Properties of precessing half-spin and double-spin states of a magnetically distorted superfluid ^{3}He–B

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The effect of anisotropy of the magnetized B phase of superfluid ^{3}He on the stability of coherently precessing half-spin and double-spin states is investigated theoretically.

1. INTRODUCTION

Superfluid phases of liquid ^{3}He are the low-temperature ordered states of the Fermi system with a spin-triplet, p-wave Cooper pairing. They are characterized by multidimensional order parameters which describe rich equilibrium and dynamic properties of superfluid ^{3}He. The rigidity of equilibrium ordered states with respect to the spatial distortions of the Goldstone degrees of freedom of the Cooper condensate generates variety of gapless collective modes. These bosonic branches can be excited against the background of the equilibrium states of superfluid phases of ^{3}He. Together with the fermionic branch, they describe the dynamic behavior of the ordered Fermi liquid.

The rigidity of ordered states of the superfluid phases of ^{3}He can be preserved far from the equilibrium, where the order parameter is a time-dependent object. Among these types of states the uniformly precessing spin states in the A- and B-phases of ^{3}He are of special interest and are studied extensively.[1–3] In the case where the precessing spin states are stable (rigid) with respect to the decay onto the long-wavelength excitations, they are observed as long-lived ordered states.

In the present study we examine some new aspects of coherently precessing spin states in ^{3}He–B. The starting point is a coherently precessing, metastable spin state, which was considered in Ref. 4 and which is characterized by a nonequilibrium spin polarization. In Ref. 4 it was shown that in the case where the spin polarization S of ^{3}He–B is considerably different from its equilibrium value $S_0=(\chi_B/g)H_0$, where χ_B is the B-phase magnetic susceptibility, g is the ^{3}He nuclear gyromagnetic ratio, and H_0 is the strength of the applied magnetic field, the new, coherently precessing spin-orbit configurations are stabilized. Especially interesting dynamic regimes are realized at $S=S_0/2$ and $S=2S_0$ [half-spin (HS) and double-spin (DS) cases, respectively]. Below we will study the effect of magnetic distortion of ^{3}He–B order parameter and of superfluid counterflows on the coherently precessing HS and DS states.

2. COHERENTLY PRECESSING HS AND DS STATES IN THE MAGNETICALLY DISTORTED ^{3}He–B

In the absence of spin-orbit coupling the spin-system in the superfluid phases of ^{3}He precesses with a Larmor frequency $\omega_0=|g|H_0$, irrespective of the tipping angle β_s of magnetization with respect to the direction of the applied magnetic field $\mathbf{H}=H_0\mathbf{z}$. This simple but fundamental property is the manifestation of the fact that in the coordinate frame rotating with the Larmor angular velocity $\boldsymbol{\omega}_0=\omega_0\mathbf{z}$ the magnetic field is eliminated and the spin system restores the isotropy.

A spin-orbit coupling (of dipole-dipole origin) has a strong effect on the spin-dynamics of superfluid phases of ^{3}He. As a result of long-range correlations of the cooper pair spins and intrinsic angular momenta in the state with spontaneously broken spin-orbit symmetry, the small (on the scale of the condensation energy) dipole-dipole interaction between nuclear magnetic moments of ^{3}He atoms (partially) lifts the degeneracy of the ordered states with respect to angular (Goldstone) degrees of freedom and the specific spin-orbit configuration is established. In this way the dipole-dipole coupling perturbs, in general, the simple Larmor precessing state and this perturbation is observed as the NMR frequency shift from ω_0.

The dipole-dipole energy density F_D depends on the components $A_{\mu i}$ of the order parameter of the superfluid state. In general, $A_{\mu i}$ can be represented in the form

$$A_{\mu i}=R^{(S)}_{\mu\nu}R^{(L)}_{ij}A^{(0)}_{\nu j}=(\hat{R}^{(S)}\hat{A}^{(0)}\hat{R}^{(L)-1})_{\mu i}, \tag{1}$$

where $\hat{R}^{(S)}$ ($\hat{R}^{(L)}$) describes the 3D rotations in spin (orbit) space, and $A^{(0)}_{\nu j}$ corresponds to a particular (say equilibrium) ordered state of liquid ^{3}He. Rotation matrix $\hat{R}^{(S)}$ ($\hat{R}^{(L)}$) can be parametrized in terms of the Eulerian angles $\alpha_S,\beta_S,\gamma_S$ ($\alpha_L,\beta_L,\gamma_L$), which for the dynamic states are time-dependent variables.

The dipole-dipole interaction potential

$$F_D=\text{const}\ \Omega^2[(\mathrm{Tr}\hat{A})^2+\mathrm{Tr}(\hat{A}^2)], \tag{2}$$

which is proportional to the square of the characteristic dipolar frequency Ω, is an intricate function of the Eulerian angles, but in the case of a strong magnetic field with $\omega_0\gg\Omega$ (which will be considered below) we can average $F_D=F_D[\hat{A}(t)]$ with respect to "fast" angular variables (on the time scale Ω^{-1}). The number of relevant variables is therefore reduced (only "slow" variables remain after averaging), and the problem of solving the equations for spin dynamics becomes manageable.[5] The key point in what follows is that the final result for the average dipole-dipole potential $\langle F_D\rangle$ depends essentially on the magnitude of spin polarization $S=pS_0$, where p is the fraction of the equilibrium value S_0. For the conventional case with $p=1$, the dynamic regime is realized when the off-diagonal spin order

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parameter executes a precessional motion around the instantaneous direction of $\delta\mathbf{S}(t)=\mathbf{S}(t)-\mathbf{S}_0$ in such a resonating way that $\dot{\gamma}_S\simeq-\dot{\alpha}_S$ and the variable $\phi=\alpha_S+\gamma_S$ turns out to be "slow". In this case $\langle F_D\rangle$ depends on the spin projection $s_z=\cos\beta_S$, the orbital angular momentum projection $1_z=\cos\beta_L$, and the phase ϕ.

A completely different, resonating, coherent, spin precessing regime develops in the case in which $p=1/2$. In this HS case we have $\dot{\gamma}_S\simeq-(1/2)\dot{\alpha}_S$ and, besides, s_z and l_z, the average $\langle F_D\rangle$, depend on the new, "slow" variable $\tilde{\phi}=\alpha_S=2\gamma_S$. This metastable HS case can be realized in the B phase[4] and in the A phase.[6] In addition, for ^{3}He–B the DS ($p=2$) coherent regime takes place with the relevant angular variable $\tilde{\phi}=2\alpha_S+\gamma_S$ (this resonance is absent in ^{3}He–A).

In Ref. 4 it was demonstrated that for the isotropic B phase the HS and DS regimes are characterized by the following average dipole-dipole potential [measured in units of $(2/15)\chi_B(\Omega_B/g)^2$]:

$$\langle f_D\rangle=\tfrac{3}{4}[1-s_z^2)(1-l_z^2)+2s_z^2l_z^2+\tfrac{2}{3}\sqrt{1-s_z^2}\times(1+s_z)\sqrt{1-l_z^2}(1+l_z)\cos\tilde{\phi}]. \qquad (3)$$

The stationary spin-orbit configurations can be determined by minimizing the thermodynamic potential

$$F'=F+\boldsymbol{\omega}\mathbf{S}, \qquad (4)$$

which is constructed in the coordinate frame rotating with the angular velocity $\boldsymbol{\omega}=\omega\hat{\mathbf{z}}$, where ω is the frequency of the transverse rf field applied to the spin system. In Eq. (4) F is the sum of the dipole-dipole potential and the Zeeman energy term:

$$F=\langle F_D\rangle-\boldsymbol{\omega}_0\mathbf{S}. \qquad (5)$$

Using dimensionless units [as in (3)], we can minimize the expression

$$f'=\langle f_D\rangle+f_\omega, \qquad (6)$$

where the so-called spectroscopic term

$$f_\omega=(\boldsymbol{\omega}-\boldsymbol{\omega}_0)\mathbf{S}/(2/15)\chi_B(\Omega_B/g)^2=ws_z \qquad (7)$$

with

$$w=(15/2)p(\omega-\omega_0)\omega_0/\Omega_B^2. \qquad (8)$$

For the case $\omega=\omega_0$ ($w=0$) the spin-orbit configurations of coherently precessing HS and DS spin states can be found by minimizing the dipole-dipole potential (3). It was shown[4] that each case (with $p=1/2$ and $p=2$) has a pair of degenerate metastable states:

$$s_z=0.75, \quad l_z=0.3, \qquad (9a)$$

$$s_z=0.3, \quad l_z=0.75, \qquad (9b)$$

which are shown in Fig. 1.

The goal of the present research is twofold. First, we shall explore the influence of anisotropic distortion of the order parameter of ^{3}He–B under the action of a strong magnetic field on the stability of HS and DS precessing states. This influence is manifested through the modification of dipole-dipole potential (3), which corresponds to the isotropic (undistorted) B phase. Second, the action of superfluid counterflows on the spin-orbit structure of HS and DS states will be elucidated.

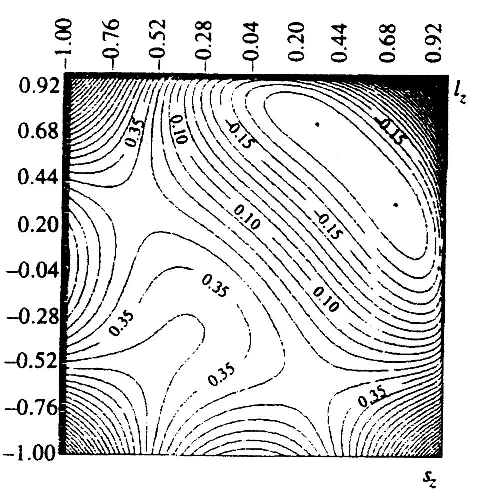

FIG. 1. The level contours of average spin-orbital potential $\langle f\rangle$ [Eq. (3) at $\tilde{\phi}_{st}=\pi$]. Two degenerate stationary points, which are seen at the upper right-hand corner, correspond to states (9a) and (9b).

The order parameter of the magnetized B phase can be represented as in (1) with

$$A_{\mu i}^{(0)}=\Delta_\parallel\hat{h}_\mu\hat{h}_i+\Delta_\perp(\delta_{\mu i}-\hat{h}_\mu\hat{h}_i), \qquad (10)$$

where $\Delta_\parallel(\Delta_\perp)$ is the energy gap component parallel (perpendicular) to the direction of applied magnetic field ($\hat{h}=\mathbf{H}/H_0=\hat{z}$). Expression for the magnetically distorted B-phase order parameter can now be written in the form

$$A_{\mu i}=\Delta_\perp[R_{\mu i}-(1-q)R_{\mu z}^{(S)}(\hat{R}^{(L)-1})_{zi}], \qquad (11)$$

where the orthogonal matrix of relative spin-orbit rotation $\hat{R}=\hat{R}^{(S)}\cdot\hat{R}^{\,(L)-1}$ and $q=\Delta_\parallel/\Delta_\perp$.

Taking into account the general expression for the dipole-dipole energy density of the B phase

$$F_D^{(B)}=\tfrac{1}{15}\chi_B(\Omega_B/g)^2\Delta_0^{-2}[(\mathrm{Tr}\hat{A})^2+\mathrm{Tr}(\hat{A}^2)], \qquad (12)$$

where $\Delta_0(T)$ is the energy gap of undistorted ^{3}He–B, and using, (11) we obtain the dimensionless dipole-dipole potential of the magnetized B phase

$$\begin{aligned}f_D&=F_D^{(B)}/(2/15)\chi_B(\Omega_B/g)^2(\Delta_\perp/\Delta_0)^2\\&=\tfrac{1}{2}[(\mathrm{Tr}\hat{R})^2+\mathrm{Tr}(\hat{R}^2)]-(1-q)[\mathrm{Tr}\hat{R})\rho_{zz}+(\hat{\rho}^2)_{zz}]\\&\quad+(1-q)^2\rho_{zz}^2.\end{aligned} \qquad (13)$$

Using the identities

$$(\mathrm{Tr}\hat{R})^2+\mathrm{Tr}(\hat{R}^2)=2[\mathrm{Tr}\hat{R}-1/2)^2-1/4], \qquad (14a)$$

$$(\hat{\rho}^2)_{zz}=(\mathrm{Tr}\hat{R})(\rho_{zz}-1)+\rho_{zz}, \qquad (14b)$$

we obtain the following expression for f_D:

$$f_D=[\mathrm{Tr}\hat{R}-1/2)-(1-q)(\rho_{zz}-1/2)]^2-\tfrac{1}{4}q^2-(1-q^2)\rho_{zz}. \qquad (15)$$

Since

$$\mathrm{Tr}\hat{R}=s_z l_z+\tfrac{1}{2}(1+s_z)(1+l_z)\cos(\alpha+\gamma) \tag{16}$$
$$+\tfrac{1}{2}(1-s_z)(1-l_z)\cos(\alpha-\gamma)$$
$$+\sqrt{1-s_z^2}\sqrt{1-l_z^2}(\cos\alpha,\cos\gamma),$$

and

$$\rho_{zz}=s_z l_z+\sqrt{1-s_z^2}\sqrt{1-l_z^2}\cos\alpha, \tag{17}$$

where $\alpha=\alpha_S-\alpha_L$ and $\gamma=\gamma_S-\gamma_L$, from (15) we can calculate the average dipole-dipole potential $\langle f_D\rangle$ for various uniformly spin precessing regimes (various values of p). For the conventional case with $p=1$, $\langle f_D\rangle$ was constructed and analyzed in Ref. 7. Here we consider the metastable spin precessing HS ($p=1/2$) and DS ($p=2$) cases. Simple calculations show that for these regimes (up to a constant additive term) we have

$$\langle f_D\rangle=\frac{3}{4}\,\frac{1+2q^2}{3}\left[(1-s_z^2)(1-l_z^2)+2s_z^2 l_z^2\right]$$
$$+\frac{1}{2}\sqrt{1-s_z^2}(1+s_z)\sqrt{1-l_z^2}(1+l_z)\begin{bmatrix}\cos(\alpha+2\gamma)\\ q\,\cos(2\alpha+\gamma)\end{bmatrix}. \tag{18}$$

On the second line in (18) the upper (lower) row in square brackets refers to the HS (DS) state.

Inspection of Eq. (18) shows that on reaching the so-called planar state ($q=0$) the resonance (i.e., the explicit dependence of $\langle f_D\rangle$ on the slow variable $\tilde{\phi}=2\alpha+\gamma$) disappears for the case of DS state and the stationary spin-orbit solution [which is realized at the minimum of (18)] is given by

$$s_z=1,\quad l_z=0, \tag{19a}$$

$$s_z=0,\quad l_z=1. \tag{19b}$$

These states are obtained from the pair of initial solutions (9a) and (9b) upon a gradual change in $q=\Delta_{\parallel}/\Delta_{\perp}$ from $q=1$ (corresponding to the undistorted B phase) up to $q=0$ (for the planar phase). An example of a magnetically deformed DS precessing state at $q=0.3$ is shown in Fig. 2. In this figure the tendency of displacement of the precessing spin-orbit configurations (9a) and (9b), shown in Fig. 1, toward the states (19a) and (19b) is clearly seen.

The absence of the $\tilde{\phi}$ resonance at $p=2$ for the planar phase ($q=0$) is analogous to the absence of a corresponding resonance for ^{3}He–A (Ref. 6) (see also Appendix I).

A completely different behavior with q of the precessing spin state has been observed for the HS case. Direct construction of $\langle f_D\rangle=f(s_z,l_z,\tilde{\phi}=\pi)$ shows that two degenerate solutions, (9a) and (9b), begin to approach each other. At $q\simeq 0.9$, they merge and upon subsequent decrease of q the minimum of $\langle f_D\rangle$ at $s_z=l_z\simeq 0.53$ progressively deepens (at $q=0$ the deepest minimum is attained). The above-mentioned changes of the precessing HS states with diminishing q are illustrated in Figs. 3 and 4.

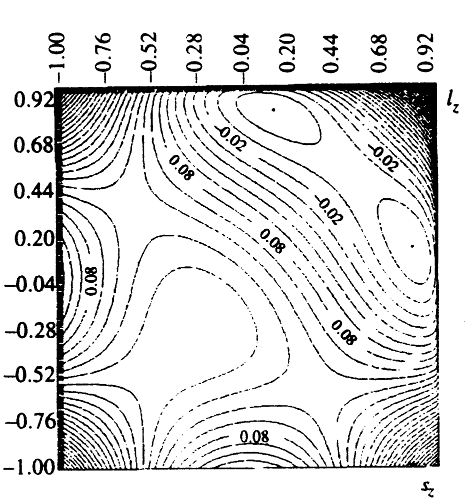

FIG. 2. The level contours of average spin-orbital potential $\langle f_D\rangle$ [Eq. (8) at $\tilde{\phi}_{st}=\pi$ and $q=0.3$] in the case of double-spin states. The tendency of the displacement of coherently precessing spin-orbital configurations toward the states (9a) and (9b) is seen.

3. EFFECT OF SUPERFLUID COUNTERFLOWS ON THE COHERENTLY PRECESSING HS AND DS STATES

The magnetic distortion of the isotropic B phase preserves spin-orbit symmetry of the dipole-dipole potential (invariance with respect to $s_z\leftrightarrow l_z$). On the other hand, the spectroscopic term f_ω violates the spin-orbit symmetry of the thermodynamic potential f' [see Eq. (6)]. Another source of violation of the $s_z\leftrightarrow l_z$ invariance is the anisotropic contribution to the hydrodynamic energy of the magnetized B phase, which is manifested in the appearance of an additional flow term f_{flow} in f'.

Uniaxial orbital anisotropy of magnetized ^{3}He–B is displayed in the tensorial nature of the superfluid density

$$\rho_{ij}^{(S)}=\rho_{\parallel}^{(S)}\hat{l}_i\hat{l}_j+\rho_{\perp}^{(S)}(\delta_{ij}-\hat{l}_i\hat{l}_j), \tag{20}$$

where $\hat{l}$ is the anisotropy axis, so that in the presence of superflow with the counterflow velocity $\mathbf{v}=\mathbf{v}_s$-$\mathbf{v}_n$ an $\hat{l}$-dependent contribution to the kinetic energy density appears:

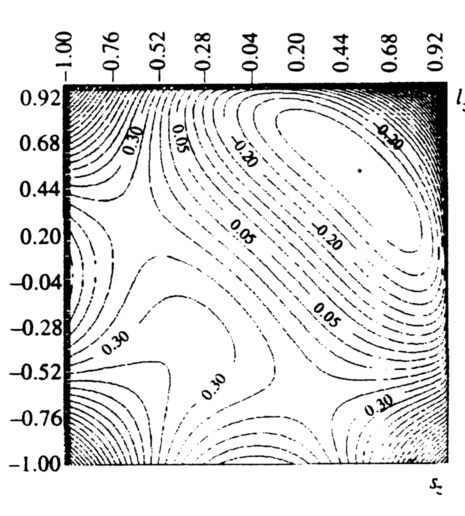

FIG. 3. The level contours of average spin-orbital potential $\langle f_D\rangle$ [Eq. (18) at $\tilde{\phi}_{st}=\pi$ and $q=0.9$] in the case of half-spin states. It is seen that at this value of the anisotropy parameter q two distinct stationary configurations for $q=0$ (shown in Fig. 1) have merged at $s_z=l_z\simeq 0.53$. Upon further decrease of q, the minimum of $\langle f_D\rangle$ deepens and reaches the largest depth for the planar phase with $q=0$ (see Fig. 4).

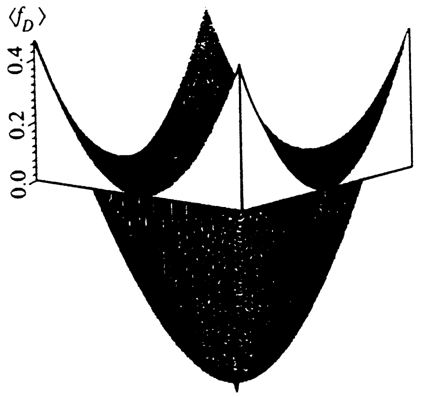

FIG. 4. The 3D picture of average spin-orbital potential $\langle f_D \rangle$ [Eq. (18) at $\bar{\phi}_{sp}=\pi$ and $q=0$] for the half-spin states. A deep minimum at $s_z=l_z\simeq 0.53$ is indicated by a tiny "leg".

$$F_{\rm flow}^{\rm (an)}=-\tfrac{1}{2}\,\delta\rho_{\rm an}^{(S)}(\mathbf{v}_S-\mathbf{v}_n)^2(\hat{l}\hat{v})^2. \tag{21}$$

Here $\delta\rho_{\rm an}^{(S)}=\rho_{\perp}^{(S)}-\rho_{\parallel}^{(S)}>0$ and $\hat{v}$ is the unit vector directed along $\mathbf{v}_s-\mathbf{v}_n$. In the case where $\hat{v}\perp\hat{z}$, the dimensionless flow term $f_{\rm flow}=ul_z^2$ with

$$u=\frac{15}{4}\,\frac{\delta\rho_{\rm an}^{(S)}v^2}{\chi_B(\Omega_B/g)^2(\Delta_\perp/\Delta_0)^2}=\delta_B(v/v_D)^2 \tag{22}$$

should be taken into account[8] [the last equality in (22) refers to the case $\delta_B=1-q\ll 1$ and v_D is the dipolar velocity on the order of 1 mm/sec]. In analyzing the possible stable spin-orbit configurations of uniformly precessing spin states we must take into account the relation

$$f'=\langle f_D\rangle+f_\omega+f_{\rm flow}=\langle f_D\rangle+ws_z+ul_z^2. \tag{23}$$

Here we explore the influence of superfluid counterflows ($u\neq 0$) in the absence of the spectroscopic term ($\omega=\omega_0$) on the HS and DS precessing spin states in the case of a weak magnetic distortion ($\delta_B\ll 1$) and $v\gg v_D$. In this case we can use $\langle f_D\rangle$ calculated for $q=1$, and the topographic profiles (level contours) of $f'=\langle f_D\rangle_{q=1}+ul_z^2$ were constructed numerically for various values of the flow parameter u. Some of the results are shown in Figs. 5 and 6. We see that the gradual increase of the superfluid counterflow destroys the spin-orbit configuration, (9b). As to the precessing state (9a), it is displaced from the initial position, although it still survives as a stationary precessing solution. It can be shown that for $w>0$ the spectroscopic term has an opposite effect on the (9a) and (9b) spin precessing states. The analysis of combined action on the spectroscopic term and the counterflows on the HS and DS states will be given in a separate publication.

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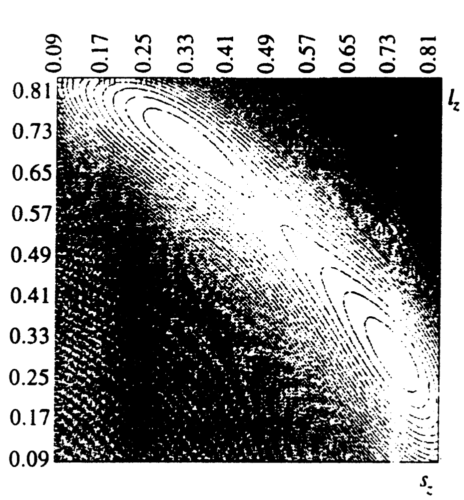

FIG. 5. A fragment of the level contours of $\langle f_D\rangle+ul_z^2$ at $q=1$ (the isotropic B phase) and $u=0.02$.

APPENDIX I

In Appendix I we consider some aspects of coherently precessing spin states in ^{3}He–A. Superfluid ^{3}He–A can be visualized as a mixture of Cooper condensates with spin configurations $\uparrow\uparrow$ and $\downarrow\downarrow$, which are characterized by the gap functions $\Delta_\uparrow$ and $\Delta_\downarrow$. In the presence of a magnetic field we have $\Delta_\uparrow\neq\Delta_\downarrow$. The corresponding order parameter is

$$A_{\mu i}=\frac{\Delta}{\sqrt{2}}(\alpha_+\hat{d}_1+\alpha_-\hat{d}_2)_\mu(\hat{u}_1+\hat{u}_2)_i, \tag{A1}$$

where $\Delta^2=(\Delta_\uparrow^2+\Delta_\downarrow^2)/2$, and $\alpha_\pm=(\Delta_\uparrow\pm\Delta_\downarrow)/2\Delta$. In (Al) the pairs of orthogonal unit vectors $(\hat{d}_1,\hat{d}_2)$ and $(\hat{u}_1,\hat{u}_2)$ define the quantization axes $\hat{s}=\hat{d}_1x\hat{d}_2$ and $\hat{l}=\hat{u}_1x\hat{u}_2$ of, respectively, the spin and orbital angular momenta of the Cooper pairs.

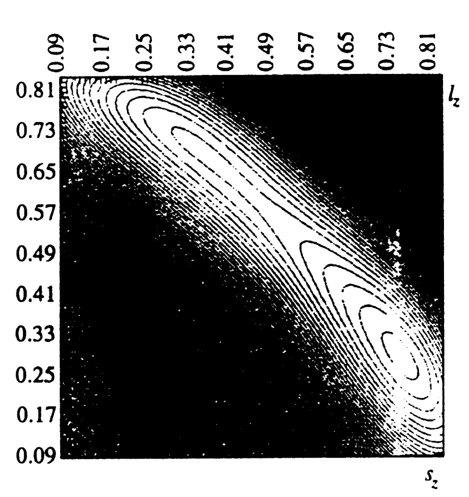

FIG. 6. A fragment of the level contours of $\langle f_D\rangle+ul_z^2$ at $q=1$ and $u=0.035$. For this value and for higher values of the superflow parameter u only one stationary spin-orbital configuration survives (as shown at the lower right-hand corner of the figure).

For the superfluid A_2 phase (A1) the dipole-dipole energy density is

$$F_D=-\tfrac{1}{2}\chi_N(\Omega_A/g)^2[\tfrac{1}{2}(1+\beta)(\hat{d}_1\hat{l})^2+\tfrac{1}{2}(1-\beta)\times(\hat{d}_1\hat{l})^2], \tag{A2}$$

where $\beta=\Delta_\uparrow\Delta_\downarrow/\Delta^2$. This parameter has the meaning of the internal Josephson coupling of the $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper condensates ($0\leqslant\beta\leqslant 1$). At $\beta=0$, the A_1 phase with the Cooper pairing in the single spin projection configuration is realized.

Using the parametrization

$$\hat{l}=\hat{z}\cos\beta_L+\hat{x}\sin\beta_L, \tag{A3a}$$

$$\hat{d}_1=\overleftrightarrow{R}^{(S)}\hat{x},\quad \hat{d}_2=\overleftrightarrow{R}^{(S)}\hat{y}, \tag{A3b}$$

taking into account, as before, the case with $\omega\,0\gg\Omega_A$, and averaging over the "fast" Eulerian variables, we can easily construct a dimensionless dipole-dipole potential

$$\langle f_D\rangle=\langle F_D\rangle/(1/8)\chi_N(\Omega_A/g)^2. \tag{A4}$$

For the case in which $S=S_0$ $(p=1)$ we have[6]

$$\langle f_D\rangle=-[s_z^2+l_z^2-3s_z^2l_z^2+\tfrac{1}{2}\beta(1+s_z)^2(1-l_z^2)\cos 2\phi] \tag{A5}$$

with the "slow" variable $\phi=\alpha_s+\gamma_s$. Note that, in contrast with ^{3}He–B, the dipole-dipole potential of ^{3}He–A has nospin-orbit symmetry for the stationary values $\phi_{\mathrm{st}}=0,\pi$ we have

$$\langle f_D\rangle=-(a_0+a_1s_z+a_2s_z^2)=-(b_0+b_1l_z+b_2l_z^2), \tag{A6}$$

where

$$\begin{cases} a_1(l_z)=\beta(1-l_z^2)\geqslant 0,\\ a_2(l_z)=\tfrac{1}{2}[(2+\beta)-(6+\beta)l_z^2]' \end{cases} \tag{A7}$$

and

$$\{b_1\equiv 0,\quad b_2(s_z)=1-3s_z^2-\tfrac{1}{2}\beta(1+s_z)^2.$$

The stability condition $a_2(l_z)<0$ of a nonuniformly precessing spin state is realized for (see also Ref. 9)

$$l_z^2>\frac{2+\beta}{6+\beta}, \tag{A8}$$

so that the Leggett orbital configuration, $l_z=0$, is unstable with respect to the decay onto the long-wavelength spin excitations of uniform spin precession.[10]

The stationary value of l_z depends on the sign of the coefficient $b_2(s_z)$. For $b_2<0$, which is realized at

$$s_z>\frac{2-\beta}{\sqrt{\beta^2+(6+\beta)(2-\beta)}+\beta}, \tag{A9}$$

the minimum of (A6) is at $l_z=0$ (an unstable case). On the other hand, for $b_2>0$ the minimum is reached at $l_z=\pm 1$, which is within the stability region, (A8). This occurs in the "window" of s_z values

$$-\frac{2-\beta}{\sqrt{\beta^2+(6+\beta)(2-\beta)}-1}<s_z<\frac{2-\beta}{\sqrt{\beta^2+(6+\beta)(2-\beta)}+1}. \tag{A10}$$

Relation (A10) extends the result obtained in Ref. 11 to the A_2 phase with $\beta<1$.

Turning to the HS state $(p=1/2)$, we find that (for $\beta=1$)

$$\langle f_D\rangle=-[s_z^2+l_z^2-3s_z^2l_z^2-2l_z\sqrt{1-l_z^2}\times(1+s_z)\sqrt{1-s_z^2}\cos\phi], \tag{A11}$$

where $\tilde{\phi}=\alpha_s+2\gamma_s$. For $l_z<0(>0)$ we have $\tilde{\phi}_{\mathrm{st}}=0(\pi)$, and the stationary spin-orbit configurations can be obtained by minimizing

$$-[s_z^2+l_z^2-3s_z^2l_z^2+2|l_z|\sqrt{1-l_z^2}(1+s_z)\sqrt{1-s_z^2}]. \tag{A12}$$

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