Electromagnetic excitation of ultrasound in crystals with helical magnetic structure

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A theoretical study is made of the generation of ultrasound by an electromagnetic wave in the helical phase of a hexagonal magnetic material in an external basal-plane magnetic field. Expressions for the amplitudes of the excited elastic waves and for the coefficient of transformation of electromagnetic into elastic waves are derived for weak magnetic fields, in which a slight distortion of the antiferromagnetic helix occurs, and for strong magnetic fields in the domain of existence of the fan phase. It is shown that in weak fields, longitudinal and transverse ultrasound are excited on an equal footing, whereas in strong fields only transverse ultrasound is excited. Results agree well with the available experimental data on ultrasound excitation in dysprosium. From this comparison it is concluded that domain walls exist between the helical and fan phases.

1. INTRODUCTION

There has recently been much research on the electromagnetic excitation of ultrasound in the magnetic rare-earth metals over a wide range of temperatures and magnetic fields.[1] References 2 and 3, for example, present an experimental study of the efficiency of electromagnetic–acoustic transformation (EMAT) in gadolinium and dysprosium single crystals. It has been shown that the EMAT, depending as it does on the magnetoelastic interaction, is most effective near Curie points and in the spin reorientation region, i.e., at sharp changes in the magnetization and susceptibility of the magnetic medium. Theoretically, this question has been studied[2,3] in the region of orientational phase transitions (OPT) only for the collinear phase of these complex magnetic substances. For the helical phase, the vicinity of the OPT has not yet been explored.

It is known that in most rare-earth magnetics there exist helical magnetic structures at certain temperatures.[4] The ground state of such magnetic phases is characterized by the occurrence of an inhomogeneous magnetization and elastic strains over the entire volume of the crystal.[5,6] On placing such a crystal into an external magnetic field perpendicular to the helical axis, a field-directed magnetization occurs. In weak fields there is a slight distortion of the helical structure and the total magnetization is small. In strong fields, the magnetic develops a fan structure and the magnetization grows dramatically and rapidly approaches its saturation level as the magnetic field is increased. In intermediate fields a phase transition takes place, from the distorted simple helix to the fan magnetic structure.[4] Thus, the behavior of the magnetization—and hence the susceptibility—of rare-earth magnetics in an external magnetic field is substantially different for the helical magnetic structure and the ferromagnetic phase. It should therefore be expected that EMAT processes will exhibit some special features in helical magnetics.

In this paper, a theoretical study is made of the magnetoelasticity-assisted electromagnetic excitation of ultrasound in the helical phases of magnetic rare-earth metals subject to a basal-plane magnetic field. The results are compared with the available experimental data.[3]

2. ENERGY AND THE GROUND STATE

The rare-earth metals have hexagonal crystal structure. The free energy of such a magnetic has the form[1]

$$W=\frac{1}{V}\int_V dV\left\{\frac{1}{2}a\mathbf{M}^2+\frac{1}{4}b\mathbf{M}^4-\frac{1}{2}\beta_1M_z^2-\frac{1}{4}\beta_2M_z^4+\frac{1}{2}\beta_6(M_+^6+M_-^6)+\frac{1}{2}\alpha_\parallel\left(\frac{\partial\mathbf{M}}{\partial Z}\right)^2+\frac{1}{2}\alpha_\perp\left[\left(\frac{\partial\mathbf{M}}{\partial X}\right)^2+\left(\frac{\partial\mathbf{M}}{\partial Y}\right)^2\right]+\frac{1}{2}w\left(\frac{\partial^2\mathbf{M}}{\partial Z^2}\right)^2-\mathbf{M}(\mathbf{H}+\mathbf{h})+\gamma_{ijkl}M_iM_jU_{kl}+c_{ijkl}U_{ij}U_{kl}\right\}. \quad (1)$$

Here $\mathbf{M}$ is the magnetization and $M_\pm=M_x\pm iM_y$ are its circular components; V is the volume of the crystal; a, b, α, w, β, γ, and c are the uniform-exchange, nonuniform-exchange, anisotropy, magnetostriction, and elastic constants, respectively; $\mathbf{H}$ ($\mathbf{h}$) is the static (alternating) external magnetic field; U_{ij} is the strain tensor.

As an example, consider the following mutual orientation of the magnetic fields strengths ($\mathbf{H}$ and $\mathbf{h}$) and the wave vector of the excited waves ($\mathbf{k}$) relative the crystal axes a, b, c of the magnetic (c being the six-fold symmetry axis):

$$\mathbf{k}\|\mathbf{y}\|\mathbf{b}\|\mathbf{n},\mathbf{H}\|\mathbf{h}\|\mathbf{x}\|\mathbf{a}\,,$$

where the vector $\mathbf{n}$ is normal to the surface of the crystal occupying the half-space $y\geqslant 0$.

Suppose that in its ground state the magnetic has magnetization and strain nonuniformities only along the $\mathbf{Z}\|\ \mathbf{c}$ axis, i.e., $\mathbf{M}=\mathbf{M}(Z)$, $U_{ij}=U_{ij}(Z)$. In this case the equilibrium values of $\mathbf{M}$ and $U_{ij}(Z)$ can be found by the method developed in Ref. 5. Let us determine the ground

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state of the crystal by neglecting its basal plane anisotropy ($\beta_6=0$). The equilibrium values of the magnetization components can be written in the form

$$M_z=0, \quad M_{\pm}=\sum_{n=-\infty}^{+\infty} M_n e^{\pm iqnz}, \tag{2}$$

where q is the wave vector of the magnetic helix. In the region of weak magnetic fields, it suffices to keep the terms $n=0$, 1, 2 in the series (2) ($M_0 \sim M_2 \sim H, M_1 \gg M_0, M_2$), whereas in strong fields we can restrict ourselves to the $n=0$, ± 1 terms ($M_{-1}=-M_1, M_0 \gg M_{\pm 1}$).

Because of the difficulty in finding the solution for intermediate fields when all terms of the series (2) must be considered, we restrict our consideration to the cases of weak and strong fields.

Expressions for q and M_n can be obtained from the condition that the total energy (1) have a minimum.

For weak fields

$$q^2=q_0^2(1-12M_2^2/M_1^2), \quad q_0^2=-\alpha_{\parallel}/2w,$$

$$M_1^2=-L(q)/\tilde{b},$$

$$M_0=\frac{H[L(2q)+2b^*M_1^2]}{(a+2b^*M_1^2)(L(2q)+2b^*M_1^2)-\tilde{b}^2M_1^4},$$

$$M_2=-\tilde{b}M_0M_1^2/[L(2q)+2b^*M_1^2], \tag{3}$$

where

$$L(q)=a+\alpha_{11}q^2+wq^4, \quad \tilde{b}=b-\gamma_1^2/2c_{33},$$

$$b^*=\tilde{b}-c_{33}\tilde{\gamma}^2/2\Delta, \quad \Delta=c_{33}(c_{11}+c_{12})-2c_{13}^2,$$

$$\tilde{\gamma}=\gamma_{11}-\gamma_{12}+\gamma_0-c_{13}\gamma_1/c_{33} \tag{4}$$

γ_0, γ_1, and γ_{ij} are the exchange-magnetostriction and relativistic-magnetostriction constants, respectively. For the elastic and relativistic-magnetostriction constants, the conventional two-index notation is used.[7]

For strong fields

$$q^2=q_0^2, \quad aM_0+[b^*-(\gamma_{11}-\gamma_{12})^2/(c_{11}-c_{12})]M_0^3=H,$$

$$M_1^2=-\frac{L(q)+[b^*-(\gamma_{11}-\gamma_{12})^2/(c_{11}-c_{12})]M_0^3}{3\tilde{b}-2c_{33}\tilde{\gamma}^2/\Delta-(\gamma_{11}-\gamma_{12})^2/(c_{11}-c_{12})}. \tag{5}$$

The ground-state equilibrium strains are given by

$$U_{xx}^{(0)}-U_{yy}^{(0)}=-\frac{\gamma_{11}-\gamma_{12}}{c_{11}-c_{12}}\sum_n M_nM_{-n},$$

$$U_{xx}^{(0)}+U_{yy}^{(0)}=-\frac{c_{33}}{\Delta}\tilde{\gamma}\sum_n M_n^2,$$

$$U_{zz}^{(0)}=-\frac{c_{13}}{c_{33}}(U_{xx}^{(0)}+U_{yy}^{(0)})-\frac{\gamma_1}{2c_{33}}M_+M_-,$$

$$U_{xy}^{(0)}=U_{yz}^{(0)}=U_{zx}^{(0)}=0. \tag{6}$$

3. DISPERSION RELATIONS

Now let us consider the oscillations of magnetization, elastic displacement, and electromagnetic field in the vicinity of the above equilibrium position. To this end we express these quantities in the form[4]

$$F=F^{(0)}+f, \quad f=e^{iky}\cdot\sum_{n=-\infty}^{n=+\infty} f_n e^{iqnz}, \tag{7}$$

where $F^{(0)}=M^{(0)}$, $U^{(0)}$, $H^{(0)}$, and $f=m,u,h$ are the small deviations of these quantities from their equilibrium values. Upon linearization near the equilibrium position (2)–(6), the coupled system of the Landau–Lifshitz, elasticity, and Maxwell equations, assuming the zero harmonic amplitudes (f_0) are largest, has the following form:

for weak fields

$$\omega_1 m_0^z+(1/2)g\gamma_{44}kM_1\cdot(M_0-M_2)\cdot u_0^z-gM_1h_0^z=0,$$

$$\omega_2 m_0^+-g\cdot(\gamma_{11}-\gamma_{12})\cdot kM_0M_1\cdot(u_0^x+iu_0^y)$$
$$-(1/2)gM_1h_0^x=0,$$

$$(\omega^2-S_{1t}k^2)\cdot u_0^x+(1/2\rho)\cdot k\cdot(\gamma_{11}-\gamma_{12})\cdot M_0m_0^+=0,$$

$$(\omega^2-S_{1t}k^2)\cdot u_0^y+(i/2\rho)\cdot k\cdot(\gamma_{11}-\gamma_{12})\cdot M_0m_0^+=0,$$

$$(\omega^2-S_{2t}k^2)\cdot u_0^z=0,$$

$$(k^2-2i/\delta^2)\cdot h_0^x-4\pi i m_0^+/\delta^2=0,$$

$$(k^2-2i/\delta^2)\cdot h_0^x-8\pi i m_0^z/\delta^2=0, \tag{8}$$

for strong fields

$$\omega_1 m_0^z+\omega\cdot m_0^+-gM_0h_0^z=0, \quad \omega_2 m_0^++\omega\cdot m_0^z-gkM_0^2$$
$$\cdot(\gamma_{11}-\gamma_{12})\cdot(1-2M_1^2/M_0^2)\cdot u_0^x=0,$$

$$(\omega^2-S_{1t}k^2)\cdot u_0^x+(1/\rho)k\cdot(\gamma_{11}-\gamma_{12})\cdot M_0m_0^+$$
$$\cdot(1-4M_1^2/M_0^2)=0,$$

$$(\omega^2-S_{1t}k^2)\cdot u_0^y-(4i/\rho)\cdot k\cdot(\gamma_{11}-\gamma_{12})$$
$$\cdot M_1^2m_0^+/M_0=0, \quad (\omega^2-S_{2t}k^2)\cdot u_0^z=0,$$

$$(k^2-2i/\delta^2)\cdot h_0^x+16\pi iM_1^2m_0^+/\delta^2M_0^2=0,$$

$$(k^2-2i/\delta^2)\cdot h_0^z-8\pi i m_0^z/\delta^2=0. \tag{9}$$

In Eqs. (8) and (9), δ is the thickness of the skin layer in a nonmagnetic metal, ρ is the density of the magnetic, g the gyromagnetic ratio; $S_{1t}^2=(c_{11}-c_{12})/2\rho$, $S_{11}^2=c_{11}/\rho$, $S_{2t}^2=c_{44}/\rho$ are the squares of the velocities of the noninteracting sound waves; ω is the frequency of the electromagnetic wave incident on the magnetic [$h\sim\exp(-i\omega t)$]. Expressions for the spin oscillation frequencies ω_1 and ω_2 are of the form:

in weak fields

$$\omega_1=gM_1\cdot(\tilde{\beta}_1-\alpha_{\parallel}q^2-wq^4+\alpha_{\perp}k^2+\gamma_{44}^2M_1^2/c_{44}),$$

$$\omega_2=gM_1\cdot(\alpha_{\parallel}q^2+7wq^4+\alpha_{\perp}k^2+\pi), \tag{10}$$

in strong fields

$$\omega_1=gM_0\cdot(\tilde{\beta}_1-\alpha_\perp k^2+H/M_0+\gamma_{44}^2M_0^2/c_{44}),$$

$$\omega_2=gM_0\cdot[\alpha_\perp k^2+4\pi\cdot(1-2M_1^2/M_0^2)+H/M_0+2\cdot(\gamma_{11}-\gamma_{12})\cdot M_0^2/(c_{11}-c_{12})]. \quad (11)$$

Here $\tilde{\beta}_1$ is the magnetostriction-renormalized first constant for uniaxial magnetic anisotropy.[1]

From the system (8) it follows that in a weak magnetic field the electromagnetic wave h_x (polarization along the x axis) excites spin oscillations m_0^+ and ultrasound oscillations u_0^y (longitudinal) and (u_0^x) (transverse) in the magnetic, whereas the electromagnetic wave h_z (polarization along z) excites only spin oscillations m_0^z. The transverse z-polarized ultrasound in this case is not excited. From (8) one can also see that in weak fields the EMAT efficiencies are comparable for longitudinal and transverse ultrasound.

In strong fields [Eqs. (9)], to a first approximation in powers of $M_1/M_0\ll 1$, the electromagnetic wave h_x does not excite spin or elastic oscillations in the magnetic. It is only in the second approximation in powers of M_1/M_0 that this component of the alternating magnetic field may have an influence on the EMAT processes. Spin and elastic vibrations (the latter appearing as transverse ultrasound u_0^x) are in this case excited by the electromagnetic wave h_z.

The systems of equations (8) and (9) determine the dispersion relations of the coupled electromagnetic, spin, and elastic waves. Under the assumption that the frequency of the excited oscillations is much less than the frequencies of the uniform precession of magnetization ($\omega\ll\omega_{1,2},\sqrt{\omega_1\omega_2}$), and neglecting the spin wave dispersion ($\alpha_\perp k^2\ll\tilde{\beta}_1,\pi$), the dispersion relations have the following form:

in the weak-field case

$$(k^2-k_t^2)(k^2-k_l^2)(k^2-k_e^2)-\xi_lk^2(k^2-k_t^2)(k^2-k_e^2)-\xi k^2(k^2-k_l^2)(k^2k_e^2)-\pi\chi k_e^2(k^2-k_t^2)(k^2-k_l^2)=0, \quad (12)$$

where

$$k_{l,t}=\omega/S_{1l,1t},\quad k_e^2=2i/\delta^2,\quad \chi=gM_1/\omega_2,$$

$$\xi_{t,l}=(\gamma_{11}-\gamma_{12})^2M_0^2\chi/2\rho S_{1t,1l}^2. \quad (13)$$

In the case of strong fields, to a first approximation in powers of M_1/M_0,

$$(k^2-k_t^2)(k^2-k_e^2)-\xi_tk^2(k^2-k_e^2)-4\pi\chi k_e^2(k^2-k_t^2)+4\pi\xi_t\chi k^2k_e^2=0, \quad (14)$$

where

$$\chi=gM_0/\omega_1,\quad \xi_t=(\gamma_{11}-\gamma_{12})^2M_0^2\chi\omega_1/\rho S_{1t}^2\omega_2. \quad (15)$$

The solutions of the dispersion relations are the following wave number values:

weak fields

$$k_{1,2}^2=k_{t,l}^2\begin{cases}(\mu-\xi_{t,l})/(\mu-\xi), & \beta_{t,l}\ll 1\\ (1-\xi_{t,l})/(1-\xi), & \beta_{t,l}\gg 1\end{cases},$$

$$k_3^2=k_e^2\begin{cases}(\mu-\xi)/(1-\xi), & \beta_{t,l}\ll 1\\ \mu, & \beta_{t,l}\gg 1\end{cases}, \quad (16)$$

where $\mu=1+\pi\chi$, $\xi=\xi_l+\xi_t$, $\beta_{t,l}=\beta^2\omega^2/S_{1t,1l}^2$. In writing expressions (16) it has been taken into account that the magnetoelastic coupling parameters $\xi_{t,l}$ are small in weak fields, i.e., $\xi_{t,l}\ll 1$.

Strong fields

$$k_1^2=k_t^2/(1-\xi_t),\quad k_2^2=\mu k_e^2, \quad (17)$$

where $\mu=1+4\pi\chi$. The wave numbers $k_{1,2}$ in (16) and k_1 in (17) correspond to quasielastic waves, while k_3 in (16) and k_2 in (17) correspond to quasielectromagnetic waves.

4. EMAT COEFFICIENTS AND DISCUSSION OF RESULTS

The systems (8) and (9) together with the boundary conditions for the stress tensor and the electromagnetic-field and magnetization vectors at the free surface $Y=0$ of the magnetic [the magnetization condition can be ignored because the adopted approximation neglects the nonuniform exchange in the energy expression (1)] enable the amplitudes of the excited ultrasound oscillations to be expressed in terms of the amplitudes of electromagnetic waves h_{0x} and h_{0z} incident on the magnetic. For weak fields and $\beta\ll 1$ (we consider here this case only, as the one usually realized experimentally),

$$u_0^{x,y}=\frac{\delta^2M_0(\gamma_{11}-\gamma_{12})h_{0x}k_{1,2}(k_3^2-k_8^2)(k_{l,t}^2-k_{1,2}^2)(k_3^2-k_{2,1}^2)}{4\pi\rho S_{1t,1l}^2(k_{l,t}^2-k_3^2)(k_{t,l}^2-k_3^2)(k_{2,1}^2-k_{1,2}^2)}. \quad (18)$$

In strong fields

$$|u_0^x|=\frac{\omega}{\omega_1}\left(\frac{c}{S_{1t}}\right)^2\frac{(\gamma_{11}-\gamma_{12})\chi M_0h_{0z}}{2\pi\rho\sigma S_{1t}\mu\cdot(1-\xi_t)^{3/2}(1+\tilde{\beta}_t)^{1/2}}, \quad (19)$$

where $\tilde{\beta}_t=\beta_t/\mu^{1/2}$ and σ is the conductivity of the magnetic.

The coefficient of transformation of electromagnetic into ultrasound waves (EMAT efficiency) will be defined as the ratio of the acoustic to electromagnetic power fluxes at the magnetic boundary $Y=0$ (Ref. 1). In weak fields for the EMAT efficiency we obtain

$$\eta_{t,l}=\frac{\pi}{4}\frac{\delta^4\omega^3M_0^2(\gamma_{11}-\gamma_{12})^2\chi^2}{\rho cS_{1t,1l}^4}\frac{k_{1,2}(k_{l,t}^2-k_{1,2}^2)}{(k_{2,1}^2-k_{1,2}^2)}\times\begin{cases}(\mu-\xi)^{-2}, & \beta_{t,l}\ll 1\\ \dfrac{k_{1,2}^2}{\delta^4\mu^2k_{t,l}^8}, & \beta_{t,l}\gg 1\end{cases}. \quad (20)$$

In the strong-field case, the transformation coefficient has the form

$$\eta_t=\left(\frac{c}{S_{1t}}\right)^2\left(\frac{\omega}{\omega_1}\right)^2\frac{(\gamma_{11}-\gamma_{12})^2\chi^2M_0^2\omega^2}{\rho\sigma^2S_{1t}^2\mu^2(1-\xi_t)^{5/2}(1+\tilde{\beta}_t)}. \quad (21)$$

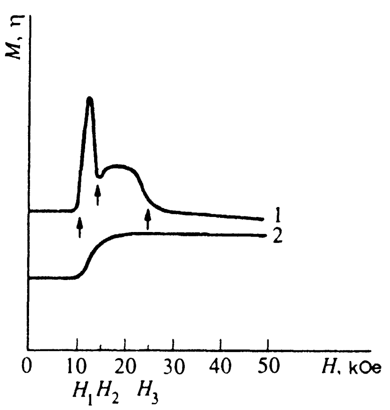

FIG. 1. Experimental dependences of the EMAT efficiency η (curve 1) and of the magnetization M (curve 2) on the magnetic field in dysprosium at $T=173$ K (Ref. 3). The magnetic fields marked by arrows correspond to the phase transitions "simple helix"–intermediate (domain) phase (H_1); intermediate phase–fan (H_2); and fan–collinear phase (H_3).

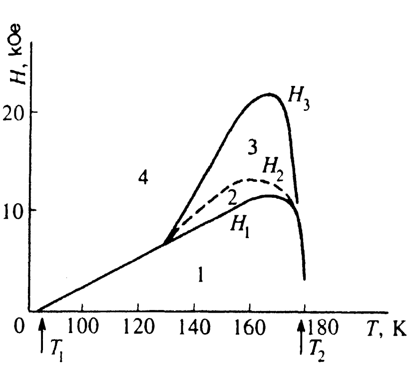

FIG. 2. Phase $H-T$ diagram for Dy for H|| **a**. Phases: 1) "simple helix"; 2) intermediate (domain) phase; 3) fan; 4) collinear phase. Curves H_1, H_2, H_3 correspond to the phase transitions.

From (20) it follows that the weak-field EMAT efficiency is directly proportional to the square of M_0, which is in turn proportional to the magnetic field intensity, Eq. (3). Thus, in weak magnetic fields the coefficient of transformation of electromagnetic into ultrasound waves, Eq. (20), is small. The EMAT efficiencies for longitudinal and transverse ultrasound turn out to be comparable in weak fields. The experimental dependence $\eta(H)$ for dysprosium in the domain of existence of its helical phase[3] (Fig. 1) shows that up to fields of 5–10 kOe, both the magnetization and the EMAT efficiency are small for the longitudinal sound. The experimental data are described well by the formula (20).

In strong magnetic fields, we have only the generation of transverse ultrasound, Eq. (21). Longitudinal ultrasound is not excited in this case. This theoretical result is also consistent with the experimental data,[3] which show that the dysprosium EMAT efficiency for longitudinal ultrasound is close to zero for $H>20$ kOe (Fig. 1). From (21) it can be seen that while the transverse-ultrasound EMAT efficiency is nonzero for strong fields, it is still low because of the small parameters $(\omega/\omega_1)^2 \ll 1$ and $\chi \ll 1$ in this formula. It follows that in strong fields the generation of transverse ultrasound will also be ineffective (note that experiments on transverse sound generation are not available).

In intermediate fields, it has been noted earlier that an analytic expression for $\eta(H)$ is not possible. However, if one assumes that (20) holds even for fields at which the transition from the (intermediate) (Fig. 2) phase of distorted simple helix to the fan phase occurs, and the incipient fan phase region is also covered, then this formula makes it possible to explain the longitudinal ultrasound generation peak observed experimentally in intermediate fields in dysprosium.[3]

To see this, note that according to (20), the field dependence of the EMAT efficiency is basically determined by the H dependences of the zeroth magnetization harmonic, M_0, and the dynamic susceptibility, χ. Experimental data[3] indicate a sharp increase of M_0 in intermediate fields (Fig. 1). On the other hand, the weak-field dynamic susceptibility is virtually independent of the magnetic field [Eqs. (10), (13)]. However, for intermediate and strong fields, χ depends on H and decreases as H is increased [see the expression (11) for ω_2]. Thus, for intermediate fields one expects that as H is increased, the EMAT should first sharply increase owing to the rapid growth of M_0, and then, in strong fields, should fall off because of the decrease in the dynamic susceptibility χ. This agrees well with the experimental results of Ref. 3. Note that the discussion above refers, in our view, to the second peak in the $\eta(H)$ dependence (Fig. 1). As for the first peak, this is presumably due to the effective generation of ultrasound owing to the displacement of boundaries between the domains separating the distorted helical structure from the fan phase. An analogous peak associated with domain wall displacements in the collinear ferromagnetic phase in dysprosium has been observed experimentally and explained theoretically in Ref. 3 [see formula (16) therein]. The existence of domain walls between the helical and fan phases was predicted theoretically in Ref. 8. Experimentally, though, these walls have not been observed. It follows that the first peak in the dysprosium $\eta(H)$ dependences of Ref. 3 may be accounted for by the ultrasound generation due to the domain wall displacements between the helical and fan phases. Thus, the occurrence of this peak may serve as a confirmation of the very existence of domain walls between the helical and fan phases in helical magnetics. Note that we can employ Eq. (16) of Ref. 3 to qualitatively describe the peak due to domain wall displacements in the helical phase of dysprosium, because the derivation of this formula is carried out without specifying any particular domain structure.

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