for the parameters entering into (41), (42), and also for the resonance angle of incidence

$$\theta_{res}=\arcsin(v_1/v_1), \tag{47}$$

corresponding to the condition $v=v_1$.

In accordance with (32) we have

$$\mathbf{L}_\alpha'\approx\mathbf{L}_\alpha-ikh(\hat{E}_{LA}^{(1)}\mathbf{A}_\alpha+\hat{E}_{LL}^{(1)}\mathbf{L}_\alpha)-0.5(kh)^2(\hat{E}_{LA}^{(2)}\mathbf{A}_\alpha+\hat{E}_{LL}^{(2)}\mathbf{L}_\alpha), \tag{48}$$

$$\hat{E}_{LA}^{(1)}=\sum_{\beta=1}^{6}p_\beta^f\mathbf{L}_\beta^f\otimes\mathbf{L}_\beta^f,\quad \hat{E}_{LL}^{(1)}=\sum_{\beta=1}^{6}p_\beta^f\mathbf{L}_\beta^f\otimes\mathbf{A}_\beta^f,$$

$$\hat{E}_{LA}^{(2)}=\sum_{B=1}^{6} p_\beta^{f2}\mathbf{L}_\beta^f\otimes\mathbf{L}_\beta^f,\quad \hat{E}_{LL}^{(2)}=\sum_{\beta=1}^{6} p_\beta^{f2}\mathbf{L}_\beta^f\otimes\mathbf{A}_\beta^f,\qquad (49)$$

while by virtue of (8), (9),

$$\hat{E}_{LA}^{(2)}=\hat{E}_{LA}^{(1)}(\hat{E}_{LL}^{(1)})^T+\hat{E}_{LL}^{(1)}\hat{E}_{LA}^{(1)},\quad \hat{E}_{LL}^{(2)}=\hat{E}_{LA}^{(1)}\hat{E}_{AL}^{(1)}+(\hat{E}_{LL}^{(1)})^2,\qquad (50)$$

where

$$\hat{E}_{AL}^{(1)}=\sum_{\beta=1}^{6} p_\beta^f\mathbf{A}_\beta^f\otimes\mathbf{A}_\beta^f;\qquad (51)$$

and $(E_{LL}^{(1)})^T$ is the transposed matrix of $E_{LL}^{(1)}$.

The matrices $E_{LA}^{(1)}$, $E_{LL}^{(1)}$, and $E_{AL}^{(1)}$ are directly expressed in terms of the elastic moduli of the layer C_{nmkl}^f in the $x_1x_2x_3$ system of coordinates and its density ρ_f. By the same token, taking into account (8), (9), it is easy to verify that

$$(\hat{E}_{LL}^{(1)})^T\mathbf{A}_\beta^f+\hat{E}_{AL}^{(1)}\mathbf{L}_\beta^f=p_\beta^f\mathbf{A}_\beta^f,$$
$$\hat{E}_{LA}^{(1)}\mathbf{A}_\beta^f+\hat{E}_{LL}^{(1)}\mathbf{L}_\beta^f=p_\beta^f\mathbf{L}_\beta^f.\qquad (52)$$

Now, if we write $p_\beta^f\mathbf{A}_\beta^f$, $p_\beta^f\mathbf{L}_\beta^f$ using Eq. (4) and the ratio (7), and then compare the results obtained with (52), we find

$$(\hat{E}_{LL}^{(1)})_{ql}=-C_{1qk2}^f[C_{2nm2}^f]_{kl}^{-1},$$
$$(\hat{E}_{LA}^{(1)})_{ql}=-C_{1qk2}^f[C_{2nm2}^f]_{kp}^{-1}C_{2pl1}^f+C_{1ql1}^f-\rho_f v^2\delta_{ql},$$
$$(\hat{E}_{AL}^{(1)})_{ql}=-[C_{2nm2}^f]_{ql}^{-1},\qquad (53)$$

where the repeated indices p,k are summed over: $p,k=1,2,3$, and $[C_{2nm2}^f]^{-1}$ is the inverse of the matrix with elements C_{2nm2}^f, n, $m=1,2,3$; $[C_{2nm2}^f]^{-1}$ always exists, since the requirement of thermodynamic stability implies that C_{2nm2}^f must be a positive definite matrix.[1)]

Substituting (17)–(20), (48) into (34), (35) and using the relations (10), (50), (53), after rather tedious transformations we are led from (34), (35) to Eqs. (41), (42), where

$$v_1\approx v_R[1+a_{11}(k_Rh)+a_{12}(k_Rh)^2],\qquad (54)$$

$$a_{11}=\frac{v_R^2[(\hat{E}_{LA}^{(1)})_{11}q_2+(\hat{E}_{LA}^{(1)})_{22}q_3]}{c_{66}v_2^2p_r^{(0)}g},\qquad (55)$$

$$a_{12}=-a_{11}^2\left[1+0.5\left(\frac{q_2q_3}{p_r^{(0)}}\right)^{1/2}\frac{G}{g}\right]+\frac{v_R^2(q_2q_3)^{1/2}}{c_{66}v_2^2g}\Bigg\{(\hat{E}_{LL}^{(1)}\hat{E}_{LA}^{(1)})_{21}-(\hat{E}_{LL}^{(1)}\hat{E}_{LA}^{(1)})_{12}+\frac{2a_{11}v_R^2}{(q_2q_3)^{1/2}}\left[\frac{v_2^2-v_3^2}{4v_2^2v_3^2(q_2q_3)^2}((\hat{E}_{LA}^{(1)})_{11}q_2-(\hat{E}_{LA}^{(1)})_{22}q_3)-\rho_f(q_2+q_3)\right]+\frac{3+q_2^2}{2c_{66}(1+q_2^2)}[(\hat{E}_{LA}^{(1)})_{11}(\hat{E}_{LA}^{(1)})_{22}-(\hat{E}_{LA}^{(1)})_{12}^2]\Bigg\}+\frac{a_{11}^2v_R^2}{2(v_R^2-v_1^2)},\qquad (56)$$

$$v_1'\approx v_Ra_1'(k_Rh)^2,\quad a_1'=\frac{v_R^2[(\hat{E}_{LA}^{(1)})_{13}^2q_2+(\hat{E}_{LA}^{(1)})_{23}^2q_3]}{c_{44}c_{66}v_2^2p_r^{(0)}g},\qquad (57)$$

$$T_\alpha^{(0)}=4\sqrt{\frac{q_2q_3}{g}}\exp(i\psi_\alpha),\quad \alpha=2,3,$$

$$\psi=-\pi/4+\operatorname{arctg}\left(\frac{(\hat{E}_{LA}^{(1)})_{23}}{(\hat{E}_{LA}^{(1)})_{13}}\sqrt{\frac{q_3}{q_2}}\right),\quad \psi_3=\psi_2+\pi/2,\qquad (58)$$

$$g=v_R(\partial f_R(v)/\partial v)|_{v=v_R},\quad G=v_R^4(\partial^2F_R(v)/\partial v^2)|_{v=v_R},$$

$$F_R(v)=\left[\frac{(v^2/v_1^2-1)}{(1-v^2/v_2^2)(1-v^2/v_3^2)}\right]^{1/4}\frac{f_R(v)}{v^2},$$

$$q_2=\sqrt{1-v_R^2/v_2^2},\quad q_3=\sqrt{1-v_R^2/v_3^2},$$

$$p_r^{(0)}=\sqrt{v_R^2/v_1^2-1},$$

$$(\hat{E}_{LL}^{(1)}\hat{E}_{LA}^{(1)})_{ij}=(\hat{E}_{LL}^{(1)})_{ik}(\hat{E}_{LA}^{(1)})_{kj}.\qquad (59)$$

Note that the parameter a_1' in (57) is greater than zero, because g is a positive quantity. For the layer orientation we have chosen, $(E_{LA}^{(1)})_{13}$,$(E_{LA}^{(1)})_{23}$ do not simultaneously vanish.

To order $(k_Rh)^2$ we obtain an approximate expression for the resonance angle of incidence (40):

$$\theta_{\rm res}\approx\arcsin(v_1/v_R)-\frac{a_{11}}{p_r^{(0)}}k_Rh+\left[\frac{a_{11}^2(2p_r^{(0)2}+1)}{2p_r^{(0)2}}-a_{12}\right]\frac{v_1}{v_R}(k_Rh)^2.\qquad (60)$$

We find the characteristic angular width $\Delta\theta_{\rm res}$ of the interval within which the rapid changes of $R(v,h)$,$T_\alpha(v,h)$ take place from the condition

$$v_1'=\frac{v_1}{\sin(\theta_{\rm res}-\Delta\theta_{\rm res}/2)},\qquad (61)$$

from which we obtain, again to order $(k_Rh)^2$,

$$\Delta\theta_{\rm res}\approx\frac{2v_1'}{v_Rp_r^{(0)}}=\frac{2a_1'}{p_r^{(0)}}(k_Rh)^2.\qquad (62)$$

Recall that in the course of our calculations we assumed that the vectors $\mathbf{A}_\alpha$,$\mathbf{L}_\alpha$ were normalized in accordance with (9). When we go over to the more traditional normalization $|\mathbf{A}_\alpha|=1$, the expression for $R(v,h)$ does not change if we set $\mathbf{A}_i=(0,0,1)$, $\mathbf{A}_r=(0,0,-i)$, but additional factors appear in $T_\alpha^{(0)}$:

$$\frac{|\mathbf{A}_\alpha|}{|\mathbf{A}_i|}=\frac{v_2}{v_R}\sqrt{\frac{c_{44}p_r^{(0)}(1+q_\alpha^2)}{c_{66}q_\alpha}},\quad \alpha=2,3.\qquad (63)$$

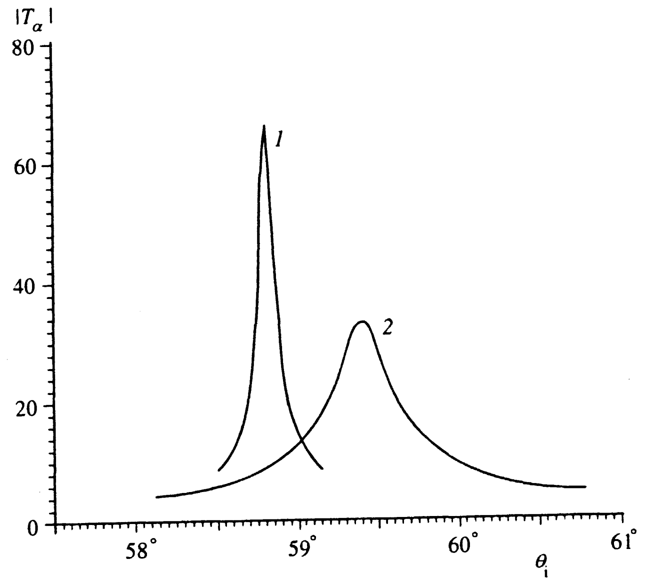

FIG. 2. Absolute value of the coefficient of excitation of nonuniform modes vs. the angle of incidence for resonance reflection. *1*—$k_R h = 0.05$, *2*—$k_R h = 0.1$.

7. CONCLUSION

In this paper we have shown that the presence of a thin elastically anisotropic film at the surface of a hexagonal crystal radically changes the nature of reflection of shear waves in the sagittal plane of a "supersonic" Rayleigh wave. A narrow interval of incident angles emerges in which the phase of the reflection coefficient changes abruptly, and reflection is accompanied by excitation of surface modes with an amplitude much greater than the amplitude of the incident wave. In Fig. 2 we show as an illustration plots of the absolute value of the coefficient of excitation of surface modes (42) versus the angle of incidence for $k_R h = 0.05$ and 0.1 for a Zn crystal with a layer deposited on its surface (also of Zn). In the layer the sixfold axis of symmetry is parallel to the surface at an angle of 50° to the x_3 axis. For crystals of Zn, the factors (63) are equal: $|\mathbf{A}_2/\mathbf{A}_i| = 1.11$, $|\mathbf{A}_3/\mathbf{A}_i| = 0.96$.

Anomalous behavior of the conversion coefficients occurs at angles of incidence such that the velocity of the incident wave along the surface is close to the velocity of the leaky wave, which evolves from the Rayleigh wave due to anisotropy of the elastic properties of the film; therefore, we can interpret the appearance of these peculiarities to be result of resonance excitation of the leaky wave. We note that a similar resonance effect occurs when the plane of incidence is rotated with respect to the sagittal plane of the "supersonic" surface wave;[11–15] in this case, the leaky wave arises from the characteristic anisotropy of the crystals as long as the angle of deviation is small.

According to (60), (62), the value of the resonant angle of incidence and the width of the resonant interval depend on the parameter $k_R h$. This allows us to control the effect by varying the frequency of the incident wave. An acoustic pulse incident on the boundary will thus suffer a significant distortion. Likewise, the abrupt dependence of the phase of the reflection coefficient on the angle of incidence gives rise to strong "nonspecular" reflection of a wave disturbance with a finite spatial spectrum (an acoustic beam), i.e., it leads to a lateral shift of the disturbance over a distance of the same order as its width, as well as a considerable distortion of its shape (see, for example, Refs. 9, 13, 16).

It is noteworthy that "supersonic" surface waves can occur in crystals with symmetries other that hexagonal. For example, it is well known that the Rayleigh wave in cubic crystals is often found to be "supersonic";[17] the Gulyaev-Bleustein waves in piezo-electric crystals can also become "supersonic".[12] Accordingly, an anisotropic layer can create conditions for resonance reflection in these cases as well. Note also that, the analogous phenomenon, i.e., exceptionally strong influence of a thin anisotropic film on reflection, can occur in optics as well.

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[1)]Thus, there is no need to compute $p^f_\beta, \mathbf{A}^f_\beta, \mathbf{L}^f_\beta$. When the orientation of the layer is "nonsymmetric," as is the case here, such computations can only rarely be done (see, e.g., Ref. 10) without the help of numerical calculations.

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