A superconducting twinning plane in a magnetic field

K. V. Samokhin

L. D. Landau Institute of Theoretical Physics, Moscow, 142432 Chernogolovka, Moscow Region, Russia
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In this paper the magnetic field at which the normal state of a superconducting twinning plane becomes absolutely unstable is calculated, taking into account the most general type of boundary conditions for the order parameter at the twinning plane. Various types of localization of the superconductivity at the twinning plane are considered. The experimental phase diagram is discussed for superconductivity in tin. A mechanism is proposed that leads to the appearance of a nonuniform state in the twinning plane, which is responsible for delaying the transition to bulk superconductivity.

1. Superconductivity in twinning planes has been the subject of a considerable amount of experimental and theoretical work in the last 10 years (see, e.g., the review Ref. 1). Additional interest in this topic has been stimulated by the discovery that the high-temperature superconductor $YBa_2Cu_3O_{7-\delta}$ develops a twinning structure (Ref. 2). However, at this point a rather large number of questions in the theory of ordinary superconductivity with twinning planes remain unresolved. In particular there is no explanation for the phase diagram of superconducting twinning planes in tin subjected to a magnetic field,[3] which we reproduce in Fig. 1. Here $H_d(T)$ is the critical magnetic field for thermodynamic equilibrium between the normal and superconducting states of the twinning plane, $H_m(T)$ and $H_b(T)$ are critical fields for supercooling (i.e., the field for absolute instability) of the normal state of the twinning plane, and $H^*(T)$ is the critical field for supercooling of a sample with a superconducting twinning plane. In the figure we also show the bulk critical field $H_c(T)$ and the critical field for surface superconductivity $H_{c3}(T)$ for tin (which is a type-I superconductor). The temperature T_b is obtained by extrapolating $H_b(T)$ until it intersects the temperature axis.

Certain features of this phase diagram are worth noting: 1) the bulk phase transition in the presence of a localized superconducting seed is delayed until a field $H^*<H_c$ is reached, i.e., even though bulk superconductivity is energetically advantageous, seeds from the twinning plane cannot freely "germinate" in the bulk of the crystal; 2) the curve for the upper critical field H_m is parallel to H_{c3}.

2. We will use a phenomenological Ginzburg–Landau functional to describe the magnetic behavior of superconductivity at a twinning boundary, taking into account the local increase in T_c near a twinning plane and the change in boundary conditions for the superconducting order parameter at the twinning plane.

Let us consider a crystal consisting of two parts that are mirror twins, separated by a boundary at the plane $z=0$. We will neglect effects connected with the crystalline anisotropy, and write the Ginzburg–Landau functional for the free energy of the superconductor $\mathscr{F}=\mathscr{F}_v+\mathscr{F}_s$ in the following form:[1,4–6]

$$\mathscr{F}_v=\int dV\left\{a_0\tau|\psi|^2+\frac{b}{2}|\psi|^4+\frac{1}{4m}|D_k\psi|^2+\frac{1}{8\pi}(\mathbf{B}-\mathbf{H})^2\right\},$$
$$\mathscr{F}_s=\int dV\left\{-\gamma(|\psi_+|^2+|\psi_-|^2)+\frac{1}{4m\alpha}\left|\psi_+-\psi_-\right|^2\right\}\delta(z), \qquad (1)$$

where

$$\tau=\frac{T-T_c}{T_c},\quad D_k=\nabla_k-\frac{2ie}{c}A_k,\quad \psi_\pm=\psi(x,y,z=\pm 0).$$

The term with γ describes the change in the superconducting coupling constant near the twinning plane. We consider the case $\gamma>0$, for which the temperature T_d at which superconductivity localized at the twinning plane appears is such that $T_d>T_c$ (Ref. 1). The second term in the surface energy, which includes the superconducting coupling between the twins in a phenomenological way, was first proposed by Andreev.[4] In the microscopic theory the constant α is determined by the transparency of the twinning plane to electrons.[5,7]

These features of the phase diagram can be obtained in a consistent way within the framework of model (1). Let us first note that one possible explanation for the existence of a supercooling field H^* could perhaps be the fact that the superconducting state of the twinning plane is nonuniform along the twinning plane itself with a characteristic dimension L that is smaller than the correlation length.[1] In this case, the growth of seeds from the superconducting twinning plane into the bulk can in fact be suppressed as the sample is cooled below $H_c(T)$, due to the increased surface energy (for a uniform state of the superconducting twinning plane, there is no energy barrier to the growth of a planar seed). Up to now it has been unclear what the reason for the appearance of such a nonuniformity might be. We will show that the model (1) leads to a periodic

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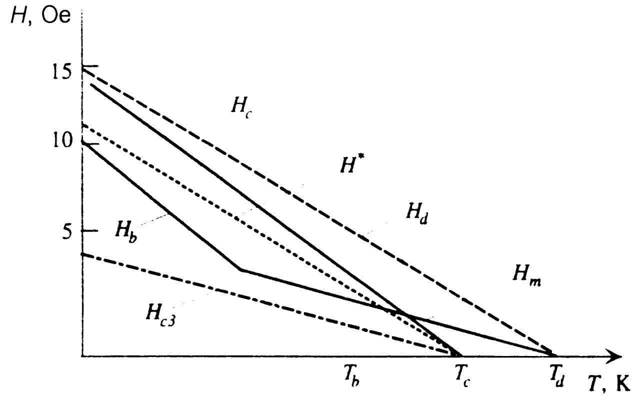

FIG. 1. Experimental phase diagram for tin with a twinning plane in a magnetic field.[3] The angle at the peak of the twinning wedge is $\alpha=1.4\cdot 10^{-3}$.

modulation of the superconducting state of the twinning plane even in the case of fields that are parallel to the twinning boundary.

Converting to dimensionless variables,[6] we rewrite the Ginzburg–Landau functional as follows:

$$\mathscr{F}_v=\int dV\{t|\tilde{\Psi}|^2+\tfrac{1}{2}|\tilde{\Psi}|^4+|(-i\tilde{\nabla}_k-a_k)\tilde{\Psi}|^2+\kappa^2(\mathbf{b}-\mathbf{h})^2\}, \qquad (2)$$

where

$$t=\frac{T-T_c}{T_d-T_c},\quad \tilde{\Psi}=\frac{\psi}{\psi_d},\quad \tilde{x}_k=\frac{x_k}{\xi_d},$$

$$a_k=A_k\frac{2\pi\xi_d\tau_d}{\phi_0},\quad b_k=B_k\frac{2\pi\xi_d^2}{\phi_0},$$

$$\psi_d=\sqrt{\frac{a_0\tau_d}{b}},\quad \tau_d=\frac{T_d-T_c}{T_c},\quad \xi_d=\frac{\xi_0}{\sqrt{\tau_d}},\quad \phi_0=\frac{\pi\hbar c}{e},$$

here h is the dimensionless external field, and κ is the Ginzburg–Landau parameter (in what follows we will omit the tilde from the notation). We will vary the boundary conditions via the dimensionless temperature-independent quantity $r=\alpha/\xi_d$, which in the phenomenological theory takes on values from $-\infty$ to $+\infty$. A zero value of r corresponds to a twinning plane that is transparent to electrons, while an infinite value of r defines a twinning plane that is completely opaque.

3. Our problem is to calculate the field at which the normal state of the twinning plane is absolutely unstable. However, we must first determine the possible types of localized superconductivity, which we derive from symmetry considerations.

Let the magnetic field be directed in the plane of the twinning boundary (along the y axis); then the symmetry group of the system described by the functional (1) is $G=G_0\times U(1)$. Here $G_0=\{E,\sigma_zR\}$, where σ_z is a reflection in the plane $z=0$, R is the time reversal operation, and $U(1)$ is a group of gauge transformations. In order to list all the possible superconducting phases it is necessary to find the subgroups H of the group G (i.e., the superconducting classes).[8] The order parameter of each phase remains invariant under the action of all the elements of the corresponding subgroup H. Each of the groups H is isomorphic to one of the subgroups H_0 of the group G_0, and consists of the transformations of H_0 combined with various gauge transformations from $U(1)$ (i.e., multiplication by phase factors).

In our case G_0 is isomorphic to the group C_s (Ref. 5), so that it will have a total of two superconducting classes: $C_s=\{E,\sigma_zR\}$ and $C_s(E)=\{E,e^{i\pi}\sigma_zR\}$. Two possible superconducting phases ("even" and "odd") correspond to these two classes. Using the terminology of Volovik *et al.*,[8] we will say that the odd phase is a form of nontrivial superconductivity.

Returning to the question of the upper critical field, we can easily see that the normal state can be reached directly from either superconducting phase. Actually, since the phase transition from the normal to the superconducting state in a magnetic field is second-order, it can be discussed within the framework of the Landau classification, according to which the order parameter for the ordered (superconducting) phase that arises below the transition temperature should transform according to one of the irreducible representations of the group G_0. The group C_s has two one-dimensional irreducible representations with different parity;[5] consequently, either the even or odd phase can appear immediately below the transition field. We will need this symmetry-related fact below in analyzing the admissible solutions to the Ginzburg–Landau equation.

4. We should point out here that Koshelev *et al.*[6] have already calculated the upper critical field for the even superconducting phase using the model (1). However, their paper contains some erroneous assertions; therefore, their calculation will be repeated here using a different mathematical apparatus.

Let the magnetic field be applied along the y axis. In the gauge $\mathbf{a}=(a(z),0,0)$, $a(0)=0$, the linearized Ginzburg–Landau equation (for either of the twins) has the form

$$\left[-\frac{\partial^2}{\partial z^2}+\left(-i\frac{\partial}{\partial x}-hz\right)^2\right]\psi(x,z)+t\psi(x,z)=0. \qquad (3)$$

The boundary condition for ψ is obtained from the surface part of the free energy $\mathscr{F}_s$ in (1):

$$\frac{\partial\psi_+}{\partial z}=-\psi_++\frac{1}{r}(\psi_+-\psi_-),$$
$$\frac{\partial\psi_-}{\partial z}=\psi_-+\frac{1}{r}(\psi_+-\psi_-). \qquad (4)$$

Let us seek a solution for the order parameter in the following form:

$$\psi_{z_0}(x,z)=f_{z_0}(z)\exp(ihz_0x). \qquad (5)$$

The function $f_{z_0}(z)$ satisfies the equation

$$-\frac{d^2 f_{z_0}(z)}{dz^2}+h^2(z-z_0)^2 f_{z_0}(z)=-t f_{z_0}(z),$$

the solution to this equation that decreases at infinity has the form:

$$f_{z_0}(z)=\begin{cases} C_+\exp\left[-\frac{h}{2}(z-z_0)^2\right]H_\nu(\sqrt{h}(z-z_0)), \\ \quad z>0, \\ C_-\exp\left[-\frac{h}{2}(z-z_0)^2\right]H_\nu(-\sqrt{h}(z-z_0)), \\ \quad z<0, \end{cases} \tag{6}$$

where $H_\nu(x)$ is a Hermite polynomial[10] and $\nu=-1/2(1+t/h)$.

Substituting Eq. (5) into (4) and making use of $H'_\nu(x)=2\nu H_{\nu-1}(x)$ (see Ref. 10) leads to a homogeneous linear system for C_+ and C_-. Setting the determinant of this system equal to zero, we obtain the following transcendental equation for $h(t,z_0)$:

$$\Phi(t,h,z_0)\Phi(t,h,-z_0)-\frac{1}{r^2}=0, \tag{7}$$

where

$$\Phi(h,t,z_0)=1-\frac{1}{r}+hz_0+2\nu\sqrt{h}\,\frac{H_{\nu-1}(-\sqrt{h}z_0)}{H_\nu(-\sqrt{h}z_0)}.$$

The field $h_m(t)$ at which the normal state is absolutely unstable is determined by the maximum value of the function $h(t,z_0)$ with respect to z_0. However, before we analyze the solution to (7) for arbitrary h, let us find the phase transition temperature in zero field, for which we pass in (7) to the limit $h\to 0$. In this case the left-hand portion of the equation can be factored:

$$\left[1+2\nu\sqrt{h}\,\frac{H_{\nu-1}(0)}{H_\nu(0)}\right]\left[1-\frac{2}{r}+2\nu\sqrt{h}\,\frac{H_{\nu-1}(0)}{H_\nu(0)}\right]=0.$$

Using the property $H_\nu(0)=\Gamma(-\nu/2)/2\Gamma(-\nu)$ of the Hermite polynomials (Ref. 10), we are led to the following two equations that determine the two possible types of behavior of the system as $h\to 0$:

$$B\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{t}{h}\right)\right)=2\sqrt{\pi h}, \tag{8a}$$

and

$$B\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{t}{h}\right)\right)=\frac{2\sqrt{\pi h}}{1-2/r} \tag{8b}$$

[here $B(x,y)$ is the beta function].

The first equation, which is well-known in the theory of superconductivity in twinning planes, has been used previously[1] to describe the behavior of a twinning plane that is transparent to electrons (i.e., $r=0$) in a magnetic field. It follows from (8a) that the transition temperature for a plane in zero field is given by $t_{c1}=1$. To verify that (8a) corresponds to the even superconducting state of the twinning plane, it is enough to compare the critical temperature for the even phase with the value of t_{c1} obtained from (8a). At $h=0$, the order parameter for the even phase near the critical temperature has the form

$$f_0(z)=C\exp(-k|z|).$$

Substituting into the boundary condition (4), we have $k=1$, from which $t_{c,\mathrm{even}}=k^2=1=t_{c1}$.

Equation (8b) leads to a different value of the transition temperature in zero field, namely $t_{c2}=(1-2/r)^2$. This case corresponds to the odd superconducting state. In point of fact, for the even phase we have at $h=0$

$$f_0(z>0)=f(z)=C\exp(-kz),$$

$$f_0(z<0)=-f(-z),$$

and from (8b) we obtain $k=1-2/r$; hence, $t_{c,\mathrm{odd}}=(1-2/r)^2=t_{c2}$. We see, first of all, that stable odd solutions exist only for $k>0$, i.e., for $r>2$ or $r<0$; and, secondly, that when $r<0$ the critical temperature for the odd phase exceeds the critical temperature for the even phase. Note that the odd phase, which belongs to the superconducting class $C_s(E)$, corresponds to the "exotic" type of twinning-plane superconductivity proposed by Andreev,[4] in which the order parameter has a phase jump of π at the twinning plane.

We now turn to finding $h_m(t)$ from Eq. (7), for which we require that the following conditions be fulfilled:

$$\frac{\partial h(t,z_0)}{\partial z_0}=0,\quad \frac{\partial^2 h(t,z_0)}{\partial z_0^2}>0.$$

The first condition gives

$$\Phi_1(z_0)\Phi(-z_0)-\Phi(z_0)\Phi_1(-z_0)=0,$$

where

$$\Phi_1(h,t,z_0)=1-4\nu(\nu-1)\frac{H_{\nu-2}(-\sqrt{h}z_0)}{H_\nu(-\sqrt{h}z_0)}+4\nu^2\frac{H^2_{\nu-1}(-\sqrt{h}z_0)}{H^2_\nu(-\sqrt{h}z_0)}.$$

This equation is obviously satisfied when $z_0=0$, i.e., when $z_0=0$ the function $h(t,z_0)$ has an extremum. We now must verify the second condition. Omitting the tedious manipulations, we present the results, which turn out to differ for the two admissible types of localized superconductivity.

For the even phase, using (8a), we have

$$\frac{\partial^2 h(t,0)}{\partial z_0^2}\propto(1-t)\left(t-1+\frac{2}{r}\right),$$

while for the odd phase [including (8b)]:

$$\frac{\partial^2 h(t,0)}{\partial z_0^2}\propto(1-t)\left[t-\left(1-\frac{2}{r}\right)^2\right].$$

Thus, we obtain the following picture. For the even phase, when $r>0$: if $1<t<t_{c1}=1-2/r$ (small fields), then the maximum of $h(t,z_0)$ is reached at $z_0=0$ and the temperature dependence of the field for absolute instability $h_m(t)$ is determined by Eq. (8a). If $t<t_{c1}$ (large fields), then the maximum of the critical field corresponds to a nonzero

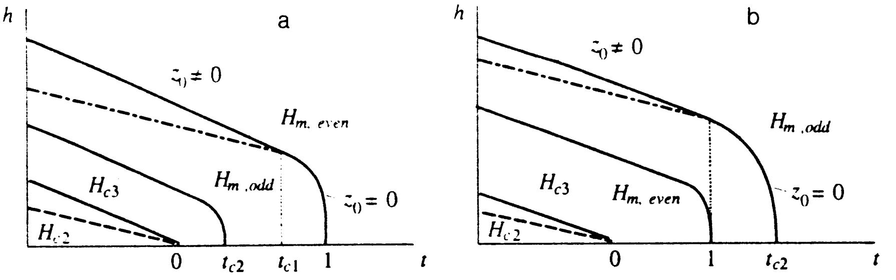

FIG. 2. Critical fields for superconducting twinning planes: a) $r>2$, b) $r<0$. $H_{m,\mathrm{even}}$ and $H_{m,\mathrm{odd}}$ are upper critical fields for the even and odd phases respectively; for the critical temperatures t_{c1} and t_{c2} see the text; H_{c2} and H_{c3} are the bulk critical fields. The dot-dash curves show the solutions to Eqs. (8a) and (8b).

value of z_0 and the function $h_m(t)$ is determined by the general equation (7). When $r<0$ we have $t_{c1}>1$, and $z_0 \neq 0$ everywhere on the curve $h_m(t)$.

For the odd phase when $r<0$: if $1<t<t_{c2}=(1-2/r)^2$ (small fields), then the maximum of $h(t,z_0)$ is reached for $z_0=0$ and the function $h_m(t)$ is determined by Eq. (8b). For $t<1$ (large fields) $z_0 \neq 0$ and we return to Eq. (7).

However, if $r>2$, then for any t, the field for absolute instability of the odd phase is given by (7) with nonzero z_0.

5. Let us now investigate the asymptotic behavior of the critical field $h_m(t)$ obtained from Eq. (7).

In the limit of high transparency of the twinning plane ($r \to 0$), the terms of order $1/r^2$ in (7) cancel out. Setting the coefficient equal to zero for $1/r$, we return to Eq. (8a). We emphasize once again that for twinning planes with finite transparency, Eq. (8a), which has been used previously[1] to describe the magnetic behavior of superconducting twinning planes in arbitrary fields, is useful only for the even phase in the range of small fields.

The limit of a twinning plane with low transparency ($r \to \infty$) was investigated previously by Mineev.[11]. In this case $t_{c1}=t_{c2}=1$ and $z_0 \neq 0$ in any fields. For both phases the function $h_m(t)$ is determined by the equation

$$\frac{H_{\nu-1}(-\sqrt{h}z_0)}{H_\nu(-\sqrt{h}z_0)}=\frac{1+hz_0}{(1+t/h)\sqrt{h}}$$

and has a square-root form for $h \to 0$, while for $h \gg 1$ the curve $H_m(T)$ becomes parallel to the curve $H_{c3}(T)$.

For arbitrary transparency, in the low-field range $h \to 0$ we find that Eqs. (8a) and (8b) imply square-root behavior of the upper critical field: $h_{m,\mathrm{even}}(t) \propto \sqrt{1-t}$ (Ref. 1) and $h_{m,\mathrm{odd}}(t) \propto \sqrt{t_{c2}-t}$.

In the high-field range, however, $h \gg h_{c1}=h_{m,\mathrm{even}}(t_{c1})$ (for the even phase) or $h \gg h_{c2}=h_{m,\mathrm{odd}}(t=1)$ (for the odd phase), and we have from (7) the equation

$$2\nu\frac{H_{\nu-1}(-\sqrt{h}z_0)}{H_\nu(-\sqrt{h}z_0)}=-\sqrt{h}z_0, \tag{9}$$

which describes the dependence of the field for surface superconductivity[12] $H_{c3}(T)$. We note that the solution to Eqs. (8a) and (8b) has a completely different asymptotic behavior in large fields, i.e., the curve $H_m(T)$ is parallel to the curve $H_{c2}(T)$ for both the even[1] and the odd phase.

To summarize the results we have obtained, we have derived the functions $h_m(t)$ shown in Fig. 2 (for $0<r<2$ in Fig. 2a, the curve $H_{m,\mathrm{odd}}$ is absent). A comparison with the experimental data (Fig. 1) probably indicates low transparency of the twinning planes in tin for electrons (a large value of r). Actually, in the opposite case of not too large fields $h_{c1} \gtrsim h \gtrsim 1$ (when $h \lesssim 1$ we are in the range of square-root behavior, which is not observed in experiment), the curve $H_1(T)$ should include a segment parallel to the curve $H_{c2}(T)$ according to Eq. (8a) or (8b). Because there is no such segment, we can assert that $h_{c1} \lesssim 1$, i.e., $r \gg 1$.

6. Let us discuss in more detail the properties of the order parameter for both phases in the high-field (low-temperature) range $h>h_{c1}$ (or $h>h_{c2}$). In this range $z_0 \neq 0$ and the solutions to Eq. (3) are doubly degenerate ($z_0 \leftrightarrow -z_0$), which physically corresponds to the possibility that superconducting seeds can be localized in either of the twin regions. The general solution for the order parameter is a linear combination of the solutions (5):

$$\psi(z,x)=C_1\psi_{z_0}(z,x)+C_2\psi_{-z_0}(z,x). \tag{10}$$

Without loss of generality, we may choose the function $f_{z_0}(z)$ to satisfy the condition $f_{-z_0}(z)=f_{z_0}(z)$.

The coefficients C_1 and C_2 are determined by solving the nonlinear problem below the transition temperature, which in general cannot be done analytically. However, we can avoid this difficulty to some extent by making use of the symmetry considerations discussed in paragraph 3 in determining the relative values of C_1 and C_2. Specifically, we start with the requirement that the order parameter, which appears on the transition curve, transform either according to the even (A_g) or the odd (A_u) representation of the group C_s (Ref. 9); we then find immediately from (10) that there are only two possibilities: $C_1=C_2$ or $C_1=-C_2$. The first implies that the even phase appears at the field of absolute instability, and the second, the odd phase. Limiting ourselves in what follows to a discussion of the first possibility (the generalization to the odd case is obvious), we rewrite expression (10) in the following way:

$$\psi(x,z)=f_{z_0}(z)\exp(ihz_0x)+f_{z_0}(-z)\exp(-ihz_0x), \tag{11}$$

where $f_{z_0}(z)$ is defined in (6).

Note that Koshelev *et al.*[6] actually wrote down an order parameter of the form (11); however, these authors identified the region of existence of this phase incorrectly.

It is easy to see that the order parameter (11) is nonuniform along the twinning plane. Its amplitude is periodically modulated in the twinning plane (for either of the twinned regions):

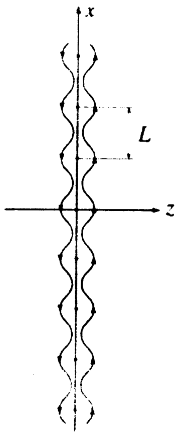

FIG. 3. Lines of current flow for a chain of "soft" vortices localized at the twinning plane in the high-field range (see Sec. 6).

$$|\psi(x,z)|^2=f_{z_0}^2(z)+f_{z_0}^2(-z)+2f_{z_0}(z)f_{z_0}(-z)\cos(hz_0x).$$

The same modulation with period $L=2\pi/hz_0$ is also shared by the surface energy density:

$$F_s=[f_{z_0}(+0)-f_{z_0}(-0)]^2\left[1+\frac{2}{r}\sin^2(hz_0x)\right]+4f_{z_0}(+0)f_{-z_0}(-0)\cos^2(hz_0x).$$

Currents flow in opposite directions along the x axis on the two sides of the twinning plane:

$$j_x(x,z)=hz_0[f_{z_0}^2(z)-f_{z_0}^2(-z)]-hz[f_{z_0}^2(z)+f_{z_0}^2(-z)+2f_{z_0}(z)f_{z_0}(-z)\cos(2hz_0x)].$$

The z component of the current is nonzero:

$$j_z(x,z)=\left[f_{z_0}(-z)\frac{df_{z_0}(z)}{dz}-f_{z_0}(z)\frac{df_{z_0}(-z)}{dz}\right]\sin(2hz_0x).$$

Thus, in sufficiently high fields, the superconducting state of a twinning plane consists of a chain of "soft" vortices localized at the twinning plane (Fig. 3). The fact that we can view the twinning plane as a planar Josephson junction[4] (of a special kind due to the local enhancement of the superconductivity) suggests that these vortices are in a certain sense analogous to Ferrel–Prange vortices in wide junctions.[12]

In the high-field limit $h\gg h_{c1}$, i.e., where the upper critical field is parallel to $H_{c3}(T)$, we have from (9) that $z_0\sim 1/\sqrt{h}$, and the period of the vortex structure $L=2\pi/hz_0\sim 1/\sqrt{h}\ll 1$ (in units of ξ_d). If a nonuniform current-carrying seed of this kind is not free to grow into the depth of the superconductor, thereby mediating a transition to uniform bulk superconductivity (see paragraph 2), the system will exhibit the observed supercooling of the superconducting twinning plane.

Thus, the distinctive features mentioned in Sec. 1 of the phase diagram of a superconducting twinning plane in tin can be explained within the framework of model (1) by assuming that the twinning planes have low transparency. However, a number of open questions remain, in particular, the origin of the kink in the H_m-H_b curves.

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