# Monomolecular exciton creation processes in polar semiconductors in a strong magnetic field

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The rate of creation of Wannier–Mott excitons in the presence of a strong magnetic field during continuous irradiation of semiconductors has been calculated by a graphical technique. A monomolecular process is considered, i.e., one in which an electron and a hole, created by light at the same point in the crystal, are joined together to form an exciton. The formation of an exciton with kinetic energy in the interval from 0 to $\hbar\omega_{LO}$ is accompanied by the emission of $N$ LO-phonons where $N-1$ of the phonons are emitted by an electron-hole pair of finite volume, and the $N$th phonon, during the binding of the pair into an exciton at the final stage of the process. An abrupt increase in the exciton creation rate when a strong magnetic field is turned on is predicted, specifically by a factor of $\alpha^{-1}$ for $N=2$ and $\alpha^{-2}$ for $N\geqslant 4$, where $\alpha$ is the Fröhlich coupling constant of the electrons with the LO-phonons. Experiments are proposed for detecting the predicted effect. Oscillations in the exciton luminescence excitation spectra in the presence of a strong magnetic field are considered.

**1.** When semiconductors are irradiated by light in their natural absorption region, Wannier–Mott excitons are formed, as a result of which it is possible, for example, to observe exciton luminescence from the bottom of the exciton band, etc.

Two types of exciton creation processes are possible. In the first type the rate of exciton creation is linear in the intensity of the exciting light, and in the second, it is quadratic. In the quadratic, or bimolecular, case the excitons are formed as a result of bonding between the electrons and the holes, which are distributed independently of each other in space. References 1–8 are dedicated to a theoretical description of such processes. In order to avoid misunderstanding, we emphasize that the term "bimolecular exciton formation" has no relation to a biexciton, i.e., to the analog of the hydrogen molecule, consisting of two excitons. We will not consider biexcitons here.

Monomolecular exciton formation takes place if the electron and hole created by the light at one point retain a memory of each other up to the moment of bonding together into an exciton. In other words, there is a spatial correlation between the electrons and holes, and an electron-hole pair (EHP) occupies a finite volume in space from the moment of formation to the moment of bonding. A well-known example of a monomolecular process is indirect creation of hot excitons with subsequent "ejection" of kinetic energy, described in Refs. 9–10.

However, there is another variant of monomolecular exciton formation, first described theoretically in Refs. 11 and 12. The essence of this process is that the intermediate states of the system are not hot excitons, but electron-hole pairs occupying a finite volume from the moment of their creation by light up to the moment when they bond together into an exciton. The finite volume of the pairs is due not to Coulomb forces, but to the interaction of the electrons and holes with phonons. The idea of finite volume of an electron-hole pair created in a crystal by exciting light was first advanced in Refs. 13 and 14, where a theory of multiphonon resonance Raman scattering (MRRS) in polar semiconductors with the participation of free electron-hole pairs as the intermediate states of the crystal was constructed. The contribution to the rate of exciton creation, linear in the intensity of the exciting light and due to the finite dimensions of the electron-hole pairs, was calculated in Refs. 11 and 12 for the case of zero magnetic field ($H=0$). It was shown that the rate of exciton creation $d\mathscr{W}_{\mathrm{exc}N}/d\omega_1$ per unit frequency interval of the exciting light is proportional to $\alpha$ for $N=1$, $\alpha^2$ for $N=2$, and $\alpha^3$ for $N\geqslant 4$, where $N$ is the number of LO-phonons emitted, and $\alpha$ is the Fröhlich coupling constant[15] for longitudinal optical (LO) lattice vibrations. The dependence of $d\mathscr{W}_{\mathrm{exc}N}/d\omega_1 \propto \alpha^3$ means that the rate of exciton formation is inversely proportional to the volume of the EHP, $V_{\mathrm{EHP}}$, which in turn is proportional to $\lambda^3 \propto \alpha^{-3}$, where $\lambda$ is the mean free path of an electron, limited by scattering by LO-phonons. As the number of emitted phonons $N$ increases for $N\geqslant 4$, the volume $V_{\mathrm{EHP}}$ grows only numerically, which leads to numerical fall-off of the magnitude of $d\mathscr{W}_{\mathrm{exc}N}/d\omega_1$ with the growth of $N$. Note that for simplicity, as in the previous papers, the influence of the Coulomb interaction between electrons and holes on continuum EHP states is not taken into account. The corresponding condition for $H=0$ has the form

$$e^2/(\lambda\kappa_0) \ll \hbar^2 k^2/2\mu, \tag{1}$$

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where $\kappa_0$ is the static dielectric constant, $\hbar k$ is the quasimomentum of the relative motion of the electron and hole, and $\mu$ is the reduced mass. With

$$a=\hbar^2\kappa_0/\mu e^2 \tag{2}$$

as the radius of the exciton in the ground state and the formula

$$\lambda=\hbar k/\mu\gamma, \tag{3}$$

and also the estimate $\gamma\simeq\alpha\omega_{\rm LO}$, inequality (1) transforms to

$$\hbar\omega_{\rm LO}\alpha\ll\frac{\hbar^2k^2}{2\mu}ak. \tag{4}$$

This new inequality is satisfied in any case under the condition of small $\alpha$.

A theory of MRRS in a strong magnetic field (SMF) was constructed in Refs. 16–18. It was predicted that turning on the SMF should lead to an abrupt increase in the intensity of MRRS due to a decrease of the volume of the EHP as a consequence of the transition to quasi-one-dimensional motion. In the presence of a SMF the volume occupied by the pair is proportional to the first, and not the third, power of the Fröhlich constant $\alpha$. This means that the intensity of MRRS should go as $\alpha^{-1}$ for $N=2$ and as $\alpha^{-2}$ for $N\geqslant 4$. Experiments[19] on MRRS in the presence of a strong magnetic field have confirmed this prediction. The analogy between multiphonon light scattering and monomolecular exciton creation makes it possible to predict an abrupt increase in the rate of the latter process when the SMF is turned on. Below we will present results of actual calculations that confirm this supposition.

**2.** Let us consider the model of a direct-band semiconductor with isotropic effective masses of the electrons and holes. It is assumed that the temperature is much lower than the Debye temperature, so that absorption of LO-phonons is negligible. Electron spin is not taken into account. We assume that the Fröhlich constant

$$\alpha\ll 1, \tag{5}$$

but that the interaction of the electrons and holes with the LO-phonons is strong in comparison with all other types of interactions, e.g., with acoustic phonons, impurities, etc.

Let us consider the following process: the exciting light with frequency $\omega_l$ creates an electron-hole pair as a result of a direct transition, and this pair then emits $N-1$ phonons in succession. Afterwards the pair binds as an exciton with simultaneous emission of the last ($N$th) phonon.

We define the criterion of a strong magnetic field for the free electrons and holes as

$$\omega_H\tau\gg 1, \tag{6}$$

where

$$\omega_H=|e|H/mc \tag{7}$$

is the cyclotron frequency, $m$ is the effective mass, $e$ is the charge of an electron, and $\tau$ is the relaxation time of an electron (hole).

Let us consider Wannier–Mott excitons in a SMF, for which

$$\hbar\omega_H/2\gg\Delta E, \tag{8}$$

where $\Delta E$ is the binding energy of the exciton at $H=0$, given by

$$\Delta E=\mu e^4/(2\kappa_0^2\hbar^2). \tag{9}$$

Condition (8) is equivalent to

$$a^2\gg a_H^2, \tag{10}$$

where $a$ is the radius of the exciton at $H=0$, given by formula (2), and

$$a_H=(c\hbar/eH)^{1/2} \tag{11}$$

is the magnetic length. The theory of the Wannier–Mott exciton in a SMF was developed in Refs. 20–24 [see also Ref. 25 (review)].

We use the magnetic field vector potential in the gauge

$$\mathbf{A}=\mathbf{A}(0, xH, 0). \tag{12}$$

The rate of exciton creation can be calculated by the same method as that used in Ref. 11 for the case $H=0$, i.e., using the Feynman techniques for the fourth-order light scattering tensor $S_{\alpha\gamma\beta\lambda}$. Direct calculation shows that this method is completely equivalent to the following approach. The number of excitons created per unit time in the volume $V_0$ after the emission of $N$ phonons, normalized to one photon of the exciting radiation, is

$$W_{\text{exc }N}(K_\perp, |K_z|)=\frac{2\pi}{\hbar}\sum_{\mathbf{q}_1,\ldots,\mathbf{q}_{N-1}}|\langle f|\mathscr{H}_{\rm int}|i\rangle|^2\delta \times(\hbar\omega_l-N\hbar\omega_{\rm LO}-E_{\rm exc}(K_\perp, |K_z|)), \tag{13}$$

where $H_{\rm int}$ is the Fröhlich interaction Hamiltonian of the electrons and holes with the LO-phonons, $E_{\rm exc}(K_\perp, |K_z|)$ is the exciton energy reckoned from the ground state of the crystal, $K_\perp$ and $K_z$ are the transverse and longitudinal (along the direction of the magnetic field) components of the wave vector of the exciton.[22] The wave function of the initial state

$$\Psi_i=\Psi_{\mathbf{q}_1,\mathbf{q}_2,\ldots\mathbf{q}_{N-1}}(\mathbf{r}_1,\mathbf{r}_2) \tag{14}$$

is the wave function of the free electron-hole pair produced by the exciting light and having emitted $N-1$ phonons with wave vectors $\mathbf{q}_1$, $\mathbf{q}_2$,..., $\mathbf{q}_{N-1}$.[18] The wave function of the final state

$$\Psi_f=\Psi_{\text{exc }\mathbf{K}_\perp,\mathbf{K}_z}(\mathbf{r}_1,\mathbf{r}_2) \tag{15}$$

is the wave function of the ground state of the exciton in the SMF, given in Ref. 22.[1)]

We write out specific expressions for the quantities entering into Eq. (13). For $H_{\rm int}$, taking account of the term that corresponds only to the emission of LO-phonons, we have

$$H_{\rm int}=\sum_{\mathbf{q}}C_{\mathbf{q}}^*(\exp(-i\mathbf{q}\mathbf{r}_1)-\exp(-i\mathbf{q}\mathbf{r}_2))\mathbf{b}_{\mathbf{q}}^+, \tag{16}$$

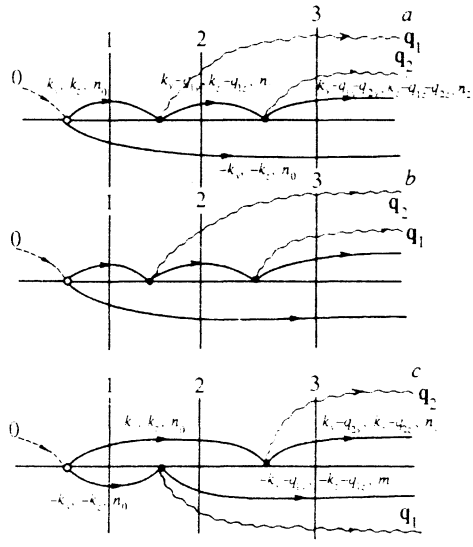

FIG. 1. Three of the set of diagrams which describe the wave function of an electron-hole pair created by light and having emitted two LO-phonons with wave vectors $\mathbf{q}_1$ and $\mathbf{q}_2$.

where

$$C_{\mathbf{q}}=-i\hbar\omega_{\mathrm{LO}}\left(\frac{4\pi\alpha l^3}{V_0}\right)^{1/2}(ql)^{-1},\quad l=(\hbar/2m_e\omega_{\mathrm{LO}})^{1/2},$$
$$\alpha=\frac{e^2}{2\hbar\omega_{\mathrm{LO}}\,l}(\kappa_\infty^{-1}-\kappa_0^{-1}), \tag{17}$$

$V_0$ is the normalization volume, $\kappa_\infty$ is the high-frequency dielectric constant, $m_e$ is the effective mass of the electron, $\mathbf{r}_1$ ($\mathbf{r}_2$) is the radius vector of the electron (hole), $b_{\mathbf{q}}^+$ is the creation operator of an LO-phonon with wave vector $\mathbf{q}$. Since the magnitude of the Fröhlich interaction does not depend on the effective mass, the interaction of holes with LO-phonons differs from the corresponding interaction of electrons only by a change of sign, which is reflected on the right-hand side of Eq. (16).

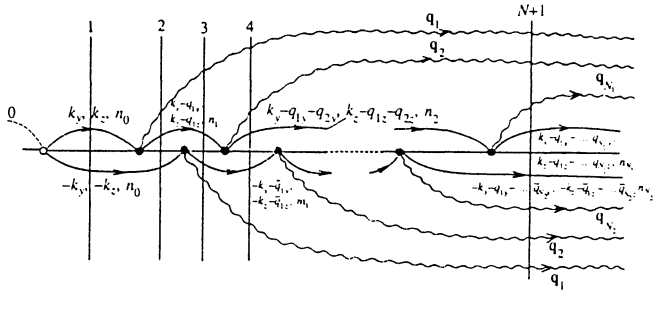

FIG. 2. One of the diagrams that describe the wave function of an electron-hole pair created by light and having emitted $N$ LO-phonons with fixed wave vectors. The electron (hole) has emitted $N_1$ ($N_2$) phonons with $N_1+N_2=N$.

3. To calculate the wave function of the pair $\Psi_{\mathbf{q}_1,\ldots\mathbf{q}_N}(\mathbf{r}_1\mathbf{r}_2)$, a diagrammatic technique has been developed, whose rules are expounded in Ref. 18. Figure 1 shows samples of some diagrams for $N=2$ (the pair releases to photons). The wave function is calculated in Ref. 18 for the case $N=3$. We present here the most general form of the wave function, corresponding to the case in which $N_1$ phonons are emitted by the electron and $N_2$ phonons by the hole, i.e., in all there are

$$N_1+N_2=N \tag{18}$$

LO-phonons. One of the diagrams is shown in Fig. 2. It corresponds to the wave function

$$\Psi^{\alpha}_{\mathbf{q}_1,\mathbf{q}_2\ldots\mathbf{q}_{N_1},\bar{\mathbf{q}}_1,\bar{\mathbf{q}}_2,\ldots,\bar{\mathbf{q}}_{N_2}}$$

$$=(-i)^{N_1}(i)^{N_2}M_l/2\pi\hbar^{N+1}a_H^2L_z\left(\prod_{i=1}^{N_1}C^*_{\mathbf{q}_i}\right)\left(\prod_{j=1}^{N_2}C^*_{\bar{\mathbf{q}}_j}\right)$$
$$\times\exp\left(i(c+c')+i\chi(\mathbf{R},y,\mathbf{Q})\right.$$
$$\left.+i\phi(\mathbf{r}_\perp,\mathbf{Q}_\perp)\right)\sum_\beta K_{n_{N_1},m_{N_2}}\left[\frac{\mathbf{r}_\perp-\mathbf{r}_\perp(-\mathbf{Q}_\perp)}{a_H}\right]$$
$$\times K_{n_0,n_1}(a_Hq_{1y},-a_Hq_{1x})K_{n_1,n_2}(a_Hq_{2y},-a_Hq_{2x})\ldots$$
$$\times K_{n_{N_1-1},n_{N_1}}(a_Hq_{N_1y},-a_Hq_{N_1x})$$
$$\times K_{n_0,m_1}(-a_H\bar{q}_{1y},-a_H\bar{q}_{1x})$$
$$\times K_{m_1,m_2}(-a_H\bar{q}_{2y},-a_H\bar{q}_{2x})\ldots$$
$$\times K_{m_{N_2-1},m_{N_2}}(-a_H\bar{q}_{N_2y},-a_H\bar{q}_{N_2x})\sum_{K_z}\exp(ik_{\mathrm{rel}}z)$$
$$\times\Omega^\alpha_\beta(k_z,q_{1z},q_{2z},\ldots,q_{N_1z},\bar{q}_{1z},\bar{q}_{2z},\ldots,\bar{q}_{N_2z}). \tag{19}$$

The following notation has been introduced in Eq. (19):

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad \mathbf{R}=(m_e\mathbf{r}_1+m_h\mathbf{r}_2)/(m_e+m_h), \tag{20}$$

and the subscript $\perp$ corresponds to the projection of the vector onto the $xy$ plane, i.e., the plane perpendicular to the magnetic field

$$M_l=-(2\pi\hbar/V_0)^{1/2}(e/m_0)(\mathbf{e}_l\mathbf{p}_{cv})(u_l/cn_l\omega_l)^{1/2} \tag{21}$$

is the matrix element of the interaction of the radiation with the semiconductor, corresponding to the creation of an electron-hole pair upon absorption of a single photon, and $m_0$ is the mass of a free electron, $\mathbf{p}_{cv}$ is the interband matrix element of the momentum operator, $\mathbf{e}_l$, $u_l$, and $n_l$ are respectively the polarization vector, the group velocity, and the refractive index of the exciting light, and $c$ is the speed of light in vacuum. The phase factors on the right-hand side of Eq. (19) are defined by

$$c+c'=\frac{a_H^2}{2}\left[\sum_{i<j}[\mathbf{q}_i\mathbf{q}_j]_z-\sum_{i<j}[\bar{\mathbf{q}}_i\bar{\mathbf{q}}_j]_z+\sum_{i,j}[\bar{\mathbf{q}}_i\mathbf{q}_j]_z\right],$$

$$\chi(\mathbf{R},y,\mathbf{Q})=-\left(Q_x+\frac{y}{a_H^2}\right)R_x-Q_yR_y-Q_zR_z, \qquad (22)$$

$$\phi(\mathbf{r}_\perp,\mathbf{Q}_\perp)=\left(\frac{m_e-m_h}{2M}\right)\left(Q_xx+Q_yy+\frac{xy}{a_H^2}\right),$$

where

$$M=m_e+m_h, \quad \mathbf{Q}=\sum_{i=1}^{N_1}\mathbf{q}_i+\sum_{i=1}^{N_2}\bar{\mathbf{q}}_i. \qquad (23)$$

The sum over $\beta$ denotes summation over the quantum numbers $n_0$, $n_1$,..., $n_{N_1}$, $m_1$,..., $m_{N_2}$. The notation

$$K_{nm}(\mathbf{p})=i^{|n-m|}\left[\frac{\min(n!,m!)}{\max(n!,m!)}\right]^{1/2}\exp\left(-\frac{p^2}{4}\right)\times\left(\frac{p^2}{2}\right)^{|n-m|/2}\exp(\phi-\pi/2)(n-m)\times L_{\min(n,m)}^{|n-m|}\left(\frac{p^2}{2}\right), \qquad (24)$$

was also used, where $\mathbf{p}$ is a vector in the plane described in polar coordinates by

$$p=\sqrt{p_x^2+p_y^2}, \quad \phi=\operatorname{arctg}(p_y/p_x),$$

and $L_m^n(x)$ are the associated Laguerre polynomials.

The vector $\mathbf{r}_\perp(-\mathbf{Q}_\perp)$ is defined in terms of its components as follows:

$$r_{\perp x}(-\mathbf{Q}_\perp)=a_H^2Q_y, \quad r_{\perp y}(-\mathbf{Q}_\perp)=-a_H^2Q_x$$

or

$$\mathbf{r}_\perp(-\mathbf{Q}_\perp)=-(a_H^2/H)[\mathbf{H}\mathbf{Q}_\perp]. \qquad (25)$$

The quantity $k_{\mathrm{relz}}$ is the $z$ component of the wave vector of the relative motion of the electron and the hole in the final state of the system, i.e., in the state corresponding to the $(N+1)$th vertical cross section:

$$k_{\mathrm{relz}}=(m_hk_{ez}-m_ek_{hz})/M. \qquad (26)$$

Substituting the wave vectors of the electron $k_{ez}=k_z-\Sigma_{i=1}^{N_1}q_{iz}$ and hole $k_{hz}=-k_z-\Sigma_{i=1}^{N_2}\bar{q}_{iz}$, in expression (26), we obtain

$$k_{\mathrm{relz}}=k_z-\frac{m_h}{M}\sum_{i=1}^{N_1}q_{iz}+\frac{m_e}{M}\sum_{i=1}^{N_2}\bar{q}_{iz}. \qquad (27)$$

Finally, the quantity $\Omega_\beta^\alpha(k_z,q_{1z},...,q_{N_1z},\bar{q}_{1z},...,\bar{q}_{N_2z})$ is given by

$$\Omega_\beta^\alpha=\prod_{i=1}^{N+1}\left(\omega_l-\frac{E_g}{\hbar}-\omega_i+\frac{i\gamma_i}{2}\right)^{-1}, \qquad (28)$$

where $E_g$ is the width of the band gap, $\hbar\omega_1$ is the sum of the energies corresponding to all the lines intersecting the given cross section, and $\gamma_i$ is the sum of the inverse lifetimes. The electron line with indices $k_z$, $n$ corresponds to energy $\hbar\omega_{eH}(n+1/2)+\hbar^2k_z^2/2m_e$ and inverse lifetime $\gamma_e(n,|k_z|)$, the hole line to energy $\hbar\omega_{hH}(n+1/2)+\hbar^2k_z^2/2m_h$ and inverse lifetime $\gamma_h(n,|k_z|)$, and the phonon line to energy $\hbar\omega_{\mathrm{LO}}$; in all of this we neglect phonon attenuation. The index $\alpha$ on the right- and left-hand sides of Eq. (19) denotes the disposition of the black dots in the diagrams. These black dots correspond to the emission of phonons by electrons and holes. On the right-hand side of Eq. (19) only the quantity $\Omega^\alpha$ depends on $\alpha$.

For example, the diagram in Fig. 1a corresponds to

$$\begin{aligned}\Omega^a_{n_0,n_1,n_2}(k_z,q_{1z},q_{2z})=&\left\{\omega_l-\frac{E_{gH}}{\hbar}-\omega_{\mu H}n_0-\frac{\hbar^2k_z^2}{2\mu}+\frac{i}{2}(\gamma_e(n_0,|k_z|)\right.\\&\left.+\gamma_h(n_0,|k_z|))\right\}^{-1}\left\{\omega_l-\frac{E_{gH}}{\hbar}-\omega_{eH}n_1-\omega_{hH}n_0\right.\\&-\omega_{\mathrm{LO}}-\hbar^2\frac{(k_z-q_{1z})^2}{2m_e}-\frac{\hbar^2k_z^2}{2m_h}+\frac{i}{2}(\gamma_e(n_1,|k_z\\&\left.-q_{1z}|)+\gamma_h(n_0,|k_z|))\right\}^{-1}\left\{\omega_l-\frac{E_{gH}}{\hbar}-\omega_{eH}n_2\right.\\&-\omega_{hH}n_0-\hbar^2\frac{(k_z-q_{1z}-q_{2z})^2}{2m_e}-\frac{\hbar^2k_z^2}{2m_h}-2\omega_{\mathrm{LO}}\\&\left.+\frac{i}{2}(\gamma_e(n_2,|k_z-q_{1z}-q_{2z}|)+\gamma_h(n_0,|k_z|))\right\}^{-1},\end{aligned} \qquad (29)$$

where we have introduced the notation

$$\omega_{\mu H}=\omega_{eH}+\omega_{hH}=\frac{eH}{\mu c}, \qquad (30)$$

$$E_{gH}=E_g+\tfrac{1}{2}\hbar\omega_{\mu H}. \qquad (31)$$

The latter quantity, $E_{gH}$, is the width of the band gap of the semiconductor in a magnetic field.

Let us consider some interesting properties of function (19). In Ref. 18 it was shown that it is an eigenfunction of the momentum operator $\mathbf{P}_{\mathrm{exc}}$ of the electron-hole pair, whose form was given in Ref. 22, and it corresponds to the eigenvalue

$$\hbar\mathbf{K}=-\hbar\mathbf{Q} \qquad (32)$$

of this operator. Relation (32) reflects the law of conservation of momentum after the pair has emitted $N$ phonons.

From Eq. (19) it follows that for each prescribed set of indices $\beta$ the function can be expressed as a product of two factors, one of which describes motion in the $xy$ plane, and the other, motion in the direction of the magnetic field. The presence of the function

$$K_{n_{N_1},m_{N_2}}\left[\frac{\mathbf{r}_\perp-\mathbf{r}_\perp(-\mathbf{Q}_\perp)}{a_H}\right]$$

in the first factor indicates that if the mean displacement between the electron and the hole in the $xy$ plane is equal to zero up until emission of the phonons, then after the emission of $n$ phonons it is equal to

$$\mathbf{r}_\perp(-\mathbf{Q}_\perp) = -(a_H^2/H)[\mathbf{H}\mathbf{Q}_\perp]$$

regardless of whether the phonons were emitted by the electron or the hole.

In classical language this situation can be described in the following way. Upon absorption of a photon of the exciting light, an electron and a hole are created at one point and they move in the $xy$ plane in opposite directions along the same orbit (the equality of the quantum numbers $n_0$ for the electron and the hole in the diagrams attests to this latter fact). Upon emission by the electron of a phonon with projection $\mathbf{q}_\perp$ of its wave vector in the $xy$ plane, the center of the electron orbit shifts by

$$\Delta\mathbf{r}_{\perp e} = -\frac{a_H^2}{H}[\mathbf{H}\mathbf{q}_\perp], \tag{33}$$

and upon emission of the same phonon by the hole, the center of the orbit of the hole shifts by the amount

$$\Delta\mathbf{r}_{\perp h} = \frac{a_H^2}{H}[\mathbf{H}\mathbf{q}_\perp], \tag{34}$$

which is simply the value (33) with the opposite sign. After the emission of $N_1$ phonons by the electron, the center of the electron orbit is located at the point

$$\mathbf{r}_{\perp e} = -\frac{a_H^2}{H}\sum_{i=1}^{N_1}[\mathbf{H}\mathbf{q}_{\perp i}], \tag{35}$$

and the center of the hole orbit, at

$$\mathbf{r}_{\perp h} = \frac{a_H^2}{H}\sum_{i=1}^{N_2}[\mathbf{H}\bar{\mathbf{q}}_{\perp i}]. \tag{36}$$

As a result, the distance between the center of the electron orbit and that of the hole becomes

$$\mathbf{r}_{\perp e} - \mathbf{r}_{\perp h} = -\frac{a_H^2}{H}[\mathbf{H}\mathbf{Q}] = \mathbf{r}_\perp(-\mathbf{Q}). \tag{37}$$

The motion of the electron and hole orbits is schematically depicted in Fig. 3. The radii of the orbits are arbitrary, since the phonon energy $\hbar\omega_{LO}$ lost by the electron or hole in each scattering event can be distributed differently between longitudinal and transverse motion.[2)] In the language of quantum mechanics, this corresponds to the different possible values of the quantum numbers $n_1$, $m_1$, $n_2$, $m_2$, etc.

At this point we can indicate the difference between MRRS processes[18] and the binding of an electron-hole pair into an exciton. In the first case $\mathbf{Q}=0$, and the centers of the electron and hole orbits converge to one point before they annihilate (with the emission of light). In exciton creation, $\mathbf{Q}\neq 0$, since

$$-\mathbf{Q} - \mathbf{q}_{fin} = \mathbf{K},$$

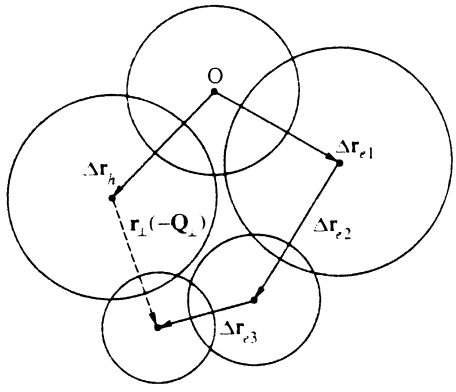


FIG. 3. Displacement of the centers of the orbits of the electron and hole upon emission of phonons. After creation of the electron-hole pair, the electron and hole move in opposite directions about the same orbit with center $O$. Displacement of the center of the electron's orbit $\Delta\mathbf{r}_{ei} = -(a_H^2/H)[\mathbf{H}\mathbf{q}_i]$, displacement of the center of the hole's orbit $\Delta\mathbf{r}_{hi} = (a_H^2/H)[\mathbf{H}\mathbf{q}_i]$, where $\mathbf{q}_i$ is the wave vector of the emitted phonon. In the figure the electron has emitted three phonons, and the hole—one. The vector $\mathbf{r}_\perp(-\mathbf{Q}_\perp)$ is the position vector of the center of the electron's last orbit relative to the center of the hole's last orbit. For $\mathbf{Q}=\Sigma_{i=1}\mathbf{q}_i=\mathbf{0}$ the centers of the last orbits coincide, which takes place in MRRS processes. When the electron and hole are bound together into an exciton, the vector $\mathbf{r}_\perp(-\mathbf{Q}_\perp)$ is nonzero.

where $\mathbf{q}_{fin}$ is the wave vector of the last phonon, the emission of which accompanies the binding of the pair into an exciton, and $\mathbf{K}$ is the wave vector of the exciton.

The wave function (15) of the final exciton state is

$$\Psi_{\mathrm{exc}\mathbf{K}_\perp, K_z} = [(2\pi)^{3/4} a_H a_\parallel^2 \sqrt{V_0}]^{-1} \times \exp\{i\chi(\mathbf{R}, y, -\mathbf{K}) + i\Phi(\mathbf{r}_\perp, -\mathbf{K}_\perp)\} \times K_{00}\left[\frac{\mathbf{r}_\perp - \mathbf{r}_\perp(\mathbf{K}_\perp)}{a_H}\right]\exp\left(\frac{-z^2}{4a_\parallel^2}\right), \tag{38}$$

where $a_\parallel$ is the dimension of the exciton in a strong magnetic field in the direction of the field.[21] According to Eq. (24)

$$K_{00}(p) = \exp(-p^2/4). \tag{39}$$

Function (38) is an eigenfunction of the exciton momentum operator, and it corresponds to the eigenvalue $\hbar\mathbf{K}$.

**4.** We calculated the probability of exciton creation according to formula (13), using as the wave function $\Psi_i$ of the initial state the function corresponding to the diagram in Fig. 1a. The form of this function is easily determined from the general expression (19):

$$W^a_{\text{exc3}}(K_\perp, |K_z|)$$
$$= 2^{3/2}\pi^{1/2}\left[\frac{L_x^2 L_y^2 a_\parallel |M_l|^2}{V_0 \hbar^8}\right]\delta\left(\omega_l - 3\omega_{\text{LO}} - \frac{E_{\text{exc}}(K_\perp, |K_z|)}{\hbar}\right)\sum_{\mathbf{q}_1,\mathbf{q}_2} |C_{\mathbf{q}_1}|^2 |C_{\mathbf{q}_2}|^2 |C_{\mathbf{q}_3}|^2$$
$$\times \left| \sum_{n_0,n_1,n_2,k_z} K_{n_0,n_1}(a_H q_{1y}, -a_H q_{1x}) K_{n_1,n_2}(a_H q_{2y}, -a_H q_{2x}) R_{n_0,n_2}(k_z) \Omega^a_{n_0,n_1,n_2}(k_z, q_{1z}, q_{2z}) \right|^2, \quad (40)$$

where

$$\mathbf{q}_3 = \mathbf{q}_{\text{fin}} = -\mathbf{q}_1 - \mathbf{q}_2 - \mathbf{K}, \quad (41)$$

$$R_{n_0,n_2}(k_z) = \delta_{n_0,0} K_{n_2,0}(a_H q_{3y}, -a_H q_{3x}) \exp(i\varphi) \times \exp\left[-a_\parallel^2 \left(k_z + \frac{m_h}{M} K_z\right)^2\right] - \delta_{n_2,0} K_{n_0,0} \times (-a_H q_{3y}, -a_H q_{3x}) \exp(-i\varphi) \times \exp\left[-a_\parallel^2 \left(k_z - q_{1z} - q_{2z} - \frac{m_e}{M} K_z\right)^2\right], \quad (42)$$

$$\varphi = \frac{a_H^2}{2}[\mathbf{q}_3 \mathbf{K}]_z = \frac{a_H^2}{2}(q_{3x} K_y - q_{3y} K_x). \quad (43)$$

The first term on the right-hand side of Eq. (42) is due to the interaction of the electron, and the second, to the interaction of the hole [compare Eq. (16)], with the LO-phonons. In calculating the matrix elements $\langle f | H_{\text{int}} | i \rangle$ we used the formulas

$$\int dx dy \exp\left(-\frac{i}{2}(Q_x x + Q_y y)\right) K_{n',n}(x - x_0, y - y_0) \times K_{m,m'}(x - x_0', y - y_0') = 2\pi(-1)^{n+m'} \times \delta_{n,m} \exp\left(\frac{i}{2}(x_0 y_0' - x_0' y_0)\right) \times K_{n',m'}(Q_y, -Q_x); \quad (44)$$

$$\int dx dy \exp\left(\frac{i}{2}(Q_x x + Q_y y)\right) K_{n',n}(x - x_0, y - y_0) \times K_{m,m'}(x - x_0', y - y_0') = 2\pi(-1)^{n'+m} \delta_{n',m'} \exp\left(-\frac{i}{2}(x_0 y_0' - x_0' y_0)\right) \times K_{n,m}(-Q_y, -Q_x), \quad (45)$$

if

$$Q_x = y_0 - y_0', \quad Q_y = -(x_0 - x_0').$$

**5.** On the right-hand side of Eq. (40) stands the square of the modulus of the sum over $n_0$, $n_1$, $n_2$, $k_z$, $n_0'$, $n_1'$, $n_2'$, $k_z'$. As will become clear, the largest contribution to $W$, the probability of exciton creation, comes from terms with

$$n_0 = n_0', \quad n_1 = n_1', \quad n_2 = n_2'. \quad (46)$$

Condition (46) corresponds to the contribution in $W$ that we will call "diagonal," in contrast to the interference contribution, for which at least one pair of $n_i$ and $n_i'$ do not coincide. These interference terms include the term with "crossed" phonon lines $\mathbf{q}_1$ and $\mathbf{q}_2$, which corresponds to interference of the wave functions in Figs. 1a and b. These contributions are also small in comparison with the diagonal terms. Let us introduce new integration variables in place of $k_z$, $k_z'$, $q_{1z}$, and $q_{2z}$, specifically

$$p = k_z - k_z', \quad k_{0z} = (k_z + k_z')/2, \quad k_{1z} = k_{0z} - q_{1z}, \quad k_{2z} = k_{0z} - q_{1z} - q_{2z}. \quad (47)$$

Then for the "diagonal" contribution (see the diagram in Fig. 1a) to the probability of exciton creation we obtain

$$W^a_{\text{exc3diag}}(K_\perp, |K_z|) = (8\pi)^{1/2}\left[\frac{L_x^2 L_y^2 a_\parallel |M_1|^2}{V_0 \hbar^8}\right] \times \delta\left(\omega_l - 3\omega_{\text{LO}} - \frac{E_{\text{exc}}(K_\perp, |K_z|)}{\hbar}\right) \times \sum_{n_0,n_1,n_2,\mathbf{q}_{1\perp},\mathbf{q}_{2\perp}} B_{n_0,n_1}(q_{1\perp}) B_{n_1,n_2}(q_{2\perp}) \times \sum_{p,k_{0z},k_{1z},k_{2z}} |C_{\mathbf{q}_1}|^2 |C_{\mathbf{q}_2}|^2 |C_{\mathbf{q}_3}|^2 R_{n_0,n_2}(p) \times R^*_{n_0',n_2'}(-p) \Omega^a_{n_0,n_1,n_2} \times (k_{0z}, k_{1z}, k_{2z}, p) \Omega^{a*}_{n_0',n_1,n_2} \times (k_{0z}, k_{1z}, k_{2z}, -p), \quad (48)$$

where

$$B_{n,n'}(q_\perp) = |K_{n,n'}(a_H q_{y'} - a_H q_x)|^2 = T_{n,n'}\left(a_H^2 \frac{q_\perp^2}{2}\right), \quad (49)$$

$$T_{n,n'}(x) = \frac{\min(n!, n'!)}{\max(n!, n'!)} \exp(-x) x^{|n-n'|} [L^{|n-n'|}_{\min(n_i,n_i')}(x)]^2. \quad (50)$$

Transforming Eq. (42) to the new variables defined in (47), we find that the dependence of the quantities $R_{n_0}$ and $R_{n_2}$ on $\mathbf{p}$ is through the exponentials

$$\exp\left(-a_\parallel^2 \left(k_{0z} + K_z\left(\frac{m_h}{M}\right) + \frac{p}{2}\right)^2\right), \quad \exp\left(-a_\parallel^2 \left(k_{2z} - K_z\left(\frac{m_e}{M}\right) + \frac{p}{2}\right)^2\right).$$

We neglect the dependence of these exponentials on $\mathbf{p}$ for the following reasons.[3)] Below we will show that in the integral over $p$ only quantities of order

$$p \simeq \lambda^{-1}$$

are important, where $\lambda$ is the electron mean free path along the $z$ axis, which is bounded by the possibility of LO-phonon emission. The mean free path $\lambda$ is inversely proportional to $\alpha$, and consequently $p \propto \alpha$. Since $\alpha \ll 1$, we take $\lambda$ to exceed all the other lengths figuring in the theory, in particular, the longitudinal dimension of the exciton $a_{\parallel}$.

We carry out the integration on the right-hand side of Eq. (48) over $k_{0z}$, $k_{1z}$, $k_{2z}$, and $p$. First we calculate

$$S_\beta(z) = \sum_{p,k_{0z},k_{1z},k_{2z}} \exp(ipz) F_\beta(k_{0z},k_{1z},k_{2z}) \times \Omega_\beta(k_{0z},k_{1z},k_{2z},p) \times \Omega_\beta^*(k_{0z},k_{1z},k_{2z},-p), \tag{51}$$

where $\beta$ is the set of indices $n_0$, $n_1$, $n_2$, and the probability of exciton creation is proportional to

$$\sum_\beta S_\beta(z=0).$$

In the interest of greater clarity and simplicity, in writing out the expression for $\Omega_\beta(k_{0z},k_{1z},k_{2z},p)$ we use the approximation

$$m_h \to \infty, \tag{52}$$

which is tenable if

$$m_h \gg m_e; \tag{53}$$

this holds for a number of III–V polar semiconductors, in particular. This means discarding the terms $\omega_{hH} \times n_0$ and $\gamma_h(n_0, |k_z|)$ on the right-hand side of Eq. (29).[4] We then obtain

$$\Omega_\nu(k_{0z},k_{1z},k_{2z},p) = \left(\frac{2m_e}{\hbar}\right)^3 \left[\left(k_{0z}+\frac{p}{2}\right)^2 - K_0^2 - iQ_0^2\right]^{-1} \times \left[\left(k_{1z}+\frac{p}{2}\right)^2 - K_1^2 - iQ_1^2\right]^{-1} \times \left[\left(k_{2z}+\frac{p}{2}\right)^2 - K_2^2 - iQ_2^2\right]^{-1}, \tag{54}$$

with

$$K_j = \left(\frac{2m_e}{\hbar}\right)^{1/2} \left(\omega_l - \frac{E_{gH}}{\hbar} - \omega_{eH} n_j - j\omega_{\mathrm{LO}}\right)^{1/2}, \tag{55}$$

$$Q_j^2 = \frac{m_e}{\hbar} \gamma_e(n_j,K_j). \tag{56}$$

We integrate over $k_{0z}$, $k_{1z}$, $k_{2z}$, and $p$ on the right-hand side of Eq. (51) using the method of Ref. 18 to calculate the MRRS cross sections. Here we assume *a priori* that

$$p \ll K_j \quad \text{and} \quad Q_j \ll K_j. \tag{57}$$

Condition (57) is satisfied if the quantities $K_j$ are not too small, i.e., at some distance from the resonance values $K_j = 0$.

We obtain the result

$$S_\beta(z) = L_z^4 \left(\frac{m_e}{\hbar}\right)^6 \frac{\lambda_0\lambda_1\lambda_2}{K_0^2K_1^2K_2^2} \{F(K_0,K_1,K_2) f^{+++}(z) + F(-K_0,K_1,K_2) f^{-++}(z) + \ldots + F(-K_0,-K_1,-K_2) f^{---}(z)\}, \tag{58}$$

where we have introduced the functions $f(z)$, normalized to unity, for example

$$f^{-++}(z) = (\lambda_0\lambda_1\lambda_2)^{-1} \int_{-\infty}^{+\infty} dz_0 \int_{-\infty}^{+\infty} dz_1 \mathscr{Y}^-(z_0/\lambda_0) \times \mathscr{Y}^+\left(z_1 - \frac{z_0}{\lambda_1}\right) \mathscr{Y}^+\left(z - \frac{z_1}{\lambda_2}\right), \tag{59}$$

where

$$\lambda_j = \hbar K_j / m_e \gamma_e(n_j,K_j), \tag{60}$$

and the functions $\mathscr{Y}^\pm(t)$ are defined as

$$\mathscr{Y}^+(t) = \begin{cases} \exp(-t), & t>0 \\ 0, & t<0 \end{cases}, \qquad \mathscr{Y}^-(t) = \mathscr{Y}^+(-t) = \begin{cases} 0, & t>0 \\ \exp(t), & t<0 \end{cases}. \tag{61}$$

Note that

$$\mathscr{Y}^+(t) + \mathscr{Y}^-(t) = \exp(-|t|). \tag{62}$$

The functions $f(z)$ have a clear physical meaning. They describe the $z$-distribution, normalized to unity, of the electrons created by the light at the point $z=0$ after two phonons have been emitted. The hole, under the condition $m_h \to \infty$, remains immobile at the point $z=0$. Obviously, both in the creation of the pair and in each subsequent emission of a phonon, two opposite directions of motion of the electron are possible along the $z$ axis, to which correspond the $+$ and $-$ signs adorning the distribution $f^{\pm\pm\pm}(z)$.

Since the probability of exciton creation is proportional to

$$\sum_\beta S_\beta(z=0),$$

it is made up of a sum of contributions of various "trajectories" of the electron over the system of Landau levels, where the contribution of each "trajectory" contains the factor $f^\nu(z=0)$. The physical meaning of this result is clear if it be recalled that we have neglected the dimension $a_{\parallel}$ of the exciton along the $z$ axis in comparison with lengths of order $\lambda$. Then for creation of an exciton it is necessary that the electron return to the point $z=0$ where it was created and which is now occupied by the immobile hole. Figure 4 shows some examples of "trajectories" of an electron that has just emitted two phonons. We stress that besides the set of $+$ and $-$ signs, the trajectory $\nu$ is characterized by the set $\beta$ of quantum numbers $n_0$, $n_1$, $n_2$. It is easy to obtain the following results:

$$f^{+++}(z=0) = f^{---}(z=0) = 0, \tag{63}$$

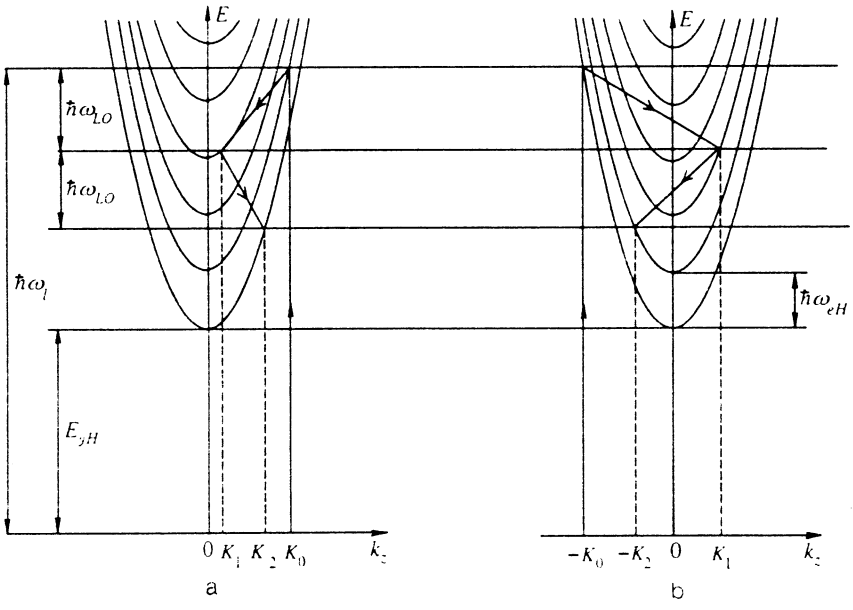


FIG. 4. Examples of possible variants of translation over the Landau bands of an electron created by light and having emitted two LO-phonons. The hole band corresponding to infinite effective mass of the hole coincides with the horizontal axis. Variant *a* does not contribute to the process of exciton formation, since return of the electron to the creation point of the electron-hole pair is ruled out.

$$f^{+--}(z=0)=f^{-++}(z=0)=\frac{\lambda_0}{(\lambda_0+\lambda_1)(\lambda_0+\lambda_2)},$$

$$f^{+-+}(z=0)=f^{-+-}(z=0)=\frac{\lambda_1}{(\lambda_1+\lambda_0)(\lambda_1+\lambda_2)}, \tag{64}$$

$$f^{++-}(z=0)=f^{--+}(z=0)=\frac{\lambda_2}{(\lambda_2+\lambda_0)(\lambda_2+\lambda_1)}.$$

The physical meaning of equality (63) is that for motion of the electron in one direction, return to the point $z=0$ is impossible.

Using Eqs. (17), (58), (63), and (64), we obtain from Eq. (48)[5)]

$$W_{\text{exc3},m_h\to\infty}(|K_z|,K_\perp)$$
$$=\frac{1}{4(2\pi)^{1/2}}\frac{\alpha^3|M_\perp|^2 a_\parallel a_H^4}{\hbar^2}\delta\left(\omega_l-3\omega_{\text{LO}}\right.$$
$$-\frac{E_{\text{exc}}(K_\perp,|K_z|)}{\hbar}\sum_{n_0,n_1,n_2}{}'\frac{\lambda_0\lambda_1\lambda_2}{l^9K_0^2K_1^2K_2^2}\int d\mathbf{q}_{1\perp}$$
$$\times\int d\mathbf{q}_{2\perp}\;T_{n_0,n_1}\left(\frac{a_Hq_{1\perp}}{2}\right)T_{n_1,n_2}\left(\frac{a_Hq_{2\perp}}{2}\right)T_{n_2,n_0}$$
$$\times\left(\frac{a_Hq_{3\perp}}{2}\right)\{[\mathcal{G}(K_0,-K_1,-K_2,K_z)$$
$$+\mathcal{G}(-K_0,K_1,K_2,K_z)]f^{+--}(z=0)$$
$$+[\mathcal{G}(K_0,-K_1,K_2,K_z)+\mathcal{G}(-K_0,K_1,-K_2,K_z)]$$
$$\times f^{+-+}(z=0)+[\mathcal{G}(K_0,K_1,-K_2,K_z)$$
$$+\mathcal{G}(-K_0,-K_1,K_2,K_z)]f^{++-}(z=0)\}, \tag{65}$$

where

$$\mathbf{q}_{3\perp}=-\mathbf{q}_{1\perp}-\mathbf{q}_{2\perp}-\mathbf{K}$$

and we have introduced the notation

$$\mathcal{G}(K_0,K_1,K_2,K_z)$$
$$=(a_H^6(q_{1\perp}^2+(K_0-K_1)^2)(q_{2\perp}^2+(K_1-K_2)^2)(q_{3\perp}^2$$
$$+(K_2-K_0-K_z)^2))^{-1}\left\{\delta_{n,0}\exp\left(-2a_\parallel^2\left(K_0\right.\right.\right.$$
$$\left.\left.+\frac{m_h}{M}K_z\right)^2\right)+\delta_{n_2,0}\exp\left(-2a_\parallel^2\left(K_2-\frac{m_e}{M}K_z\right)^2\right)$$
$$-2\delta_{n,0}\delta_{n_2,0}\cos[a_H^2(q_{3x}K_y-q_{3y}K_x)]$$
$$\times\exp\left[-a_\parallel^2\left(\left(K_0+\frac{m_h}{M}K_z\right)^2\right.\right.$$
$$\left.\left.\left.+\left(K_2-\frac{m_e}{M}K_z\right)^2\right)\right]\right\}. \tag{66}$$

The prime in the sum over $n_0$, $n_1$, $n_2$ means that the summation is limited to those values of these indices for which the wave vectors $K_0$, $K_1$, $K_2$ are real, i.e., they correspond to real processes (those for which energy is conserved) of emission of phonons by the electron. Permissible values of the indices $n_0$, $n_1$, $n_2$ are shown, for example, in Fig. 4.

From definition (66),

$$\mathcal{G}(-K_0,-K_1,-K_2,-K_z)=\mathcal{G}(K_0,K_1,K_2,K_z). \tag{67}$$

Using Eqs. (65) and (67), we conclude that the rate of exciton creation depends on $|K_z|$, which was assumed beforehand.

We also present here the result, analogous to (65), for a two-phonon process. We have

$$W_{\text{exc2},m_h\to\infty}(|K_z|,K_\perp)$$

$$=\left(\frac{\pi}{2}\right)^{1/2}\frac{\alpha^2|M_1|^2a_\parallel a_H^2}{\hbar^2}\delta(\omega_l-2\omega_{\text{LO}})$$

$$-E_{\text{exc}}(K_\perp,|K_z|)\sum_{n_0,n_1}{}'\frac{\lambda_0\lambda_1}{f^6K_0^2K_1^2}\int d\mathbf{q}_{1\perp}$$

$$\times T_{n_0,n_1}\left(\frac{a_Hq_{1\perp}}{2}\right)T_{n_1,n_0}\left(\frac{a_Hq_{2\perp}}{2}\right)[\mathscr{Y}(K_0,$$

$$-K_1,K_z)+\mathscr{Y}(-K_0,K_1,K_z)]f^{+-}(z=0), \qquad (68)$$

where

$$\mathbf{q}_{2\perp}=-\mathbf{q}_{1\perp}-\mathbf{K}_\perp,\quad f^{+-}(z=0)=(\lambda_0+\lambda_1)^{-1}, \qquad (69)$$

$$\mathscr{Y}(K_0,K_1,K_z)$$

$$=\{a_H^4(q_{1\perp}^2+(K_0-K_1)^2)(q_{2\perp}^2+(K_1-K_0$$

$$-K_z)^2)\}^{-1}\delta_{n_0,0}\exp\left(-2a_\parallel^2\left(K_0+\frac{m_h}{M}K_z\right)^2\right)$$

$$+\delta_{n_1,0}\exp\left(-2a_\parallel^2\left(K_1-\frac{m_e}{M}K_z\right)^2\right)$$

$$-2\delta_{n_0,0}\delta_{n_1,0}\cos[a_H^2(q_{2x}K_y-q_{2y}K_x)]$$

$$\times\exp\left(-a_\parallel^2\left[\left(K_0+\frac{m_h}{M}K_z\right)^2+\left(K_1-\frac{m_e}{M}K_z\right)^2\right]\right). \qquad (70)$$

Comparing Eqs. (68) and (65), it is easy to see that these results generalize to the case of an arbitrary number of phonons $N-1$, emitted by the electron just before it binds to a hole to form an exciton, if $m_h\to\infty$.

Let us calculate the inverse relaxation times $\gamma_e(n_j,K_j)$ of the electron that enter into expression (60) (the expression for $\lambda_j$). If the inverse electron lifetime $\gamma_e(n,K_z)$ is determined by the probability of emission of an LO-phonon, then a simple perturbation theory calculation gives

$$\gamma_e(n,|k_z|)=\alpha\omega_{\text{LO}}\sum_{n'}{}'\left(\frac{\omega_{\text{LO}}}{\hbar k_z^2/2m+\omega_H(n-n')-\omega_{\text{LO}}}\right)^{1/2}$$

$$\times\int_0^\infty dxT_{n,n'}(x)\left(\frac{1}{2x+a_H^2(k_z-k_z')^2}\right.$$

$$\left.+\frac{1}{2x+a_H^2(k_z+k_z')^2}\right), \qquad (71)$$

where we have used the dimensionless integration variable $x=a_H^2q_\perp^2/2$.

Substituting the values of $K_0$, $K_1$, $K_2$ given by Eq. (55) into Eq. (71) in place of $|k_z|$, we have

$$\gamma_e(n_0,K_0)=\alpha\omega_{\text{LO}}\sum_{n_1'}{}'\left(\frac{\omega_{\text{LO}}}{\omega_l-E_{gH}/\hbar-\omega_{eH}n_1'-\omega_{\text{LO}}}\right)^{1/2}$$

$$\times\int_0^\infty dxT_{n,n_1'}(x)\left(\frac{1}{2x+a_\parallel^2(K_0-K_1')^2}\right.$$

$$\left.+\frac{1}{2x+a_H^2(K_0+K_1')^2}\right); \qquad (72)$$

$$\gamma_e(n_1,K_1)=\alpha\omega_{\text{LO}}\sum_{n_2'}{}'\left(\frac{\omega_{\text{LO}}}{\omega_l-E_{gH}/\hbar-\omega_{eH}n_2'-2\omega_{\text{LO}}}\right)^{1/2}$$

$$\times\int_0^\infty dxT_{n_1,n_2'}(x)\left(\frac{1}{2x+a_\parallel^2(K_1-K_2')^2}\right.$$

$$\left.+\frac{1}{2x+a_H^2(K_1+K_2')^2}\right); \qquad (73)$$

$$\gamma_e(n_2,K_2)=\alpha\omega_{\text{LO}}\sum_{n_3'}{}'\left(\frac{\omega_{\text{LO}}}{\omega_l-E_{gH}/\hbar-\omega_{eH}n_3'-3\omega_{\text{LO}}}\right)^{1/2}$$

$$\times\int_0^\infty dxT_{n_2,n_3'}(x)\left(\frac{1}{2x+a_\parallel^2(K_2-K_3')^2}\right.$$

$$\left.+\frac{1}{2x+a_H^2(K_2+K_3')^2}\right), \qquad (74)$$

where

$$K_j'=\left[\frac{2m_e}{\hbar}\left(\omega_1-\frac{E_{gH}}{\hbar}-\omega_{eH}n_j'-j\omega_{\text{LO}}\right)\right]^{1/2}.$$

6. We introduce the quantity $d\mathscr{W}_{\text{exc}N}/d\omega_l$—the number of excitons created per unit time per unit volume per unit interval of frequency $\omega_l$. According to Ref. 11, we have

$$\frac{d\mathscr{W}_{\text{exc}N}}{d\omega_l}=(\hbar\omega_lu_l)^{-1}\sum_{K_\perp,K_z}W_{\text{exc}N}(K_\perp,|K_z|)\frac{dI_l}{d\omega_l}, \qquad (75)$$

where $dI_l/d\omega_l$ is the intensity of the exciting light per unit interval of frequency $\omega_l$.

According to Eq. (13), upon irradiation of the crystal by light of frequency $\omega_l$, after the emission of $N$ LO-phonons an exciton with the fixed energy

$$E_{\text{exc}}(K_\perp,|K_z|)=\hbar\omega_l-N\omega_{\text{LO}} \qquad (76)$$

is created. The energy of this exciton, reckoned from the ground state energy of the crystal, depends on the momentum components $K_\perp$ and $K_z$ in the following way:

$$E_{\text{exc}}(K_\perp,|K_z|)=E_{gH}+E(K_\perp)+\hbar^2K_z^2/2M, \qquad (77)$$

where $E(K_\perp)$ is the contribution to the exciton energy that depends on the transverse component of the momentum. According to Ref. 22, if

$$K_\perp\ll a/a_H^2 \qquad (78)$$

the function $E(K_\perp)$ is

$$E(K_\perp)=-\Delta E\xi^2, \qquad (79)$$

where $\xi$ is a dimensionless quantity that depends on $H$ and $K_\perp$, for which an equation was derived in Ref. 22. For $K_\perp=0$ we have

$$\xi(K_\perp=0)=\xi_0,\quad E(K_\perp=0)=-\Delta E_H=-\Delta E\xi_0^2, \tag{80}$$

where $\Delta E_H$ is the binding energy of the exciton in a strong magnetic field.

We are interested in the creation of excitons with energy

$$E(K_\perp)+\Delta E_H+\hbar^2K_z^2/2M,$$

(reckoned from the level $E_{gH}-\Delta E_H$) in the range from 0 to $\hbar\omega_{\rm LO}$, i.e., excitons capable of emitting a real LO-phonon. If an exciton with energy between 0 and $\hbar\omega_{\rm LO}$ is to be created after the emission of $N$ LO-phonons, then the frequency of the exciting light should lie in the range

$$E_{1H}/\hbar+N\omega_{\rm LO}<\omega_l<E_{1H}/\hbar+(N+1)\omega_{\rm LO}, \tag{81}$$

where

$$E_{1H}=E_{gH}-\Delta E_H \tag{82}$$

is the ground state energy of the exciton in a magnetic field.

The energy range (81) divides into two ranges (a and b), for which two different formulas obtain when we calculate the exciton (75) creation rate. These ranges are defined by the inequalities[6)]

$$E_{1H}/\hbar+N\omega_{\rm LO}<\omega_l<E_{gH}/\hbar+N\omega_{\rm LO}, \tag{83a}$$

$$E_{gH}/\hbar+N\omega_{\rm LO}<\omega_l<E_{1H}/\hbar+(N+1)\omega_{\rm LO}. \tag{83b}$$

For irradiation in range a, excitons are created whose kinetic energy, equal to $E(K_\perp)+\Delta E_H+\hbar^2K_z^2/2M$, is less than the binding energy of the exciton $\Delta E_H$.

According to Eqs. (76) and (77), the absolute value of $K_z$ for the created exciton is

$$K_{z0}=[(2M/\hbar^2)(\hbar\omega_l-E_{gH}-N\hbar\omega_{\rm LO}-E(K_\perp))]^{1/2}, \tag{84}$$

where $E(K_\perp)$ is negative and varies from $-\Delta E_H$ at $K_\perp=0$ to 0 in the limit $K_\perp\to\infty$.[7)] It follows from Eq. (83) that in range a, values of $K_\perp$ from 0 to $K_{\perp\,\max}$ are admissible, where $K_{\perp\,\max}$ is the solution of the equation

$$\hbar\omega_l-E_{gH}-N\hbar\omega_{\rm LO}=E(K_{\perp\,\max}). \tag{85}$$

In range b the integration over $x$ is from 0 to $\infty$.

The second difference between range a and range b is that in the former, the inverse lifetime $\gamma_e(n_{N-1},K_{N-1})$ tends to zero since the energy of an electron in the state $n=n_{N-1}$, $|k_z|=K_{N-1}$ is not high enough to emit an LO-phonon [see Eq. (73)]. This means that the $\gamma_e(n_{N-1},K_{N-1})$ must be replaced by

$$\Gamma\ll\gamma, \tag{86}$$

where $\Gamma$ defines the inverse lifetime of the electron in the given state over a range of kinetic energies that are so low that it can no longer emit a real LO-phonon. The quantity $\Gamma$ does not enter into the final expression for the exciton creation rate (in range a).

Introducing the dimensionless integration variables

$$x_1=(a_Hq_{1\perp})^2/2,\quad x_2=(a_Hq_{2\perp})^2/2,\quad x=(a_HK_\perp)^2/2, \tag{87}$$

and denoting by $\theta$ the angle between the vectors $\mathbf{q}_{1\perp}$ and $\mathbf{q}_{2\perp}$, and by $\chi$ the angle between $\mathbf{K}_\perp$ and $\mathbf{q}_{1\perp}+\mathbf{q}_{2\perp}$, we obtain

$$\frac{d\mathscr{W}^{\rm a(b)}_{\rm exc3,m_h\to\infty}}{d\omega_l}=\left(\frac{\pi}{2}\right)^{1/2}(\alpha^3Me^2/\hbar^3\omega_l^2m_0^2)\,|\mathbf{e}_l\mathbf{p}_{cv}|^2\times(a_\parallel/n_lca_\parallel^2)\left(\frac{dI_l}{d\omega_l}\right)\times\sum_{n_0,n_1,n_2}{}'(\lambda_0\lambda_1\lambda_2/l^9K_0^2K_1^2K_2^2)\Lambda^{\rm a(b)}_{n_0,n_1,n_2}, \tag{88}$$

where the indices a(b) correspond to the frequency ranges defined by (83).

The quantity $\Lambda^{\rm a}_{n_0,n_1,n_2}$ is given by

$$\Lambda^{\rm a}_{n_0,n_1,n_2}=\int_0^\infty dx_1T_{n_0,n_1}(x_1)\int_0^\infty dx_2T_{n_1,n_2}(x_2)\times\int_0^{2\pi}\frac{d\theta}{2\pi}\int_0^{2\pi}\frac{d\chi}{2\pi}\int_0^{x_0}dxT_{n_2,n_0}(x_3)\times K_{z0}^{-1}(x)\Big[(\mathscr{G}(K_0,-K_1,-K_2,K_{z0})+\mathscr{G}(K_0,-K_1,-K_2,-K_{z0}))\left(\frac{\lambda_0}{\lambda_0+\lambda_1}\right)+(\mathscr{G}(K_0,-K_1,K_2,K_{z0})+\mathscr{G}(K_0,-K_1,K_2,-K_{z0}))\left(\frac{\lambda_1}{\lambda_1+\lambda_0}\right)+(\mathscr{G}(K_0,K_1,-K_2,K_{z0})+\mathscr{G}(K_0,K_1,-K_2,-K_{z0}))\Big]\lambda_2^{-1}, \tag{89}$$

where

$$x_3=\frac{a_H^2q_3^2}{2}=x_1+x_2+x+2(x_1x_2)^{1/2}\cos(\theta)+2\{x(x_1+x_2+2(x_1x_2)^{1/2}\cos(\theta))\}^{1/2}\cos(\chi), \tag{90}$$

$$x_0=(a_H^2K_{\perp\,\max}^2/2), \tag{91}$$

and the expression for $K_{z0}$ can be obtained from (84) by taking account of the transformation of variables (87); correspondingly, the expression for $\mathscr{G}(K_0,K_1,K_2,K_{z0})$ can be obtained from expression (66), also taking Eqs. (87) and (90) into account, where

$$\cos(a_H^2(q_{3x}K_y-q_{3y}K_x))=\cos[2\{x(x_1+x_2+2(x_1x_2)^{1/2}\cos(\chi)\}^{1/2}\sin\theta].$$

For range b we obtain

$$\Lambda^{b}_{n_0,n_1,n_2}=\int_0^\infty dx_1 T_{n_0,n_1}(x_1)\int_0^\infty dx_2 T_{n_1,n_2}(x_2)$$
$$\times\int_0^{2\pi}\frac{d\theta}{2\pi}\int_0^{2\pi}\frac{d\chi}{2\pi}\int_0^\infty dx T_{n_2,n_0}(x_3)$$
$$\times K_{z0}^{-1}(x)\Big[(\mathcal{G}(K_0,-K_1,-K_2,K_{z0})$$
$$+\mathcal{G}(K_0,-K_1,-K_2,-K_{z0}))$$
$$\times\frac{\lambda_0}{(\lambda_0+\lambda_1)(\lambda_0+\lambda_2)}$$
$$+(\mathcal{G}(K_0,-K_1,K_2,K_{z0})+\mathcal{G}(K_0,-K_1,K_2,$$
$$-K_{z0}))\frac{\lambda_1}{(\lambda_1+\lambda_0)(\lambda_1+\lambda_2)}+(\mathcal{G}(K_0,K_1,$$
$$-K_2,K_{z0})+\mathcal{G}(K_0,K_1,-K_2,$$
$$-K_{z0}))\frac{\lambda_2}{(\lambda_2+\lambda_0)(\lambda_2+\lambda_1)}\Big]. \tag{92}$$

7. Let us consider the main results that have been obtained and their applicability.

From Eq. (76) it follows that the theory can be applied to the emission of $N$ phonons under the condition

$$\omega_1-E_{gH}/\hbar-N\omega_{LO}+\Delta E_H\geqslant 0,\quad N\geqslant 2. \tag{93}$$

The results for $N=3$, $m_h\to\infty$ can easily be generalized to any $N$ and arbitrary effective masses of the electron and hole.

It has been shown that the rate of exciton formation $d\mathcal{W}_{excN}/d\omega_1$, which is linear in the intensity of the exciting light, should grow abruptly in the presence of a strong magnetic field, since turning on a strong magnetic field leads to a change in the power of the Fröhlich constant in the expression for $d\mathcal{W}_{excN}/d\omega_1$, specifically

$$d\mathcal{W}_{excN}/d\omega_1\propto\alpha\quad\text{for } N\geqslant 2. \tag{94}$$

Thus, in the presence of a strong magnetic field, the rate of exciton formation $d\mathcal{W}_{excN}/d\omega_1$ increases by a factor of $\alpha^{-1}$ for $N=2$ and $\alpha^{-2}$ for $N\geqslant 4$ in comparison with the case for $H=0$.

Correspondingly, the exciton luminescence intensity should increase if the latter is due to monomolecular exciton creation. An appropriate object of study here would be, for example, a crystal of InSb ($\alpha=0.014$) or GaAs ($\alpha=0.06$). With increasing $N$, the exciton creation rate decreases only numerically. However, at very large $N$, or equivalently, when the frequency $\omega_1$ is much greater than the width of the band gap, the probability of monomolecular exciton creation nevertheless becomes smaller than the probability of bimolecular binding of independent electrons and holes. It is completely clear that the emission of a very large number of phonons results in the almost complete disappearance of spatial correlation of the electrons and holes.

It has actually been experimentally observed[9,10] that when semiconductors are irradiated by light at a frequency much higher than the natural absorption region, the intensity of exciton luminescence is quadratic in the intensity of the exciting light. In Refs. 9 and 10 it was suggested that under such conditions the creation of excitons which give rise to exciton luminescence is due to binding of thermalized independent electrons and holes.

Let us discuss the applicability of the approximations

$$a\gg a_H,\quad m_h\gg m_e \tag{95}$$

in the calculation of the exciton creation rate in crystals of InSb.

The parameters of InSb are

$$\Delta E=5\cdot 10^{-4}\text{ eV},\quad \hbar\omega_{LO}=2.5\cdot 10^{-2}\text{ eV},$$
$$a=8.4\cdot 10^{-6}\text{ cm},\quad m_2/m_1=10.6. \tag{96}$$

From these we obtain

$$m_e=0.012m_0,\quad m_h=0.127m_0. \tag{97}$$

Consider magnetic fields from 40 to 80 kOe, as in Ref. 26. For fields in this range the parameter $z=a/\sqrt{2}a_H$ varies from 4.67 to 6.58, so the condition of a strong magnetic field for excitons is satisfied. The quantities $\hbar\omega_{eH}$ and $\hbar\omega_{hH}$ vary respectively from $3.83\cdot 10^{-2}$ eV to $7.66\cdot 10^{-2}$ eV and from $3.18\cdot 10^{-3}$ eV to $7.2\cdot 10^{-3}$ eV. We find then that

$$\Delta E_H<\hbar\omega_{LO},\quad \omega_{eH}>\omega_{LO},\quad \omega_{hH}<\omega_{LO}. \tag{98}$$

It is significant that if we consider the creation of excitons with kinetic energies from 0 to $\hbar\omega_{LO}$ (here the number of phonons $N$ depends on the interval in which $\omega_1$ lies [see Eq. (81)], then from $N=2$ to $N=10$ inclusively emission of even one phonon by a hole is impossible because of an insufficiency of energy. Therefore, first of all, the main contribution comes only from a diagram of the type in Fig. 1a, where all $N-1$ phonons are emitted by the electron, and, secondly, the inverse lifetime of the hole $\gamma_h$ in the given state is determined not by the emission of an LO-phonon, but by other processes; therefore $\gamma_h\ll\gamma_e$, as was supposed. Thus, the approximation $m_h\gg m_e$ should apply to crystals of InSb all the way to $N=10$.

8. Let us consider the extrema that show up in the analysis of expression (88).

The right-hand side of Eq. (88) contains the factor

$$\frac{\lambda_0\lambda_1\lambda_2}{l^9K_0^2K_1^2K_2^2}=\frac{2^3\omega_{LO}^3}{\gamma_0\gamma_1\gamma_2}\cdot\frac{1}{l^3K_0K_1K_2}. \tag{99}$$

Using expressions (56) and (73), we find that the above factor diverges at $K_0=0$, i.e.,

$$\omega_{l,\max,m_h\to\infty}(n)\simeq E_{gH}/\hbar+\omega_{1H}n. \tag{100}$$

In the case of finite hole mass, in place of Eq. (100) we obtain

$$\omega_{l,\max}(n)=E_{gH}/\hbar+(\omega_{1H}+\omega_{2H})n. \tag{101}$$

This resonance should exist for any number $N$ of emitted phonons. Condition (101) means that the light creates an electron and a hole near the bottom of each of the Landau bands. Near the points (101), maxima should be observed in the exciton luminescence.

It should be possible to observe these maxima not only by varying the frequency $\omega_l$ for a fixed value of $H$, but also by varying the magnitude of the magnetic field for fixed $\omega_l$. Solving Eq. (101) for $H$, we find that maxima exist at the points

$$H_{\max}(n)=\frac{\mu c}{e}\frac{\omega_l-E_g/\hbar}{n+1/2}. \tag{102}$$

Using Eqs. (56) and (73), we find that for $K_1=0$ and $K_2=0$ the factor (99) does not have any singularities, since the zeroes in the denominator are cancelled by the product $\gamma_0\gamma_1$. So for $K_1=0$ and $K_2=0$ there are no extrema.

However, the case $N=2$ is an exception. This follows from Eqs. (69) and (70). Indeed, the factor

$$f^{+-}(z=0)=(\lambda_0+\lambda_1)^{-1}$$

diverges at $K_1=0$, as was noted in Ref. 18. Therefore, for a two-phonon process under the condition

$$\omega'_{l,\max,m_h\to\infty}(n)\cong E_{gh}/\hbar+\omega_{1H}n+\omega_{\mathrm{LO}} \tag{103}$$

the exciton creation probability does have a maximum.

As $\gamma_2\to\infty$ expression (99) tends to zero, which corresponds to a minimum in the exciton creation probability and a minimum in the exciton luminescence due to monomolecular processes. For arbitrary $N$ we obtain $\gamma_{N-1}\to\infty$, which is satisfied at the points

$$\omega_{l\,\min,m_h\to\infty}(n,N)=E_{gH}/\hbar+n\omega_{1H}+N\omega_{\mathrm{LO}}. \tag{104}$$

Exciton creation involving the participation of $N$ phonons is significant in the frequency range (81). The minimum of (104) at $n=0$ corresponds to the boundary of ranges $a$ and $b$, defined by inequalities (83), so in range $a$ there are no minima. With further increase of $\omega_l$ within the limits of range $b$, a series of minima appear at the points (104) for $n=1$, $n=2$, etc., where $n$ is bounded by the condition

$$n<(-\Delta E_H/\hbar+\omega_{\mathrm{LO}})/\omega_{1H}, \tag{105}$$

which is determined by the upper limit of range $b$. Increasing $\omega_l$ still further, we arrive at processes in which $N+1$ phonons participate, in which a new series of minima $\omega_{l\,\min,m_h\to\infty}(n,N+1)$ begins at the boundary of regions $a$ and $b$. We stress again that the minima at the points (104) exist when $m_h\gg m_e$.

**9.** The construction of a theory of exciton creation in the presence of a strong magnetic field was stimulated by the appearance of the experimental work of Ref. 26, which reports the detection of the flareup in a magnetic field of the Raman emission phonon repetition from a diamagnetic exciton in InSb, and Refs. 27, which report the detection of the anomalous flareup in a magnetic field of the polariton luminescence in superclean epitaxial layers of GaAs. However, in Refs. 26 and 27, bimolecular exciton creation is probably important, since the frequencies $\omega_l$ (fixed) are such that the excess of $\hbar\omega_l-E_g$ over the edge of the band gap is of the order of 1 eV, which is around 50 $\hbar\omega_{\mathrm{LO}}$. So the flareup of exciton luminescence may be due to other factors (see Ref. 27).

Let us enumerate some indications by which it is possible to distinguish exciton emission due to monomolecular or bimolecular processes of exciton creation.

First, the intensity of the secondary emission due to bimolecular processes should be quadratic in the intensity $dI_l/d\omega_l$ of the exciting light at small values of the latter, i.e., with increasing intensity from 0 in the initial segment, but with further growth $dI_l/d\omega_l$ should slow down into a linear dependence, as was shown in Ref. 28. The intensity of the secondary emission due to monomolecular processes should be linear in $dI_l/d\omega_l$ at all values of $dI_l/d\omega_l$. Second, when one looks at the exciton emission excitation spectra, i.e., the dependence of the intensity of emission on the frequency $\omega_l$ of the exciting light, monomolecular and bimolecular exciton creation are seen to lead to oscillations with different periods: in the case of monomolecular processes the period corresponds to $\omega_{\mathrm{LO}}$, while for bimolecular processes it corresponds to $\omega_{\mathrm{LO}}(1+m_e/m_h)$. The latter sort of oscillation was observed in Ref. 28. The intensity maxima correspond to the subsidence of the free electrons created by the light to the bottom of the conduction band after emitting some number of LO-phonons. We will discuss the occurrence of oscillations with period $\omega_{\mathrm{LO}}$ in more detail below. They were first observed experimentally in Refs. 9 and 10. In these works exciton emission in crystals of CdS was due to monomolecular exciton creation.

References 9 and 10 investigated the so-called "excitation spectra" of free emission of excitons in crystals of CdS at 4.4 K and 77 K for three phonon-free lines ($n=1A$, $n=2B$, and $n=2A$) and for exciton emission lines with simultaneous emission of an LO-phonon ($A_1-\mathrm{LO}$). For all of the lines investigated, a pronounced oscillatory structure was observed in the excitation spectra. The intensity maxima of the phonon-free lines are observed at

$$\omega_{l,\max}(N)=E_g/\hbar-\Delta E/\hbar+N\omega_{\mathrm{LO}}. \tag{106}$$

Pronounced oscillations were observed all the way to $N=7$ (the $n=1A$ line). As to the excitation spectra of the line $A_1-\mathrm{LO}$, the maxima in these spectra are located at the frequencies

$$\omega'_{l,\max}(N)=E_g/\hbar-\Delta E/\hbar+E'/\hbar+N\omega_{\mathrm{LO}}, \tag{107}$$

where $E'$ is the kinetic energy of the annihilating exciton, which can be assigned by studying the excitation spectra at certain frequencies $\omega_s$ of the secondary radiation within the limits of the wider lines of the ($A_1$—LO) radiation.

The authors of Ref. 9 explain the oscillations in the excitation spectra using the concept of hot excitons, i.e., by assuming that excitons with kinetic energy up to 0.2 eV take part in the processes in question. They propose the following physical picture: light with frequency $\omega_l$ as a result of an indirect transition with simultaneous emission of one LO-phonon creates an exciton with kinetic energy $\hbar\omega_l-E_g+\Delta E-\hbar\omega_{\mathrm{LO}}$. This hot exciton then emits phonons until its energy is less than $\hbar\omega_{\mathrm{LO}}$. In all, it emits $N-1$ phonons. An exciton with fixed kinetic energy

$$0<\hbar\omega_l-E_g+\Delta E-N\hbar\omega_{\mathrm{LO}}<\hbar\omega_{\mathrm{LO}} \tag{108}$$

is thus formed. The subsequent fate of the exciton is determined by two factors: nonradiative annihilation processes and interaction with acoustic phonons. In the case of incomplete thermalization, i.e., for high enough probability of nonradiative annihilation, the intensity of the exciton luminescence line should be maximum under either of conditions (106) and (107).[9,10] Indeed, for continuous irradiation of a crystal in the case of incomplete thermalization, the exciton kinetic energy distribution function in the range 0 to $\hbar\omega_{LO}$ has a maximum at the energies (108). The actual form of this function was calculated in Ref. 29.

If radiative annihilation of excitons with kinetic energy $E'$ ($E'=0$ for the bottom of the exciton band is observed), then the intensity of the radiation should grow as the maximum of the exciton distribution function approaches $E'$. This means that intensity maxima will be observed when (106) or (107) hold, explaining the oscillations in the excitation spectra. In the analysis of their experiments, the authors of Refs. 9 and 10 do not take account of the influence of exciton decay on the processes taking place. Reference 11 argues against the idea of hot excitons, noting the destructive role of exciton decay on the free electron and the free hole. Here it is necessary to add the following remark. The decay of excitons whose kinetic energy exceeds the binding energy $\Delta E$ (or $\Delta E_H$ in a strong magnetic field) can be due to interaction with acoustic phonons. Therefore the fate of the excitons created in ranges $a$ and $b$ is different [see inequalities (83)]. Excitons created in ranges $a$ have kinetic energy less than $\Delta E$, and are not subject to decay. They can lose their energy gradually, interacting with acoustic phonons over and over again, and finally fall to the bottom of the exciton band. Excitons created in range $b$ have a short lifetime because of decay as a result of interaction with acoustic phonons. Their distribution function differs from zero in a narrow interval near each of the values in the range 0 to $\hbar\omega_{LO}$ (108). In general they cannot fall to the bottom of the exciton band. Therefore the $b$ ranges should give zero segments in the exciton emission excitation spectra extending up from the bottom of the exciton band if we are talking about monomolecular processes. For irradiation in the $b$ ranges, the phonon repetition lines should be comparatively narrow, and they can be classified as MRRS lines of order $N+1$. Note that in the case of CdS the $a$ ranges are quite large since $\hbar\omega_{LO}=0.038$ eV and $\Delta E=0.028$ eV.

In Ref. 11 we suggested that the experiments reported in Refs. 9 and 10 can be explained with the help of the monomolecular exciton creation process that we have proposed, in which electron-hole pairs of finite volume play a role, rather than hot excitons with energies greater than $\hbar\omega_{LO}$.

The mechanism that we have proposed also leads to a linear dependence of the intensity of the secondary radiation on the intensity of the exciting light, as was observed in Refs. 9 and 10 at frequencies $\omega_1$ exceeding the width of the forbidden band by a few phonons, and to the creation of excitons with fixed energy (108), which keeps in force all of the considerations about the subsequent fate of the excitons and about the oscillations of the excitation spectra.

To verify the hypothesis of the participation of electron-hole pairs of finite volume in the processes involved in the experiments reported in Refs. 9 and 10, we propose to carry out experiments of the type described in Refs. 9 and 10 in the presence of strong magnetic fields. Upon turning on the strong magnetic fields, the excitation spectra should reflect the growth of the total intensity by a factor of $\alpha^{-2}$ (for $N\geqslant 4$). The oscillations observed in Refs. 9 and 10 should be preserved, but in the expressions (106) and (107) for $\omega_{1,\max}(N)$, $E_g$ should be replaced by $E_{gH}$ and $\Delta E$ by $\Delta E_H$, i.e., instead of expression (106), for example, we now have

$$\omega_{1,\max}(N,H)=E_{gH}/\hbar-\Delta E_H/\hbar+N\omega_{LO}. \qquad (109)$$

It should also be possible to observe the extrema (109) at fixed frequency $\omega_1$ while varying the magnetic field.

Against the background of oscillations with period $\omega_{LO}$ in the excitation spectrum in a strong magnetic field (101), there should appear maxima with the period $\omega_{H\mu}=eH/\mu c$. Both types of oscillations should be manifested at any ratio of the effective masses of the electron and the hole.

Crystals of InSb and GaAs are preferable to CdS due to their small values of $\alpha$, which correspond to a stronger dependence on the magnitude of the magnetic field. A drawback of these crystals is small values of $\Delta E$ and $\Delta E_H$, which lead to narrow $a$ ranges.

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[1)]Reference 22 uses a vector potential gauge different from (12), which must be taken into account when defining the form of the wave function $\Psi_f$.

[2)]Below we show that the electron and hole phonon emission processes are real, i.e., they conserve energy.

[3)]The theory can easily be generalized by not neglecting $p$ in the exponential, since the integral over $p$ can be calculated using the residue theorem.

[4)]If the effective mass of the hole is very large, the kinetic energy of the hole will be insufficient for real (energy conserving) emission of an LO-phonon. This means that the inverse lifetime of the hole $\gamma_h$ will be much less than the inverse lifetime of the electron.

[5)]Below we omit subscripts "$a$" and "diag" from the $W_{3,m_h\to\infty}$, as the contributions of other diagrams (apart from Fig. 1a) and off-diagonal terms are small.

[6)]It is understood that $\Delta E_H<\hbar\omega_{LO}$.

[7)]In reality, for $a_H^2K_1^2\cong a^2/a_H^2$ in a strong magnetic field the exciton ceases to exist, but if the variable $x=a_H^2K_1^2/2\leqslant 1$ in the integral over $x$, it is possible to assume that $x$ varies within the limits from 0 to $\infty$.

[8)]It is assumed that $m_h\gg m_e$ as in the case of GaAs.[28]

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