Effect of radiative friction on the phase space of a beam moving in a free electron laser

A. V. Serov

P. N. Lebedev Physical Institute, Russian Academy of Sciences, 117924 Moscow, Russia

(Submitted 20 April 1993; resubmitted 1 December 1993)
Zh. Eksp. Teor. Fiz. **105**, 494–498 (March 1994)

The effect of the reaction forces due to spontaneous emission on the particle dynamics in a free electron laser is considered. It is shown that the function which describes the effective radiative friction is proportional to the derivative with respect to energy (or frequency) of the laser gain or the second derivative with respect to the spectral intensity of the spontaneous emission. It is shown that to second order the reduction in area of each element of phase space produced by radiative friction is independent of its position in the beam phase space.

The interaction between an electron beam and an electromagnetic wave in a free electron laser (FEL) not only reduces the average energy of the beam but also substantially changes the electron phase and energy distributions. Generally speaking,[1] these distributions are largely responsible for determining the efficiency with which electron energy is converted into radiation, and in particular, determine in principle whether it is possible to use a single circulating beam repeatedly in a FEL.

It is usual in describing the action of a FEL to neglect the effect of radiative reaction on the particle motion.[1] However, the radiative friction, although relatively small, introduces certain features into the electron dynamics which (aside from their theoretical interest) can turn out to be of practical importance in certain cases.

It is natural that including the radiative forces is important in those cases in which they are comparable with the action of the external electromagnetic field, e.g., in the initial stage when lasing begins in the FEL, when the wave field in the laser cavity is weak. Assume that a relativistic electron beam with charge e and energy γmc^2 propagates through a undulator in the field of an external electromagnetic wave. We assume the electrons to be initially uniformly distributed in phase, and we assume the energy distribution to be quasi-monoenergetic with a width $\Delta\gamma \ll \gamma$. When the beam interacts with an external wave the electrons undergo acceleration or deceleration, depending on the initial phase of the field, which gives rise to bunching. The effect on the phase motion of the radiative interaction of the particles which have already been grouped in bunches with longitudinal dimensions on the order of or less than the wavelength of the laser radiation has been studied by Serov.[2]

We will consider the initial stage of electron motion in a FEL and study the effect of radiative friction produced by spontaneous emission. Here we will neglect the change in the radiative friction due to particle bunching. The change in the energy of an electron moving through the FEL is always much less than its initial energy, so that the radiative force can be expanded in powers of the difference between the particle energy and the equilibrium value. Since the decelerating force due to the radiation of a relativistic particle is proportional to the square of its energy, electrons with larger values of energy are decelerated more rapidly. The effect of the radiative friction force acting on a particle with a relative energy shift $\mu = 4\pi N(\gamma - \gamma_r)/\gamma_r$, where γ_r is the electron equilibrium energy and N is the number of undulator periods, can be written in the form $w_r(\mu) = w_r(0) + \mu(dw_r/d\mu)$.

Since we have $w_r(0) \gg \mu dw_r/d\mu$, we can assume that the second term is small. The equations which describe the change in the electron energy when it interacts with the wave field, taking into account the radiative friction forces, assume the form

$$\frac{d\mu}{d\tau} = -w_r - \Omega^2(\sin\varphi + \mu\alpha), \qquad \frac{d\varphi}{d\tau} = \mu, \tag{1}$$

where φ is the phase, $\tau = ct/N\lambda$ is the dimensionless time, λ is the spatial period of the undulator, $\Omega = e\lambda\sqrt{E_w H_u}/mc^2\gamma$ is the frequency of small oscillations in phase, E_w is the strength of the electromagnetic field, H_u is the strength of the magnetic field of the undulator, and w_r is the work done by the radiative friction forces and in this treatment is proportional to the work done by the radiation force when a single electron moves through the undulator: $w_{r1} = (8\pi/3)e^4 H_u^2 N^2 \lambda\gamma^2/m^3c^6\gamma_r$, and $\alpha = \Omega^{-2} dw_r/d\mu$. Equations (1) differ from the standard FEL equations[3–5] in the terms $-w_r$ and $-\Omega^2\mu\alpha$ on the right-hand side of the first equation, which take into account the effects of the radiative reaction forces. In most cases we have $\Omega^2 \ll 1$, and this quantity can be used as a small expansion parameter in studying Eqs. (1).

The solutions of equations of the form (1), neglecting radiative friction, describe the change in energy μ and phase φ of an individual electron in the zeroth approximation $(\mu^{(0)},\varphi^{(0)})$, the first approximation $(\mu^{(1)},\varphi^{(1)})$, and the second approximation $(\mu^{(2)},\varphi^{(2)})$. Only in second order does the average change in beam energy (with respect to the initial phase φ_0) over the length of the undulator $(\tau = 1)$ differ from zero; it is given by the expression

$$\langle\mu\rangle=\Omega^4\left(\frac{\cos\mu_0-1+\frac{\mu_0}{2}\sin\mu_0}{\mu_0^3}\right), \qquad (2)$$

where μ_0 is the initial value of the energy mismatch. The expression in parentheses, which relates the change in beam energy to the initial electron energy, determines the laser gain $G\propto\langle\mu\rangle$. In Refs. 6 and 7 it is shown that

$$G\propto dE_\omega/d\mu_0, \qquad (3)$$

where E_ω is the spectral density of the spontaneous emission.

In evaluating the effect of the radiative forces on the particle motion we will retain the term $w_r(0)$ in finding the solutions in the zeroth approximation and the term $\mu(dw_r/d\mu)$ in finding the higher-order solutions. Including the action of the forces of radiative friction in solving Eqs. (1) leads to additional terms in the expressions which describe the electron trajectory in phase space. We write down these additional terms only for solutions of zeroth and first order:

$$\mu_r^{(0)}=-w_r\tau,\quad \varphi_r^{(0)}=-w_r\tau^2/2,$$

$$\mu_r^{(1)}=\Omega^2 w_r(0)\left\{\frac{\tau}{\mu_0^2}\cos(\mu_0\tau+\varphi_0)+\frac{\tau^2}{2\mu_0}\sin(\mu_0\tau+\varphi)\right.$$
$$\left.-\frac{1}{\mu_0^3}\left[\sin(\mu_0\tau+\varphi_0)-\sin\varphi_0\right]\right\}-\alpha\mu_0\Omega^2\tau, \qquad (4)$$

$$\varphi_r^{(1)}=\Omega^2 w_r(0)\left\{\frac{3}{\mu_0^4}\left[\cos(\mu_0\tau+\varphi_0)-\cos\varphi_0\right]\right.$$
$$+\frac{2\tau}{\mu_0^3}\sin(\mu_0\tau+\varphi_0)-\frac{\tau^2}{2\mu_0^2}\cos(\mu_0\tau+\varphi_0)$$
$$\left.+\frac{\tau}{\mu_0^3}\sin\varphi_0\right\}-\frac{1}{2}\alpha\mu_0\Omega^2\tau^2.$$

In expressions (4) we have retained only terms containing $w_r(0)$ and $\alpha\mu_0$ to the first power and have dropped terms containing squares of these quantities and their products. The average change in the beam energy (with respect to the initial phase φ_0) over the length of the undulator is nonzero in zeroth and second orders. Omitting the lengthy intermediate steps, we write down the expression for the change in the average beam energy:

$$\langle\mu\rangle=-w_r(\mu)\tau+\Omega^4 f_0(\mu_0)+\Omega^4 w_r(\mu)\tau f_r(\mu_0), \qquad (5)$$

where

$$f_0=\frac{1}{\mu_0^3}\left[\cos(\mu_0\tau)-1+\frac{\tau\mu_0}{2}\sin(\mu_0\tau)\right], \qquad (6)$$

$$f_r=\frac{1}{\mu_0^4}\left[\frac{3}{2}\cos(\mu_0\tau)-3/2+\tau\mu_0\sin(\mu_0\tau)-\frac{\mu_0^2\tau^2}{4}\cos(\mu_0\tau)\right] \qquad (7)$$

are the gain function f_0 and a new function f_r which describes the effect of the radiative friction. A plot of the radiative friction function (for $\tau=1$) is shown in Fig. 1 (curve 2), where for comparison we have also plotted the function f_0 (curve 1). From (5) it can be seen that the effect of the radiative friction on the particle dynamics is determined not only by the power of the spontaneous beam emission but also by the electron phase motion in the FEL. The first term in expression (5) is a function only of the emitted power due to the undulator oscillations of the particle, and the third term relates the radiation to the particle phase motion. If the power of the spontaneous emission decreases, then the amount of work done by the radiative friction forces approaches zero and expression (5) goes over to (2).

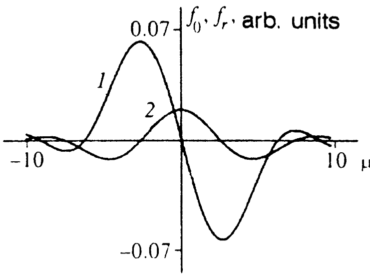

FIG. 1. Gain function f_0 (curve 1) and radiative friction function f_r (curve 2).

Comparison of expressions (6) and (7) shows that the gain and radiative friction functions in this case are related by

$$-2f_r(\mu_0,\tau)=\frac{df_0(\mu_0,\tau)}{d\mu_0}. \qquad (8)$$

Thus, the radiative friction function is related to the spontaneous emission spectral density by

$$f_r(\mu_0,\tau)\propto\frac{d^2E_\omega}{d\mu_0^2}. \qquad (9)$$

From (5) it follows that treating the radiative friction forces (independent of phase) gives rise to an additional term ΔG in the expression for the amplification of the radiation. This term is proportional to the work done by the radiative friction forces and the second derivative of the spontaneous emission spectral density. Depending on the initial value of the energy mismatch μ_0, the radiation force either increases or decreases the gain G. The effect of this term is large when the quantity w_r becomes comparable with unity. For example, for a continuous electron beam radiating incoherently as it moves through an undulator with period $\lambda=1.6$ cm, number of periods $N=750$, and optimum magnetic field $H=7.5$ kOe, the work done by the radiative frictional force is equal to $w_{r1}=5\cdot10^{-5}\gamma$. This force can evidently have a significant effect on the lasing process only for large electron energies $\gamma>10^3$.

However, the effect of radiative deceleration can enter at significantly lower electron energies if we use preformed electron bunches. In the general case, electrons emit spontaneously as they move through the undulator over a broad spectral range containing not only the fundamental frequency generated in the laser but also low-frequency radiation. In the low-frequency part of the spectrum the wavelength can become comparable with the dimensions of a bunch. The electrons in the bunch then can radiate quasicoherently, and the emitted power increases by up to a factor of n compared with the case of incoherent radiation, where n is the number of particles in a bunch. The power of this radiation can even exceed the power radiated at the fundamental wavelength. Consequently, the force of radiative deceleration acting on the FEL gain can be determined by spontaneous coherent low-frequency radiation from the bunch.

Another instance in which bunches are slowed down by spontaneous coherent long-wavelength radiation can be observed when a FEL radiates at its harmonics.[8] The magnetic field in the undulator of such a laser is much larger than the optimum value and the laser radiation is generated at higher harmonics of the fundamental frequency of the undulator radiation. Bunches can be used with parameters such that the radiation at the fundamental frequency is quasicoherent. Generally speaking, the reaction of such quasicoherent radiation is distributed in a complicated manner over the bunch and changes considerably in magnitude over a distance typically equal to the wavelength of the fundamental radiation.[9] At distances comparable to the radiation wavelength generated in such a FEL, the magnitude of the radiative force will change negligibly.

Expressions describing the change μ in energy and φ in phase of an individual electron are exact transformations of the variables μ_0, φ_0 into the variables μ, φ. They yield a description of the change in the beam phase space. A phase space element $d\mu_0 d\varphi_0$ is transformed into the element $d\mu d\varphi$, and the factor by which the area of the element changes is given by the Jacobian

$$\frac{\partial(\mu,\varphi)}{\partial(\mu_0,\varphi_0)}=D. \tag{10}$$

Substituting the expressions which describe the electron dynamics in phase space into Eq. (10), differentiating, and grouping terms of the same order, we find the value of D as a series in the parameter Ω^2. If we evaluate the Jacobian by substituting into (10) expressions describing the change in energy μ and phase φ neglecting the radiative reaction, it yields a value equal to unity, which implies that the area of the beam phase space is conserved. This result follows directly from the fact that the system of equations describing the electron dynamics in a FEL when radiative reaction is disregarded can be derived from a Hamiltonian,[4] which is obviously independent of time. If we take into account the additional radiation terms (4), then the determinant of the Jacobian takes the form

$$D^{(0)}=1, \quad D^{(1)}(\Omega^2)=-\alpha\Omega^2\tau, \quad D^{(2)}(\Omega^4)=0. \tag{11}$$

This result implies that the radiative friction forces reduce the area of phase space occupied by the particles. Note that the reduction in area is identical for all elements, regardless of their location in the beam phase space (the reduction in phase space does not depend on the initial phase and the initial energy mismatch), although both the change in energy and the change in the phase duration of each beam element depend on φ_0 and μ_0.

Thus, this calculation of the beam dynamics, taking into account the effect of the radiative frictional forces, enables us to clarify the relation between the change in the average beam energy and the properties of the spontaneous and induced radiation. We have studied the change in the area of the beam phase space resulting from the radiative friction.

I am grateful to A. N. Lebedev for posing this problem, and to G. D. Bogomolov for discussing the results and the writeup.

[1] T. Marshall, Free Electron Lasers, Macmillan, London (1985).

[2] A. V. Serov, Kratk. Soobshch. Fiz. FIAN No. 5-6, 7 (1992).

[3] A. A. Kolomenskii and A. N. Lebedev, Kvantovaya Élektron. (Moscow) **5**, 1543 (1978) [Sov. J. Quantum Electron. **8**, 879 (1978)].

[4] A. A. Varfolomeev, *Free Electron Lasers and Prospects for Developing Them* [in Russian], Kurchatov Atomic Energy Institute Press, Moscow (1980).

[5] M. V. Fedorov, Usp. Fiz. Nauk **135**, 213 (1981) [Sov. Phys. Usp. **24**, 801 (1981)].

[6] A. A. Kolomenskii and A. N. Lebedev, in Proc. 10th Intern. Conf. on High-Energy Charged-Particle Accelerators, Vol. 2, Protvino (1977), p. 446.

[7] J. M. J. Madey, Nuovo Cim. B **50**, 64 (1979).

[8] S. V. Benson and J. M. J. Madey, Phys. Rev. A **39**, 1579 (1989).

[9] A. N. Lebedev and A. V. Serov, Zh. Tekh. Fiz. **62** (8), 147 (1992) [Sov. Phys. Tech. Phys., **37**, 887 (1992)].

Translated by David L. Book