The polarization operator and the three-photon vertex in QED_{2+1} in a dense medium

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The polarization operator and the three-photon vertex in the one-loop approximation in QED_{2+1} in the presence of a chemical potential are calculated. It is shown that the presence of the dense medium leads to renormalization of the Chern–Simons mass. The magnetic attraction between two-dimensional charges due to the effective three-photon interaction is found to be modified.

1. INTRODUCTION

In recent years $2+1$-dimensional versions of quantum field theory (QFT) have attracted considerable attention. This is connected first of all with attempts at a theoretical description of such phenomena as high-temperature superconductivity (HTSC)[1] and the quantum Hall effect (QHE),[2] which occur in planar structures. In addition, in view of such nontrivial properties as the fractional charge[3] and anomalous statistics,[4] $2+1$-dimensional QFT is an interesting subject to study for its own sake. However HTSC and QHE have another significant feature in addition to their two-dimensional nature. These phenomena are macroscopic and are possible when the average value of the particle number is nonzero. Under these conditions interaction effects due to quantum fluctuations in the presence of a dense medium, could turn out to be significant. This work aims at the calculation and study of the properties of the polarization operator and the three-photon vertex in $2+1$-dimensional quantum electrodynamics (QED) in a dense medium described by a chemical potential μ. As will be seen in what follows, the terms in the one-loop expansion for the effective action of the electromagnetic field due to the three-photon diagram vanish in a dense medium. Consequently the number of calculated functions suffices to construct the effective action of the electromagnetic field in a dense medium.

2. POLARIZATION OPERATOR

We consider the partition function for QED_{2+1} with a chemical potential in Euclidean space

$$Z=\frac{1}{N}\int D\bar{\psi}\,D\psi\,D A\exp\left(-\int L_E dx_E\right), \tag{1}$$

where

$$L_E=\frac{1}{4\gamma}F_{\mu\nu}^2+\bar{\psi}(\hat{\partial}_E+m)\psi-ie\bar{\psi}\hat{A}\psi\,. \tag{2}$$

After integration over the fermions we obtain for the partition function the expression

$$Z=\frac{1}{N}\int D A\exp[-W_{\rm eff}(A)], \tag{3}$$

in which $W_{\rm eff}$ can be represented as a sum of one-loop diagrams,[5] Fig. 1. In this expansion, as a result of Furry's theorem, only the statistical part is nonvanishing in the first and third terms, and the quantum-field part of the second term is known and well-studied.[6] Consequently in what follows we will be only interested in the statistical part of $W_{\rm eff}(A)$.

The analytic expression, corresponding to the first term of the expansion under study, has the form

$$\Pi_\nu=-\frac{ie}{(2\pi)^3}\,{\rm Tr}\int\sigma_\nu G(p^*)d^3p, \tag{4}$$

where $\nu=1, 2, 3$, $G(p^*)$ is the electron propagator

$$G(p^*)=\frac{-i\hat{p}^*+m}{p^2+m^2}; \tag{5}$$

$$p_\alpha^*=\begin{cases} p_\alpha, & \alpha=1,\ 2\\ p_3-i\mu, & \alpha=3\end{cases} \tag{6}$$

(for three-dimensional Euclidean space $p_3=ip_0$), μ is the chemical potential and σ_1, σ_2, σ_3 are the Pauli matrices.

After evaluation of the trace and integration we obtain

$$\Pi_i=0,\qquad \Pi_3=-\frac{ie}{2\pi}(\mu^2-m^2)\theta(\mu^2-m^2), \tag{7}$$

where $i=1$, 2, and $\theta(\mu^2-m^2)$ is the Heaviside step function. We now go over to the calculation of the statistical part of the polarization operator $\Pi_{\alpha\beta}$. Its expression is of the form

$$\Delta\Pi_{\alpha\beta}(\mu)=\frac{e^2}{(2\pi)^3}\,{\rm Tr}\int d\mathbf{p}\{\theta(\mu-\varepsilon_p){\rm res}[\sigma_\alpha G(p^*)\times\sigma_\beta G(p^*+k)]|_{p_3=-i\varepsilon_p}+\theta(\mu-\varepsilon_{p+k})\times{\rm res}[\sigma_\alpha G(p^*)\sigma_\beta G(p^*+k)]|_{p_3=-i\varepsilon_{p+k}}\}, \tag{8}$$

where $\varepsilon_p=\sqrt{\mathbf{p}^2+m^2}$, $\varepsilon_{p+k}=\sqrt{(\mathbf{p}^2+\mathbf{k}^2)+m^2}$.

As a result of the calculations we obtain in the static limit ($k_3=0$) the following expressions for the components of $\Delta\Pi_{\alpha\beta(\mu)}$:

$$\Delta\Pi_{33}=\frac{e^2}{(2\pi)^2}\theta(\mu^2-m^2)[2I_3(\mathbf{k})-I_2+\mathbf{k}^2I_1(\mathbf{k})], \tag{9}$$

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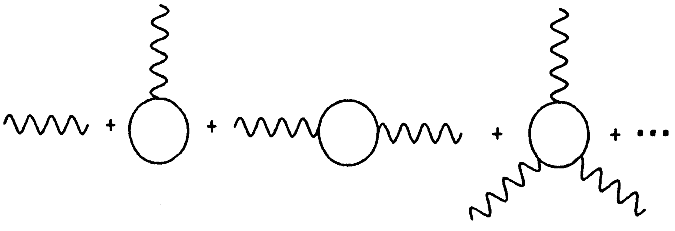

FIG. 1.

$$\Delta\Pi_{ij}=\frac{e^2}{(2\pi)^2}\,\theta(\mu^2-m^2)[I_2-\mathbf{k}^2I_1(\mathbf{k})]\left(\frac{k_ik_j}{\mathbf{k}^2}-\delta_{ij}\right), \tag{10}$$

$$\Delta\Pi_{i3}=\frac{e^2}{(2\pi)^2}\,\theta(\mu^2-m^2)2I_1(\mathbf{k})\varepsilon_{i3l}k_l, \tag{11}$$

where $I_1(\mathbf{k})$, $I_3(\mathbf{k})$, I_2 are scalar functions of the external momentum, the chemical potential μ, and the mass m, whose explicit form is given in the Appendix A; ε_{ijk} is the totally antisymmetric unit tensor, $i, j, k=1, 2$.

As can be seen, in a dense medium all nonzero components of the polarization operator acquire statistical corrections, including also the component $\Delta\Pi_{i3}(\mu)$, which corresponds to the induced Chern–Simons term.[7] This implies renormalization of the Chern–Simons mass for $\mu>m$.

Let us investigate the asymptotic behavior of $\Delta\Pi_{ij}(\mu)$ for $\mathbf{k}\to 0$ and $\mathbf{k}\to\infty$. In the first case we obtain from (9)–(11)

$$\Delta\Pi_{33}|_{\mathbf{k}\to 0}=-\frac{e^2}{2\pi}\,\theta(\mu^2-m^2)(\mu-m), \tag{12}$$

$$\Delta\Pi_{ij}|_{\mathbf{k}\to 0}=\frac{e^2}{2\pi}\,\theta(\mu^2-m^2)(\mu-m)\left(\frac{k_ik_j}{\mathbf{k}^2}-\delta_{ij}\right), \tag{13}$$

$$\Delta\Pi_{i3}|_{\mathbf{k}\to 0}=0. \tag{14}$$

For large momentum transfers the asymptotic expressions have the form

$$\Delta\Pi_{33}|_{\mathbf{k}\to\infty}=-\frac{e^2}{4\pi}\,\theta(\mu^2-m^2)(\mu-m), \tag{15}$$

$$\Delta\Pi_{ij}|_{\mathbf{k}\to\infty}=\frac{e^2}{2\pi}\,\theta(\mu^2-m^2)(\mu-m)\left(\frac{k_ik_j}{\mathbf{k}^2}-\delta_{ij}\right), \tag{16}$$

$$\Delta\Pi_{i3}|_{\mathbf{k}\to\infty}=0. \tag{17}$$

From (12) we can obtain an approximate value for the Debye radius $\lambda^{-1}\approx e^2/2\pi(\mu-m)$. In this way, since the statistical part of the Chern–Simons mass tends to zero at large distances, the screening of the charge at high densities of the medium is completely determined by the chemical potential.

3. THE THREE-PHOTON VERTEX

To calculate the three-photon vertex in the one-loop approximation we write its analytic form for a medium at rest

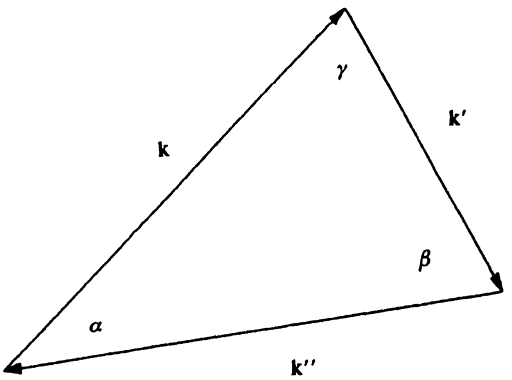

FIG. 2.

$$\begin{aligned}\Pi_{\alpha\beta\gamma}(k,k',k'')&=\Pi^{(1)}_{\alpha\beta\gamma}+\Pi^{(2)}_{\alpha\beta\gamma}\\&=\delta^3(k+k'+k'')\,\frac{ie^3}{(2\pi)^3}\,\mathrm{Tr}\\&\quad\times\int d^3p[\sigma_\alpha G(p^*+k)\sigma_\beta G(p^*-k')\\&\quad\times\sigma_\gamma G(p^*)+\sigma_\alpha G(p^*)\sigma_\beta G(p^*+k')\\&\quad\times\sigma\gamma G(p^*-k)],\end{aligned} \tag{18}$$

where the notation is the same as was used in writing down expressions (4) and (8).

In evaluating the integrals in (18) we find exact expressions for the static case when $k_3=0$. The corresponding expressions for the components of the tensor $\Pi_{\alpha\beta\gamma}$ have the form

$$\begin{aligned}\Pi_{333}=&-\frac{ie^3}{4\pi}\,\theta(\mu^2-m^2)\{[(\mathbf{kk}')-\mathbf{k}''^2\cot^2\gamma-4m^2]\\&\times[J_1(\mathbf{k},\mathbf{k}')+J_1(\mathbf{k},\mathbf{k}'')+J_1(\mathbf{k}',\mathbf{k}'')]\\&-4J_2(\mathbf{k})\cot\gamma\cot\alpha-4J_2(\mathbf{k}')\cot\gamma\cot\beta\\&-4J_2(\mathbf{k}'')\cot\alpha\cot\beta\},\end{aligned} \tag{19}$$

$$\begin{aligned}\Pi_{i3j}=&-\frac{ie^3}{4\pi}\,\theta(\mu^2-m^2)\Bigg\{[k_i'k_j-(\mathbf{kk}')\delta_{ij}][J_1(\mathbf{k},\mathbf{k}')\\&+J_1(\mathbf{k},\mathbf{k}'')+J_1(\mathbf{k}',\mathbf{k}'')]+2J_2(\mathbf{k}'')\\&\times\left[\frac{k_ik_j\mathbf{k}'^2+k_i'k_j'\mathbf{k}^2-k_ik_j'(\mathbf{kk}')}{\mathbf{k}^2\mathbf{k}'^2}-\delta_{ij}\right]\Bigg\},\end{aligned} \tag{20}$$

$$\begin{aligned}\Pi_{i33}=&\frac{ie^3}{4\pi}\,\theta(\mu^2-m^2)m\varepsilon_{i3l}k_l[J_1(\mathbf{k},\mathbf{k}')+J_1(\mathbf{k},\mathbf{k}'')\\&+J_1(\mathbf{k}',\mathbf{k}'')],\end{aligned} \tag{21}$$

where J_1, J_2 are functions of the external momenta $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{k}''$, the chemical potential μ, and the mass m, whose explicit form is given in Appendix A; α, β, γ are the angles between the corresponding vectors $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{k}''$ (Fig. 2), $i, j, l=1, 2$.

The components Π_{ij3} and Π_{3ij} are obtained from (20) by the replacement $\mathbf{k}'\to\mathbf{k}''$ for Π_{ij3} and $\mathbf{k}\to\mathbf{k}''$, $\mathbf{k}''\to\mathbf{k}'$ for Π_{3ij}. Π_{3i3} and Π_{33i} are obtained from (21) by the replacements $\mathbf{k}\to\mathbf{k}'$ and $\mathbf{k}\to\mathbf{k}''$ respectively. As can be seen from the expressions (20) (21) the components Π_{ij3} and Π_{i33} are manifestly transverse ($\Pi_{i3j}k_i=0$, $\Pi_{i3j}k'_j = 0$, $\Pi_{i33}k_i=0$), which guarantees gauge invariance of $W_{\text{eff}}(A)$ in this approximation.

In studying the asymptotic behavior of the tensor $\Pi_{\alpha\beta\gamma}$ we find that all its components vanish for $\mathbf{k}, \mathbf{k}'\to\infty$. In the region of low momenta in the symmetric limit ($|\mathbf{k}|/|\mathbf{k}'|\to 1$) we find for the components $\Pi_{\alpha\beta\gamma}$

$$\Pi_{333}=-\frac{ie^3}{4\pi}\,\theta(\mu^2-m^2)\,\frac{1}{|\mathbf{k}|\,|\mathbf{k}'|}\times\left\{|\,\mathbf{k}||\mathbf{k}'|\left(2\pi\cot\gamma\tan^2\frac{\gamma}{2}-4\right)-\frac{4m^2}{\sin\gamma}\right\}, \tag{22}$$

$$\Pi_{i3j}=-\frac{ie^3}{4\pi}\,\theta(\mu^2-m^2)\left\{\frac{k'_ik_j-(\mathbf{k}\mathbf{k}')\delta_{ij}}{|\mathbf{k}|\,|\mathbf{k}'|\sin\gamma}\,2\pi+\left(\frac{k_ik_j\mathbf{k}'^2+k'_ik'_j\mathbf{k}^2-k_ik'_j(\mathbf{k}\mathbf{k}')}{\mathbf{k}^2\mathbf{k}'^2}-\delta_{ij}\right)\right\}, \tag{23}$$

$$\Pi_{i33}=ie^3\theta(\mu^2-m^2)\,\frac{m}{|\mathbf{k}'|\sin\gamma}\,\varepsilon_{il3}\,\frac{k_l}{|\mathbf{k}|}\,. \tag{24}$$

Thus the three-photon interaction manifests itself at large distances and can turn out to be significant when screening of the Coulomb interaction in the dense medium is taken into account. Another important result is the presence of the nonzero Π_{i33} component, which appears only in two dimensions. This component corresponds to a term of the form $\sim e^3\phi^2 B$ in the effective action. The presence of such terms will affect the magnetic attraction between two-dimensional charges at rest. Such an attraction was discussed in the absence of a medium in Ref. 8, where it was noted that it could lead to the formation of bound electron states in high-temperature superconductors.

4. DISCUSSION OF RESULTS

Let us summarize our calculations. It turns out in the study of the one-loop contributions to the effective action of the electromagnetic field $W_{\text{eff}}(A)$ that at large distances we have $\Pi_\alpha\sim\mu^2-m^2$, $\Delta\Pi_{\alpha\beta}\sim\mu-m$, and the dependence of $\Pi_{\alpha\beta\gamma}$ on the chemical potential is expressed through the step function only, Eqs. (22)–(24). It is natural, therefore, to expect that the terms in the expansion for $W_{\text{eff}}(A)$ that follow will depend on negative powers of μ and will tend to zero for high-density media. Consequently, for a dense medium the first three terms, discussed in this work, fully determine the effective action for the electromagnetic field.

The first term is a μ-dependent constant and its contribution reduces to renormalization of the electric charge. The presence of the second term in the expansion for $W_{\text{eff}}(A)$ leads, first, to Debye screening of the charge and, second, to renormalization of the Chern–Simons mass. In the three-photon vertex two properties are most interesting. The first is the presence of the nonzero Π_{i33} component. It leads to a modification in the action of the terms responsible for the magnetic attraction between charges at rest, of the form $e^2\phi B(1+e\phi)$. Further, the three-photon interaction turns out to be long-range, which when the charges in the medium are screened could turn out to be significant for the discussion of the magnetic attraction in question and the formation of bound electron states. The study of the effect of the three-photon vertex on the magnetic attraction of charges in the medium, as well as the construction of the effective action of the electromagnetic field for 2+1-dimensional QED in a dense medium on the basis of one-loop functions will be considered in a separate paper.

APPENDIX A

$$I_1(\mathbf{k})=\frac{\pi}{2|\mathbf{k}|}\left\{\arcsin\left(\frac{\frac{\mathbf{k}^2}{4}-m^2}{\frac{\mathbf{k}^2}{4}+m^2}\right)+\frac{\pi}{2}\,\theta\left(\mu^2-m^2-\frac{\mathbf{k}^2}{4}\right)-\theta\left(\frac{\mathbf{k}^2}{4}-\mu^2+m^2\right)\arcsin\left(\frac{\frac{\mathbf{k}^2}{4}+m^2-\mu^2}{\frac{\mathbf{k}^2}{4}+m^2}\right)\right\},$$

$$I_2=2\pi(\mu-m),$$

$$I_3(\mathbf{k})=-\frac{\pi}{|\mathbf{k}|}\left\{\frac{|\mathbf{k}|m}{2}-\frac{\mathbf{k}^2+4m^2}{8}\arctan\left(\frac{4|\mathbf{k}|m}{4m^2-\mathbf{k}^2}\right)+\theta\left(\frac{\mathbf{k}^2}{4}-\mu^2+m^2\right)\left[-\mu\sqrt{\frac{\mathbf{k}^2}{4}-\mu^2+m^2}+\frac{\mathbf{k}^2+4m^2}{8}\arctan\left(\frac{2\mu\sqrt{\frac{\mathbf{k}^2}{4}-\mu^2+m^2}}{2\mu^2-m^2-\frac{\mathbf{k}^2}{4}}\right)\right]\right\},$$

$$J_1(\mathbf{k},\mathbf{k}')=\frac{1}{|\mathbf{k}|\,|\mathbf{k}'|\sin\gamma}\left\{\alpha+\beta-\theta\left(\frac{\mathbf{k}^2}{4}-\mu^2+m^2\right)\times\arctan\left(\sqrt{1-4\frac{\mu^2-m^2}{\mathbf{k}^2}}\tan\beta\right)-\theta\left(\frac{\mathbf{k}'^2}{4}-\mu^2+m^2\right)\arctan\left(\sqrt{1-4\frac{\mu^2-m^2}{\mathbf{k}'^2}}\tan\alpha\right)\right\},$$

$$J_2(\mathbf{k})=1-\theta\left(\frac{\mathbf{k}^2}{4}-\mu^2+m^2\right)\sqrt{1-4\frac{\mu^2-m^2}{\mathbf{k}'^2}}.$$

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