Stability of self-induced transparency solitons in a resonant waveguide

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We show that a three-dimensional self-induced transparency pulse is stable under periodic perturbations and perturbations localized along the propagation axis. The necessary condition for stability is the matching of the transverse distribution of the field and the concentration distribution of the absorbing atoms. We study the case of wide beams using perturbation theory with a small parameter ε, the ratio of the rate of transverse variation of the field strength to that of longitudinal variation.

1. INTRODUCTION

Beginning with the classical work of McCall and Hahn,[1] experimental observation of the propagation of self-induced transparency (SIT) solitons has been complicated by transverse effects. Comparison between the theories[1,2] based on the plane wave approximation and the experimental results requires using wide beams with a smooth transverse profile. In practice, however, one observes the self-focusing of the beam as a whole and/or the breakup of the beam into filaments when it propagates through a dense ($\alpha_0 L>5$) resonant medium.[3] This new type of resonant self-focusing was thoroughly studied in its theoretical aspects in Refs. 4–6. In contrast to ordinary resonant self-focusing, the effect occurs even when the carrier frequency of the pulse is tuned exactly to the resonant transition. The theoretical analysis in Refs. 4–6 is based on a numerical solution of the wave equation that allows for radial field variations together with the system of Bloch equations for a two-level atom. An alternative method of studying beam decay processes as applied to SIT effects has been suggested by Bol'shov, Likhanskiĭ, and Napartovich,[7,8] who analyzed the stability of a 2π pulse with a plane wavefront under transverse field perturbations. They found that a one-dimensional 2π pulse is unstable under transverse perturbations and decays after traveling several absorption lengths in the medium ($\alpha_0 L\approx 10$). It appears that small distortions of the field make the wavefront inhomogeneous. In view of the dependence of the pulse velocity on the intensity, the sections of the transverse profile with larger amplitude overtake the sections with smaller amplitude. Thus, weak perturbations become even stronger, which, in the final analysis, leads to decay of the stationary shape of the field.

Recently a new approach to obtaining stable SIT pulses with a nonplanar wavefront has been developed.[9,10] The idea originates in the expression for the velocity of a 2π pulse,

$$V_p^{-1}=c^{-1}+\frac{2\pi k d^2}{\hbar}\,n_0\tau_p^2,$$

derived on the assumption that the absorption line is homogeneously broadened and that there is exact tuning to resonance. The length of the 2π pulse is inversely proportional to the peak value of the pulse, and in the case of bounded beams with cylindrical symmetry (to be specific we assume that the beam has a Gaussian profile) $\tau_p(\rho)$ is a monotone increasing function of coordinate ρ. For a homogeneous medium, $n_0=$const, the velocity at different distances from the axis is different, and hence steady-state propagation is impossible. We assume that the absorbing particle concentration varies over the cross section of the medium, with $n_0(\rho)$ a monotone decreasing function of ρ. By selecting the distribution so that

$$n_0(\rho)\tau_p^2(\rho)=\text{const} \tag{1}$$

we make the pulse velocity the same for all values of ρ, or $V_p=$const. The tools developed in Ref. 10 allow matching conditions to be obtained for the field and atom-concentration distributions similar to Eq. (1) for the case when the carrier frequency of the field is detuned from resonance and for a medium with an inhomogeneously broadened absorption profile. For our purposes in this paper analyzing Eq. (1) is sufficient. The waveguide structure created by the variation of the medium is very similar to an optical waveguide with a refractive index smoothly varying in the direction perpendicular to the propagation axis. The essential difference from the linear case is that the intensity profile is rigidly matched with the transverse distribution of the atoms, with Eq. (1) the condition imposed by the fact that the interaction between the field and the resonant medium is essentially nonlinear.

In this paper we study the stability of a three-dimensional SIT pulse under localized and periodic perturbations in conditions where the concentration profile of the absorbing atoms is matched to the transverse profile of the field.

2. BASIC EQUATIONS

We write the system of Maxwell–Bloch equations without the relaxation terms, proportional to T_2^{-1} and T_1^{-1}, assuming that the pulse length τ_p is much shorter than T_2 and T_1:

$$i\varepsilon(\rho)\{\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)E]\}-\alpha^{-1}(\rho)\frac{\partial E}{\partial z}+\sigma(\rho)\frac{\partial E}{\partial u}=-iP, \tag{2a}$$

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$$\frac{\partial}{\partial u}P=-iNE, \quad (2b)$$

$$\frac{\partial}{\partial u}N=-\frac{i}{2}(EP^*-E^*P). \quad (2c)$$

Here we have introduced the following notation:

$$\alpha(\rho)=\frac{2\pi kd^2}{\hbar}n_0(\rho)\tau_p(\rho), \quad \sigma(\rho)=\frac{V_p^{-1}-c^{-1}}{\tau_p(\rho)},$$

$$\varepsilon(\rho)=\frac{1}{2kr_0^2}\alpha^{-1}(\rho), \quad u(\rho)=\frac{t-z/V_p}{\tau_p(\rho)}, \quad (3)$$

$$\rho=\frac{r}{r_0}, \quad \Delta_\perp=\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}.$$

The total field and polarization have the form

$$\tilde{E}(t,z,\rho)=\frac{\hbar}{d\tau_p(\rho)}E\left(\frac{t-z/V_p}{\tau_p(\rho)},z,\rho\right)\exp[-i(\omega t-kz)],$$
$$\tilde{P}(t,z,\rho)=dP\left(\frac{t-z/V_p}{\tau_p(\rho)},z,\rho\right)\exp[-i(\omega_{ab}t-kz)]. \quad (4)$$

Here in writing the expression (3) for the absorption coefficient $\alpha(\rho)$, instead of the transverse relaxation time T_2 we have used another characteristic time, the pulse length $\tau_p(\rho)$. The other quantities are designated as follows: $k=\omega/c$ is the wave vector in vacuum for the pulse carrier frequency ω, d is the dipole moment of the transition, r_0 is the characteristic radius of the transverse field distribution, c is the velocity of light in vacuum, and t, z, and r are the time and the longitudinal and transverse coordinates, respectively.

Note that in writing Eq. (2a) we introduced a dependence of the pulse and medium parameters on the transverse coordinate. From the outset we assumed that the atoms of the absorbing medium are distributed unevenly over the cross section, i.e., $n_0=n_0(\rho)$, which is the main prerequisite for the existence of a steady-state solution, a three-dimensional optical soliton.[10] The pulse length $\tau_p(\rho)$ is an increasing function of ρ, which is necessary for limiting the field in the transverse direction. This can easily be understood if one considers wide beams: in steady-state propagation the pulse area must be equal to 2π for any value of ρ. Since the field amplitude decreases as ρ grows, this condition presupposes that the pulse length increases appropriately. In reality, at a certain distance $\rho=\rho_0$ from the axis the value of $\tau_p(\rho)$ becomes close to T_2 and the employed model ceases to be valid. Bearing in mind this fact, we restrict our discussion to processes taking place in the vicinity of the axis.

For future convenience we have introduced into the wave variable u a dependence on the coordinate ρ. The parameter $\varepsilon(\rho)$ used in Eq. (2a) is the ratio of the rate of transverse field variations to that of the longitudinal field variations. Limiting our discussion to wide beams, we assume the parameter $\varepsilon(\rho)$ small and use it as the expansion parameter in perturbation theory. Below we show that $\varepsilon(\rho)\propto\tau_p(\rho)$ and hence $\varepsilon(\rho)$ increases with ρ. For $\varepsilon(0)\ll 1$ the perturbation theory constructed with $\varepsilon(\rho)$ is valid in the region near the axis.

We now present Eqs. (2) in a new form by separating the field strength and polarization into real and imaginary parts:

$$E=E_1+iE_2, \quad P=P_1+iP_2. \quad (5)$$

The result is

$$-\varepsilon(\rho)\{\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)E_2]\}-\alpha^{-1}(\rho)\frac{\partial E_1}{\partial z}+\sigma(\rho)\frac{\partial E_1}{\partial u}=P_2, \quad (6a)$$

$$\varepsilon(\rho)\{\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)E_1]\}-\alpha^{-1}(\rho)\frac{\partial E_2}{\partial z}+\sigma(\rho)\frac{\partial E_2}{\partial u}=-P_1, \quad (6b)$$

$$\frac{\partial P_1}{\partial u}=-NE_2, \quad \frac{\partial P_2}{\partial u}=NE_1, \quad (6c)$$

$$\frac{\partial N}{\partial u}=E_2P_1-E_1P_2. \quad (6d)$$

The system of differential equations (6) serves as a basis for studying the stability of the steady-state solution in the form of a three-dimensional optical soliton (E_{1st}, E_{2st}, P_{1st}, P_{2st}, N_{st}) under small perturbations.

3. STEADY-STATE SOLUTION

Steady-state solutions of the system (6) were found in Refs. 9 and 10. Here we will examine only the basic topics without going into details.

The steady-state solution for the field is a function of two variables, u and ρ, and we discard the derivatives with respect to z in Eqs. (6a) and (6b).[1)] Using the fact that the parameter $\varepsilon(\rho)$ is small near the axis, we construct a perturbation theory by expanding the unknown quantities E_{1st}, E_{2st}, P_{1st}, P_{2st}, and N_{st} in power series in $\varepsilon(\rho)$. A steady-state solution can exist only if all parts of a pulse move with the same velocity, i.e., V_p is not a function of ρ. This occurs when the distributions of field and medium are matched:

$$V_p^{-1}-c^{-1}=\alpha(\rho)\tau_p(\rho)=\text{const}, \quad \text{i.e.,} \quad n_0(\rho)\propto\tau_p^{-2}(\rho). \quad (7)$$

One can easily check that in the event of exact resonance all the phase shifts emerge because the phase front is not plane. This statement is a direct corollary of the absence of phase modulation in the steady-state solution as a 2π pulse with a plane wavefront.[11] Bearing all this in mind, we write the steady-state solution of the system of equations (6) in the form

$$\begin{aligned}
E_{1st}&=\varepsilon^0(\rho)\cdot 2\mathscr{E}_0(u,\rho)+\varepsilon^1(\rho)\cdot 0,\\
E_{2st}&=\varepsilon^0(\rho)\cdot 0+\varepsilon^1(\rho)\mathscr{E}_2(u,\rho),\\
P_{1st}&=\varepsilon^0(\rho)\cdot 0+\varepsilon^1(\rho)\mathscr{P}_1(u,\rho),\\
P_{2st}&=\varepsilon^0(\rho)\cdot 2\dot{\mathscr{E}}_0(u,\rho)+\varepsilon^1(\rho)\cdot 0,\\
N_{st}&=\varepsilon^0(\rho)[1-2\mathscr{E}_0^2(u,\rho)]+\varepsilon^1(\rho)\cdot 0.
\end{aligned} \quad (8)$$

The only terms left here were those which are at most first order in $\varepsilon(\rho)$. The dot stands for differentiation with respect to u, and the following notation has been introduced:

$$\mathscr{E}_0(u,\rho)=\text{ch}^{-1}u\ ,$$

$$\mathscr{E}_2(u,\rho)=-2\mathscr{E}_0\int^u\left\{\int^{\bar u}\mathscr{E}_0[\tau_p(\rho)\cdot\Delta_\perp(\tau_p^{-1}(\rho)\mathscr{E}_0)]'_{\bar u}d\bar u\right.$$

$$\times\frac{du}{\mathscr{E}_0^2}, \tag{9}$$

$$\mathscr{P}_1(u,\rho)=-\dot{\mathscr{E}}_2(u,\rho)-2\tau_p\Delta_\perp(\tau_p^{-1}(\rho)\mathscr{E}_0)\ .$$

Knowing the form of the steady-state solution [Eqs. (7) and (8)], we can derive the equations for the perturbations.

4. THE SYSTEM OF EQUATIONS FOR THE PERTURBATIONS

In accordance with the standard method of studying stability, we write the solution of Eqs. (6) as the sum of the steady-state solution (8) and a perturbation (E_{1p}, E_{2p}, P_{1p}, P_{2p}, and N_p). We linearize the system of equations (6) with respect to the perturbation terms by employing the fact that these terms are small. To avoid burdening the formulas by writing the independent variable of the functions explicitly, we only note that the steady-state solution is a function of two variables, u and ρ, while the perturbation is a function of three variables, u, ρ, and z.

The system of equations (6c) and (6d) has a conservation law expressing the fact that the length of the Bloch vector does not vary with time:

$$N^2+P_1^2+P_2^2=1.$$

Here we have used the following initial condition for the population difference:

$$N|_{t\to-\infty}=N|_{u\to-\infty}=-1.$$

Substituting the perturbed density-matrix elements into the conservation law yields

$$N_pN_{st}+P_{1st}P_{1p}+P_{2st}P_{2p}=0. \tag{10}$$

If we combine this with Eq. (6c) and (10), we arrive at the following equation for P_{2p}:

$$\frac{\partial}{\partial u}P_{2p}+\frac{P_{2st}E_{1st}}{N_{st}}P_{2p}=N_{st}E_{1p}-\frac{P_{1st}E_{1st}}{N_{st}}P_{1p}. \tag{11}$$

The steady-state solution has been found as a series expansion in $\varepsilon(\rho)$ [see Eqs. (8)]; a similar expansion holds for the perturbation. Here we do not carry out the expansion explicitly, bearing in mind that the accuracy of the solution is limited to second order in the expansion (up to $\varepsilon^1(\rho)$ inclusive). Then Eq. (11) can be written as

$$\frac{\partial}{\partial u}P_{2p}+\frac{4\mathscr{E}_0\dot{\mathscr{E}}_0}{1-2\mathscr{E}_0^2}P_{2p}=(1-2\mathscr{E}_0^2)E_{1p}+\varepsilon^1(\rho)\frac{2\mathscr{E}_0\mathscr{P}_1}{1-2\mathscr{E}_0^2}$$

$$\times\left(\frac{\partial}{\partial u}E_{2p}-\alpha^{-1}(\rho)\frac{\partial E_{2p}}{\partial z}\right),$$

and its solution is

$$P_{2p}=(1-2E_0^2)\left\{\int^u E_{1p}d\bar u+2\varepsilon_1(\rho)\int^u\frac{\mathscr{E}_0\mathscr{P}_1}{(1-2\mathscr{E}_0^2)^2}\right.$$

$$\left.\times\left(\frac{\partial E_{2p}}{\partial\bar u}-\alpha^{-1}(\rho)\cdot\frac{\partial E_{2p}}{\partial z}\right)d\bar u\right\}. \tag{12}$$

Now we have all we need to reduce the system of equations for the perturbation to two equations for the following quantities:

$$e_2(u,\rho,z)=E_{2p}(u,\rho,z),$$

$$e_1(u,\rho,z)=\int^u E_{1p}(\bar u,\rho,z)d\bar u. \tag{13}$$

The equations are

$$\ddot e_1-\alpha^{-1}(\rho)\frac{\partial}{\partial z}\dot e_1-(1-2\mathscr{E}_0^2)e_1$$

$$=\varepsilon^1(\rho)\left\{\tau_p(\rho)\Delta_\perp[\tau_p^{-1}e_{2p}]+2(1-2\mathscr{E}_0^2)\right.$$

$$\left.\times\int^u\frac{\mathscr{E}_0\mathscr{P}_1}{(1-2\mathscr{E}_0^2)^2}\left(\dot e_{2p}-\alpha^{-1}(\rho)\frac{\partial}{\partial z}e_{2p}\right)d\bar u\right\}, \tag{14a}$$

$$\ddot e_2-\alpha^{-1}(\rho)\frac{\partial}{\partial z}\dot e_2-(1-2E_0^2)e_2$$

$$=-\varepsilon^1(\rho)\tau_p(\rho)\Delta_\perp[\tau_p^{-1}\ddot e_1]\ . \tag{14b}$$

The coefficients in Eqs. (14) do not depend explicitly on z, which makes it possible to look for their solution via an ansatz:

$$e_1(u,\rho,z)=\tilde\varphi(u,\rho)\exp(\gamma z),\quad e_2(u,\rho,z)=\tilde\psi(u,\rho)\exp(\gamma z), \tag{15}$$

where γ is an eigenvalue that remains to be found. If $\text{Re}\gamma$ is negative, the steady-state solution is stable; if $\text{Re}\gamma$ is positive, the steady-state solution is unstable; and if $\text{Re}\gamma=0$, the steady-state solution is neutrally stable under perturbations. We introduce the following change of variables:

$$\tilde\varphi(u,\rho)=\varphi(u,\rho)\exp(\lambda u),\quad \tilde\psi(u,\rho)=\psi(u,\rho)\exp(\lambda u), \tag{16}$$

$$\lambda=\lambda(\rho)=\frac{\gamma}{2\alpha(\rho)}\ .$$

Now the right-hand sides of both equations are self-adjoint operators:

$$\ddot\varphi-(1+\lambda^2-2\mathscr{E}_0^2)\varphi=\varepsilon^1(\rho)e^{-\lambda u}\left\{\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)\psi e^{\lambda u}]\right.$$

$$+2(1-2\mathscr{E}_0^2)\int^u\frac{\mathscr{E}_0\mathscr{P}_1}{(1-2\mathscr{E}_0^2)^2}$$

$$\left.\times(\dot\psi-\lambda\psi)e^{\lambda\bar u}d\bar u\right\}, \tag{17a}$$

$$\ddot{\psi}-(1+\lambda^2-2\mathscr{E}_0^2)\psi=-\varepsilon^1(\rho)e^{-\lambda u}\frac{\partial}{\partial u}\times\{\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)(\dot{\varphi}-\lambda\varphi)e^{\lambda u}]\}. \tag{17b}$$

Following the logic of the above discussion, we look for the eigenfunctions and eigenvalues of Eqs. (17) in the form of power series in the parameter $\varepsilon(\rho)$. In zeroth order the system of equations (17) splits into two identical Schrödinger equations with potentials $U=N_{st}=1-2\mathrm{ch}^{-2}u$. The properties of this equation have been studied extensively.[12] The continuous spectrum of the equation occupies the range $\lambda^2(\rho)<-1$. Earlier we noted that our approach is meaningful in the region near the axis; to be specific we write the boundary value ρ_1 found from the condition $\varepsilon(\rho_1)=1$. Then the eigenvalues γ corresponding to the solutions of the Schrödinger equations that do not decrease at infinity occupy the interval

$$\gamma^2<-2[\min\alpha(\rho)]^2=-2\alpha^2(\rho_1),$$

i.e., the steady-state solution is neutrally stable under longitudinal perturbations.

The discrete spectrum consists of a single eigenvalue $\lambda(\rho)=0$, i.e., $\gamma=0$, and corresponds to the eigenfunction $\mathrm{ch}^{-1}u$ localized near the origin of coordinates. The eigenvalue $\gamma=0$ is doubly degenerate and corresponds to the eigenfunctions

$$\begin{cases}\varphi_0(u,\rho)=0,\\ \psi_0(u,\rho)=\mathrm{ch}^{-1}u,\end{cases}\qquad \begin{cases}\varphi_0(u,\rho)=\mathrm{ch}^{-1}u,\\ \psi_0(u,\rho)=0\end{cases}$$

which means that the pulse is insensitive to a small displacement of the initial position of the envelope and a small phase shift.

The next step involves finding the correction to the eigenvalue $\gamma=0$ caused by diffraction effects. We start by determining the correction to $\gamma=0$ for functions satisfying zero boundary conditions as $u\to\pm\infty$. The existence of nontrivial solutions of the system of inhomogeneous differential equations (17) requires the right-hand sides to be orthogonal to the solutions of unperturbed equations. By discarding the terms in the equations containing the factor $\varepsilon(\rho)\lambda^n$ $(n\geqslant 1)$, which we know to exceed the assumed order of accuracy, we can write the expression for the corrected eigenvalue as follows:[2)]

$$\lambda^4(\rho)=\varepsilon^2(\rho)\int_{-\infty}^{\infty}\mathscr{E}_0\left\{\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)\mathscr{E}_0]+2(1-2\mathscr{E}_0^2)\int^u\frac{\mathscr{E}_0\dot{\mathscr{E}}_0\mathscr{P}_1}{(1-2\mathscr{E}_0^2)^2}d\bar{u}\right\}du,$$
$$\times\int_{-\infty}^{\infty}\dot{\mathscr{E}}_0\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)\dot{\mathscr{E}}_0]\,du\times\left(\int_{-\infty}^{\infty}\mathscr{E}_0^2du\right)^{-2}. \tag{18}$$

The first cofactor in (18) vanishes. Hence the steady-state solution (8) retains neutral stability to localized perturbations.

We now return to the study of periodic perturbations. The right-hand sides of Eqs. (17a) and (17b) introduce corrections caused by diffraction into the eigenfunctions and eigenvalues of the homogeneous problem. We write the unknown eigenvalues in the form $\lambda=\lambda_0+\lambda_\varepsilon$, where $\lambda_\varepsilon\ll\lambda_0$, which makes it possible to leave terms linear in λ_ε in the equations. It seems reasonable to search for the values of λ_ε in the asymptotic behavior of Eqs. (17a) and (17b). By selecting u fairly large, $u=L\gg 1$, we can ignore terms of the form $\mathscr{E}_0^2\varphi$ on the left-hand sides. Furthermore, for u large the second term on the right-hand side of Eq. (17a) becomes negligible. The equations can be written in a simplified form as follows:

$$\ddot{\varphi}-(1+\lambda^2)\varphi=\varepsilon^1(\rho)\tau_p(\rho)\Delta_\perp[\tau_p^{-1}(\rho)\psi],$$
$$\ddot{\psi}-(1+\lambda^2)\psi=-\varepsilon^1(\rho)\tau_p(\rho)\{\lambda(\rho)\Delta_\perp[\tau_p^{-1}(\rho)\times(\dot{\varphi}-\lambda(\rho)\varphi)]+\Delta_\perp[\tau_p^{-1}(\rho)\times(\dot{\varphi}-\lambda(\rho)\dot{\varphi}]\}. \tag{19}$$

In deriving (19) we have employed the fact that λu is independent of ρ.

Each eigenvalue of the unperturbed continuous spectrum $\lambda_0^2(\rho)<-1$ is fourfold degenerate:

$$\begin{cases}\varphi_0=\cos(|\lambda_0|u),\\ \psi_0=0,\end{cases}\quad \begin{cases}\varphi_0=0,\\ \psi_0=\cos(|\lambda_0|u),\end{cases}$$
$$\begin{cases}\varphi_0=\sin(|\lambda_0|u),\\ \psi_0=0,\end{cases}\quad \begin{cases}\varphi_0=0,\\ \psi_0=\sin(|\lambda_0|u).\end{cases}$$

Developing the perturbation theory, we arrive at equations for finding the corrections λ_ε by employing the fact that the right-hand sides of Eqs. (19) are orthogonal to the solutions φ_{hom} and ψ_{hom} of the homogeneous equation:

$$-2\lambda_0\lambda_\varepsilon\int_{-L}^{L}\varphi_{\mathrm{hom}}^2du=\varepsilon^1(\rho)\tau_p(\rho)\int_{-L}^{L}\Delta_\perp[\tau_p^{-1}(\rho)\varphi_{\mathrm{hom}}]\varphi_{\mathrm{hom}}du, \tag{20a}$$

$$-2\lambda_0\lambda_\varepsilon\int_{-L}^{L}\psi_{\mathrm{hom}}^2du=-\varepsilon^1(\rho)\tau_p(\rho)\int_{-L}^{L}\{\lambda_0\Delta_\perp[\tau_p^{-1}(\rho)(\dot{\varphi}_{\mathrm{hom}}-\lambda_0\varphi_{\mathrm{hom}})]+\Delta_\perp[\tau_p^{-1}(\rho)(\ddot{\varphi}_{\mathrm{hom}}-\lambda_0\dot{\varphi}_{\mathrm{hom}})]\}\psi_{\mathrm{hom}}du. \tag{20b}$$

To estimate the values that λ_ε takes on there is no need to solve Eqs. (20a) and (20b). Note that the right-hand side of Eq. (20a) contains only real quantities, so that the left-hand side of Eq. (20a) must also be real. This necessarily leads to $\mathrm{Re}\lambda_\varepsilon=0$. Thus, a three-dimensional SIT pulse retains its neutral stability under periodic perturbations as well.

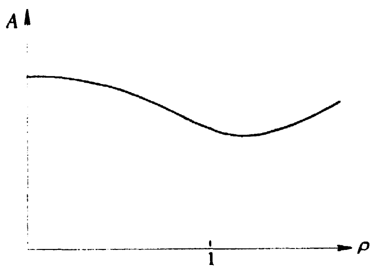

FIG. 1. The field amplitude as a function of the distance from the beam axis, $A(\rho)$ (for a 2π pulse the amplitude is proportional to τ_p^{-1}). Here ρ is the dimensionless coordinate normalized to the radius of the refractive-index distribution function $g(\rho)$, i.e., to $r_0 = 385\mu m$.

5. CONCLUSION

We have shown that the propagation of a three-dimensional SIT soliton in a system with a transverse profile of resonant atoms is stable. Experimentally such a stable propagation mode can be achieved by using fiber-optic waveguides with implanted resonant impurities with a specified concentration profile. Experiments in coherent propagation of pulses in a waveguide with erbium atoms as resonant impurities have been carried out by Nakazawa *et al.*[13] The use of radiation trapped in the waveguide made it possible to observe the SIT effect in "pure form" without the beam structure being damaged by diffraction. Nakazawa *et al.*[13] observed the processes in which the shape of the envelope became stationary, also the threshold nature of transmission, considerable delay times, and pulse splitting. One of the most astounding aspects of the experiment was the propagation of a pulse though an optical depth $\alpha_0 L > 160$ with no apparent change in the beam structure.

Nakazawa *et al.*[13] give no data on the transverse distributions of resonant and nonresonant atoms in the waveguide, nor did they record the transverse field profile at the exit from the waveguide. Also, in the experiments the condition when $\mathscr{E} \ll 1$ which is the base of the theory under study is not met. However, the convincing proof of the possibility of stabilizing diffraction instability in the self-induced transparency effect prompt a search for points of contact between the theory developed here and in Ref. 10 and the experimental data of Ref. 13 Allowing for the inhomogeneous distribution of the index of refraction of the nonresonant atoms in the plane perpendicular to the propagation axis, $\eta(\rho) = \eta_0 g(\rho)$ [here $\eta_0 = \eta|_{\rho=0}$ is the refractive index at the center of the waveguide, and $\rho = r/r_0$, with r_0 the curvature radius of the distribution function $g(\rho)$], we can write the following expression for the SIT soliton velocity valid for $\tau_p \gg T_2^*$:

$$\frac{1}{V_p} = \frac{\eta_0}{c} g(\rho) + \alpha(\rho) \frac{\tau_p(\rho)}{g(\rho)},$$
$$\alpha(\rho) = \frac{2\pi\omega d^2 n_0(\rho)}{\hbar c \eta_0} T_2^*. \tag{21}$$

In their experiments Nakazawa *et al.*[13] used a medium with an inhomogeneously broadened resonant transition ($1/T_2^*$ is the width of the absorption profile), with the result that Eqs. (21) differ from the similar expression (7) obtained on the assumption that $\tau_p \ll T_2^*$. According to our theory, steady-state propagation of a pulse is possible if the transverse profile of the pulse satisfies the following equation:

$$\tau_p^{-1}(\rho) = \frac{\alpha(\rho)}{[1/V_p - (\eta_0/c) g(\rho)] g(\rho)}. \tag{22}$$

A rough sketch of the distribution of $\tau_p^{-1}(\rho)$ for the case where $\alpha(\rho) = \text{const}$ and $g(\rho) = \exp(-\rho^2)$ is given in Fig. 1. To construct the distribution we used the following numerical values taken from Ref. 13:

$$\eta_0 = 1.72, \quad \tau_p|_{\rho=0} = 294 \text{ ps},$$
$$\alpha = 27.5 \text{m}^{-1}, \quad r_0 = 43.8 \ \mu\text{m}.$$

We find that $\tau_p^{-1}(\rho) \to \infty$ as $\rho \to \infty$, which contradicts the requirement that the field decrease at infinity. However, the waveguide size $r_w = 5\mu m$ is smaller than r_0 by a factor of one; hence the pulse interacts effectively with the waveguide material only near the axis. We can therefore assume that the periphery of the beam has only a slight effect on the nature of pulse propagation and in practice the field distribution for $r > r_w$ can be arbitrary (including distributions with decreasing wings.) The analysis given here makes one to believe that in the exeriment[13] a not-fixed regime of propagation, which leads to the impulse stabilization, has been realized.

A quick comparison of the results of theory and experiment reveals the general nature of suppression of diffraction instability: the matching of the transverse distributions of the resonant and/or nonresonant atoms to the transverse field distribution. Because of the nonlinear nature of the interaction between the medium and a SIT pulse, the waveguide formed by the atoms is capable of supporting only one distribution, in contrast to ordinary "linear" optical waveguides, which have a set of transverse modes. In this sense a resonant nonlinear waveguide can be characterized as single-mode.

We believe that it is worthwhile to develop applications of the theory to optical waveguides. The natural problem arising here is how to estimate the extent to which nonresonant atoms influence the formation of SIT solitons and, among other things, to account for the nonlinearity of the refractive index and the dispersion of the group velocity.

[1]This is true only if the field is exactly in resonance with the period, $\omega = \omega_{ab}$; otherwise the phase of the field in Eq. (4) may be a linear function of distance z.

[2]Note that when developing the perturbation theory, we must expand the eigenvalue $\lambda(\rho)$ in powers of $\sqrt{\varepsilon(\rho)}$ rather than $\varepsilon(\rho)$. In the present case this detail is unimportant.

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