Structure of the domain walls of a uniaxial ferromagnet

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An isolated domain wall is analyzed in the model of an infinite uniaxial ferromagnet. Analytic solutions describing new types of 1D domain walls are derived in the limits of large and small values of the quality factor. These walls consist of a core and two branches. The analysis is based on an equation for the trajectory of the magnetization in configuration space and an equation for the evolution of the trajectory in ordinary space. The method proposed here is quite general. It can be used to analyze a wide range of nonlinear problems. © *1995 American Institute of Physics.*

1. INTRODUCTION

The static structure of domain walls of a uniaxial ferromagnet was first described by Landau and Lifshitz.[1,2] Domain walls with the Landau-Lifshitz structure ("Bloch walls") have become the basis for interpreting experimental data. The method proposed by Landau and Lifshitz for solving the problem is the basis for the theory of domain structures.[3–5]

In this paper we derive general solutions of the Landau–Lifshitz problem in the limits of large and small values of the quality factor. In contrast with Bloch walls, in which the magnetization vector rotates in a fixed plane as we move from one domain to another, we find domain walls for which the plane in which the magnetization vector rotates is not fixed.

Our analysis is based on an equation for the trajectory of the magnetization in configuration space and an equation for the evolution of the trajectory in ordinary space. We accordingly first find an appropriate variational principle and then derive the necessary equations. In the sections of the paper which follow, these equations are used to determine the magnetization distribution in a domain wall of ferromagnets with large and small values of the quality factor.

2. VARIATIONAL PRINCIPLE

This analysis of 1D domain walls of a uniaxial ferromagnet is conducted within the framework of the problem proposed by Landau and Lifshitz.[1,2] We assume an infinite uniaxial ferromagnet in which the z axis is the easy axis, and the x axis is normal to the surface of the domain wall. We write the energy in the form

$$e=\int dx\left[A\left(\frac{d\mathbf{m}}{dx}\right)^2-Km_z^2+2\pi M^2m_x^2\right], \tag{1}$$

where A and $K(>0)$ are the constants of nonuniform exchange and uniaxial anisotropy, $\mathbf{m}=\mathbf{M}/M$, $\mathbf{M}$ is the magnetization vector, and M is the saturation magnetization. The last term is the energy of the magnetic-dipole interaction.

Within domains, the magnetization vector is uniform and is directed either parallel or antiparallel to the easy axis:[1,2]

$$\mathbf{M}_{1,2}=M(0,0,\mp 1), \tag{2}$$

where the subscripts 1 and 2 specify the domains.

We solve the problem with the help of angle variables. To avoid restricting the generality of our results, we will not specify the dependence of the unit vector $\mathbf{m}$ on the angle variables θ and φ at this point, but we do assume that once these variables have been introduced the energy of the ferromagnet is given by

$$e=\int dx(T-U), \tag{3}$$

$$T=A\left[\left(\frac{d\theta}{dx}\right)^2+\sin^2\theta\left(\frac{d\varphi}{dx}\right)^2\right],\quad U=U(\theta,\varphi). \tag{4}$$

We see that the nonuniform part of the magnetic energy is represented by the quantity T, while the uniform part is equal to minus U. The quantity U depends only on the angle variables.

The boundary conditions are determined by assumption (2)—that the magnetization is uniform within the domains.[1,2] The nonuniform part of the magnetic energy is thus zero, and the quantity U is stationary. We thus find the following results inside the domains:

$$\left.\frac{d\theta}{dx}\right|_{1,2}=0,\quad \sin\theta\left.\left(\frac{d\varphi}{dx}\right)\right|_{1,2}=0, \tag{5a}$$

$$\delta U=0, \tag{5b}$$

where δ is the variational derivative.

The magnetic energy does not depend explicitly on the spatial variable, so we have the integral

$$T+U=U_0 \tag{6}$$

in this problem (U_0 is a constant of integration), and instead of using the variational principle

$$\delta e=0 \tag{7}$$

we can describe the system by means of the variational principle

$$\delta\sigma=0,\qquad \sigma=2\int\sqrt{A(U_0-U)(d\theta^2+\sin^2\theta d\varphi^2)}, \tag{8}$$

where σ is the surface energy density of the domain wall.

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The variational principle (8) can be used to find the trajectory of the magnetization in configuration space. (The transformation from (7) to (8) can be found in Refs. 6–8, among other places.) The evolution of the trajectory in ordinary space is found with the help of integral (6), with T and U from (4).

We treat σ as an element of length in configuration space. From the geometric standpoint our problem thus reduces to one of determining geodesic lines between points in the space with a deformed spherical metric specified by means of (8). It is useful to note the relationship between the length of a line in configuration space and the energy of the domain wall.

Our solution is based on (8). The boundary conditions on θ and φ are found from (5b). The derivative $d\varphi/d\theta$ (or $d\theta/d\varphi$) is undetermined, as follows from (5a).

The particular cases θ=const and φ=const have already been analyzed,[1,3] so we will look at the general case. We will derive an equation for extrema of (8) of the form $\theta=\theta(\varphi)$ or $\varphi=\varphi(\theta)$.

3. EQUATION OF THE TRAJECTORY

A variational equation corresponding to (8) can be written

$$\frac{d^2\theta}{d\varphi^2}\,2aV+\left(\frac{d\theta}{d\varphi}\right)^3 a^2\frac{\partial V}{\partial\varphi}-\left(\frac{d\theta}{d\varphi}\right)^2\left(a\frac{\partial V}{\partial\theta}-V\frac{\partial a}{\partial\theta}\right)+\frac{d\theta}{d\varphi}\,a\frac{\partial V}{\partial\varphi}-\frac{\partial V}{\partial\theta}=0, \qquad (9)$$

where $a=1/\sin^2\theta$ and $V=(U_0-U)\sin^2\theta$.

At this point we transform Eq. (9) to an equation more convenient for analysis. For this purpose we first multiply the entire equation by $d\theta/d\varphi$. We then add to and subtract from the resulting equation a term $(1+a(d\theta/d\varphi)^2)\partial V/\partial\varphi$.

After some manipulations, the variational equation can be written

$$\frac{\partial V}{\partial\varphi}-\frac{d}{d\varphi}\,\frac{V}{\dfrac{1}{\sin^2\theta}\left(\dfrac{d\theta}{d\varphi}\right)^2+1}=0. \qquad (10)$$

Multiplying (10) by $d\varphi$, integrating the resulting expression along the magnetization trajectory L, carrying out some simple manipulations, and using the integral (6), we can write an equation for the trajectory along with an equation for the spatial evolution of the trajectory. As a result the system of equations becomes

$$\frac{d\theta^2}{\sin^2\theta\displaystyle\int_{(L)}\frac{\partial V}{\partial\theta}\,d\theta}=\frac{d\varphi^2}{\displaystyle\int_{(L)}\frac{\partial V}{\partial\varphi}\,d\varphi},$$
$$\frac{dx^2}{A}=\frac{\sin^2\theta\, d\theta^2}{\displaystyle\int_{(L)}\frac{\partial V}{\partial\theta}\,d\theta}. \qquad (11)$$

In deriving (11) we used the identity

$$V\Big|_{\theta_1,\varphi_1}^{\theta,\varphi}=\int_{(L)}dV=\int_{(L)}\left(\frac{\partial V}{\partial\theta}\,d\theta+\frac{\partial V}{\partial\varphi}\,d\varphi\right), \qquad (12)$$

which allows us to avoid distinguishing any one of the variables. The lower limit in (12) corresponds to the state of the domain (for definiteness, we are using a domain specified by the coordinates $\theta=\theta_1$ and $\varphi=\varphi_1$ as a reference point), while the upper limit is the instantaneous point of the trajectory. The limits in (11) are equivalent to those in (12).

Equations (11) constitute the fundamental system of equations in the method being proposed here for analyzing nonlinear problems. The first equation is the equation of the trajectory; the second is the equation of the spatial evolution of the trajectory. We will not carry out a general analysis of these equations in the present paper. For convenience in understanding these equations we suggest drawing on an analogy between mechanical problems and problems involving static structures of a domain wall.[1,3]

We accordingly assume that e in (3) is the action of the mechanical system, that θ and φ are generalized coordinates, and that T and U are kinetic and potential energies. The variable x plays the role of time. The integral (6) is then an energy integral, and the switch to a description of a mechanical system by means of (8) corresponds to a switch to a description of this system by means of an "abbreviated" action. Principle (8) itself is called the "Maupertuis principle"[7,8] or, after Ref. 6, the "Jacobi principle."

If we take the $\sin^2\theta$ outside the parentheses in (8) and introduce the new variable $d\kappa=d\theta/\sin\theta$, then we can discuss yet another analogous mechanical system, characterized by the generalized coordinates κ and φ. In contrast with the $\theta\varphi$ mechanical analog, the potential energy of the $\kappa\varphi$ analog is equal to V, and the mass coefficients are constants. The $\theta\varphi$ and $\kappa\varphi$ analogs have identical trajectories and differ only in the time evolution of the trajectory. The procedure for deriving the trajectory equation in the form in (11) is considerably simpler for the $\kappa\varphi$ analog: This equation can be derived in a trivial way from the Euler–Lagrange equations for the $\kappa\varphi$ system.

In mechanics, equations similar to Eqs. (11) can describe systems which are characterized by two dynamic variables and which are moving in a potential well, e.g., a 2D oscillator with initial conditions corresponding to a turning point. The quantity V is the potential energy formed by the external fields (the terms depend on only a single variable) and by the interaction fields (the terms depend on both variables). The integral over $d\theta$ is then the change in the potential energy of the θ subsystem, while the integral over $d\varphi$ is correspondingly the change in the potential energy of the φ subsystem. We will make use of this analogy in solving system (11).

4. STRUCTURE OF A DOMAIN WALL IN THE CASE OF A LARGE QUALITY FACTOR

In this section of the paper we find the static structure of 1D domain walls of a uniaxial ferromagnet in the limit of large values of the quality factor.

We assume a unit vector

$$\mathbf{m}=(\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta). \qquad (13)$$

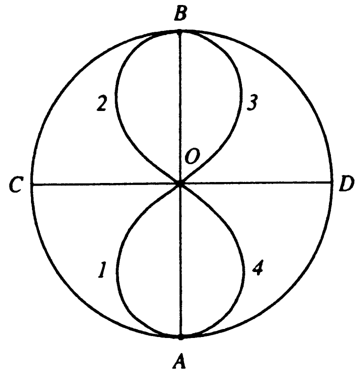

FIG. 1. Magnetization trajectory in a domain wall of a uniaxial ferromagnet in the limit $Q \gg 1$. The configuration space is a sphere with the metric given by expression (8). The angles θ and φ are defined in (13). This figure shows the projection of the "eastern" hemisphere onto a great circle. At point A we have the value $\theta=0$, while at B we have $\theta=\pi$. Lines ACB and ADB represent domain walls with the Landau–Lifshitz structure (i.e., Bloch walls). On line ACB we have $\varphi=-\pi/2$, while on ADB we have $\varphi=\pi/2$. Line AOB represents a Néel wall ($\varphi=0$). Lines $A102B$ and $A403B$ correspond to a domain wall of type I, while lines $A103B$ and $402B$ correspond to a domain wall of type II. The magnetic-dipole barrier is at a maximum along line AOB, while it is zero along ACB and ADB. Domain walls of type I "hang down" on one side of this barrier and can, accordingly, convert into Landau–Lifshitz domain walls, but the domain walls of type II lie on different slopes of the barrier and cannot convert into a Bloch state.

The energy of the ferromagnet is then given by (3) with

$$U=K\cos^2\theta-2\pi M^2\sin^2\theta\cos^2\varphi. \tag{14}$$

Using (5b), we find that the angle θ in the domains has the values

$$\theta_1=0, \quad \theta_2=\pi. \tag{15}$$

The angle φ is undetermined. The value of φ in the domains is undetermined because of the metric being used for the configuration space and because of the definition of the angle variables in (13).

We first consider two particular cases.

a) $\theta=$const. The solutions describe uniformly magnetized states and are the same as the boundary conditions on the problem.

b) $\varphi=$const. The solutions describe Bloch and Néel walls. In both such walls the magnetization vector rotates in a given plane as we cross from one domain into another, but in Bloch walls the rotation occurs in the plane of the domain wall, while in Néel walls it occurs in a plane perpendicular to the surface of the domain wall.[1–3] The functional dependence of the angle θ is specified by the Landau–Lifshitz solution (the spatial positions of the domains are fixed),

$$\theta(x)=2\arctan e^{x/\Delta}, \quad \Delta=\sqrt{A/K},$$

but in a Bloch wall the angle which specifies the rotation plane is

$$\varphi=\pi/2+\pi n, \quad n=0,\pm1,\pm2,\ldots,$$

while that in a Néel wall is

$$\varphi=\pi n, \quad n=0,\pm1,\pm2,\ldots\,.$$

In configuration space (Fig. 1) the magnetization trajectory corresponding to these domains walls consists of the following lines: Domain walls with the Landau–Lifshitz structure are shown by lines ACB ($\varphi=-\pi/2$) and ADB ($\varphi=\pi/2$), while a Néel wall is shown by line AOB ($\varphi=0$). Figure 1 shows a projection of the spherical configuration space onto the surface of a great circle. (Only the "eastern" hemisphere has been projected; the projection of the "western" hemisphere has a similar shape, but the central line AOB corresponds to a Néel wall with $\varphi=\pi$.)

We turn now to the general case. We wish to find extrema of (8) of the form $\theta=\theta(\varphi)$ or $\varphi=\varphi(\theta)$. We will first take a qualitative look at this question. The domains are represented by the points A and B. The 1D domain walls in which we are interested are represented by certain curves which connect domain states. It is clear just from geometric considerations that there exist curves of a general type which connect these points; accordingly, 1D domain walls with $\theta=\theta(\varphi)$ also exist. From the physical standpoint, the existence of domain walls with $\theta=\theta(\varphi)$ is associated with a magnetic-dipole barrier.

The energy of the magnetic-dipole interaction is zero along ACB and ADB, which represent Bloch walls. This energy is at a maximum for each fixed θ along line AOB, i.e., along a Néel extremum. There is accordingly a "ridge" along AOB on the surface of the magnetic-dipole energy. As we descend from this ridge, the magnetic-dipole energy decreases monotonically, vanishing on ACB and ADB. At point O the energy of the magnetic-dipole interaction is stationary, as is the entire uniform part of the magnetic energy of the ferromagnet.

This shape of the magnetic-dipole barrier separating two Bloch states is such that two topologically nonequivalent trajectories with $\theta=\theta(\varphi)$ can exist. A magnetization trajectory which begins at, say, point A may, as it moves to point B, either run entirely on one side of the ridge (as shown by curves such as $A102B$ and $A403B$ in Fig. 1) or lie on both sides of the ridge (as shown by curves such as $A103B$ and $A402B$ in Fig. 1).

Understandably, domain walls of the first type may be topologically unstable with respect to a Landau–Lifshitz transition, since they can simply "slide down" the slope toward the base of the magnetic-dipole barrier and thus transform into Bloch walls. Domain walls of the second type are topologically stable with respect to a transition with the Landau–Lifshitz structure, since the curve corresponding to these walls simultaneously tends to slide down into the ravines running along the opposite slopes of the magnetic-dipole barrier. Because of the tendency of a domain wall of the second type to have a minimum energy (or a minimum length) and to be at equilibrium, the function $\theta=\theta(\varphi)$ for them becomes antisymmetric: $\theta(\varphi)=-\theta(-\varphi)$. The opposite parts of a wall will then "hang over" from the slopes of the magnetic-dipole barrier to an identical extent.

We wish to repeat that the existence of 1D domain walls in uniaxial ferromagnets within which both angles vary is a consequence of the magnetic dipole interaction. If this interaction were turned off, the shape of the potential barrier would permit the existence of domain walls only with the Landau–Lifshitz structure.

We now wish to derive an analytic solution of the problem. In our case we have

$$V=(U_0-U)\sin^2\theta=K(1+\varepsilon\cos^2\varphi)\sin^4\theta,$$

so Eqs. (11) become

$$\frac{d\theta^2}{\sin^2\theta\int_{(L)}\frac{\partial}{\partial\theta}(1+\varepsilon\cos^2\varphi)\sin^4\theta d\theta}=\frac{d\varphi^2}{\varepsilon\int_{(L)}\frac{\partial}{\partial\varphi}\sin^4\theta\cos^2\varphi d\varphi},$$
$$(dx')^2=\frac{\sin^2\theta d\theta^2}{\int_{(L)}\frac{\partial}{\partial\theta}(1+\varepsilon\cos^2\varphi)\sin^4\theta d\theta}, \tag{16}$$

where $x'=x/\Delta$, $\Delta=\sqrt{A/K}$, and $\varepsilon=1/Q=2\pi M^2/K$.

We assume $\varepsilon\ll 1$ (this is the case of large values of the quality factor). We can then ignore the contribution of the "interaction" energy to the energy of the θ subsystem (this contribution is on the order of ε, and the energy of this subsystem in an external field is of order unity). The change in the energy of the φ subsystem can be assumed equal to the total change in the interaction energy of the entire system (i.e., we add to the term of order ε one more term of order ε, thereby turning the integrand into a total differential). In other words, in our approximation, because of the weak correlation between the variables θ and φ, we are assuming that the functional dependence $\theta=\theta(x)$ is determined without the involvement of φ, which in turn simply "adjusts" by means of the interaction potential to accommodate the corresponding value of $\theta=\theta(x)$ at each point x.

In our approximation, Eqs. (16) become the system of equations

$$\frac{d\theta^2}{\sin^6\theta}=\frac{d\varphi^2}{\varepsilon\sin^4\theta\cos^2\varphi},$$
$$dx'^2=\frac{\sin^2\theta d\theta^2}{\sin^4\theta}. \tag{17}$$

Solving Eqs. (17), we find the magnetization trajectory to be

$$\left|\tan\frac{\theta}{2}\right|^{\mp\sqrt{\varepsilon}}=R\left|\tan\left(\frac{\varphi}{2}+\frac{\pi}{4}\right)\right|, \tag{18}$$

while the spatial evolution of the trajectory is described by

$$\mp(x'-x_0')=\ln\left|\tan\frac{\theta}{2}\right|, \tag{19}$$

where x_0' and R (>0) are constants.

Note that the magnetization trajectory is determined more accurately than to within terms on the order of ε. The reason is that "global" approximation for an arbitrary part of the trajectory which is determined by the estimated value of the integral is refined even further locally, since we are solving a differential equation. The spatial evolution of the trajectory is determined within terms of zeroth order in ε, but by using trajectory (18) we can refine the accuracy of the solution where necessary.

The set of equations (18), (19) thus constitutes a general solution which describes domain walls with $\theta=\theta(\varphi)$. (Accordingly, when we take the particular cases θ=const and φ=const into account, we find a complete solution of the Landau–Lifshitz problem for large values of the quality factor.) The boundary conditions are not sufficient for determining the constants x_0' and R. The constant x_0' characterizes the position of the center of the wall, and in the static case it is customary to assume $x_0'=0$, i.e., to assume that the center of the wall is at the origin of coordinates. The simplest way to determine the constant R is to use the results of a qualitative analysis of the problem, from which it follows that we have $\theta(\varphi)=-\theta(-\varphi)$ for domain walls of the second type, and the magnetization trajectory necessarily passes through the point $\theta=\pi/2$, $\varphi=\theta$. Using this fact and (18), we find $R=1$. There is yet another way to determine R: substitute (18) into (8) and minimize σ with respect to this parameter.

A unit value for the parameter R implies that the centers of the spatial distribution coincide for the two angle variables and, correspondingly, for all components of the magnetization vector. This is a natural result in the static case; in the dynamic case, the centers of the distribution may undergo a relative shift, so the parameter R or some function of it would characterize the dynamic response of the domain wall. A detailed examination of that topic goes beyond the scope of the present paper.

We are now in a position to write the final expressions for the magnetization trajectory,

$$\left|\tan\frac{\theta}{2}\right|^{\mp\sqrt{\varepsilon}}=\left|\tan\left(\frac{\varphi}{2}+\frac{\pi}{4}\right)\right|, \tag{20}$$

and for the spatial evolution of the trajectory,

$$\mp x'=\ln\left|\tan\frac{\theta}{2}\right|. \tag{21}$$

In (20) we have written the entire sheaf of geodesic lines of $\theta=\theta(\varphi)$ emerging from the point $\theta=0$ or the point $\theta=\pi$. We recall that in determining the trajectory we used only a single domain state. Noting that geodesic lines connecting the singular points $\theta=0$ and $\theta=\pi$ correspond to a domain wall, and using (20), we can determine the magnetization trajectory in a domain wall with $\theta=\theta(\varphi)$. The spatial evolution for each type of trajectory is given by (21).

Figure 1 shows a magnetization trajectory. Here we see both domain walls of the first type (DWIs) and domain walls of the second type (DWIIs). The qualitative analysis carried out above shows that the DWIs may be unstable against conversion into a domain wall with the Landau–Lifshitz structure. Domain walls of the second type are, according to the structure of the potential barrier, topologically stable against conversion into Bloch walls.

We wish to stress that these conclusions regarding the stability of DWIs and DWIIs are only qualitative. A systematic study of the stability of the micromagnetic solution derived here with respect to nonuniform perturbations runs into serious mathematical difficulties, because of the nonlocal nature of the magnetic-dipole interaction. We will not go into such an analysis in the present paper. We should, on the other hand, point out that a magnetization trajectory in either a DWI or a DWII passes through the point O, at which the uniform part of the magnetic energy is stationary. In the mechanical sense, the resultant force acting on the trajectory at this point is zero. This point is thus immobile. Since the point O is immobile, we cannot draw an unambiguous conclusion regarding the topological instability of DWIs against conversion into Bloch walls (one point of the trajectory cannot slide down into the ravine). Both types of domain walls may thus be stable.

In the symmetry classification of the domain walls of a uniaxial ferromagnetic which was carried out by Bar'yakhtar *et al.*[9] the DWIIs fall in class $G^{(11)}$. The components of the magnetization vector in this symmetry class have the following transformation properties:

$$M_x(x)=M_x(-x)M_y(x)=-M_y(-x)M_z(x)=-M_z(-x).$$

The DWIs belong to class $G^{(10)}$, and from the symmetry standpoint they differ from the domain walls of class $G^{(11)}$ in the symmetry of the function $M_y=M_y(x)$ (the transformation properties of other magnetization components are unchanged).

Using expressions (20) and (21), we can write functional dependences for the angle variables for DWIs and DWIIs. We begin with the DWIIs. The structure of these walls is described by the following pairs of functions:

$$\text{a)}\quad \theta=2\arctan e^{x'},\quad \varphi=-\frac{\pi}{2}+2\arctan e^{\sqrt{\varepsilon}x'};$$

$$\text{b)}\quad \theta=2\arctan e^{x'},\quad \varphi=-\frac{\pi}{2}+2\arctan e^{-\sqrt{\varepsilon}x'};$$

$$\text{c)}\quad \theta=2\arctan e^{x'},\quad \varphi=-\frac{3\pi}{2}+2\arctan e^{\sqrt{\varepsilon}x'};$$

$$\text{d)}\quad \theta=2\arctan e^{x'},\quad \varphi=-\frac{3\pi}{2}+2\arctan e^{-\sqrt{\varepsilon}x'}.$$

At the center of domain walls described by expressions a) and b), we have a vector $\mathbf{m}=(1,0,0)$; at the center of domain walls described by c) and d) we instead have $\mathbf{m}=(-1,0,0)$. In the expression for the DWIIs, the positions of the domains were fixed.

The structure of a DWI for fixed positions of the domains is also specified by four pairs of functions, but here we will write out only a single pair. It is simple to construct the other pairs by comparison with the preceding case:

a) $\theta=2\arctan e^{x'}$,

$$\varphi=\begin{cases} -\dfrac{\pi}{2}+2\arctan e^{\sqrt{\varepsilon}x'}, & x'<0,\\[2mm] -\dfrac{\pi}{2}+2\arctan e^{-\sqrt{\varepsilon}x'}, & x'>0.\end{cases}$$

At the center of a DWI, the derivative $d\varphi/dx$ is discontinuous.

We see that the new domain walls consist of a core and two branches. The angle φ changes sharply in the branches, while the angle θ remains essentially constant. Accordingly, it is primarily the components m_x and m_y which change in the branches. In the central part there is a sharp change in the angle θ, which changes all components of the magnetization vector. At the center of the DWIs and DWIIs the magnetization vector is directed perpendicular to the surface of the domain wall. In other word, the central part of the new domain walls is similar in structure to Néel walls.

The existence of this structure for the new walls is a consequence of the different scales of the typical changes in the angles θ and φ, as follows from the magnetization trajectory (20) and from the spatial evolution of this trajectory in (21). Using (20), (21), (3), and (8), we can find both the typical sizes of the constituent "parts" of the domain wall and the surface energy density of the wall. We will not actually go through this procedure in the present paper. We should point out, on the other hand, that the energy of the new walls is greater than the energy of Bloch walls but lower than the energy of Néel walls, as is easily seen from (8).

It is currently believed that a Bloch wall in a magnetic field directed perpendicular to the easy axis converts into a Néel wall at certain values of the field.[3,4] It can be seen from Fig. 1 that the trajectory of a Néel wall is completely blocked by the trajectory of the new walls. As the magnetic field increases from zero, a Bloch wall thus necessarily converts into one of the new domain walls. This new wall is favored over a Néel wall from the energy standpoint because the functional in (8) reaches a maximum on a Néel extremum. The new wall (which is one of the DWIs) vanishes in fields on the order of the uniaxial anisotropy, at which points A and B (i.e., domain states) merge to form a single point at O, and the ferromagnet goes into a uniformly magnetized state. When the magnetic field is varied in the opposite direction, we may find a situation in which DWIIs are nucleated, and, because of their topology, these walls cannot convert into Bloch walls in a zero field. The presence of the new walls thus suggests a hysteresis in the evolution of the system in a magnetic field. There is a hysteresis in the average value of the component M_x as a function of the magnetic field H_x.

It is believed that, in the absence of a magnetic field, the static structure of cylindrical magnetic domains (magnetic bubbles) with Bloch lines consists of Bloch segments separated by Néel regions.[4] The existence of the new domain walls requires a modification of that model. Since their energy is smaller than the energy of Néel walls, it is preferable from the energy standpoint that the regions separating the Bloch segments have the structure of the new domain walls. If a hysteresis is possible in the domain walls, the model of Bloch lines requires qualitative changes.

5. DOMAIN WALLS FOR SMALL VALUES OF THE QUALITY FACTOR

In this section of the paper we find the static structure of 1D domain walls of a uniaxial ferromagnetic in the limit of small values of the quality factor.

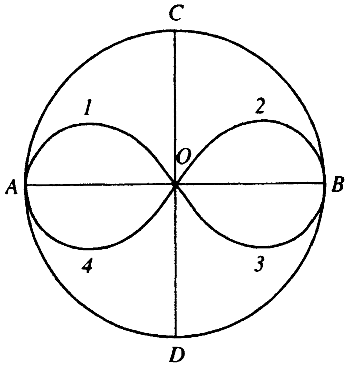

FIG. 2. Magnetization trajectory in a domain wall of a uniaxial ferromagnet in the limit $Q \ll 1$. The configuration space is a sphere with the metric given by expression (8). The angles θ and φ are defined in (22). This figure shows the projection of the "southern" hemisphere onto a great circle. The lines of θ=const are concentric circles centered on point O. We have $\theta=0$ at point O and $\theta=\pi/2$ on circle $ADBCA$. The angle φ has the values $\varphi_B=0$, $\varphi_C=\pi/2$, $\varphi_A=\pi$, and $\varphi_D=3\pi/2$ at points B, C, A, and D, respectively. Lines ACB and ADB represent Bloch walls. Line AOB is a Néel wall. Lines $A102B$ and $A403B$ correspond to domain walls of type I, while lines $A103B$ and $A402B$ correspond to domain walls of type II. The magnetic-dipole barrier is at a maximum along AOB, while it is zero along ACB and ADB. Domain walls of type I "hang down" on one side of this barrier and can accordingly convert into Landau–Lifshitz domain walls, but the domain walls of type II lie on different slopes of the barrier and cannot convert into a Bloch state.

We assume a unit vector

$$\mathbf{m}=(\cos\theta,\sin\theta\sin\varphi,\sin\theta\cos\varphi). \tag{22}$$

The energy is then given by expression (4) with U in the form

$$U=K\sin^2\theta\cos^2\varphi-2\pi M^2\cos^2\theta. \tag{23}$$

Minimizing the uniform part of the magnetic energy, we find that the angles θ and φ in the domains take on the values

$$\theta=\frac{\pi}{2}k,\quad k=\mp1,\mp3,\mp5\ldots;$$

$$\varphi=\frac{\pi}{2}n,\quad n=0,\mp2,\mp4,\ldots\,. \tag{24}$$

Let us find an analytic solution of the problem. As above, we first consider two particular cases.

a) θ=const. The solutions in this case describe a domain wall with the Landau–Lifshitz structure.[1,2] These domain walls were described in detail in the preceding section of this paper. However, we should recall that in the present section of the paper the angle variables are determined in a way different from that of the preceding section.

b) φ=const. In this case the solutions describe Néel walls. The structure of these domain walls was also described in detail in the preceding section of the paper. Under the condition $K>0$, the solutions correspond to states of the system with the maximum energy.

The potential barrier separating Bloch states is characterized as in the case $Q\gg1$. The existence of domain walls with $\theta=\theta(\varphi)$ is again a consequence of the structure of the potential barrier, which again allows the existence of two types of new domains walls. Figure 2 shows the projection of the "southern" hemisphere of configuration space onto a great circle. The other Néel wall is found by projecting the "northern" hemisphere.

We now wish to derive analytic expressions for extrema of the form $\theta=\theta(\varphi)$ or $\varphi=\varphi(\theta)$.

In this case we have

$$V=(U_0-U)\sin^2\theta=(K+2\pi M^2)\times\sin^2\theta(\cos^2\theta+\varepsilon\sin^2\theta\sin^2\varphi), \tag{25}$$

so Eqs. (11) can be written

$$\frac{d\theta^2}{\sin^2\theta\int_{(L)}\frac{\partial}{\partial\theta}\sin^2\theta(\cos^2\theta+\varepsilon\sin^2\theta\sin^2\varphi)d\theta}=\frac{d\varphi^2}{\varepsilon\int_{(L)}\frac{\partial}{\partial\varphi}\sin^4\theta\sin^2\varphi d\varphi},$$

$$dx'^2=\frac{\sin^2\theta d\theta^2}{\int_{(L)}\frac{\partial}{\partial\theta}\sin^2\theta(\cos^2\theta+\varepsilon\sin^2\theta\sin^2\varphi)d\theta}, \tag{26}$$

where $x'=x/\Delta$, $\Delta=\sqrt{A/(K+2\pi M^2)}$, $\varepsilon=Q/(1+Q)$, and $Q=K/2\pi M^2$.

We assume $\varepsilon\ll1$ (this is the case of small values of the quality factor). As above, we assume that the change in the "potential" energy of the θ subsystem is due to "external fields" alone, while the change in the "potential" energy of the φ subsystem is equal to the total change in the "interaction" energy of the entire $\theta\varphi$ system. It follows from the form of the potential V that for the θ subsystem this partitioning of the potential energy may not be correct near domain states, since the energy in the external field may be on the order of the interaction energy, and (strictly speaking) we cannot ignore this term in the energy of the θ subsystem.

In our method, the trajectory is constructed from the point which corresponds to the state of the domain, so an error may begin to build up even in the initial stage in our case. We will come back to a more concrete discussion of the validity of this method when we derive solutions.

In our approximation, Eqs. (26) become the following system of equations:

$$\frac{d\theta^2}{\sin^4\theta\cos^2\theta}=\frac{d\varphi^2}{\varepsilon\sin^4\theta\sin^2\varphi},\qquad dx'^2=\frac{\sin^2\theta d\theta^2}{\sin^2\varphi\cos^2\theta}. \tag{27}$$

Solving (27), we find that the trajectory of the magnetization is

$$\left|\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right|^{\mp\sqrt{\varepsilon}}=R\left|\tan\frac{\varphi}{2}\right|, \tag{28}$$

while the spatial evolution of the trajectory is described by

$$\mp(x'-x_0')=\ln\left|\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right|, \tag{29}$$

where x_0' and R (>0) are constants.

In deriving solutions (28) and (29) we assumed that the interaction does not greatly perturb the θ subsystem. Is this assumption valid along the trajectories which have been found?

A direct calculation shows that for our trajectories, and at small values of ε, the interaction energy of the θ subsystem near a domain state is on the order of the energy of the subsystem in the external field. For these domain walls, our method thus yields only the qualitative behavior of the trajectory near the entrance into a domain. Although an error is introduced in the trajectory by the initial region of the trajectory, the error does not build up, and we have derived the solution expected on the basis of physical considerations (the curve connects domain states). The part of the trajectory outside the initial region is determined, as in the preceding case, at an accuracy better than to within terms of order ε. We should also point out that the entrance into the domain (at point A, for example) occurs directly in accordance with the requirements of the problem. In order to determine the behavior of the trajectory near the domain more accurately, we would need to solve Eqs. (26), using the trajectory (28) as an initial approximation. We will not go through that procedure in the present paper.

Note that the trajectory does not simply enter a domain randomly. This behavior is embodied in the method used. Specifically, the "global" approximation regarding the shape of the trajectory presupposes that the change in the square of the velocity of motion along a selected trajectory is equal to the value of V. On the parts of the trajectory on which we have $V \to 0$, the absolute value of the velocity also tends toward zero. The absolute value of the velocity along the actual trajectories also tends toward zero in these regions. [In both cases, the reason is the conservation of integral (6).] The small value of V near domain states shows that a trial trajectory in this region must differ slightly from the actual trajectory (to the extent that V is small) and that the trial and actual trajectories coincide in the case $V=0$, i.e., at the point corresponding to the domain.

The set of relations in (28) and (29) thus constitutes a general solution which describes domain walls with $\theta=\theta(\varphi)$ at small values of the quality factor. Proceeding as in the case $Q \gg 1$, we can show that we have $x_0' = 0$ and $R=1$.

We can now write final expressions for the magnetization trajectory,

$$\left|\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right|^{\mp\sqrt{\varepsilon}} = \left|\tan\frac{\varphi}{2}\right|, \tag{30}$$

and the spatial evolution of the trajectory,

$$\mp x' = \ln\left|\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)\right|. \tag{31}$$

In (30) we have written out all possible types of trajectories in the case $Q \ll 1$. More precisely, we have written here (as above) all the geodesic lines on which $\theta=\theta(\varphi)$ holds which pass through domain states.

Figure 2 shows the magnetization trajectory. The general characteristics of the properties of the domain walls for small values of the quality factor are completely analogous to the corresponding characteristics at large values of the quality factor. Accordingly, we can, as above, introduce two types of walls: DWI and DWII. We recall that from the symmetry standpoint these two types of walls differ in the transformation properties of the y component of the magnetization vector.

We now consider a mapping of the trajectory, and we write equations expressing the functional dependence of the angle variables for each type of wall. We begin with DWI. The magnetization trajectory in these walls is represented by lines $A102B$ and $A403B$ in Fig. 2. The analytic expressions for the angle variables as a function of the spatial coordinate in a DWI, for fixed positions of the domains, are given by pairs of the types

$$\text{a)}\quad \theta=\begin{cases} -\dfrac{\pi}{2}+2\arctan e^{-x'}, & x'<0, \\ -\dfrac{\pi}{2}+2\arctan e^{x'}, & x'\geqslant 0, \end{cases}$$

$$\varphi = 2\arctan e^{\sqrt{\varepsilon}x'},$$

$$\text{b)}\quad \theta=\begin{cases} -\dfrac{\pi}{2}+2\arctan e^{-x'}, & x'<0, \\ -\dfrac{\pi}{2}+2\arctan e^{x'}, & x'\geqslant 0, \end{cases}$$

$$\varphi = -2\arctan e^{\sqrt{\varepsilon}x'},$$

$$\text{c)}\quad \theta=\begin{cases} \dfrac{3\pi}{2}+2\arctan e^{-x'}, & x'<0, \\ \dfrac{3\pi}{2}-2\arctan e^{x'}, & x'\geqslant 0, \end{cases}$$

$$\varphi = 2\arctan e^{\sqrt{\varepsilon}x'},$$

$$\text{d)}\quad \theta=\begin{cases} \dfrac{3\pi}{2}+2\arctan e^{-x'}, & x'<0, \\ \dfrac{3\pi}{2}-2\arctan e^{x'}, & x'\geqslant 0, \end{cases}$$

$$\varphi = -2\arctan e^{\sqrt{\varepsilon}x'}.$$

The derivative $d\theta/dx$ has a discontinuity at the center of a DWI.

The magnetization trajectory in the DWII in Fig. 2 is represented by the curves $A103B$ and $A402B$. The dependence of the angle variables on the spatial coordinate in a DWII for fixed positions of the domains is represented by pairs of the type

$$\text{a)}\quad \theta=\begin{cases} -\dfrac{\pi}{2}+2\arctan e^{-x'}, & x'<0, \\ -\dfrac{\pi}{2}+2\arctan e^{x'}, & x'\geqslant 0, \end{cases}$$

$$\varphi=\begin{cases} 2\arctan e^{\sqrt{\varepsilon}x'}, & x'<0, \\ 2\pi-2\arctan e^{\sqrt{\varepsilon}x'}, & x'\geqslant 0, \end{cases}$$

$$\text{b)}\ \theta=\begin{cases} -\frac{\pi}{2}+2\arctan e^{-x'}, & x'<0,\\ -\frac{\pi}{2}+2\arctan e^{x'}, & x'\geqslant 0,\end{cases}$$

$$\varphi=\begin{cases} -2\arctan e^{\sqrt{\varepsilon}x'}, & x'<0,\\ -2\pi+2\arctan e^{\sqrt{\varepsilon}x'}, & x'\geqslant 0,\end{cases}$$

$$\text{c)}\ \theta=\begin{cases} \frac{3\pi}{2}-2\arctan e^{-x'}, & x'<0,\\ \frac{3\pi}{2}-2\arctan e^{x'}, & x'\geqslant 0,\end{cases}$$

$$\varphi=\begin{cases} 2\arctan e^{\sqrt{\varepsilon}x'}, & x'<0,\\ 2\pi-2\arctan e^{\sqrt{\varepsilon}x'}, & x'\geqslant 0,\end{cases}$$

$$\text{d)}\ \theta=\begin{cases} \frac{3\pi}{2}-2\arctan e^{-x'}, & x'<0,\\ \frac{3\pi}{2}-2\arctan e^{x'}, & x'\geqslant 0,\end{cases}$$

$$\varphi=\begin{cases} -2\arctan e^{\sqrt{\varepsilon}x'}, & x'<0,\\ -2\pi+2\arctan e^{\sqrt{\varepsilon}x'}, & x'\geqslant 0.\end{cases}$$

In a DWII, the functions $\theta=\theta(x)$ and $\varphi=\varphi(x)$ are continuous, as are their derivatives.

We see that the new domain walls also consists of a core and two branches in the case $Q\ll 1$. In the branches, the angle φ changes sharply, while θ remains essentially constant. It is thus the components m_y and m_z which undergo the basic changes in the branches. In the central region, there is a sharp change in the angle θ, which causes a change in all the components of the magnetization vector. The structure of the new walls in this region is similar to that of Néel walls. Comparing the results found for the cases $Q\gg 1$ and $Q\ll 1$, we can assert that the branches in the new domain walls are restructured when the quality factor changes.

The existence of such a structure in the new walls for small values of the quality factor is again due to the different scales of the typical changes in the angles θ and φ, as follows from magnetization trajectory (30) and the spatial evolution of the trajectory, (31). Using (30), (31), (3), and (8), we could find both the typical sizes of the constituent parts of a domain wall and the surface energy density of the wall. We will not actually do that in the present paper. We do wish to point out that the properties of the new domain walls described at the end of the preceding section of this paper are exactly the same as the properties of the new domains walls again the case $Q\ll 1$.

6. CONCLUSION

A result found here is that a trajectory which begins at a branch point of the solution (in a domain state) necessarily passes through all the nearest branch points, some of which are domain states with oppositely directed magnetization vectors (opposite with respect to that of the original state). The other branch points, as we will see below, in a study of the evolution of domain walls in arbitrary magnetic points, serve as "custodians" of information about the system: Each of these points is a nucleating region from which all allowed state of the system can develop under the appropriate conditions. The use of a branch point as an information "storage place" is quite natural, since the limiting case of the trajectory is a branch point. The fixed positions of the branch points on the trajectory and the nature of the behavior of the trajectory at these points also make it possible to draw a qualitative picture of the trajectory for an arbitrary form of the uniform part of the magnetic energy. This capability is important for numerical calculations.

The existence in ferromagnetic samples of domain walls with a structure differing from that of Bloch walls was predicted by Néel, who showed that surface scattering fields strongly influence domain walls in thin ferromagnetic films (see Ref. 3 and the papers cited in that book). We accordingly note that the domain walls described in the present paper differ qualitatively from the walls predicted by Néel, since they can exist even in bulk ferromagnets.

The existence of features of the Bloch-line type in the new domains walls is favored energetically. When the degree of degeneracy of these domain walls is taken into account, the features have quite diverse structures, which require a separate study. We note in this connection that the Slonczewski model,[4] which is widely used in research on Bloch lines, cannot be used in the case at hand. A formal indication of this situation is the existence of a relationship between the angle variables.

In analyzing the problem of this paper we adhered to the formulation of the problem proposed by Landau and Lifshitz, but we see that that formulation is overly detailed for our approach. For our purposes, in a study of isolated domain walls, it is quite sufficient to define a domain as a state in which the nonuniform part of the magnetic energy is zero. Knowing the state of only one domain (i.e., knowing the initial "velocities" and "coordinates"), we are in a position to study the extrema, which describe both uniformly magnetized ferromagnets and ferromagnets with a domain structure. We discuss periodic domain structures when we are dealing with the complete magnetization equation (in the case at hand, we are working on a $\mathbf{M}^2$=const surface). Just which extremum is realized under certain conditions or others depends on the magnetic history of the magnetic material.

The solutions derived here also describe extrema for which the initial and final states are the same (e.g., the curves $A104A$ and $B203B$ in Fig. 1). We are not concerned with those solutions, since they correspond to "pure" soliton solutions,[10] which go beyond the scope of the present paper. Using the expressions for the trajectory and for the spatial evolution of the trajectory, we can also, and easily, find descriptions of those solutions. Those solutions also pass through point O, so we cannot say for sure whether these curves shrink to a point on the basis of qualitative considerations. Resolving this question will require a systematic study of the stability of these solitons. In contrast with the soliton solutions described in Ref. 10, our solitons are static.

It would be a bit premature to attempt to use the results derived here for comparisons with experimental data, but we believe that the existence of Bloch lines and the structure of the domain walls in real magnetic materials (a dense central part and widely spaced branches)[3,4] are evidence that the new domain walls are realized in practical situations.

Clearly, our approach is a general method for studying nonlinear equations of certain class of variational problems. We will broaden this class of problems in the future, but even at this point we see that the method is fairly general, since functionals like (1) often arise in physical problems. For an analytic solution of the problem, of course, the canonical version of a quadratic form in the gradients of generalized coordinates is not the only important point; the vanishing of this quadratic form under the initial conditions is also important.

In conclusion we would like to stress that we used different configuration spaces (i.e., different coordinate systems) in studying the limiting case $Q \gg 1$ and $Q \ll 1$. It is necessary to change the coordinate system when the interaction constants change because of the very existence of a trajectory equation which, in certain limiting cases, becomes the condition for a relationship between configuration variables. The existence of a relationship in a system is equivalent to a reduction of the dimensionality of the configuration space or to a decrease in the number of degrees of freedom of the system. The latter change requires a change in the method used to describe the system; a redefinition of the configuration space makes it possible to avoid this difficulty: We simply retain the 2D nature of the configuration space.

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[1] L. D. Landau, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1984).
[2] L. D. Landau, *Collected Works* [in Russian] (Nauka, Moscow, 1969), p. 128.
[3] A. Hubert, *Theory of Domain Walls in Ordered Media* [Russian translation] (Mir, Moscow, 1977).
[4] A. P. Malozemoff and J. C. Slonczewski, *Magnetic Domain Walls in Bubble Materials* (Applied Solid State Science, Supplement I) (Academic Press, New York, 1979).
[5] V. G. Bar'yakhtar, N. B. Bogdanov, and D. A. Yablonskiĭ, Usp. Fiz. Nauk **156**, 47 (1988) [Sov. Phys. Usp. **31**, 810 (1988)].
[6] C. Lanczos, *Variational Principles in Mechanics* [Russian translation] (Mir, Moscow, 1965).
[7] L. D. Landau and E. M. Lifshitz, *Mechanics* (Pergamon, New York, 1976).
[8] D. A. Dubrovin, S. P. Novikov, and A. T. Fomenko, *Modern Geometry* [in Russian] (Nauka, Moscow, 1979).
[9] V. G. Bar'yakhtar, V. A. L'vov, and D. A. Yablonskiĭ, "Magnetic symmetry and electrical polarization of domain walls in magnetically ordered crystals" [in Russian], Preprint ITF-84-41P, ITF, Ukrainian Academy of Sciences, Kiev, 1984.
[10] A. M. Kosevich, B. A. Ivanov, and A. S. Kovalev, *Nonlinear Magnetization Waves: Dynamic and Topological Solitons* [in Russian] (Naukova Dumka, Kiev, 1983).

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