Hollow beams of electromagnetic radiation: formation and nonlinear propagation in plasma

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A theoretical and experimental study has been made of the time-independent structure of beams of electromagnetic radiation formed by radial (axicon) and azimuthal (phase screw) phase transformations. It is shown that if the phase screw parameter s assumes integral values ($s=m>0$), then the radial profile of the radiation intensity in the linear dissipationless case is described by a Bessel function of order m. The effects of radiation absorption and the nonlinearity of the medium on the electric field profile of such beams propagating in a plasma is considered. The radiation power needed for self-modulation to occur is determined as a function of m. It is shown that in contrast to beams with $m=0$, the instability begins to grow at other than the first maximum of the radial profile of the field.

1. INTRODUCTION

The propagation of beams of electromagnetic radiation focused by an axicon[1,2] in media with different types of nonlinearity[3–5] has been studied with a view to explaining the structure of the plasmas that develop when gas is broken down by laser radiation. The radial profile of the electric field of such beams is characterized by the presence of an intensity maximum on the axis of symmetry, and near the axis it is described by a Bessel function of order zero $J_0(kr\sin\gamma)$, where γ is the angle between the rays and the axis of the axicon and k is the wave number of the radiation. Based on the details of the radial structure, Andreev *et al.*[5] classified them as a special category of "Bessel beams." It is of particular interest to study beams formed by an azimuthal phase transformer (a phase screw), rather than an axicon, which is essentially a radial phase transformer. Such beams have fields whose radial structure is described by a Bessel function $J_m(kr\sin\gamma)$ of order $m>0$ equal to the parameter of the phase screw. In contrast to Bessel beams, their maximum is displaced from the axis of symmetry, and near the axis there is a region where the electric field essentially vanishes, which justifies calling them "hollow Bessel beams." Beams with such a "null region" were previously studied, e.g., by Valyaev and Krivoshlykov,[6] who discussed the solution of the wave equation describing highly collimated nondiffractive beams, and Azimov *et al.*,[7] in connection with the time-independent self-focusing of annular laser beams, and also by Margolin *et al.*,[8] who proposed a device for producing such beams. In the present work we consider the formation of hollow Bessel beams and their propagation in the plasma produced when gas is broken down by laser radiation.

2. FORMATION OF HOLLOW BESSEL BEAMS

Hollow beams can be produced by focusing radiation with an optical system (Fig. 1) consisting of a phase screw, which deflects rays in the azimuthal direction, and an axicon. After passing through a phase plate of thickness $h(\varphi)$ that varies linearly with azimuthal angle according to $h(\varphi)=(h_0/2\pi)\varphi$, $h_0/2\pi\ll 1$, the phase of a radiation field incident on the plate in the direction of the z axis undergoes a change given in the geometric-optics approximation (neglecting scattering by the discontinuity at $\varphi=0$) by

$$\Delta\psi_p=k(n_p-1)\frac{h_0}{2\pi}\varphi, \tag{1}$$

where $k=(\omega/c)\sqrt{\varepsilon_0}$ is the absolute value of the wave vector of the radiation with frequency ω in the medium with permittivity ε_0 surrounding the phase transformer, n_p is the relative index of refraction of the plate, and h_0 is its maximum thickness. The phase of the field focused by an axicon with a rectilinear generatrix (inclined at an angle $\alpha\ll 1$ with respect to the base perpendicular to the z axis) varies linearly with the radius:

$$\Delta\psi_a=-k(n_a-1)\alpha r, \tag{2}$$

where n_a is the relative index of refraction of the axicon. Hence the expression for the complex amplitude $E(\varphi,r,z)$ of the radiation electric field,

$$\mathbf{E}(r,t)=\mathbf{e}\,\mathrm{Re}\{E(\varphi,r,z)\exp(-i(\omega t-kz)\},$$

propagating in the z direction from an initially planar phase front and passing through the optical system from the phase screw (1) and axicon (2), can be described in view of the smallness of the angle α in the following form (the coordinate origin is at the vertex of the axicon):

$$E(\varphi,r,z=0)=E_{in}(r)\exp(i\,[s\varphi-kr\sin\gamma)], \tag{3}$$

where $E_{in}(r>R)=0$ (R is the aperture of the focusing system). For $r\leqslant R$ it is determined by the radial intensity profile in the incident beam, e.g., a hyper-Gaussian

$$E_{in}(r)=E_{in}^0\exp\left(-\left(\frac{r}{r_0}\right)^{2N}\right),\quad N>1. \tag{4}$$

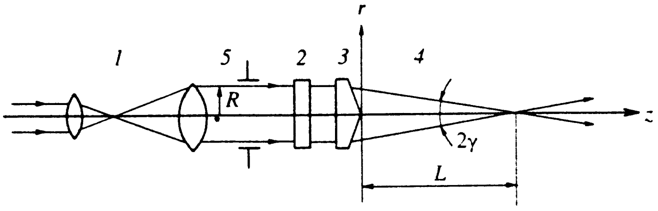

FIG. 1. Layout of the measurements: *1*) telescope; *2*) phase screw; *3*) axicon; *4*) beam-focusing region (caustic); *5*) diaphragm.

Here $s=k(n_p-1)h_0/2\pi$ and $\gamma=(n_a-1)\alpha$ are the phase screw parameter and the angle between the rays and the axis of symmetry of radiation focused by the axicon, respectively.

To describe the structure of the beam field formed by the focusing system for $z>0$ we use the wave equation in the parabolic approximation

$$i2k\frac{\partial E}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right)+\frac{1}{r}\frac{\partial^2 E}{\partial \varphi^2}+\left(\frac{\omega}{c}\right)^2(i\varepsilon''+\varepsilon_{NL}(|E|^2)E=0 \quad (5)$$

imposing the boundary condition (3) at $z=0$ and requiring that the solution vanish asymptotically in the limit $r\to\infty$ and be bounded at the axis. Here ε'', $\varepsilon_{NL}(|E|)$ are the components of the permittivity tensor of the medium in which the beam is propagating.

Expanding the field as a sum of azimuthal harmonics

$$E(\varphi,r,z)=\sum_{n=-\infty}^{\infty}E_n(r,z)\exp(in\varphi) \quad (6)$$

and using the Fourier–Bessel transformation of order n to determine the amplitudes $E_n(r,z)$ of the harmonics, we can write the linear ($\varepsilon_{NL}(|E|^2)=0$) solution of Eq. (5) in a nonabsorbing medium ($\varepsilon''=0$) using (3) in the form

$$E^{(0)}(\varphi,r,z)=-i\frac{k}{z}\exp\left(i\frac{kr}{2z}\right)\sum_{n=-\infty}^{\infty}A_n(s)\times\exp\left(in\left(\varphi-\frac{\pi}{2}\right)\right)\int_0^R E_{in}(r')J_n\left(k\frac{r}{z}r'\right)\times\exp[i\psi(r')]r'dr', \quad (7)$$

where we have written $A_n(s)=[\exp(2\pi i(s-n)]-1)/2\pi i(s-n)$, $\psi(r')=kr'^2/2z-kr'\sin\gamma$, and $J_n(rr'k/z)$ is the Bessel function of order n. From (7) it follows that when the phase screw parameter is an integer $s=m$, the solution corresponds to a single azimuthal mode with index determined by the value of m: $A_n(s=m)=\delta_{mn}$. Hence in the region near the axis ($r<z\sin\gamma$, $kr^2<z$) of the focal section $L\simeq R/\tan\gamma$, for $1/k\sin\gamma<z<L$ we have the following asymptotic expression:

$$E^{(0)}(\varphi,r,z)=\left(E_0J_m(kr\sin\gamma)\exp\left(ik\frac{r^2+z^2\sin^2\gamma}{2z}\right)+E_aJ_m\left(k\frac{r}{z}R\right)\exp\left(ik\frac{r^2+R^2}{2z}\right)\right)\times\exp\left(im\left(\varphi-\frac{\pi}{2}\right)\right), \quad (8)$$

in which

$$E_0(z)=\sqrt{2\pi kz}\sin\gamma\cdot E_{in}(z\sin\gamma)\exp\left(-i\frac{\pi}{4}\right),$$

$$E_a=\frac{1}{1-z/L}E_{in}(R)\exp(-ikR\sin\gamma).$$

The first term in this expression is the change in the complex amplitude of the electric field of an undiffracted beam. The second term describes diffraction at the periphery of the axicon and is small in comparison with the first over the distance of the focal section when z is not close to L, in relation to $\sqrt{\lambda/z}$, and λ is the wavelength of the radiation. From this it follows that the radial profile of the field strength in this region is approximately

$$|E^{(0)}(r,z)|^2\simeq|E_0|^2\cdot|J_m(kr\sin\gamma)|^2. \quad (9)$$

The possibility of creating a hollow Bessel beam has been verified experimentally using the scheme proposed by Margolin *et al.*[8] (see Fig. 1). A beam of radiation from a helium-neon laser, after spreading by the telescope *1*, was transformed by the phase screw *2* and axicon *3*. The diameter of the diaphragm *5* corresponded to the diameter of the phase screw, which had the minimum aperture, and was equal to 5 mm. The phase screw was fabricated of photoresist on a glass substrate. Its optical thickness increased linearly as a function of the azimuthal angle φ from 0 to 2π, independent of the radius, and changed discontinuously by $\varphi=2\pi$ at 0.63 μm. The axicon was fabricated of K-8 glass and had a base angle of 2°.

After focusing by the axicon alone, and also by the axicon together with the phase screw, the elongation of the caustic was equal to about 15 cm. Figure 2 shows typical photos of the field profile in the transverse cross section of the beam in these two cases. They provide systems of alternating concentric light and dark rings, essentially unvarying along the caustic. In the axicon case (Fig. 2a), as expected,[9] a Bessel beam of order zero was formed with maximum intensity on the axis. In the case where the axicon and phase screw acted together (Fig. 2b) a hollow beam with minimum intensity on the axis developed. The dimensions of the observed rings are in good agreement with the locations of the maxima of the Bessel function of order unity having the corresponding scale. This enables us to conclude that the use of an axicon together with a phase screw having a parameter (pitch) equal to the wavelengths gives rise to a first-order hollow Bessel beam, in complete agreement with (9).

Thus, we can confirm that for integer values of the phase transformer parameter $s=m$, an essentially

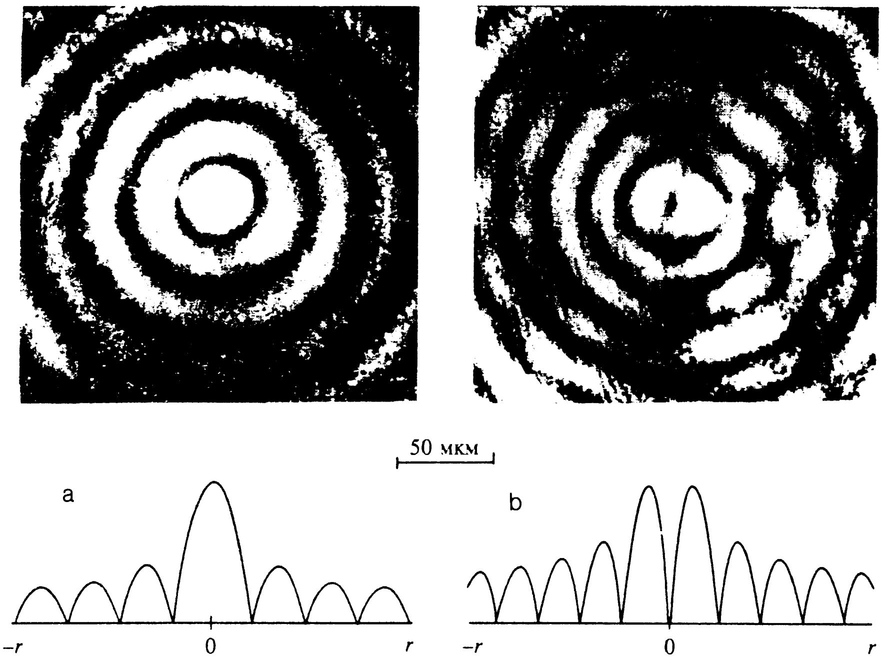

FIG. 2. Typical photographs of the intensity profile in the transverse cross section of a Bessel beam of order 0 (a) and 1 (b); below, the corresponding calculated dependences.

diffraction-free beam forms over the larger part of the focal section. The radial profile of its electric field is described by a Bessel function of order equal to the phase transformer parameter.

3. PROPAGATION OF A HOLLOW BESSEL BEAM IN PLASMA

We will analyze the effect of the medium on the propagation of a focused beam using the dielectric function ε, which we write in the form

$$\varepsilon=\varepsilon_0+i\varepsilon''+\varepsilon_{NL}(|E|^2),$$

where $\varepsilon_0,\varepsilon''$ are the real and imaginary parts of the linear dielectric function of the plasma, incorporating respectively the difference between the electromagnetic properties of the medium and those of vacuum and the absorption of radiation; $\varepsilon_{NL}(|E|^2)$ is the nonlinear correction determined by the dependence of the plasma parameters on the strength of the beam electric field.

The interaction between the beam and the nonlinear medium, together with the behavior of the beam propagating in the medium, are determined by the nonlinear part of the dielectric function and the nature of its dependence on the field strength (local or nonlocal, with or without saturation). In the incompletely ionized plasma that results from gas breakdown due to laser radiation, the change in the electron temperature is determined by the balance between the energy acquired by the electrons from the radiation and that lost mainly through collisions with neutral particles. If the typical scale L_E of the variation in the field substantially exceeds the electron mean free path $l_e/L_E < \sqrt{\delta_{en}}$ (here δ_{en} is the part of the energy transferred by the electron to neutrals in collisions), the nonlinear part of the dielectric function determined by the deviation $\delta n_e=n_e-n_{e0}$ in the electron density from its initial value n_{e0} is given by[4,5]

$$\varepsilon_{NL}=\frac{n_{e0}|E|^2}{n_{ec}E_p^2},$$

where $E_p=\sqrt{12\pi\delta_{en}n_{ec}T_{e0}}$ is the characteristic plasma electric field for the thermal nonlinearity, $n_{ec}=m_e\omega^2/4\pi e^2$ is the critical electron density, and T_{e0} is the initial electron temperature.

A similar expression results for ε_{NL} in the case of a hot plasma as well,[5] when the electron mean free path is longer than the scale of the electric field variation, $l_e>L_E$, and the electron density perturbation, $|\delta n_e/n_{e0}|<1$, is determined by the ponderomotive force, except that here $E_p=\sqrt{16\pi n_{ec}T_{e0}}$ is the characteristic plasma field for the ponderomotive nonlinearity.

Next we consider beams which correspond to the single-mode solution (8), resulting from a phase screw with parameter $s=m$, $m=1, 2,\dots$. Introducing the dimensionless amplitude $\mathscr{E}_m=E_m/E_p$ of the corresponding azimuthal harmonic and the dimensionless cylindrical coordinates

$$z'=\tfrac{1}{2}kz\sin^2\gamma,\quad r'=kr\sin\gamma, \tag{10}$$

we find an equation for $\mathscr{E}_m$ in the form (from now on we omit the prime; all quantities in what follows are dimensionless):

$$i\frac{\partial\mathscr{E}_m}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\mathscr{E}_m}{\partial r}\right)+\left(i\Gamma+\beta\left|\mathscr{E}_m\right|^2-\frac{m^2}{r^2}\right)\mathscr{E}_m=0, \tag{11}$$

where $\Gamma=\varepsilon''/\varepsilon_0\sin^2\gamma$ is the dimensionless absorption rate and $\beta=n_{e0}/n_{ec}\varepsilon_0\sin^2\gamma$ is the dimensionless nonlinearity coefficient.

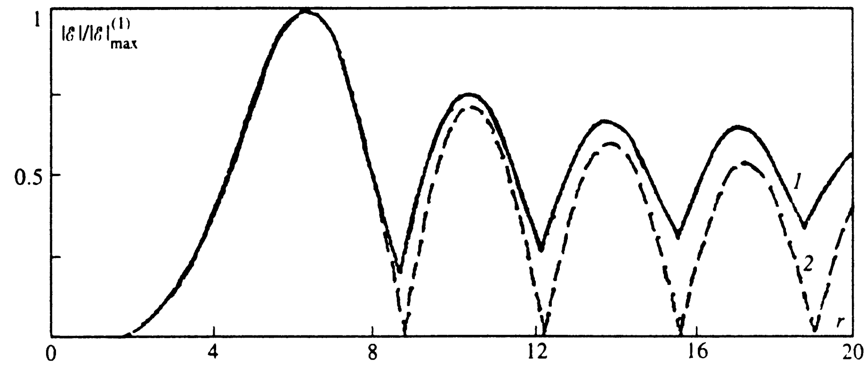

FIG. 3. The effect on the radial structure of a field produced by absorption of radiation for $\Gamma=0.1$, $m=5$, $z=20$; $1-|\mathscr{E}|/|\mathscr{E}|^{(1)}_{max}$, $2-|J_5|/|J_5|^{(1)}_{max}$.

This equation was solved numerically by tridiagonal inversion in the radial direction; the accuracy of the calculation was checked by means of the conservation of energy. The boundary condition (3) for $z=0$ corresponds to focusing of the hyper-Gaussian beam (4) with $N=8$; its dimensionless amplitude $\mathscr{E}_{in}(r)=E_{in}/E_p$ is shaped so that over most of the focal section $(1 \ll z < R/2)$ the dimensionless amplitude $\mathscr{E}_0=E_0/E_p$ of the linear solution (8) remained constant.

The beam-plasma interaction was treated in two stages: first we treated the absorption of the radiation, and then the effects associated with the nonlinearity of the medium.

The change in the structure of the beam electric field associated with dissipation was adjusted in a model of the cylindrical absorption region with a Gaussian profile for the absorption rate:

$$\Gamma=\Gamma_a \exp\left(-\left(\frac{r}{r_a}\right)^2\right),$$

where Γ_a is a coefficient that determines the absorption rate and r_a is the length scale of the absorbing region. Calculations carried out for various modes at $r_a=20$ and in the range where Γ_a varied from 0 to 0.1 revealed that for $r<r_c$, where r_c is the typical value of the radius as a function of the order of the mode (e.g., for $m=5$ we have $r_c\simeq 1.2r^{(1)}_{max}$, where $r^{(1)}_{max}$ is the radius corresponding to the first maximum in the magnitude of the amplitude $|\mathscr{E}|^{(1)}_{max}$), the radial profile of the field is essentially the same as that in the absence of absorption. For r greater than r_c, the Bessel structure of the beam is smeared out, as can be seen from Fig. 3, which displays the profile $|\mathscr{E}|/|\mathscr{E}|^{(1)}_{max}$ for $z=20$, $0< r <20$ with $\Gamma_a=0.1$ and $m=5$, together with the normalized Bessel function of the same order. As shown by the calculations, the amount of "smearing" for a given mode increases with the value of Γ_a.

The propagation of radiation in a nonlinear medium is accompanied by the appearance of a number of specific effects for which there is no analog in the linear case. As shown in Refs. 3–5, in a medium with a local cubic nonlinearity (which includes the plasma in the present approximation), Bessel beams exhibit self-modulation of the electric field with a threshold. The parameter which determines the nonlinear self-focusing adjustment is the power enclosed in each annular region of the radial profile (9), which in the dimensionless variables (10) is proportional to the maximum intensity of the field (9):

$$|\mathscr{E}^{(0)}|^2_{max}=\frac{|E^{(0)}(r^{(1)}_{max},z)|^2}{E_p^2}.$$

Calculations carried out for hollow Bessel beams reveal that the self-modulation effect arises, as in the case of the Bessel beams of Refs. 3–5, at a power close to the critical value for self-focusing:

$$|\mathscr{E}^{(0)}|^2_{max}=|\mathscr{E}|^2_c=\frac{1}{\beta},$$

where β is the dimensionless nonlinearity coefficient in Eq. (11). Figure 4 displays the dependence of the threshold parameter P_{th} for the self-modulation effect,

$$P=\frac{|\mathscr{E}^{(0)}|^2}{|\mathscr{E}|^2_c}=\beta|\mathscr{E}^{(0)}|^2$$

as a function of the order m of the mode. This dependence shows that the power enclosed in the annular regions of the beam needed for self-modulation to occur increases monotonically with the order of the mode.

The principal difference between the growth of the instability in hollow Bessel beams and in Bessel beams should be pointed out: while self-modulation in the latter

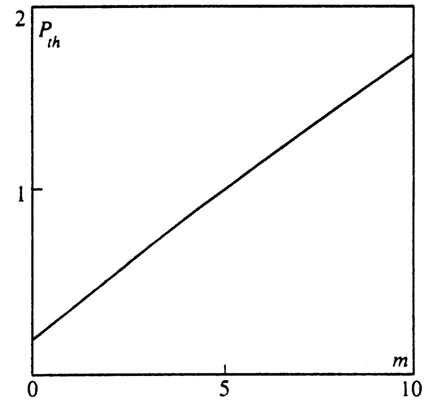

FIG. 4. Threshold value of the parameter P (beam power) as a function of the mode order m.

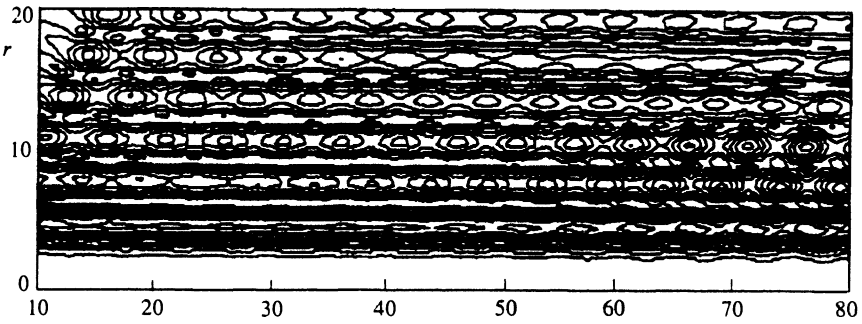

FIG. 5. Spatial profile of the field (contours of $|\mathscr{E}|/|\mathscr{E}|_{max}$) for $m=5$ and $P/P_{th}=1.25$.

initially occurs at the beam axis and propagates toward the peripheral region as the power increases, in hollow Bessel beams the opposite pattern is observed; as the power increases the instability develops at the beam periphery and propagates toward the center, and the strongest modulation is observed not near the principal maximum of the field as in Bessel beams, but is displaced toward the higher-order maxima. In Fig. 5, which displays contours of constant $|\mathscr{E}|/|\mathscr{E}|_{max}$ (here $|\mathscr{E}|_{max}$ is the maximum value of the amplitude in a given range of r and z), it is clear that for $m=5$ and $P/P_{th}=1.25$, the region where the modulation is largest in the present case is near $r\simeq 11$ and corresponds to the location of the third maximum of the field.

The nonlinearity of the medium exerts a strong influence on the spatial structure of the electric field of the beam, substantially changing both the radial and the longitudinal profiles of the amplitude. From Fig. 6, which shows $|\mathscr{E}|/|\mathscr{E}|^{(1)}_{max}$ as a function of radius for $m=5$, it is clear in particular that the radial beam profile no longer has a Bessel structure, although the general behavior of the profile (quasiperiodicity) is retained. This figure also shows that in a nonlinear medium there is a displacement of the first maximum of the field toward smaller values of r; the magnitude of this shift is larger, the higher the order of the mode. An illustration of the quantitative nature of this displacement is seen in Fig. 7a, where $r^{(1)}_{max}$ is plotted versus the mode number for $z=20$ in the linear ($P=0$) case as trace *1*, and the nonlinear case (close to the threshold $P\simeq P_{th}$ for self-modulation) is plotted as trace 2. Figure 7b shows $r^{(1)}_{max}$ as a function of P for $m=5$ and 10, from which it follows that with increasing P (or, what is the same thing, beam power), a monotonic shift $r^{(1)}_{max}$ toward smaller values of the radius occurs for both modes, which implies that there is no threshold (in P) for the change in the radial structure of the field.

4. CONCLUSION

On the basis of the foregoing analysis we can assert that an optical system consisting of a phase screw and axicon permits beams of electromagnetic radiation to form with a radial electric-field profile in a linear nondissipative medium which is described by the Bessel function $J_m(kr\sin\gamma)$ of order $m>0$ equal to the parameter of the phase screw. These are the so-called hollow Bessel beams. If the medium is absorbing and nonlinear, then this brings about a change in the Bessel structure of the beam radial profile. However, the characteristic behavior of such beams (the presence near the axis of electric fields that are close to zero), which fundamentally distinguishes them from Bessel beams (whose field peaks on axis), is also preserved in this case, at least in the range of the parameters P and Γ_a treated, corresponding to the beam power and radiation absorption respectively. Self-modulation of the electric field occurs for hollow Bessel beams at higher power levels than for Bessel beams, and in contrast to the latter the greatest modulation is observed not at the principal maximum of the amplitude, but is displaced toward the higher-order maxima.

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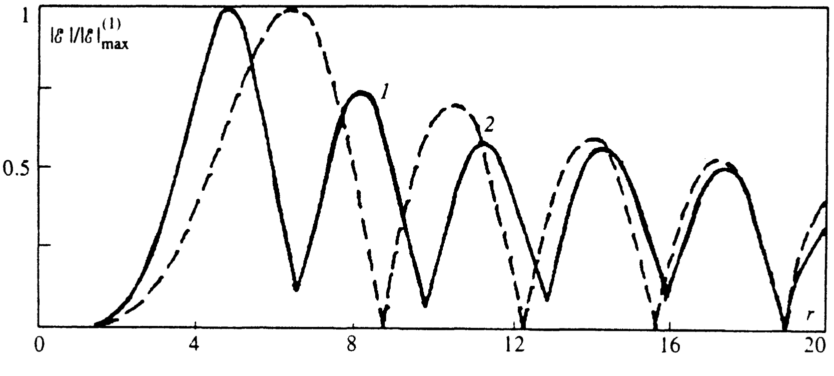

FIG. 6. Radial profile of the field near the self-modulation threshold $P\simeq P_{th}$, for $m=5$, $z=20$; *1*—$|\mathscr{E}|/|\mathscr{E}|_{max}$, *2*—$|J_5|/|J_5|^{(1)}_{max}$

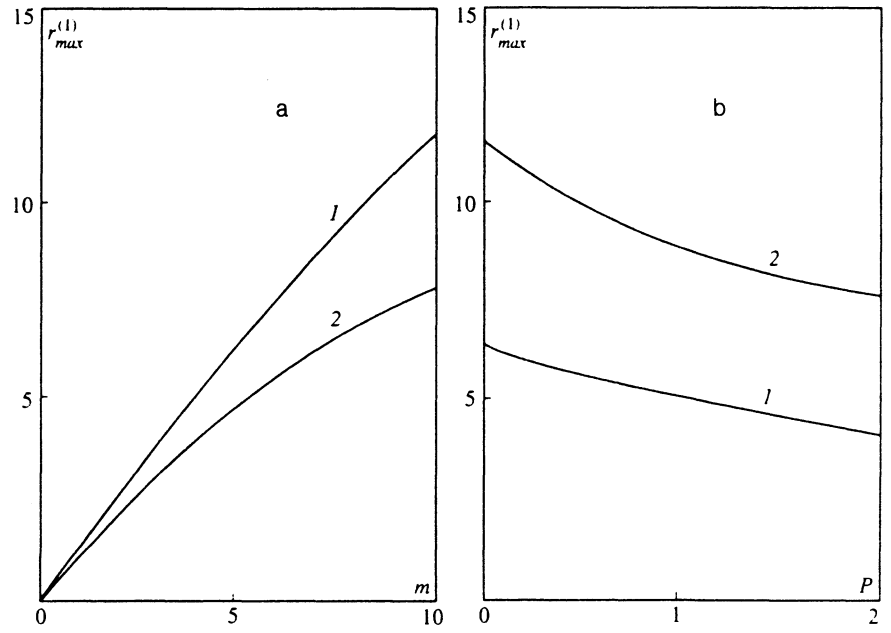

FIG. 7. Radial location of the first field maximum as a function of the order of the mode (a) and of the parameter P (b); *a*: *1*) linear solution of Eq. (9); *2*) nonlinear solution close to the self-focusing threshold $P \simeq P_{th}$, b: $1-m=5$, $2-m=10$.

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