Nonlinear diffusion of magnetic flux in type-II superconductors

V. V. Bryksin and S. N. Dorogovtsev

A. F. Ioffe Physicotechnical Institute, Russian Academy of Sciences, 171036 St. Petersburg, Russia
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Nonlinear diffusion of the magnetic flux in a soft superconductor is investigated for times before the flux creep becomes significant. Situations in which linearization of the equations of motion for magnetic induction is impossible are discussed. It is shown that this problem differs significantly from the classical nonlinear-diffusion problems that arise, for example, in combustion theory. The solutions of the magnetic-induction evolution equations are described. The IVC characteristics of a type-II superconductor are obtained with allowance for the inhomogeneous distribution of the magnetic vortices over the superconductor. It is shown that a constant magnetic induction B_∞ sets in the interior of the sample in the case of an external magnetic field $H_a(t)=H_0+H_1 \sin \omega t$, with $|B_\infty| \gg |H_0|$ if $|H_1| \gg |H_0|$. Moreover, an external periodic magnetic field can induce a nonzero B_∞ even if the constant component of the external field is zero. The evolution of the spatial distributions of the magnetic flux inside a superconductor following application of an external field $H_a(t)=H_1 \sin(\omega t+\varphi)$ is considered. It is shown that the magnitude and sign of magnetic flux m penetrating into the interior of the sample is determined by the phase φ of the external field at the instant of its application. The asymptotic values are here $m \propto t^{-1}$ for a slab with a parallel applied field and $m \propto t^{-1/4}$ for a semi-infinite specimen.

1. INTRODUCTION

The discovery of high-temperature superconductivity has increased greatly the number of experimental and theoretical investigations of the diffusion of the magnetic flux (magnetic vortices) in type-II superconductors (see Refs. 1–3 and the citations therein). Generally speaking, the problem of nonlinear diffusion is a classical one and can be formulated for a great variety of physical systems (e.g., combustion[4]). Its various aspects were touched upon in many studies, it was included in many textbooks, and the solutions of the corresponding equations are the subject of several monographs (see Refs. 4–9 and the citations therein). Similar problems are encountered also in the investigation of self-organization processes,[10] in the problem of strewing a heap of sand,[11,12] etc.

The references cited involved in fact the solution of an equation of the type

$$\frac{\partial n}{\partial t}=D_N \frac{\partial}{\partial x}\left(n^\gamma \frac{\partial n}{\partial x}\right), \tag{1}$$

where n is a positive quantity (e.g., density), $\gamma=\text{const}>0$, D_N is the coefficient of "nonlinear diffusion," t the time, and x the coordinate. We consider for simplicity only the one-dimensional problem and disregard the sources.

Nonlinear magnetic-flux diffusion in type-II superconductors differs in at least one respect from the aforementioned problems described by Eq. (1). The point is the local magnetic induction $\mathbf{B}(\mathbf{r},t)$, unlike n, is a vector quantity and the evolution equation for it should differ most substantially from the classical equation (1). We obtain this equation in the next section in the simplest and most lucid form for a soft superconductor in one-dimensional geometry (semi-infinite specimen or plate) with a one-component magnetic induction[13] $B_z(x,t)\equiv B(x,t)$

$$\frac{\partial B}{\partial t}=D_B \frac{\partial}{\partial x}\left(B \frac{\partial B}{\partial x} \operatorname{sign} B\right), \tag{2}$$

where D_B is the corresponding coefficient. Evidently Eq. (2) differs from (1) only by the function sign B in the right-hand side. We shall see that this difference is extremely important in those situations in which $B(x,t)$ can reverse sign. It must be stated that Eq. (2) can be modernized and generalized to include the case of a hard superconductor, creep, etc.

Note that the magnetic-flux diffusion in superconductors was considered in the available papers only in problems where the equation for the magnetic induction can be linearized,[1,3] or where the equation is not linearized but there is no need to consider the vector character of $\mathbf{B}$. (The last case involves, for example, investigation of the creep of the flux—Refs. 14 and 15 and the citations therein). We, on the contrary, consider just situations in which the difference between Eqs. (1) and (2) is important. In particular, when an external alternating magnetic field is turned on. The experimental evolution of the spatial distribution of the magnetic flux in a an HTSC upon application of an oscillating low-frequency magnetic field was investigated in Ref. 16 (see also Ref. 17 and the citations in both). Here, however, since we are interested only in those principal singularities of nonlinear diffusion which are connected with the presence of the function sign B in Eq. (2), we shall discuss only the simplest case of a one-dimensional soft superconductor (i.e., a zero critical current). We confine ourselves also to diffusion over times for which the creep of the flux need not be taken into account. These constraints still prevent us from describing a real experi-

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ment, but we shall show that the behavior of even this simplified system is not trivial. In view of the complexity of the corresponding evolution equations for the magnetic induction, the solution of the posed problems calls for numerical investigations. We shall therefore not pay serious attention to these questions.

In Sec. 3 we describe the general structure of the solutions of Eq. (2) in the case of an applied (or reversed) dc magnetic field or electric current. These solutions, in particular, will be used next to consider more complicated situations. In Sec. 4 we discuss the IVC (current-voltage characteristics) obtained for type-II superconductors from Eq. (2). We shall demonstrate that owing to the inhomogeneous distribution of the vortices across a current-carrying slab, the IVC should differ greatly, even for the thinnest slabs, from those obtained in the single-particle approximation (for a soft superconductor this would mean proportionality of the voltage to the current along the slab). It will be shown that application of a strong enough dc magnetic field alters substantially the form of the IVC. We shall also touch upon the IVC of hard superconductors.

We shall show next that in the case of a low-frequency alternating magnetic field of frequency ω and amplitude $H (H_{c1} < H < H_{c2}$ where H_{c1} and H_{c2} are the lower and upper critical field) there appears in the problem a characteristic length

$$l = (H\Phi_0/4\pi\eta\omega)^{1/2} = (H\rho_n/4\pi H_{c2}\omega)^{1/2},$$

where Φ_0 is the magnetic-flux quantum, η is the viscosity coefficient for Abrikosov vortices, and ρ_n is the resistivity in the normal phase. The oscillations of the magnetic field penetrate into a soft superconductor to a depth of the order of l. In Sec. 5 we assess the magnetic induction B_∞ produced in the interior of a soft superconductor (i.e., at distances from the surface much larger than l) when an alternating magnetic field is applied. Clearly, for an applied field $H_a(t) = H_1 \sin \omega t$ we have $B_\infty = 0$. We shall show that in the case $H_a(t) = H_0 + H_1 \sin \omega t$ the magnetic induction is $B_\infty = 2|H_0 H_1/\pi|^{1/2} \operatorname{sign} H_0$ if $|H_0| \ll |H_1|$, i.e., $|B_\infty|$ can be much larger than $|H_0|$. It will be shown that, in general, an external periodic magnetic field can induce a nonzero B_∞ even if the time-averaged magnetic field, i.e., its zeroth Fourier component H_0, is zero. For example, for $H_a(t) = H_1 \sin \omega t + H_2 \sin(2\omega t + \varphi)$ at $|H_1| \gg |H_2|$ we have $B_\infty = -2 \operatorname{sign}(H_1 H_2 \sin \varphi) |H_1 H_2 \sin \varphi/3\pi|^{1/2}$, i.e., the answer depends on the relative phases of the harmonics of the external field. A periodic external field can thus pump a dc magnetic field into the considered nonlinear system.

In Sec. 6 we consider the transient that follows the application of a periodic external magnetic field, namely, the evolution of spatial distributions of the magnetic flux in the interior of the superconductor, i.e., at distances exceeding the characteristic length l from the surface. In the case $H_a(t) = H \sin(\omega t + \varphi)$, where φ is the phase of the field at the instant of the application, we shall show that the magnetic-flux distribution in the interior of the superconductor has the following behavior. Immediately after the application, within a time of the order of one period of the field (i.e., ω^{-1}), a flux is injected into the interior of the sample (at a distance of the order of l from the surface). The sign and value of this flux depend on the initial phase φ of the field. This flux starts to diffuse into the interior of the sample, and also leaves it through the surface. We shall see that the spatial distribution of the flux in the interior of the superconductor evolve as if the external magnetic field were zero. It will be shown as a result that for a semi-infinite sample the coordinate of the distribution front has an asymptote $x_f \propto t^{1/4}$, and the total magnetic flux m in the superconductor, averaged over the time within the limits of the external-field periods, behaves like $m \propto t^{-1/4}$. The system loses, in power-law fashion, the information on the initial phase (or on the phase collapse) of the applied field.

2. EQUATION OF NONLINEAR DIFFUSION OF MAGNETIC FLUX

We begin with a derivation of the equation for the evolution of the magnetic induction $B(r,t)$ in a type-II superconductor for the simplest case of a one-dimensional geometry, a slab or semi-infinite sample with an external magnetic field $H_a(t)$ parallel to the surface (the yz plane), so that $B_z(x,t) = B(x,t)$. For the time being, however, we confine ourselves to the assumption that the vector $\mathbf{B}$ is parallel (or antiparallel) to the z axis, and $\mathbf{r}$ is a vector in the xy plane. We start with a model of a two-component magnetic gas. Its first component has a density $n^1(\mathbf{r},t)$ and consists of particles with magnetic moment $\mathbf{\Phi}$. The vector $\mathbf{\Phi}$ is directed along the z axis, and $|\mathbf{\Phi}| = \Phi_0$ is the magnetic-flux quantum. The second component, with density $n_2(\mathbf{r},t)$ consists of particles with a magnetic moment $-\mathbf{\Phi}$. The system is spatially homogeneous along the z axis.

The equations of motion are

$$\begin{aligned}&\frac{\partial n_1}{\partial t} + \operatorname{div}(n_1\mathbf{v}_1) + \frac{n_1 n_2}{\vartheta} = 0,\\ &\frac{\partial n_2}{\partial t} + \operatorname{div}(n_2\mathbf{v}_2) + \frac{n_1 n_2}{\vartheta} = 0.\end{aligned} \tag{3}$$

Here $\mathbf{v}_i$ is the velocity of the particles of the ith component, the term $n_1 n_2/\vartheta$ describes the recombination (cf. electron-hole gas), and ϑ is a dimensional constant. Note that the last terms of Eqs. (3) can in principle be chosen in another form. We, however, have used the simplest possibility. By virtue of symmetry we have $\mathbf{v}_1 = -\mathbf{v}_2 \equiv \mathbf{v}$. We proceed now to the equations of motion for $\nu = n_1 - n_2$ and $n = n_1 + n_2$

$$\begin{aligned}&\frac{\partial \nu}{\partial t} + \operatorname{div}(n\mathbf{v}) = 0,\\ &\frac{\partial n}{\partial t} + \operatorname{div}(\nu\mathbf{v}) + \frac{n^2 - \nu^2}{2\vartheta} = 0.\end{aligned} \tag{4}$$

Let us find an expression for $\mathbf{v}$. Assuming viscous motion of the vortices, we have

$$\eta\mathbf{v} = -\nabla U, \tag{5}$$

where η is the viscosity coefficient for vortices and is connected with the resistivity ρ_n by the known relation[12]

$$\Phi_0/\eta = \rho_n c^2/H_{c2},$$

c is the speed of light, and U is the potential energy of a vortex belonging to the first component, i.e., a vortex with a magnetic moment directed upward. For soft superconductors (i.e., with pinning disregarded) we have

$$U = (\mathbf{\Phi B})/4\pi,$$

where $\mathbf{B}$ is the magnetic induction, equal to $\mathbf{\Phi}\nu$ in the system considered. Consequently

$$\mathbf{v} = -\frac{\Phi_0^2}{4\pi\eta}\nabla\nu. \tag{6}$$

Equations (4) and (6) (together with the corresponding initial and boundary conditions which we shall discuss later) determine the kinetics of the considered vortex system.

It is easy to express in terms of ν and n the electric field $\mathbf{E}$ induced by the moving vortices. To this end we multiply the first equation of (4) by $\mathbf{\Phi}$. This yields

$$\frac{\partial B}{\partial t} + \mathrm{rot}([\mathbf{\Phi v}]n) = 0 \tag{7}$$

(the curl has here only a z-component). Comparing this relation with Maxwell's equation, we get

$$\mathbf{E} = \frac{n}{c}[\mathbf{\Phi v}] = -\frac{\Phi_0^2}{4\pi c\eta} n[\mathbf{\Phi}\nabla\nu] - \nabla\varphi. \tag{8}$$

The scalar potential is obtained from the condition div $\mathbf{E}=0$:

$$\Delta\varphi = -\frac{\Phi_0^2}{4\pi c\eta}([\nabla\nu\nabla n]\mathbf{\Phi}). \tag{9}$$

In one-dimensional geometry we have $\varphi = 0$.

The system (3) or (4) differs from the equations used to describe chemical reactions between diffusing components,[19] primarily the nonlinearity of terms with spatial derivatives. We shall not investigate here the most interesting solutions for nonzero ϑ, but consider only equations that are considerably simpler in the limit of very intense recombination, when $\vartheta \to 0$. In this limit it follows from the second equation of the system (4) that $n^2 = \nu^2$, i.e., $n = |\nu|$. As a result, the first equation of (4) yields

$$\frac{\partial\nu}{\partial t} = \frac{\Phi_0^2}{4\pi\eta}\mathrm{div}(|\nu|\nabla\nu), \tag{10}$$

and an expression for the electric field is obtained from (8) by replacing n with ν. The final equation for the evolution of the magnetic induction in a soft superconductor in the case of one-dimensional geometry is:

$$\frac{\partial B}{\partial t} = \frac{\Phi_0}{4\pi\eta}\frac{\partial}{\partial x}\left(B\frac{\partial B}{\partial x}\,\mathrm{sign}\,B\right). \tag{11}$$

Just this equation will be used henceforth. The electric field has in this case only one component

$$E_y = -\frac{\Phi_0}{4\pi\eta c}B\frac{\partial B}{\partial x}\,\mathrm{sign}\,B. \tag{12}$$

E_y is proportional to the vortex flux $J_v(\partial\nu/\partial t + \partial J_v/\partial x = 0)$, which has only an x-component.

Note, without a rigorous proof, that Eq. (11) can be generalized to include a linear Josephson structure—an extended Josephson junction having periodically disposed pinning centers and a nonzero critical-current density j_c (see Refs. 20–22)—in the following manner: We replace $\partial B/\partial x$ in (11) by the expression

$$[(\partial B/\partial x)^2 - (4\pi j_c/c)^2]^{1/2}\,\mathrm{sign}(\partial B/\partial x),$$

if $|\partial B/\partial x| > 4\pi j_c/c$ and by zero if $|\partial B/\partial x| < 4\pi j_c/c$. (E. H. Brandt[3] attempted to use a similar expression for the average vortex velocity to describe magnetic-flux diffusion in hard superconductors.)

We shall assume that it is possible to set on the boundary the magnetic induction equal to the external magnetic field (parallel to the surface): $B(x_b,t) = H_a(t)$. We disregard here the surface currents which are important for a more complex sample geometry[1,3] than in the present paper (we have shown[22] that, particularly for H_a considerably larger than H_{c1}, the difference between $B(x_b)$ and H_a, due to the surface current, is insignificant for the system considered).

3. SIMPLEST SOLUTIONS OF THE MAGNETIC-INDUCTION EVOLUTION EQUATION

Let us examine briefly the form of the solutions of Eq. (11) in the simplest case of a constant or zero external magnetic field, since we shall need some of these solutions later. We seek solutions in the self-similar form

$$B(x,t) = At^{-\beta}g(x/Ct^{\alpha}), \tag{13}$$

where A and C are dimensional constants, such that $A/C^2 = 4\pi\eta/\Phi_0$, and $\xi = x/Ct^{\alpha}$ is the dimensionless coordinate. Substituting (13) in Eq. (11) and comparing the powers t of different terms of the equation obtained, we find it necessary to have $\beta = 1-2\alpha$ in the self-similar solution. The equation for g takes thus the form

$$\partial(|g|\partial g/\partial x)/\partial\xi + \alpha\xi\partial g/\partial\xi + (1-2\alpha)g = 0. \tag{14}$$

In contrast to the linear diffusion equations, the equations considered here have solutions with distinct fronts.[5] It is easy to verify that the solutions of Eq. (14), which vanish at the finite points ($g(\xi_0)=0$, $|\xi_0| < \infty$), can be only linear or square-root functions near ξ_0 (i.e., near the front): (a) $g \propto (\xi - \xi_0)$ and (b) $g \propto |\xi - \xi_0|^{1/2}$. It is evident from Eq. (12) that in the first case the electric field E_y and the vortex flux J_v vanish at the point ξ_0, while in the second case E_y and J_v differ from zero.

Thus, if $g(\xi)=0$ and hence $E_y=0$ on one side of the point ξ_0, whereas $g(\xi) \neq 0$ on the other side, the case (a) should be realized on account of the continuity of the electric field. If the external field differs from zero and vortices flow into (or out of) the sample (e.g., at the point $x=0$), meaning $J_v \neq 0$, it can be seen from (12) that we should have $g = a + b\xi$, where a and b are constants. If, however, vortices flow through the surface when the external field is turned off, the solution near the surface should take the form (b). Finally, if $g(\xi)>0$ on one side of ξ_0 and $g(\xi)<0$ on the other, it is readily seen that at this point the flux J_v (as well as E_y) differs from zero, and $g(\xi)$ on both sides of ξ_0 takes the form (b): $g(\xi\to\xi_0-0)=c_-(\xi_0-\xi)^{1/2}$, $g(\xi\to\xi_0+0)=c_+(\xi-\xi_0)^{1/2}$. Since the electric field is continuous, the relation between the coefficients in these expressions is $c_+=-c_-$.

The exponent α (meaning also β) in (13) can be determined by stipulating that the value of $g(\xi)$ near the surface and the front be expressed by one of the dependences listed above. The number of variants is limited, and so is correspondingly the set of values of α. We present the results directly, without detailed derivations.

We begin with an already known problem.[5,9] Let a flux $\int dx B(x,t=0)=0$ be injected at the instant $t=0$ into an infinite sample (it is assumed that $B(x,t<0)=0$). The quantity $\int dx B(x,t)$ is, naturally, independent of time. The question is: how does $B(x,t)$ over a long time? In this situation we have in the self similar case $\alpha=1/3=\beta$, and the solution has a symmetric form with a maximum value $B_{\max}\propto t^{1/3}$ in a distribution center with coordinate $\bar{x}$, and the coordinate diverge on both sides of the fronts $|x_f\pm\bar{x}|\propto t^{1/3}$.

We have so far not encountered any solutions that reverse sign in the considered interval. Such solutions can appear, in particular, when a constant external magnetic field H is applied [the sample is considered in this case to be semi-infinite $(x>0)$, so that $B(x=0,t>0)=H$]. Assume that prior to the application of the external field the vortex distribution $B(x>0,t<0)=H$ inside the sample is homogeneous. It is easily seen that in the self-similar solutions (13) one should have in this problem $\alpha=1/2$ and $\beta=0$ (since $B(x=0,t>0)$ is fixed on the boundary). All the possible self-similar solutions obtained in this case are shown schematically in Fig. 1 (we assume without loss of generality that $H>0$). We do not present here actual expressions for such solutions. Note that near the surface such solutions are linear functions of the coordinate. If $H_-=0$, then $g(\xi)$ is a linear function of the coordinate in the vicinity of the front $x_f(t)$ is a linear function of the coordinate. If $H_-<0$ the solution reverses sign at the point $x_f(t)\propto t^{1/2}$. In its vicinity, as explained above, we have

$$B\propto |x-x_f(t)|^{1/2}\,\mathrm{sign}[x-x_f(t)].$$

We shall need below to know the behavior of the distribution of the magnetic flux

$$\int dx B(x,t=0)\neq 0,$$

injected into a superconductor in which $B(x,t<0)=0$ in a zero external field $H_a(t)=0$. Two situations are possible in this case. In the first the flux is injected into a slab of finite thickness and flows out of it over long times through both surfaces. It is readily seen then that $\alpha=0$ $(\beta=1)$, and the scaling function $g(\xi)$ (curve *1* of Fig. 2) is symmetric and has a square-root variation in the vicinities of both zeros (i.e., on the surfaces). Thus, $B(x,t)\propto t^{-1}g(x/L)$. Note that the general form of the spatiotemporal distribution of the magnetic induction over long times is independent of the actual form of the initial distribution of the injected flux, i.e., self-organization is observed.

In the second situation a portion of the magnetic flux is injected at the instant $t=0$ into a semi-infinite sample in a zero external field. The flux escapes gradually from the sample through a unit surface $(x=0)$. In addition, the $B(x,t)$ distribution has a front that moves away from the surface. The self-similar solution has thus a square-root form near the surface and a linear one near the front. It can

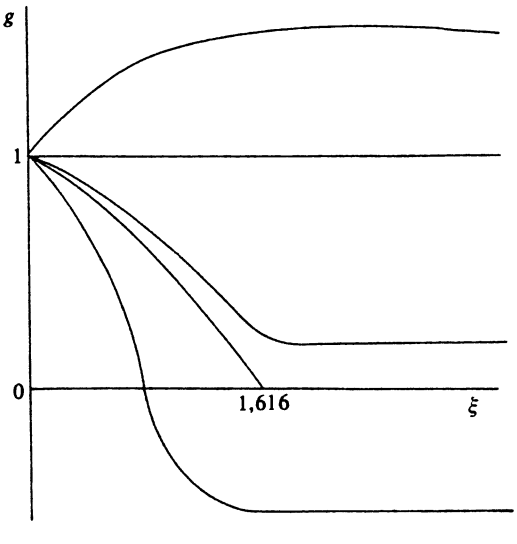

FIG. 1. Schematic form of the scaling functions $g(\xi)$ [see (13) and (14)] upon application of a constant external magnetic field $H>0$, if the vortices are uniformly distributed at the initial instant over a semi-infinite soft superconductor. Each curve corresponds to a separate value of the magnetic induction in the specimen at the initial instant.

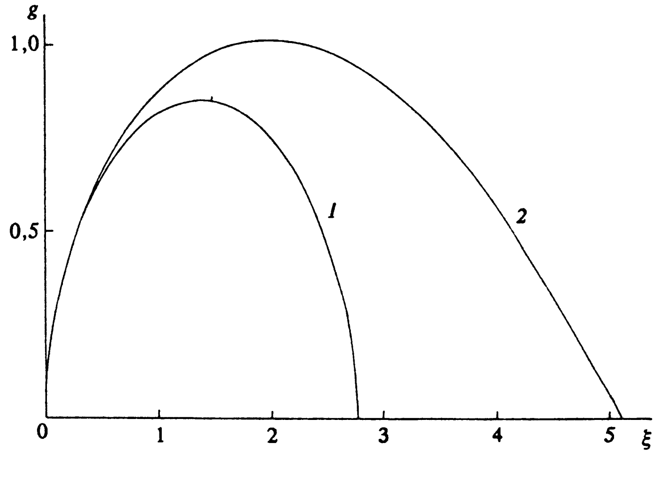

FIG. 2. Scaling functions $g(\xi)$ describing the evolution of the magnetic flux injected into a superconductor in a zero external field [see (13)]. *1*—$\alpha=0$, $\beta=1$—for a slab. *2*—$\alpha=1/4$, $\beta=1/2$—scaling function for a flux injected into a semi-infinite specimen. The scales of the axes can be consistently changed (see the structure of Eq. (14) for g).

be verified that in this case $\alpha=1/4$ ($\beta=1/2$). The dependence of the front coordinate on the time has the form

$$x_f \propto t^{1/4}.$$

As a result $B(x,t) \propto t^{-1/2} g(x/Ct^{1/4})$, where C is a dimensional constant that depends on the initial distribution. In this case, the total magnetic flux in the superconductor is $m \propto t^{-1/4}$.

4. CURRENT-VOLTAGE CHARACTERISTICS (IVC) OF TYPE-II SUPERCONDUCTORS—ALLOWANCE FOR INHOMOGENEOUS VORTEX DISTRIBUTION

We confine ourselves for the time being to a constant external magnetic field and a constant current. Let a total current J (per unit length along the z axis) flow through a soft-superconductor slab of thickness L or, equivalently, assume that a magnetic field

$$H_{0a}=H+2\pi J/c,$$

is applied on one side of the slab, and a field

$$H_{La}=H-2\pi J/c.$$

on the other. After the end of the transient (i.e., at a sufficiently long time after the field or current is turned on), an equilibrium density distribution of moving vortices is established in the slab. It follows from Eq. (11) with boundary conditions

$$B(0)=H_{0a}, \quad B(L)=H_{La}$$

that this distribution is of the form

$$B(x)=\sqrt{|g|}\,\mathrm{sign}\, g,$$

$$g(x)\equiv B^2\,\mathrm{sign}\, B$$

$$=\frac{x}{L}H_{La}^2\,\mathrm{sign}\, H_{La}+\left(1-\frac{x}{L}\right)H_{0a}^2\,\mathrm{sign}\, H_{0a}. \quad (15)$$

This means that in the case $|H|>2\pi|J|/c$, i.e. when H_{0a} and H_{La} are of the same sign, we have

$$B(x)=\mathrm{sign}\, H\left[\left(H+\frac{2\pi}{c}J\right)^2+\frac{x}{L}\frac{4\pi}{c}HJ\right]^{1/2} \quad (16)$$

(see Fig. 3, curve *1*). If, however, $|H|<2\pi|J|/c$ and consequently $\mathrm{sign}\, H_{0a}=-\mathrm{sign}\, H_{La}=\mathrm{sign}\, J$, the $B(x)$ distribution should have a zero at the point

$$x_0=L(H+2\pi J/c)^2/(H^2+(2\pi J/c)^2)$$

and take the form

$$B(x)=\mathrm{sign}\, J\,\mathrm{sign}\left[\left(H+\frac{2\pi}{c}J\right)^2-\frac{x}{L}\left(H^2+\left(\frac{2\pi}{c}J\right)^2\right)\right]$$

$$\times\left|\left(H+\frac{2\pi}{c}J\right)^2-\frac{x}{L}\left(H^2+\left(\frac{2\pi}{c}J\right)^2\right)\right|^{1/2}, \quad (17)$$

i.e., it consists of two square-root functions joined at the point x_0 (see Fig. 3, curve *2*). The magnetic vortices move then from one side of the slab to the other, producing an

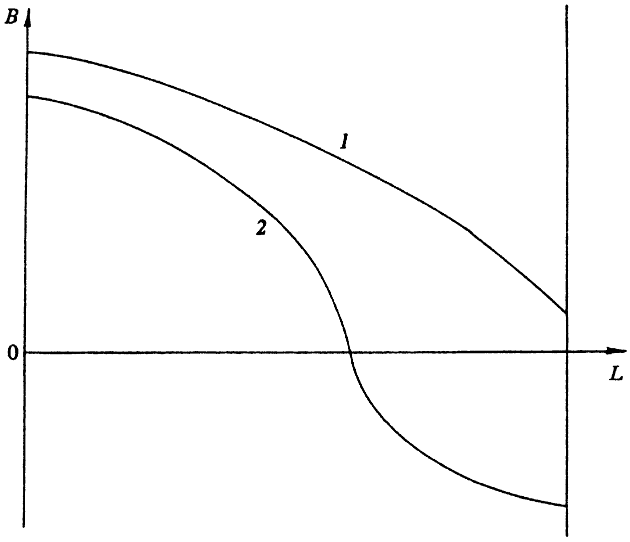

FIG. 3. Schematic diagram of distribution of vortices across a slab in the presence of a constant external magnetic field H and a current J. *1*—$|H|>2\pi|J|/c$, *2*—$|H|<2\pi|J|/c$.

electric field. (In the case $|H|<2\pi|J|/c$ there exist also antivortices moving in the opposite direction and annihilated by the vortices at the point x_0.)

It follows from Eqs. (11) and (12) that the electric field produced by motion of vortices in the steady state is constant in the cross section of the slab [i.e., $E_y(x)=\mathrm{const}$] and coincides, naturally with the value $\bar{E}_y$ obtainable by integrating (12) over x:

$$E_y=\bar{E}_y$$

$$=\frac{\Phi_0}{8\pi c\eta L}(H_{0a}^2\,\mathrm{sign}\, H_{0a}-H_{La}^2\,\mathrm{sign}\, H_{La})$$

$$=\frac{\Phi_0}{8\pi c\eta L}\left\{\left|H+\frac{2\pi}{c}J\right|\left(H+\frac{2\pi}{c}J\right)\right.$$

$$\left.-\left|H-\frac{2\pi}{c}J\right|\left(H-\frac{2\pi}{c}J\right)\right\}. \quad (18)$$

We have used here the continuity of $B(x)$. (Note that it follows also from the form of (12) that in a soft superconductor, in contrast to a hard one, $\bar{E}_y(t)$ is determined only by the values of $H_{0a}(t)$ and $H_{La}(t)$ at the very same instants of time, and is therefore an "inertialess" quantity independent of the preceding evolution of the system. The value of $\bar{E}_y$ sets in therefore immediately after H and J are turned on, with no transient process, while Eq. (18) for $\bar{E}_y$ turns out to be valid also for time-dependent $H(t)$ and J or $J(t)$. It follows from the last relation that if $|H|>2\pi|J|/c$ (Fig. 3, curve *1*), then

$$E_y=\bar{E}_y=\frac{\Phi_0}{c^2\eta L}|H|J=\rho_n\frac{|H|}{H_{c2}}j, \quad (19)$$

where $J=jL$, and for $|H|<2\pi|J|/c$ (i.e., in the situation illustrated by curve *2* of Fig. 3), we have

$$E_y=\bar{E}_y$$
$$=\frac{\Phi_0}{4\pi c\eta L}\operatorname{sign}J\left(H^2+\frac{4\pi^2}{c^2}J^2\right)$$
$$=\rho_n\frac{|H|}{2H_{c2}}j\left(\frac{c}{2\pi}\left|\frac{H}{J}\right|+\frac{2\pi}{c}\left|\frac{J}{H}\right|\right), \quad (20)$$

and the complete form of the IVC is far from linear. The IVC break at the point $|H|=2\pi|J|/c$ is due to a transition to a one-component system of vortices (or antivortices) at fields $|H|>2\pi|J|/c$, meaning due the vanishing of their annihilation. Note that in the case $H<2\pi|J|/c$ the resultant electric field depends already not only on the current density j, but also on the total current J through the sample i.e., on the slab thickness L if j is given). In particular, for $H=0$ we have

$$E_y=\bar{E}_y=\frac{\pi}{c}\frac{L}{H_{c2}}j|j|. \quad (21)$$

The equations obtained can be generalized to include the case of a linear osephson structure (extended Josephson junction with periodically placed pinning center, having naturally a nonzero critical-current density j_c). From the analogs of Eqs. (11) and (12) for such Josephson structures (see the comments at the end of Sec. 2) it follows, in particular, that for $|H|>2\pi|J|/c$

$$E_y=\frac{\Phi_0}{Lc^2\eta}(J^2-J_c^2)^{1/2}\left(H^2-\left(\frac{2\pi}{c}J_c\right)^2\right)^{1/2}\operatorname{sign}J, \quad (22)$$

where $J_c=j_cL$, and for $H=0$

$$E_y=\frac{\Phi_0}{4\pi c\eta L}\left(\frac{2\pi}{c}\right)^2(J^2-J_c^2)\operatorname{sign}J. \quad (23)$$

5. PUMPING A CONSTANT MAGNETIC FIELD BY AN ALTERNATING ONE AND RELATED EFFECTS

We consider now a regime that is stationary even in the case of alternating external fields. For the time being, let the total current be zero. If a field $H_a(t)=H\sin\omega t$, is applied, Eq. (11) can be rewritten in the nondimensionalized form

$$\frac{\partial h}{\partial\tau}=\frac{\partial}{\partial u}\left(h\frac{\partial}{\partial u}\operatorname{sign}h\right), \quad (24)$$

where $h=B/H$, $\tau=\omega t$, and $u=x/(H\Phi_0/4\pi\eta\omega)^{1/2}$. We can introduce thus a characteristic length

$$l=(H\Phi_0/4\pi\eta\omega)^{1/2}=(H\rho_n/4\pi H_{c2}\omega)^{1/2}. \quad (25)$$

For $H/H_{c2}\sim10^{-3}$, $\omega\sim10^2$ Hz, $\rho_n\sim10^{-7}$–10^{-4} $\Omega\cdot$cm [the resistivity in the normal phase for the largest classes of superconductors) the range of l is 10^{-2}–0.3 cm. Using, for example, a numerical solution of Eq. (11)] [or (24)] it is easy to verify that the magnetic-field oscillations penetrate from the surface into the interior of the sample to a depth on the order of l (see Fig. 4a).

Let now the applied field be

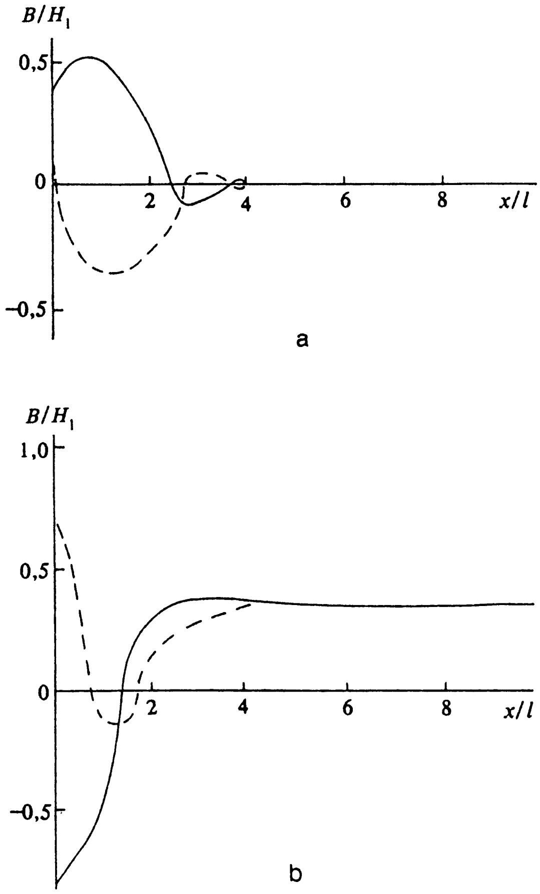

FIG. 4. Instantaneous spatial distributions of the magnetic induction in a semi-infinite specimen ($x>0$) in the case of a periodic external magnetic field $H_a(t)$—results of a numerical solution of Eq. (11). The solid and dashed lines correspond to different instants of time following the end of the initial relaxation. a—$H_a(t)=H_1\sin\omega t$, b—$H_a(t)=H_0+H_1\sin\omega t$, $H_0/H_1=0.\ 1$. Note that in the last case B_∞ is considerably stronger than H_0.

$$H_a(t)=H_0+\sum_{m=1}^{\infty}H_m\sin(\omega mt+\varphi_m). \quad (26)$$

We consider a semi-infinite specimen ($x>0$). (Strictly speaking, we would actually have to deal here with a slab of thickness $L\gg l$ or, when examining the state of the superconductor in the interior of the specimen, the coordinate of the corresponding point should be large compared with l but finite. In practice we shall not touch upon these fine points. We note only that relations $L\gg l$ and (25) lead to constraints on the considered frequency ω.) We use the boundary condition $B(x=0,t)=H_a(t)$.

In the presence of several harmonics in (26) (as follows again, e.g., from the numerical solution of Eq. (11)—see Fig. 4b) the magnetic induction $B(x=\infty)=B_\infty$ in the interior of the specimen after the termination of the transient is is independent of time and is not necessarily zero. The last statement remains in force also if $H_a(t)$ contains harmonics with not only commensurate but also noncommensurate frequencies.

To obtain B_∞ we can multiply Eq. (11) by x and integrate over x from 0 to ∞. After (accurate) integration by parts we have

$$\frac{\partial}{\partial t}\int dx\,xB(x,t)=\frac{\Phi_0}{8\pi\eta}\,(H_a(t)\,|H_a(t)|-B_\infty\,|B_\infty|). \tag{27}$$

From the assumption that B is independent of x in the interior of the specimen, it follows that B_∞ is likewise independent of time [see Eq. (11)]. After the end of the initial relaxation, $B(x,t)$ becomes a periodic function of the time for any x, and the integral of Eq. (27) over the period vanishes. It follows from (12) that in this case the time-averaged electric field (i.e., the zeroth Fourier component of the field E_y) is equal to zero at any point. As a result,

$$B_\infty=\sqrt{|f|}\,\text{sign}\,f,$$

$$f=\frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt H_a(t)\,|H_a(t)|. \tag{28}$$

We have thus obtained for the induction B_∞ a very simple expression which is in fact independent even of the external-field frequency. (What depend on the frequency are the duration of the transient and the thickness of the surface layer in which B depends on the coordinate and oscillates with time.) If $H_a(t)$ given by (26) has only one harmonic $m\neq 0$, we have B_∞ from (28). For the only harmonic $m=0$, the induction B_∞ is equal to H_0.

We turn now the interactions between different harmonics. We discuss first the simple case $H_a(t)=H_0+H_1\sin\omega t$. The direct integration of (28) yields directly

$$B_\infty=\text{sign}\,H_0\begin{cases}(H_0^2+H_1^2/2)^{1/2}, & |H_0|\geqslant|H_1|,\\ \pi^{-1/2}\left[(2H_0^2+H_1^2)\arcsin\left|\dfrac{H_0}{H_1}\right|+3\left|H_0\right|(H_1^2-H_0^2)^{1/2}\right]^{1/2}, & |H_0|\leqslant|H_1|.\end{cases} \tag{29}$$

The derivatives $\partial B_\infty/\partial H_0$ and $\partial B_\infty/\partial H_1$ are continuous at the point $|H_0|=|H_1|$ (see Fig. 5). In the region $|H_0|\ll|H_1|$, we have

$$B_\infty\approx 2\,|H_0H_1/\pi|^{1/2}\,\text{sign}\,H_0, \tag{30}$$

meaning a paramagnetic response with a non-analytic $B_\infty(H_0)$ dependence: $\partial B_\infty/\partial H_0\to\infty$ as $H_0\to 0$. In this region $|B_\infty|\gg|H_0|$.

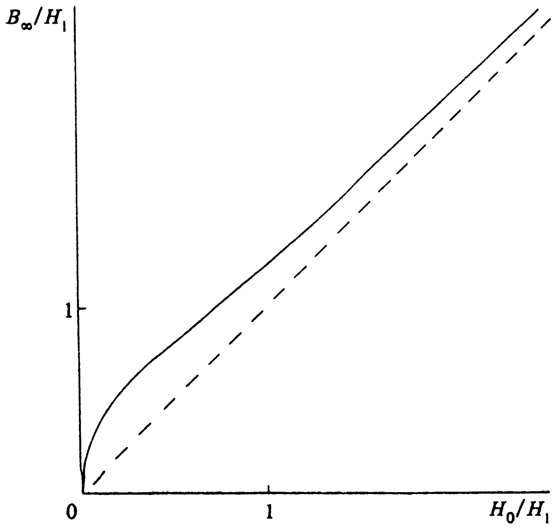

FIG. 5. Dependence of the constant magnetic induction B_∞ in the interior of a soft superconductor on the value of the constant component of the external magnetic field $H_a(t)=H_0+H_1\sin\omega t$.

It follows also from (28) that the induction B_∞ in the interior of the sample can differ from zero even if the *dc* component of the external field H_0 is zero, meaning "rectification" of the alternating external field or pumping of a constant magnetic field by a variable one. A simple analytic expression for B_∞ can be obtained if the first harmonic in (26) is considerably larger than all others: $|H_1|\gg|H_m|$, $m\neq 1$. Without loss of generality we can put $\varphi_1=0$. Breaking up the integral in (28) into integrals from 0 to π/ω and from π/ω to $2\pi/\omega$, we obtain

$$f=H_1^2\left[F\left(\frac{H_0}{H_1},\frac{H_{\{M\geqslant 2\}}}{H_1}\right)-F\left(-\frac{H_0}{H_1},(-1)^{m+1}\frac{H_{\{m\geqslant 2\}}}{H_1}\right)\right], \tag{31}$$

where

$$F(\alpha_0,\alpha_{\{m\geqslant 2\}})=\frac{1}{2\pi}\int_0^{\pi} d\tau\left[\alpha_0+\sin\tau+\sum_{m\geqslant 2}\alpha_m\sin(m\tau+\varphi_m)\right]\left|\alpha_0+\sin\tau+\sum_{m\geqslant 2}\alpha_m\sin(m\tau+\varphi_m)\right|. \tag{32}$$

Therefore

$$f\approx 2H_1^2\sum_{n=1}^{\infty}\frac{H_{2n}}{H_1}\frac{\partial F(0,0,\ldots)}{\partial(H_{2n}/H_1)}. \tag{33}$$

Substituting (33) in (28) we obtain

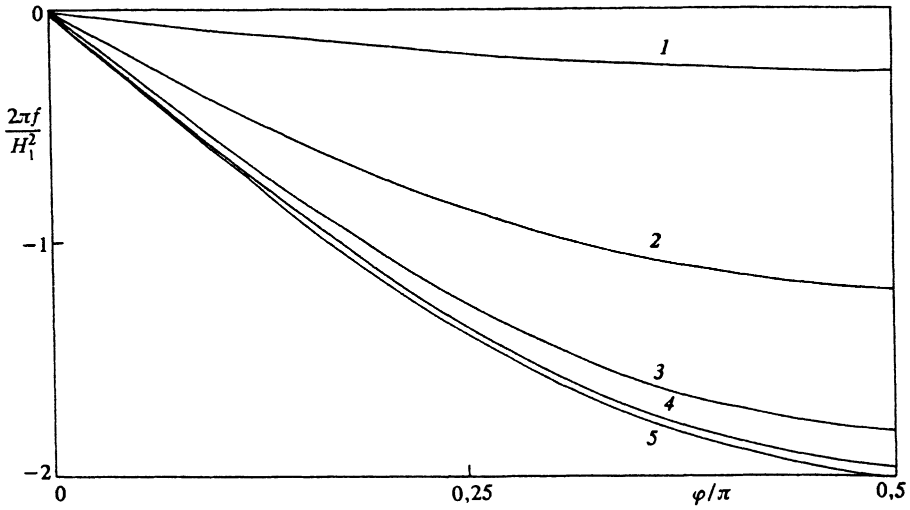

FIG. 6. Dependence of f [see (28)] on φ for $H_a(t)=H_1\sin\omega t+H_2\sin(2\omega t+\varphi)$, $H_2/H_1=0.1, 0.5, 1, 2, 10$—curves *1, 2, 3, 4*, and *5*, respectively.

$$B_\infty=2\,\mathrm{sign}\left[H_1\left(H_0-\sum_{n=1}^{\infty}\frac{H_{2n}\sin\varphi_{2n}}{4n^2-1}\right)\right]\times\left|H_1\left(H_0-\sum_{n=1}^{\infty}\frac{H_{2n}\sin\varphi_{2n}}{4n^2-1}\right)\right|^{1/2}. \quad (34)$$

It is evident from (34) that $B_\infty=0$ if $H_a(t)$ has only odd harmonics. To this end it is not even necessary to satisfy the condition $|H_1|\gg|H_m|$, $m\neq1$. Indeed, for any relation between the amplitudes of the harmonics in the integral (28) we have

$$\int_0^{\pi/\omega}=-\int_{\pi/\omega}^{2\pi/\omega},$$

since

$$\sin(\omega mt+\pi+\varphi_m)=-\sin(\omega mt+\varphi_m)$$

for odd m, hence $f=0$. If Eq. (26) contains only the first and second harmonics and $|H_1|\gg|H_2|$, it follows from (34) that

$$B_\infty=-2\,\mathrm{sign}(H_1H_2\sin\varphi_2)\,|H_1H_2\sin\varphi_2/3\pi|^{1/2}. \quad (35)$$

Therefore the sign and magnitude of B_∞ depend on the relative phase between the harmonics. Note also that in this case $B_\infty\propto|H_1H_2|^{1/2}$(cf. Eq. (30) for $|H_1|\gg|H_0|$).

Numerical calculation using (28) shows that for an arbitrary relation between the amplitudes of the harmonics the dependence of f on the relative phases of the harmonic differs only insignificantly from sinusoidal (Fig. 6). This difference, naturally, is extremely abrupt for B_∞ [see Eq. (28)].

If (26) has only a first and second harmonic and $|H_2|\gg|H_1|$, we obtain, proceeding as in the derivation of (31),

$$f=H_2^2\left[F\left(\frac{H_1}{H_2}\right)+F\left(-\frac{H_1}{H_2}\right)\right]\approx H_2^2 2\left(\frac{H_1}{H_2}\right)^2\frac{\partial^2F(0)}{\partial(H_1/H_2)^2}, \quad (36)$$

where

$$F(d)=\frac{1}{2\pi}\int_0^{\pi}d\tau[d\sin\tau+\sin(2\tau+\varphi_2)]\,|d\sin\tau+\sin(2\tau+\varphi_2)|. \quad (37)$$

We ultimately obtain for $|H_2|\gg|H_1|$:

$$B_\infty=-\mathrm{sign}(H_1H_2\sin\varphi_2)\,|H_1|\,|\sin\varphi_2/\pi|^{1/2}. \quad (38)$$

The induction thus saturates as the amplitude of the second harmonic increases (Fig. 6). In the case of two harmonics with numbers 2 and $2n+1$, if $|H_2|\gg|H_{2n+1}|$ (where $n=0,1,2,...$ and $\varphi_2=0$), we readily obtain

$$B_\infty=\mathrm{sign}(H_2H_{2n+1}\,\mathrm{sign}\,\varphi_{2n+1})\,|H_{2n+1}||\times|\sin\varphi_{2n+1}/\pi(2n+1)|^{1/2}. \quad (39)$$

Other combinations of harmonics can be considered similarly.

We proceed now to the case of a nonzero total current $J(t)$ through a specimen, a slab of thickness L $(0<x<L)$. Let $J(t)$ be a periodic function with a period commensurate with that of the external magnetic field $H(t)$ (which, as above, has only a z component, with the current flowing along the y axis). Following the procedure used to derive (28) and recognizing that we have on the boundaries

$$B(x=0)=H_{0a}=H+2\pi J/c,$$

$$B(x=L)=H_{La}=H-2\pi J/c,$$

we obtain for a point with coordinate x in the interior of the slab

$$B^2\,\mathrm{sign}\,B=\frac{x}{L}T^{-1}\int_0^T dtH_{La}|H_{La}|+\left(1-\frac{x}{L}\right)T^{-1}\int_0^T dtH_{0a}|H_{0a}|\equiv g(x), \quad (40)$$

where T is the period of the function $H_{0a}(t)$ and $H_{La}(t)$, and ultimately

$$B(x)=\sqrt{|g|}\operatorname{sign} g. \tag{41}$$

It is easily seen that (40) and (41) are each a direct generalization of Eq. (15) obtained for a static external field and a static current.

As already mentioned, expression (18) [meaning also (21)] given for $\bar{E}_y$ of the preceding section is valid also for time-dependent external magnetic fields and currents. Integrating both sides of Eq. (11) over time, one readily sees that for periodic $H(t)$ and $J(t)$ the zeroth Fourier component of the resultant electric field

$$E_y^{(0)}=T^{-1}\int_0^T dtE_y(x,t)$$

is independent of the field. Thus, $E_y^{(0)}$ coincides with the time-averaged left-hand side of (18), where H and J are already time-dependent quantities. In particular, for $H=0$ we have

$$E_y^{(0)}=\rho_n\frac{\pi}{c}\frac{L}{H_{c2}}T^{-1}\int_0^T dtj|j| \tag{42}$$

[cf. Eq. (28) for $J=0$]. The rectification effects considered above occur thus when current flows through a soft superconductor. The onset of a nonzero *dc* component of the electric field means here the presence of a constant vortex flow from one edge of the slab to the other.

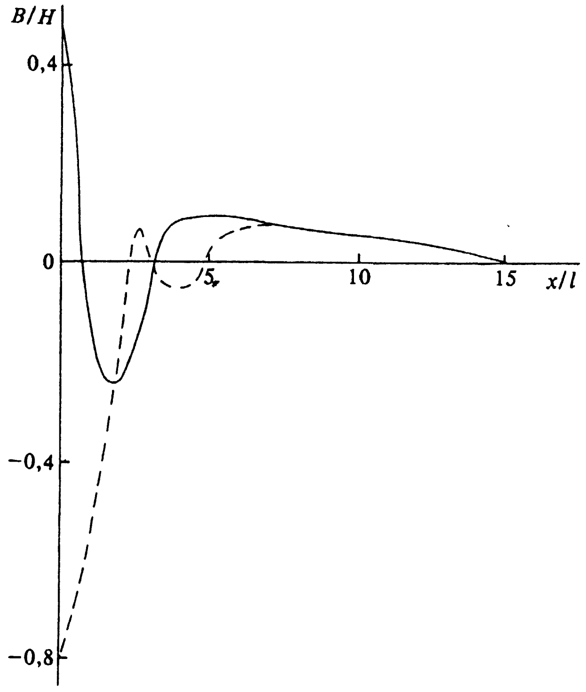

FIG. 7. Coordinate dependence of $B(x,t)$ in a semi-infinite specimen at the instant $\omega t=195.25$ after application of an external field $H\sin\omega t$ ($t>0$). The dashed line shows the distribution at a close instant $\omega t=192.6$. The two distributions almost coincide in the interior of the specimen.

6. EVOLUTION OF SPATIAL DISTRIBUTIONS OF THE MAGNETIC FLUX AFTER APPLICATION OF AN ALTERNATING MAGNETIC FIELD

Whereas in the preceding section we discussed the steady dynamic equilibrium of an electrodynamic field produced after application of an external field, we are interested here in the transition to this state. We assume for simplicity that at $t<0$ there is no external magnetic field and the magnetic induction of the specimen $B(x,t<0)$ is zero, while at $t>0$ the external field is $H_a(t>0)=H\sin(\omega t+\varphi)$. The frequency ω is assumed low enough and the field is turned on at $t=0$ within a time much shorter than ω^{-1}. We use now the results of a numerical solution of Eq. (11) and compare the resultant spatiotemporal distribution of $B(x,t)$ in the interior of the specimen [i.e., outside the surface layer—see (25)] with the data given in Sec. 3 for a zero external field.

We shall not dwell here on the peculiarities of the numerical calculations made in accordance with the standard schemes[8,23] and specially monitored. We begin with the case of a semi-infinite specimen ($x>0$).

The magnetic-induction spatial distributions obtained for certain instants of time $t\gg\omega^{-1}$ are shown in Fig. 7, where two regions can be distinguished. The oscillating part of the magnetic field is concentrated in a surface region having an approximate thickness l. This part tends to zero after averaging over time within the limits of individual periods. The inner part of the distribution does not oscillate and varies slowly with time, so that changes in a time of the order of a period are hardly noticeable. This internal part of the distribution (together with its sign) depends on the initial phase φ of the applied field. As stated in the preceding section, this part of B tends to zero with time. Thus, only a near-surface oscillating distributions remains after a long time (see Fig. 8, which shows the coordinate dependence of the amplitude of the oscillations of the induction in the region $t\to\infty$). The $B(x,t)$ distribution has a distinct front $x_f(t)$ [$B(x_f(t),t)=0$], near which $B(x,t)$ is a linear function:

$$B(x,t)\propto(x_f(t)-x).$$

As seen from Fig. 9, $u_f\propto\tau^{1/4}$ if $\tau\ll1$, i.e., $x_f\propto H^{1/2}\omega^{-1/4}t^{-1/4}$ for $t\gg\omega^{-1}$.

On the whole, disregarding near-surface oscillations, the complete picture is the following. Following application of an external field, after a time of the order of one period, a magnetic field with a strength (and sign) dependent on φ is injected into the superconductor to a depth of the order of l. The flux then begins to "spread" over the specimen and flow out of it, so that ultimately no vortices are left in the interior of the superconductor.

Moreover, the numerical procedure leads to the asymptotic form of the magnetic-flux distribution in the interior of the specimen

$$B(x,t)=At^{-1/2}g(x/Ct^{1/4}),$$

where g is the scaling function while A and C are dimensional constants. Recall that in Sec. 3 we have already encountered just such an asymptotic form of the spa-

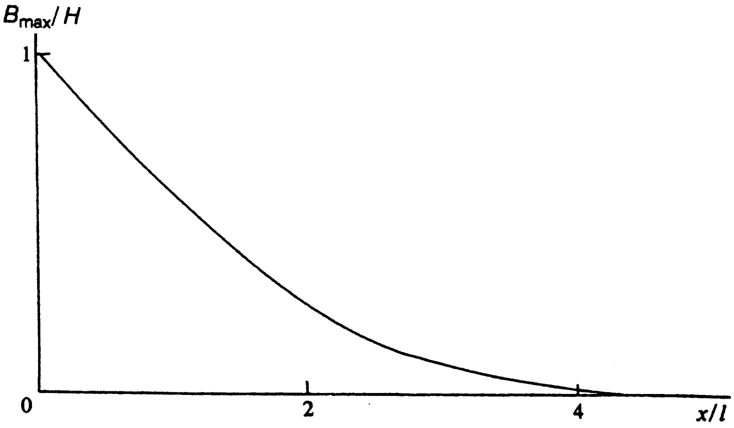

FIG. 8. Coordinate dependence of $\max_t B(x,t)$ as $t\to\infty$, i.e. amplitude of induction oscillations in a semi-infinite specimen in the steady state. $H_a(t)=H\sin\omega t$.

tiotemporal distribution of the magnetic flux injected into a semi-infinite superconductor in a zero external field (see Fig. 2, curve *2*). Thus, the distribution $B(x>l,t>\omega^{-1})$ behaves as if, after the first period, the magnetic field were altogether zero after injection of the vortices. We shall not change here to a rough temporal variable, but prove this statement rigorously.

We did not touch above upon the role of the initial phase φ. We shall now consider it briefly, again without rigorous proofs. Numerical solution of Eq. (11) for various φ leads to the following conclusion: the non-oscillating (i.e., internal) part of the spatiotemporal distribution of $B(x,t;H^*(\varphi),0)$ for a zero initial phase and for an external field with amplitude $H^*(\varphi)=HF(\varphi)$. We have introduced here the function $F(\varphi)$ which shows how the phase φ effectively renormalizes the amplitude of the external field.

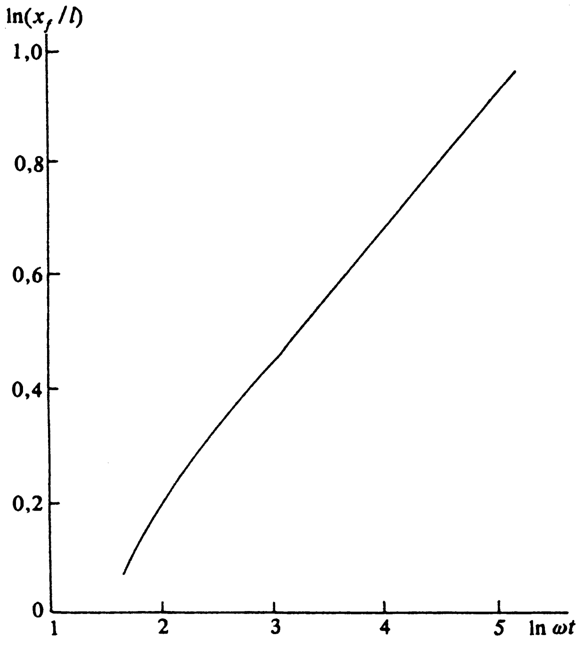

FIG. 9. Plot of the front coordinate vs. time. An external field $H\sin\omega t$ is applied at the instant $t=0$.

Thus, when the external field is switched on there is injected into the specimen a magnetic flux

$$\int B dx \sim H^* l^*,$$

where

$$l^* - l|H^*/H|^{1/2} = l|F(\varphi)|^{1/2}$$

[see Eq. (25)]. Subsequently the flux evolves as if $H_a=0$. We ultimately obtain for long times

$$B(x,t)\sim H^*(\omega t)^{-1/2} g\{(x/l^*)(\omega t)^{-1/4}\}. \quad (43)$$

A similar distribution results also from the drop of the external-field phase to φ. The function $F(\varphi)$ has, naturally, a period 2π. It follows from our numerical calculation that

$$F(\varphi=\pm\pi/2)\approx 0, \quad |F(\varphi\approx 0,\pi/2)|=1,$$

so that it is similar to but not exactly a cosine. The asymptote of the time dependence of the distribution-front coordinate is of the form

$$x_f\sim(\omega t)^{1/4} l^* = \omega^{-1/4} t^{1/4} c (H\rho_n |F(\varphi)|/4\pi H_{c2})^{1/2}. \quad (44)$$

We proceed now to the case of a slab of thickness $L\gg l^*$, to which an external field, $H_a(t>0)=H\sin(\omega t+\varphi)$ is applied from both sides. Following the merging of the fronts moving from both surfaces of the slab, i.e., at times $t>\omega^{-1}(L/l^*)^4$, $B(x,t)$ acquires distributions of the type shown in Fig. 10 (numerical results). Figures 11 and 12 show the time dependences of the local induction $B_c(t)$ at the center of the slab for different L and φ, with $B_c(t)=0$ for $t<t_c<\omega^{-1}(L/2l^*)^4$. This is followed at the center of the slab by a signal that depends on the phase φ (or on the phase break) of the external field. An asymptotic $B_c(t)\propto t^{-1}$ dependence can be observed for sufficiently large $L\gg l^*$ (see Fig. 13). Figure 14 shows the dependence of sign $B_c(t)\max_t B_c(t)/H$ on φ.

On the whole, the evolutions of the $B(x)$ distributions can be interpreted in this case in practically the same manner as for a semi-infinite superconductor. A flux whose value is dependent on H and φ is injected into the interior

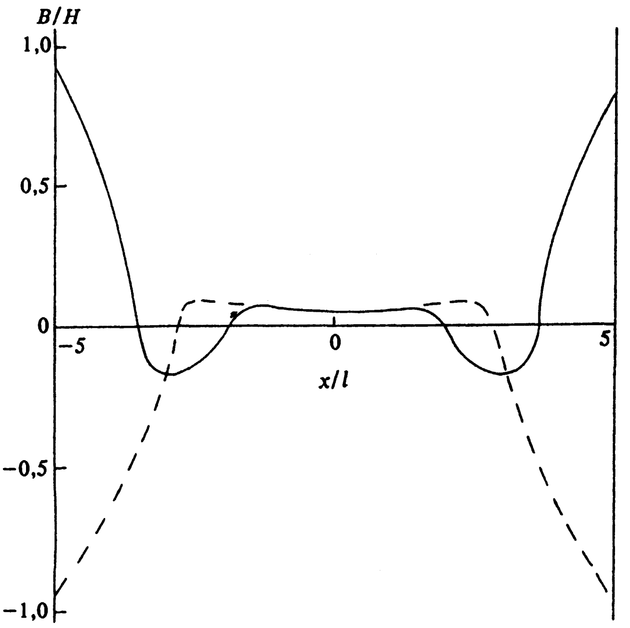

FIG. 10. Distribution of $B(x,t)$ in a slab of thickness $L=10l$ at close instants of time: $\omega t=46$ (solid line), 50 (dashed). The external field $H\sin\omega t$ is applied at the instant $t=0$.

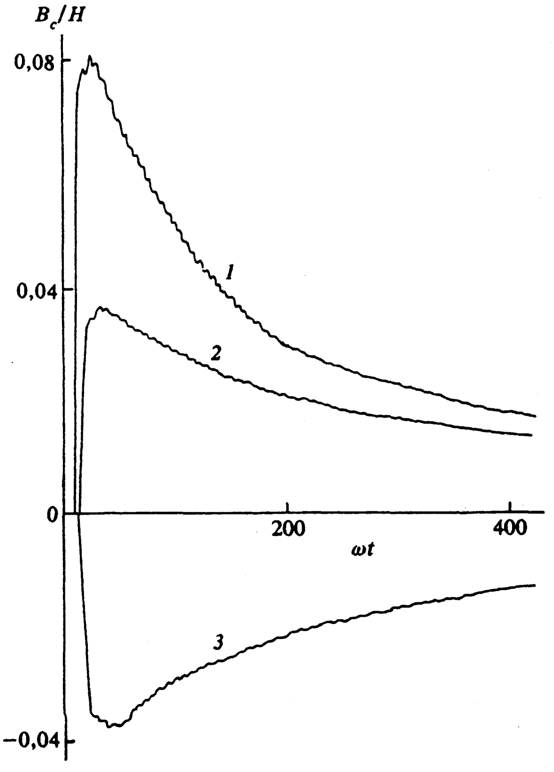

FIG. 11. Time dependence of the magnetic induction B_c at the center of the slab. $L=10l$, $t=0$—instant of application of external field $H_a=H\sin(\omega t+\varphi)$, $\varphi=0$ (*1*), $\varphi=0.4\pi$ (*2*), $\varphi=0.6\pi$ (*3*).

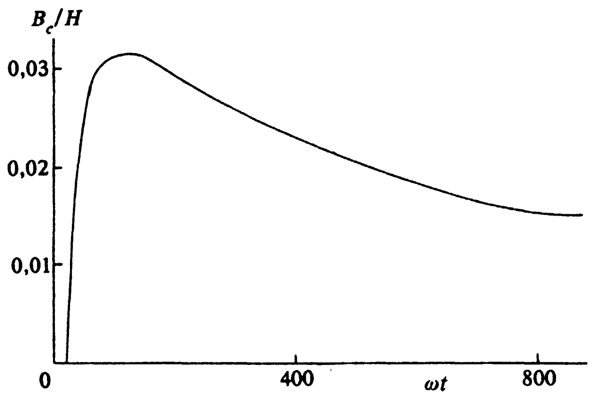

FIG. 12. Dependence of the magnetic induction B_c at the center of a slab of thickness $L=15l$ on the time, $H_a=H\sin\omega t$.

of the sample after an external alternating field is turned on. The flux spreads over the slab and escapes from it as if the applied field H_a were zero, and the asymptote at $t\gg\omega^{-1}(L/2l^*)^4$ takes the form

$$B(x,t)\sim H^*(L/l^*)^2(\omega t)^{-1}g(x/L). \tag{45}$$

The reason for the large factor $(L/l^*)^2$ in (45) is that, as follows from (43), at the instant t_c of the collision of the fronts there remains in the interior of the superconductor a flux of order $H^*l^*(\omega t_c)^{-1/4}\sim H^*l^*(l^*/L)$, and we should therefore have

$$B(x,t)\sim B(x,t_c)(L/l^*)^4/\omega t,$$

$$B(L/2,t_c)\sim H^*(l^*/L)^2,$$

which leads in fact to (45). The function g is symmetric about the center of the slab and is represented in Fig. 2 by curve *1*.

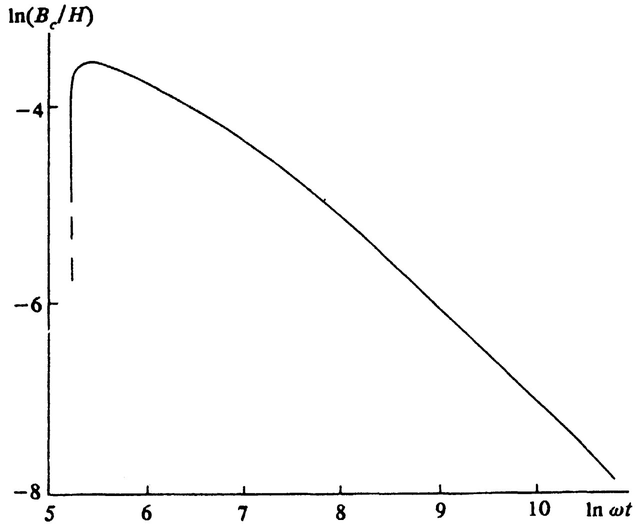

FIG. 13. Dependence of ln B_c on the time $L=15l$, $H_a=H\sin\omega t$.

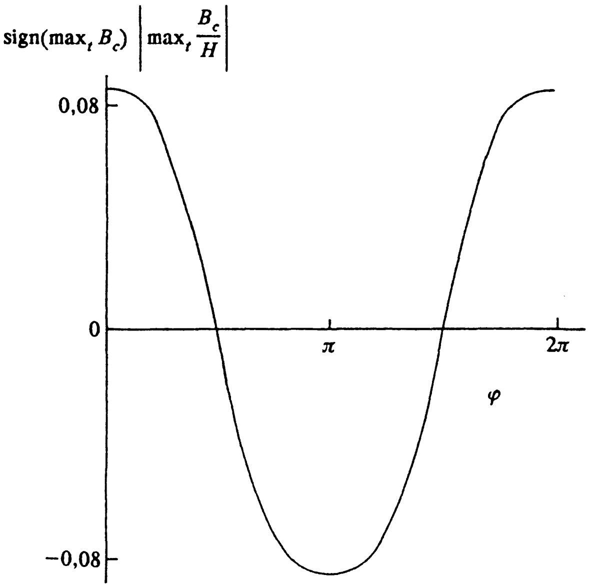

FIG. 14. Dependence of sign $B_c(t,\varphi)\max_t|B_c(t,\varphi)/H|$ on φ, $L=10l$, $H_a=H\sin(\omega t+\varphi)$.

Ultimately, in the case of a superconductor in the form of a slab, the flux and the magnetic induction in the interior of the specimen relax like t^{-1} after the phase collapse. It is interesting that a similar relation ($B(x,t\gg\omega^{-1}(L/l^*)^4)$) is obtained also if an external field $H_a(t)=H\sin(\omega t+\varphi)$ is applied to only one side of the slab, and the field on the other size is zero all the time.

7. CONCLUSIONS

We have thus demonstrated that the nonlinear diffusion of the flux in a soft superconductor is characterized, even in the simplest case of one-dimensional geometry, by a number of nontrivial peculiarities. Such a complicated behavior of the system is attributed to a conjunction of two factors; the impossibility of linearization and the vector nature of the magnetic induction. When an alternating external magnetic field is applied, a type-II superconductor plays the role of a unique nonlinear element, a damper, so that the magnetic-field oscillations penetrate into the interior of the specimen only to a depth l of the order of $\omega^{-1/2}$ [see (25)]. In the interior of a soft superconductor, on the other hand, if the external field $H_a(t)$ is periodic, a stationary magnetic-induction distribution sets in with time, as if a constant magnetic field

$$H=|\langle H_a|H_a|\rangle_t|^{1/2}\text{sign}\langle H_a|H_a|\rangle_t,$$

were applied to the sample, where $\langle\cdot\rangle_t$ denotes an average over time. Therefore even an external field with a zero constant component can lead to the appearance of a nonzero magnetic induction B_∞ in the interior of the specimen, and usually $B_\infty\neq\langle H_a\rangle_t$. If a current flows through the specimen, the magnetic induction established in the interior of the superconductor is already spatially inhomogeneous and the form of the IVC can be determined only when account is taken of this fact.

The transient initiated by application of an alternating external magnetic field is characterized by power-law asymptotes of a number of quantities (coordinates of the fronts and amplitudes of the magnetic-flux distributions, the total magnetic flux in the interior of the superconductors, etc.), and the actual powers depend on the shape of the specimen (we have verified this by considering a semi-infinite specimen and a slab). If $H_a=H\sin(\omega t+\varphi)$, the magnitude and sign of the flux injected into the interior of the superconductors (and subsequently spreads over the specimen and flows out of it) is determined primarily by the phase of the external magnetic field at the instant when it is applied. In sum, we have shown that the system "forgets" the initial phase of the field (or the phase loss) in power-law fashion.

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