Onset of steady-state reflection of electromagnetic waves at a vacuum–medium interface

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A study is made of transient surface processes associated with the excitation of a reflected wave at the interface between the vacuum and a linear medium upon incidence of an electromagnetic wave with a leading edge. Estimates of the time to achieve steady-state reflection at a definite frequency (described by the Fresnel formulas) are given for a plasma and a dielectric. © *1994 American Institute of Physics.*

1. INTRODUCTION

The propagation of an electromagnetic wave with a leading edge in a linear medium with dispersion was first addressed and solved in the well-known work by L. Brillouin and A. Sommerfeld.[1] As a wave with a leading edge impinges the vacuum–medium interface, surface transient processes develop which are due to the excitation of reflected and refracted waves and determine the approach to the steady-state reflection–refraction regime.[2] Previous calculations of surface transient processes are only concerned with the properties of the reflected waves near its leading edge and do not enable one to estimate the time for the onset of steady-state reflection.

In the present work, surface transients at the interface between a vacuum and a linear medium with dispersion are described by invoking the impulse response and unit-step response functions familiar from the theory of linear systems. This enables one to avoid computational problems, and to obtain results common to all linear reflecting media by relying on causality and using the high-frequency asymptotic behavior of the dielectric constant.

Analysis of surface-transient asymptotic behavior yields the relaxation mechanism during reflection from a transparent medium, and provides time estimates for establishing Fresnel formulas for a plasma and a dielectric. The results suggest the existence of speed limitations in optical devices due to interface wave processes.

1. IMPULSE RESPONSE AND THE UNIT-STEP RESPONSE OF THE VACUUM–MEDIUM INTERFACE

Consider an electromagnetic wave with a leading edge which propagates in vacuum in the positive z direction and has the electric field

$$E(z,t)=E(\tau_-)=\begin{cases} A(\tau_-)e^{-i\omega_0\tau_-}, & \tau_->0, \\ 0, & \tau_-<0, \end{cases} \tag{1.1}$$

where t is the time, $\tau_-=t-z/c$, c is the speed of light in vacuum, $i=\sqrt{-1}$, and ω is the wave frequency. The complex amplitude $A(\tau_-)$ satisfies the condition

$$\lim_{\tau_-\to+\infty} A(\tau_-)=\text{const}=A_0 \tag{1.2}$$

and determines the onset of steady-state oscillations following the arrival of the wave leading edge at $\tau_-=0$. In the special case $A(\tau_->0)\equiv A_0$, Eq. (1.1) describes a wave with a sharp leading edge, where the amplitude jumps from zero to A_0.

If the wave (1.1) is normally incident upon the flat interface $z=0$ between the vacuum $z<0$ and the linear medium $z>0$ with index of refraction $n(\omega)$, the electric field of the reflected wave, $E_r(z,t)$, can be written in the form

$$E_r(z,t)=E_r(\tau_+)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R(\omega)E(\omega)e^{-i\omega\tau_+}d\omega, \tag{1.3}$$

where $\tau_+=t+z/c$, $z<0$, $R(\omega)=[1-n(\omega)/1+n(\omega)]$ is the amplitude reflection coefficient at frequency ω, and $E(\omega)$ is the Fourier transform of the field (1.1).

Using the familiar Fourier transform theorem for a product of two functions, Eq. (1.2) becomes

$$E_r(\tau_+)=A_r(\tau_+)e^{-i\omega_0\tau_+}, \tag{1.4}$$

where

$$A_r(\tau_+)=\int_{-\infty}^{\tau_+} G_R(\tau)A(\tau_+-\tau)e^{i\omega_0\tau}d\tau \tag{1.5}$$

is the complex amplitude of the reflected wave and

$$G_R(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty} R(\omega)e^{-i\omega\tau}d\omega \tag{1.6}$$

is the impulse response at the vacuum–medium interface. If the function $R(\omega)$ has singular points on the real ω axis, then in the integration in Eq. (1.6) we take a detour around a small semicircle above the points.

According to the causality principle, the dielectric constant $\varepsilon(\omega)$, the index of refraction $n(\omega)$, and the amplitude reflection coefficient $R(\omega)$ have no singular points in the upper half-plane.[3] The high-frequency asymptotic behavior of these functions for linear media corresponds to the response of an ensemble of free electrons,[3]

$$\varepsilon(\omega)=1-\frac{\Omega_p^2}{\omega^2},\quad R(\omega)=\frac{\Omega_p^2}{4\omega^2},\quad |\omega|\gg\Omega_p, \tag{1.7}$$

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where $\Omega_p=\sqrt{4\pi e^2 N/m}$ is the electron plasma frequency, e and m are the electron charge and mass, respectively, and N is the electron concentration of the medium.

From this it follows that for $\tau\leqslant 0$ the contour in (1.6) can be closed in the upper half-plane and the Cauchy theorem can be applied to the closed contour thus obtained. The result is

$$G_R(\tau)=0, \quad \tau\leqslant 0 \tag{1.8}$$

and

$$A_r(\tau_+>0)=\int_0^{\tau_+} G_R(\tau)A(\tau_+-\tau)e^{i\omega_0\tau}d\tau$$
$$=R(\omega_0,\tau_+)A(\tau_+)+\sum_{n=1}^{\infty}\frac{i^n}{n!}\frac{d^nA}{d\tau_+^n}\frac{d^nR(\omega_0,\tau_+)}{d\omega_0^n}. \tag{1.9}$$

Here

$$R(\omega_0,\tau_+)=\int_0^{\tau_+} G_R(\tau)e^{i\omega_0\tau}d\tau \tag{1.10}$$

is the unit-step response of the vacuum–medium interface for reflection at frequency ω_0.

The unit-step response (1.10) is the Fourier component, at frequency ω_0, of the instantaneous spectrum of the pulse response, and according to Eqs. (1.2), (1.6), and (1.8)–(1.10),

$$\lim_{\tau_+\to+\infty} R(\omega_0,\tau_+)=R(\omega_0), \qquad \lim_{\tau_+\to+\infty} E_r(\tau_+)=R(\omega_0)A_0. \tag{1.11}$$

The time to achieve steady-state reflection at frequency ω_0 can be taken to be the time needed for the unit-step response to assume its steady-state value, the amplitude reflection coefficient $R(\omega_0)$.

2. CHARACTERISTICS OF THE LEADING EDGE OF THE REFLECTED WAVE

According to Eqs. (1.8)–(1.9), for all linear media at the leading edge of the reflected wave[2]

$$A_r(0)=\left.\frac{dA_r}{d\tau_+}\right|_{\tau_+=0}=0. \tag{2.1}$$

If at the leading edge of the incident wave (1.1) the amplitude is zero, then the reflected wave obeys the additional condition

$$\left.\frac{d^2A_r}{d\tau_+^2}\right|_{\tau_+=0}=0. \tag{2.2}$$

In an incident wave with a sharp leading edge, when in Eq. (1.9) $d^nA/d\tau_+^n=0, n\geqslant 0$, we have

$$A_r(\tau_+>0)=R(\omega_0,\tau_+)A_0 \tag{2.3}$$

and all the transient characteristics are determined solely by the time constants of the reflecting medium.

Subsequent analysis is restricted to an incident wave with a sharp leading edge, for which

$$\frac{d^2A_r}{d\tau_+^2}=\left.\frac{d^2R(\omega_0,\tau_+)}{d\tau_+^2}\right|_{\tau_+=0}A_0. \tag{2.4}$$

From Eqs. (1.6), (1.10), and the relation $R(-\omega^*)=R^*(\omega)$,

$$\left.\frac{d^2R}{d\tau_+^2}\right|_{\tau_+=0}=\frac{2}{\pi}\int_0^{\infty}R''(\omega)\omega d\omega, \tag{2.5}$$

where $R(\omega)=R'(\omega)+iR''(\omega)$ and * denotes complex conjugation.

Using the analytic properties of the function $R(\omega)$ in the upper half-plane and recalling the asymptotic behavior (1.7), one can prove the validity of the Kramers–Kronig relations[3]

$$R'(\omega_0)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{R''(\omega)}{\omega-\omega_0}d\omega,$$
$$R''(\omega_0)=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{R'(\omega)}{\omega-\omega_0}d\omega, \tag{2.6}$$

where the integration in the vicinity of the singular point of the integrand, $\omega=\omega_0$, is in the principal value sense.

Using the oddness of the function $R''(\omega)$ along with (2.6), it is easily found that

$$R'(\omega_0)=\frac{2}{\pi}\int_0^{\infty}\frac{R''(\omega)\omega}{\omega^2-\omega_0^2}d\omega. \tag{2.7}$$

Since as $\omega_0\to\infty$ the quantity ω^2 in the integrand can be neglected compared with ω_0^2, Eqs. (1.7) and (2.7) imply the sum rule

$$\int_0^{\infty}R''(\omega)\omega d\omega=-\frac{\pi}{8}\Omega_p^2. \tag{2.8}$$

Substituting (2.8) into (2.5) and taking into account (2.1) together with (2.4) we obtain formulas describing the properties of the reflected wave near its leading edge ($\tau_+\ll 1/\Omega_p$):

$$R(\omega_0,\tau_+)=-\tfrac{1}{8}\Omega_p^2\tau_+^2, \quad A_r(\tau_+)=-\tfrac{1}{8}\Omega_p^2\tau_+^2A_0. \tag{2.9}$$

Thus, at the leading edge of a wave reflected from an arbitrary linear mediu, there is always a phase shift of π, and the amplitude buildup process is characterized by a lag time of order $1/\Omega_p$ (Ref. 2).

According to the Kramers–Kronig relations (2.6), the reflected wave will be causal only if the function $R(\omega)$ is complex. If the reflecting medium is an absorbing or amplifying one, then obviously the function $R(\omega)$ is complex. It is of interest to note that even in a transparent reflecting medium, when $\varepsilon''(\omega)\equiv 0$ and the dielectric constant fails to obey the Kramers–Kronig relations, the function $R(\omega)$ may still be complex and satisfy the relations (2.6). For this it is sufficient that the dielectric constant $\varepsilon(\omega)$ be negative over a certain frequency range. Over this range, the refractive index of the medium is imaginary, and the amplitude reflection coefficient obeys $|R(\omega)|=1$. This corresponds to total reflection of the normally incident plane monochromatic waves over this frequency range. A nonabsorptive plasma is an example of such a medium.

3. REFLECTION FROM A PLASMA

The amplitude reflection coefficient of a nonabsorptive plasma is

$$R(\omega)=\begin{cases} 2\dfrac{\omega^2}{\Omega_p^2}-1-2i\dfrac{\omega\sqrt{\Omega_p^2-\omega^2}}{\Omega_p^2}, & 0\leqslant\omega\leqslant\Omega_p, \\ 2\dfrac{\omega^2}{\Omega_p^2}-1-2\dfrac{\omega\sqrt{\omega^2-\Omega_p^2}}{\Omega_p^2}, & \omega>\Omega_p, \end{cases} \tag{3.1}$$

where $R(-\omega^*)=R^*(\omega)$. The function (3.1) has two branch points, $\omega=\pm\Omega_p$, which correspond to the zeros of the dielectric constant and separate the regions of total ($|\omega|\leqslant\Omega_p$) and partial ($|\omega|>\Omega_p$) reflection of normally incident plane monochromatic waves.

Substituting (3.1) into (1.6), some algebra yields the following expression for the impulse response of the vacuum–nonabsorptive plasma interface:

$$\begin{aligned} G_R(\tau>0)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}R(\omega)(e^{-i\omega\tau}-e^{i\omega\tau})d\omega \\ &=\frac{2}{\pi}\int_0^{\infty}R''(\omega)\sin\,\omega\tau d\omega \\ &=-\frac{4}{\pi\Omega_p^2}\int_0^{\Omega_p}\omega\sqrt{\Omega_p^2-\omega^2}\,\sin\,\omega\tau d\omega \\ &=-2\,\frac{J_2(\Omega_p\tau)}{\tau}. \end{aligned} \tag{3.2}$$

Here $J_2(x)$ is the Bessel function of the first kind and we have used the familiar integrals[4]

$$\int_0^1\sqrt{1-x^2}\,\cos\,px\,dx=\frac{\pi}{2p}J_1(x),$$

$$\int_0^1\frac{\cos\,px}{\sqrt{1-x^2}}dx=\frac{\pi}{2}J_0(x)$$

together with the relation $J_2(x)=2J_1(x)/x-J_0(x)$ for the Bessel function of the first kind.

The asymptotic behavior of the impulse response (3.2) for $\tau\gg1/\Omega_p$, which determines the time required to achieve steady-state reflection, has the form

$$G_R(\tau)=\frac{2\sqrt{2}\Omega_p}{\sqrt{\pi}}\frac{\cos(\Omega_p\tau-\pi/4)}{(\Omega_p\tau)^{3/2}} \tag{3.3}$$

and corresponds to damped oscillations at frequency Ω_p, the one at which the dielectric constant of a nonabsorptive plasma vanishes. Since the reflecting medium does not absorb and the refracted wave of frequency Ω_p does not transfer energy along the z axis, it follows that the damping of the oscillations is due to the transmission of the refracted wave energy across the interface into the medium from which the wave (1.1) is incident. This relaxation mechanism gives rise to the power-law decrease, with time, of the oscillation amplitude at frequency Ω_p.

According to (1.10) and (3.2), the unit-step response of the vacuum–medium interface is

$$R(\omega_0,\tau_+)=-2\int_0^{\tau_+}\frac{J_2(\Omega_p\tau)}{\tau}e^{i\omega_0\tau}d\tau. \tag{3.4}$$

For a short time $\tau_+\ll1/\omega_0$, we can set $\omega_0\tau=0$ in the integrand to obtain

$$R(\omega_0,\tau_+)\approx R(0,\tau_+)=-1+2\frac{J_1(\Omega_p\tau_+)}{\Omega_p\tau_+}, \tag{3.5}$$

describing the time-dependent reflection of a video-pulse.[5] In the immediate vicinity of the leading edge of the reflected wave, where $\tau_+\ll1/\Omega_p$, Eq. (3.5) goes over into the formula (2.9), which is a universal characteristic of all linear reflecting media.

To estimate the time $\tau_R(\omega_0)$ to achieve steady-state reflection at frequency ω_0, we use the condition

$$\left|\frac{R(\omega_0,\tau_+)-R(\omega_0)}{R(\omega_0)}\right|=\left|\int_{\tau_+}^{\infty}\frac{G_R(\tau)}{R(\omega_0)}e^{i\omega_0\tau}d\tau\right|<\beta\ll1 \tag{3.6}$$

for all $\tau_+>\tau_R(\omega_0)$. Calculations using Eqs. (3.2), (3.3), and (3.6) indicate that

$$\tau_R(\omega_0)=\begin{cases} 1/\beta^{2/3}\Omega_p, & \omega_0\ll\Omega_p, \\ 1/\beta^2\Omega_p, & \omega_0\approx\Omega_p, \\ \omega_0^{2/3}/\beta^{2/3}\Omega_p^{5/3}, & \omega_0\gg\Omega_p. \end{cases} \tag{3.7}$$

If one takes into account the weak absorption of the plasma and sets

$$\varepsilon(\omega)=1-\frac{\Omega_p^2}{\omega(\omega+i\nu_c)}, \tag{3.8}$$

where $\nu_c\ll\Omega_p$ is the ion–electron collision frequency, this obviously does not affect the reflected-wave characteristics near the leading edge at $\tau_+\ll1/\Omega_p\ll1/\nu_c$. Calculation shows that weak absorption can determine $\tau_R(\omega_0)$ only over the frequency range $\omega_0\gtrsim\Omega_p$, where the quantity $1/\nu_c$ sets the upper bound for the onset time. For example, when $\omega_0\approx\Omega_p$, $\Omega_p\gg\nu_c\gg\beta^2\Omega_p$,

$$\tau_R(\omega_0)\approx\frac{\ln(\nu_c/\beta^2\Omega_p)}{\nu_c}. \tag{3.9}$$

4. REFLECTION FROM A DIELECTRIC

Consider the reflection of a wave with a sharp leading edge from a medium with a dielectric constant

$$\varepsilon(\omega)=1-\sum_s\frac{\alpha_s^2}{\omega^2+2i\gamma_s\omega-\omega_s^2}, \tag{4.1}$$

where the summation is over all ensembles of the medium's harmonic oscillators with natural frequencies ω_s and attenuation coefficients $0<\gamma_s\ll\omega_s$. The constants α_s satisfy the relation

$$\sum_s \alpha_s^2=\frac{4\pi e^2N}{m}=\Omega_p^2, \tag{4.2}$$

where N is the electron concentration in the medium.

The dielectric constant (4.1) has poles $\omega_{sp}=\omega'_{sp}+i\omega_{sp}\approx\pm\omega_s-i\gamma_s$ and zeros $\omega_{sn}=\omega'_{sn}+i\omega''_{sn}\approx\pm\sqrt{\omega_s^2-\alpha_s^2}-i\gamma_s$, which lie in the lower half-plane and which correspond to the branch points of the amplitude reflection coefficient $R(\omega)$. In the transparent limit, when all quantities $\gamma_s=0$,the frequencies ω_{sp} and ω_{sn} lie on the real ω axis and separate the regions of partial and total reflection of plane monochromatic waves upon normal incidence.

In calculating the impulse response it is convenient to analytically continue the function $R(\omega)$ into the lower half-plane and then displace the path of integration in (1.6) by an infinite distance downward from the real ω axis. To this end, we create cuts parallel to the real ω axis that connect pairs of poles ω_{sp} and zeros ω_{sn} with the same s and the same signs of the real parts. As a result, the expression (1.6) for $\tau>0$ can be written as a sum of integrals over the closed contours $L_s(\omega)$ which correspond to the edges of the cuts,

$$G_R(\tau)=\sum_s G_R^s(\tau)=\sum_s \frac{1}{2\pi}\left[\oint_{L_s(\omega'>0)} R(\omega)e^{-i\omega\tau}d\omega + \oint_{L_s(\omega'<0)} R(\omega)e^{-i\omega\tau}d\omega\right]. \tag{4.3}$$

Since $n(-\omega^*)=n^*(\omega)$ and since $n(\omega)$ reverses sign on going from the upper edge of the cut to the lower, we have

$$G_R^{(s)}(\tau)=-\frac{2}{\pi}\int_{\omega_{sp}}^{\omega_{sn}}\frac{n(\omega)}{1-n^2(\omega)}e^{-i\omega\tau}d\omega+\text{c.c.}, \tag{4.4}$$

where the integration is carried out over the upper edge of the cut for $\omega'>0$, and "c. c." denotes complex conjugation.

For $\gamma_s\ll\omega_s$ and $\omega'_{sp}<\omega'<\omega'_{sn}$,

$$\frac{n(\omega)}{1-n^2(\omega)}\approx-\frac{2(\omega'_{sp}\omega'_{sn})^{1/2}}{\alpha_s^2}\sqrt{(\omega-\omega_{sp})(\omega-\omega_{sn})}, \tag{4.4'}$$

so, using known integrals,[4] one obtains the following expression for the impulse response of an individual oscillator ensemble:

$$G_R^{(s)}(\tau)=-2\alpha_s^2\frac{J_1\left(\frac{\omega'_{sn}-\omega'_{sp}}{2}\tau\right)e^{-\gamma_s\tau}}{(\omega'^2_{sn}-\omega'^2_{sp})\tau}\sin\left(\frac{\omega'_{sn}+\omega'_{sp}}{2}\tau\right). \tag{4.5}$$

Near the leading edge of the reflected wave [where for all s we have $(\omega'_{sn}+\omega'_{sp})\tau\ll1(\Omega_p\tau_+\ll1)$], expanding (4.5) in a power series in τ, keeping only the first term of the expansion, and using (4.2) and (4.3), we arrive again at the universal formulas (2.9). For $(\omega'_{sn}-\omega'_{sp})\tau/2\gg1$, the asymptotic behavior of (4.5) has the form

$$G_R^{(s)}(\tau)=-\frac{2}{\sqrt{\pi}}\frac{e^{-\gamma_s\tau}}{\sqrt{(\omega'_{sn}-\omega'_{sp})\tau^3}}[\sin(\omega'_{sp}\tau+3\pi/4)+\sin(\omega'_{sn}\tau-3\pi/4)], \tag{4.6}$$

using the asymptotic behavior of $J_1(x)$ for $x\gg1$ and the relation $\omega'^2_{sn}-\omega'^2_{sp}\approx\alpha_s^2$.

As in a plasma, Eq. (4.6) describes damped oscillations at the limiting frequencies ω'_{sp} and ω'_{sn}, which in the limit $\gamma_s=0$ separate the regions of total and partial reflection of normally incident plane monochromatic waves. The exponential factor in (4.6) controls damping due to absorption in the medium, and the power-law factor $\sim\tau^{-3/2}$, damping due to the transmission of the energy of the refracted waves into the medium from which the wave (1.1) is incident.

To estimate the time $\tau_R(\omega_0)$, consider the case when in Eq. (4.3) we may content ourselves with the contribution from just a single oscillator ensemble, the one with the smallest value of $|\omega_0-\omega_s|$. In this approximation for the resonant reflection region, when $|(\omega_0-\omega'_{sp})\times(\omega_0-\omega'_{sn})|\ll\gamma_s\omega_0$ and $\gamma_s\gg\beta^2(\omega'_{sn}-\omega'_{sp})=\beta^2\Delta\omega_s$, we have

$$\tau_R(\omega_0)\approx\frac{1}{\gamma_s}\ln\left(\frac{2\omega_0^2}{\beta\gamma_s^{3/2}\Delta\omega_s^{1/2}}\right), \tag{4.7}$$

and for the nonresonant reflection region, when $|(\omega_0-\omega'_{sn})\times(\omega_0-\omega'_{sp})|\gg\gamma_s\omega_0$ and $\gamma_s\ll\Delta\omega_s$,

$$\tau_R(\omega_0)\approx\left[\frac{\Delta\omega_s\omega_0}{\beta(\omega_0-\omega'_{sp})(\omega_0-\omega'_{sn})}\right]^{2/3}\frac{1}{\Delta\omega_s}. \tag{4.8}$$

Thus, the time to achieve steady-state reflection at a definite frequency in a dielectric is determined by the damping coefficient γ_s of the natural vibrations of the electrons of the medium, and by the frequency width$\Delta\omega_s$ of the total reflection region of the normally incident plane monochromatic waves.

CONCLUSION

The results obtained for normal incidence of a wave with a leading edge can be extended to an arbitrary angle of incidence θ. For example, for a wave polarized perpendicular to the plane of incidence, all one needs to do in the formulas above is make the replacements $\tau_\pm\to\tau_\pm(\theta) = t\pm(z\cos\theta\mp x\sin\theta)/c,\Omega_p\to\Omega_p/\cos\theta,\alpha_p\to\alpha_p/\cos\theta$, where the x axis is directed along the vacuum–medium interface. From this it follows that as steady-state reflection is established, the angle of reflection always remains equal to the angle of incidence.

We have considered transient processes associated with the excitation of the reflected wave. Because of the boundary conditions near the vacuum–medium interface, analogous transient processes govern the excitation of the refracted wave. The refracted-wave precursor forms at a distance of order c/Ω_p from the interface due to phase effects during propagation of the spectral harmonics.

According to the calculations, the time to achieve steady-state reflection at a given frequency, that is, the time for establishing the Fresnel formulas, depends on three parameters: (i) the absorption by the reflecting medium, (ii) the proximity of the incident wave frequency to one of the resonance frequencies of the reflecting medium, the latter being determined by the zeros and poles of the dielectric constant of the medium, and (iii) the frequency width of the total reflection region for plane monochromatic waves.

In most cases involving reflection from a plasma or a dielectric, the time to achieve steady-state reflection is determined primarily by the damping coefficient of the natural vibrations of the electrons in the medium. For a nonresonant reflection from a weakly absorbing dielectric and a plasma with $\omega_0 \ll \Omega_p$, the time to achieve steady-state reflection is determined by the frequency width of the total reflection region. Note that the results obtained for a plasma can be used to describe a time-dependent reflection from a metal.

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