Energy shift and anomalous magnetic moment of the neutrino in a constant magnetic field at finite temperature and density

V. Ch. Zhukovskiĭ, T. L. Shoniya, and P. A. Éminov

Moscow Institute of Electronic Machine Construction, 109028 Moscow, Russia
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The energy shift and anomalous magnetic moment of a massive Dirac neutrino moving in a constant magnetic field at finite temperature in a dense electron-positron medium are calculated in the Weinberg–Salam–Glashow model. It is shown that if the density of the medium is nonzero the anomalous magnetic moment of the neutrino can exceed the vacuum static magnetic moment by several orders of magnitude. The dynamical nature of the energy shift of the neutrino is investigated. The well known Wolfenstein formula is generalized to the case of a strong external field in various limiting cases.

In the standard Weinberg–Salam–Glashow model the anomalous magnetic moment (AMM) of the Dirac neutrino with mass m_ν is determined by the expression[1]

$$\mu_\nu^0=\frac{3eG_F}{8\pi^2\sqrt{2}}m_\nu\approx3\cdot10^{-19}\frac{m_\nu}{1\text{ eV}}\mu_B, \tag{1}$$

where $\mu_B=e/2m_e$ is the Bohr magneton, $c=\hbar=1$.

In extended electroweak interaction schemes the magnetic moment of the neutrino is proportional not to the mass of the neutrino but to the mass of the charged lepton,[2–4] and can reach values $\mu_\nu\sim10^{-13}\mu_B$.

The considerable interest in the problem of the compatibility[5] of the small mass and large magnetic moment of the neutrino is related, in particular, to the so-called solar-neutrino problem. The most attractive explanation of this problem is the hypothesis of resonant amplification of the neutrino oscillations in the material of the sun together with spin and spin-flavor precession.[6–9] To explain the solar-neutrino problem in the framework of this hypothesis it is required that the neutrino magnetic moment have a magnitude $\mu_\nu\gtrsim(10^{-11}\text{–}10^{-10})\mu_B$.

The investigation of the electromagnetic interactions of neutrinos with nonzero mass is also of fundamental interest for the elucidation of the question of whether the neutrino is a Dirac or a Majorana particle.[10]

The Majorana neutrino does not possess a magnetic (or, incidentally, an electric) moment, and, if the magnetic moment of the Dirac neutrino has a relatively large magnitude, the Majorana neutrino and Dirac neutrino can be distinguished, in principle, by their electromagnetic properties.

In the Weinberg–Salam–Glashow model the electromagnetic properties of the neutrino due to the radiative corrections that describe its virtual decay into charged particles have been studied in detail in Ref. 11, in which the effect of a constant electromagnetic field on the radiative mass shift and AMM of a massive Dirac neutrino was considered.

In this paper it was shown that the AMM of the neutrino depends essentially on the dynamical parameter

$$\kappa=\frac{1}{m_eB_0}\sqrt{-(F_{\mu\nu}p^\nu)^2},$$

where $F_{\mu\nu}$ is the electromagnetic-field tensor, q^ν is the four-momentum of the neutrino, and $B_0=m_e^2/e=4.4\times10^{13}$ G is the critical field.

It then turns out that in magnetic fields with intensity

$$H\ll B_0\left(\frac{M_W}{m_e}\right)^2\approx10^{24}\text{ G}$$

(M_W is the mass of the W boson) the field contribution to the AMM of the neutrino is small in comparison with its static value (1).

The magnetic moment of fermions (e.g., charged leptons) changes in heated and dense matter. The AMM of the electron at finite temperature and zero chemical potential has been calculated in a number of papers (see, e.g., Refs. 12–14), and in Ref. 14 the dynamical nature of the mass shift and AMM of the electron in a constant magnetic field at finite temperature and zero density of the medium were considered. In Ref. 15 the energy shift of a massive Dirac neutrino in a constant magnetic field at finite temperature in a zero-density medium was calculated in the nonrelativistic case.

The electromagnetic properties of the neutrino in a dense electron-positron medium at finite temperature in the free case, i.e., without allowance for the external field, have also been discussed repeatedly in the literature.[16–20] The calculation of the vertex function and electromagnetic form factors of the neutrino is performed in these papers on the basis of the real-time representation for the electron Green's function at finite temperature and density. However, except for the estimate obtained in Ref. 19 from dimensional considerations, in these papers the calculation of the magnetic moment of the neutrino is not carried to completion. We note also that the approach used by the authors of Refs. 16–20 does not make it possible to investigate the question of the dynamical nature of the energy shift and AMM of the neutrino, i.e., their dependence on the external-field intensity and the neutrino energy.

In this paper we calculate the radiative energy shift and AMM of the neutrino for finite temperature and non-

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zero density of the electron-positron medium in a constant magnetic field. In Sec. 1 we obtain for the radiative energy shift of the neutrino a closed expression that contains in explicit form the dependence on the polarization state of the massive Dirac neutrino. In Sec. 2, in the zero-temperature limit, we investigate the energy shift and AMM of a neutrino moving parallel to the magnetic field, as a function of the magnitude of the magnetic-field strength, the energy of the neutrino, and the density of the medium. In Sec. 3, in various limiting cases, we calculate the AMM of the neutrino in the case when a relativistic neutrino is moving in a direction perpendicular to the external magnetic field. In Sec. 4 we discuss the results obtained.

The principal result of the calculations performed is that, when the density of the electron-positron medium is nonzero, under certain conditions the AMM of the neutrino can substantially exceed the static moment (1).

1. ENERGY SHIFT OF THE NEUTRINO IN AN EXTERNAL MAGNETIC FIELD IN A MEDIUM WITH NONZERO TEMPERATURE AND FINITE DENSITY

The electron neutrino interacts with electrons both on account of the neutral weak current and on account of the charged weak current, whereas neutrinos of other types interact with electrons only through the neutral current. Here we shall confine ourselves to calculating the part due to the contribution of the charged weak current in the energy shift of an electron neutrino propagating in a constant magnetic field in a dense electron-positron medium at temperature T.

The contribution of the W boson to the mass operator of the neutrino in the one-loop approximation is determined by the expression[1]

$$M(x,x')=-i\frac{g^2}{8}(1-\gamma^5)\gamma^\mu S(x,x')\times\gamma^\nu(1+\gamma^5)D_{\mu\nu}(x,x'), \quad (2)$$

where $S(x,x')$ and $D_{\mu\nu}(x,x')$ are the electron and W-boson propagators in a magnetic field specified by the potential

$$A_\mu=(0,0,xH,0). \quad (3)$$

In the Feynman gauge the W-boson propagator has the form[1]

$$D_{\mu\nu}(x,x')=\frac{1}{(4\pi)^2}\int_0^\infty\frac{dt}{t^2}D_{\mu\nu}(t)\exp(-iM_W^2t)\frac{bt}{\sin(bt)}\times\exp\left\{-\frac{i}{4}\left[x_\parallel^2-\frac{bx_\perp^2}{\mathrm{tg}(bt)}-4\Omega\right]\right\}, \quad (4)$$

where we have introduced the notation

$$\Omega=-b(x_2-x_2')(x_1+x_1'),$$
$$x_\perp^2=(x_1-x_1')^2+(x_2-x_2')^2,\quad b=eH, \quad (5)$$

and in the matrix $D_{\mu\nu}(t)$ the following elements are nonzero:

$$D_{00}=-D_{33}=1,\quad D_{22}=D_{11}=-\cos 2y,$$
$$D_{12}=-D_{21}=-\sin 2y,\quad y=bt. \quad (6)$$

The electron propagator for finite temperature and density is given in the real-time representation by the expression[14]

$$S(x,x')=-\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\omega\exp[i\omega(t-t')]\times\sum_{s,\varepsilon=\pm1}\frac{\Psi_n^{(\varepsilon)}(x)\bar\Psi_n^{(\varepsilon)}(\mathbf{x}')}{\omega+\varepsilon E_n(1-i0)}+i\sum_{s,\varepsilon=\pm1}\frac{\varepsilon}{\exp[(E_n-\varepsilon\mu)/T]+1}\times\Psi_n^{(\varepsilon)}(\mathbf{x})\bar\Psi_n^{(\varepsilon)}(\mathbf{x}')\exp\{-i\varepsilon E_n(t-t')\}, \quad (7)$$

where the summation is performed over all intermediate states s, corresponding both to positive ($\varepsilon=+1$) and to negative ($\varepsilon=-1$) frequencies, μ is the chemical potential of the electron gas, and $\Psi_n(\mathbf{x})$ is the coordinate part of the wave function of the electron in the magnetic field (3), with corresponding electron energy[21]

$$E_n=\sqrt{m_e^2+2eHn+p_3^2}, \quad (8)$$

where $n=0, 1, 2,...$ is the principal quantum number and $-\infty<p_3<+\infty$. The radiative shift of the neutrino energy in the one-loop approximation under consideration is equal to (see also Refs. 22–23)

$$\Delta E_\nu(T,\mu,H)=\int dxdx'\bar\Psi_\nu(x)M(x,x')\Psi_\nu(x'), \quad (9)$$

where $\Psi_\nu(x)$ is the wave function of the massive Dirac neutrino.

As the operator determining the polarization state of the Dirac neutrino we choose the transverse-polarization operator μ_3 (Ref. 21):

$$\Psi_\nu(x)=\frac{e^{-iqx}}{2\sqrt2}\times\begin{bmatrix}\left(1+s\frac{m_\nu}{E_\nu^\perp}\right)^{1/2}\left[\left(1+\frac{q_3}{E_\nu}\right)^{1/2}+s\left(1-\frac{q_3}{E_\nu}\right)^{1/2}\right]\\ \left(1-s\frac{m_\nu}{E_\nu^\perp}\right)^{1/2}\left[-s\left(1+\frac{q_3}{E_\nu}\right)^{1/2}+\left(1-\frac{q_3}{E_\nu}\right)^{1/2}\right]\\ \left(1+s\frac{m_\nu}{E_\nu^\perp}\right)^{1/2}\left[\left(1+\frac{q_3}{E_\nu}\right)^{1/2}-s\left(1-\frac{q_3}{E_\nu}\right)^{1/2}\right]\\ \left(1-s\frac{m_\nu}{E_\nu^\perp}\right)^{1/2}\left[s\left(1+\frac{q_3}{E_\nu}\right)^{1/2}+\left(1-\frac{q_3}{E_\nu}\right)^{1/2}\right]\end{bmatrix}. \quad (10)$$

Here, $E_\nu^\perp=\sqrt{E_\nu^2-q_3^2}$, and $s=\pm1$ correspond to the orientations of the neutrino spin parallel and antiparallel to the direction of the magnetic field.

Using the explicit form of the propagators (4)–(7) and Eq. (10), we represent the neutrino-energy shift in the form

$$\Delta E_\nu=\Delta E_\nu(H,\mu=0,T=0)+\Delta E_\nu(H,T\neq0,\mu\neq0), \quad (11)$$

where ΔE_ν $(H, \mu=0, T=0)$ is the radiative energy shift of the neutrino in a magnetic field at zero temperature and zero density of the medium; this energy shift, as mentioned above, was investigated in detail in Ref. 11, and is omitted in what follows.

The second term in Eq. (11) takes into account the effects of interest that arise from the finite temperature and density of the medium, and is determined by the expression

$$\Delta E_\nu(H,\mu,T)=i\frac{g^2}{64\pi^2}\sum_{n,\varepsilon=\pm1}\int_{-\infty}^{+\infty}dk_3\int_0^\infty dt$$
$$\times\frac{\varepsilon}{\exp[(E_n-\varepsilon\mu)/T]+1}\left[i\varepsilon\frac{\sqrt{2eHn}}{E_n}ABI_{n,n-1}+A^2e^{iy}\right.$$
$$\left.\times\left(1+\varepsilon\frac{p_3}{E_n}\right)I_{n,n}+B^2e^{-iy}\left(1-\varepsilon\frac{p_3}{E_n}\right)I_{n-1,n-1}\right]$$
$$\times\exp\left\{it(q_0-\varepsilon E_n)^2-k_3^2-M^2-2eHn]\right.$$
$$\left.\times-i\frac{q_\perp^2}{eH}\sin y\cos y\right\}. \qquad (12)$$

In this formula $I_{n,n}(z)$ is a Laguerre function[21] with argument

$$z=\frac{2q_\perp^2}{eH}\sin^2 y, \qquad (13)$$

$k_3=p_3-q_3$ is the z-component of the momentum of the intermediate W boson, $q^\mu=(E_\nu,q)$ is the four-momentum of the neutrino, and the coefficients A and B, which take account of the polarization state of the neutrino, are equal to

$$A=2\sqrt{\left(1+s\frac{m_\nu}{E_\nu^\perp}\right)\left(1-\frac{q_3}{E_\nu}\right)},$$
$$B=-2\sqrt{\left(1-s\frac{m_\nu}{E_\nu^\perp}\right)\left(1+\frac{q_3}{E_\nu}\right)}. \qquad (14)$$

In the following sections the result (12) is investigated in various limiting cases.

2. ENERGY SHIFT AND AMM OF A NEUTRINO MOVING PARALLEL TO THE MAGNETIC FIELD AT ZERO TEMPERATURE

In the limiting case $T\ll E_F=\mu(T=0)$ the first term of the expansion of (12) in the parameter $(T/E_F)\ll1$ corresponds to replacing the Fermi distribution function by a θ-function:

$$\frac{1}{\exp[(E_n-\mu)/T]+1}\to\theta(\mu-E_n)=\begin{cases}1, & \mu>E_n,\\ 0, & \mu<E_n\end{cases}. \qquad (15)$$

A further simplification of Eq. (12) occurs if the neutrino moves parallel (or antiparallel) to the magnetic field, when the argument of the Laguerre function is equal to zero. In this case,

$$I_{n,m}(0)=\delta_{n,m}, \qquad (16)$$

where $\delta_{n,m}$ is the Kronecker symbol.

As a result, the radiative energy shift of the neutrino can be represented, after integration over the variables t and p_3, in the form of a sum of two parts:

$$\Delta E_\nu=\Delta E_\nu(s,H,\mu)+\Delta E_\nu(H,\mu), \qquad (17)$$

where the part of the energy shift that depends explicitly on the orientation of the neutrino spin is equal to

$$\Delta E_\nu(s,H,\mu)=\frac{g^2eHs}{16\pi^2}\sum_{n=0}^{N_0}\left\{\frac{E_\nu-q_3}{E_\nu}[F_1(n,M_1^2)\right.$$
$$+F_2(n,M_1^2)]-\frac{E_\nu+q_3}{E_\nu}[F_1(n,M_2^2)$$
$$\left.-F_2(n,M_2^2)](1-\delta_{0,n})\right\}, \qquad (18)$$

while for that part of the energy shift which does not depend on the orientation of the neutrino spin we have

$$\Delta E_\nu(M,\mu)=\frac{g^2eH}{16\pi^2}\sum_{n=0}^{N_0}\left\{\frac{E_\nu-q_3}{E_\nu}[F_1(n,M_1^2)\right.$$
$$+F_2(n,M_2^2)]+\frac{E_\nu+q_3}{E_\nu}[F_1(n,M_2^2)$$
$$\left.-F_2(n,M_2^2)](1-\delta_{0,n})\right\}. \qquad (19)$$

Explicit expressions for the integrals

$$F_1(n,M_{1,2}^2)=\int_{-a}^{a}\frac{dp_3}{M_{1,2}^2+2E_\nu E_n-2p_3q_3},$$

$$F_2(n,M_{1,2}^2)$$
$$=\int_{-a}^{a}\frac{p_3\,dp_3}{\sqrt{m_e^2+2eHn+p_3^2}[M_{1,2}^2+2E_\nu E_n-2p_3q_3]}, \qquad (20)$$

where

$$a=\sqrt{\mu^2-m^2-2eHn},$$

are given in the Appendix. In Eqs. (18) and (19), $M_{1,2}^2=M_w^2-m_e^2-m_\nu^2\pm eH$, and $N_0=[(\mu^2-m^2)/2eH]$ is the integer part of the parameter $(\mu^2-m^2)/(2eH)$. Note that the results (18) and (19) are exact in the limit of zero temperature. In relatively strong magnetic fields, when

$$2eH>\mu^2-m^2, \qquad (21)$$

the principal quantum number n can take only the value $n=0$, i.e., only the ground level is filled with electrons.

When the condition (21) is fulfilled the energy shift and AMM of the neutrino are determined by Eqs. (18) and (19), in which it is necessary to take only the term $n=0$ in the sum.

If we require also that the condition

$$E_\nu\ll\frac{M_W^2}{\mu}, \qquad (22)$$

be fulfilled, then for the part of the energy shift that does not depend on the neutrino spin we find from Eq. (19) in the first approximation:

$$\Delta E_\nu(H,\mu)=\frac{g^2 eH}{8\pi^2}\frac{E_\nu-q_3}{E_\nu}\frac{\sqrt{\mu^2-m^2}}{M_W^2}. \tag{23}$$

Taking into account next the relationship between the chemical potential and the density of the electron gas,[24]

$$n_e=\frac{eH}{2\pi^2}\sqrt{\mu^2-m^2}, \tag{24}$$

we represent the result (23) in the form

$$\Delta E_\nu(H,\mu)=\frac{g^2}{4}\frac{E_\nu-q_3}{E_\nu}\frac{n_e}{M_W^2}. \tag{25}$$

When the conditions (21) and (22) are fulfilled we obtain for the neutrino-AMM contribution due to the electron density

$$\Delta\mu_\nu=\frac{16\pi^2}{3}\frac{n_e(E_\nu-q_3)}{m_\nu^2 M_W^2}\mu_\nu^0, \tag{26}$$

where μ_ν^0 is the static magnetic moment (1) of the neutrino.

We next consider the case of a weak magnetic field, when

$$2eH\ll\mu^2-m^2. \tag{27}$$

When the condition (22) is fulfilled it follows from Eq. (19) that

$$\Delta E_\nu(H\neq0,\mu\neq0)=\frac{g^2}{4\pi^2}\frac{1}{M_W^2}\left[\frac{(\mu^2-m^2)^{3/2}}{3M_W^2}-\frac{eHq_3}{2E_\nu}\sqrt{\mu^2-m_e^2}\right], \tag{28}$$

where μ is the chemical potential of the free electron gas, related to the density n_e by the formula[25]

$$\frac{\mu}{m}=\sqrt{1+\left(\frac{3\pi^2 n_e}{m^3}\right)^{2/3}}. \tag{29}$$

Taking (29) into account, we write Eq. (28) in the form

$$\Delta E_\nu(H=0,\mu\neq0)=\sqrt{2}n_e G_F, \tag{30}$$

where $G_F=\sqrt{2}g^2/(8M_W^2)$ is the Fermi constant.

As we should expect, the result (30) coincides with the Wolfenstein formula for the energy shift of an electron neutrino moving in a dense electron medium in the absence of an external field.[26,27]

When the conditions (22) and (27) are fulfilled we have for the AMM of the neutrino the following representation:

$$\Delta\mu_\nu=-\frac{g^2}{8\pi^2}e\frac{E_\nu-q_3}{m_\nu}\frac{1}{M_W^2}(3\pi^2 n_e)^{1/3}=-\mu_\nu^0\frac{8}{3}\frac{E_\nu-q_3}{m_\nu}\frac{(3\pi^2 n_e)^{1/3}}{m_\nu}. \tag{31}$$

3. ENERGY SHIFT AND AMM OF A RELATIVISTIC NEUTRINO MOVING PERPENDICULAR TO A MAGNETIC FIELD

Suppose that a relativistic neutrino is moving perpendicular to a weak magnetic field, when

$$H\ll\frac{M_W^2}{e}\simeq10^{24}\ \text{G},\quad q_3=0,\quad q_\perp\gg M_W. \tag{32}$$

In this case the main contribution to the integral (12) is given by the region $y\ll1$.

In the first approximation in the zero-temperature limit, the part of the energy shift that depends explicitly on the orientation of the neutrino spin takes the form

$$\Delta E_\nu(s,H,\mu)=i\frac{g^2}{8\pi^2}\frac{eHsm_\nu}{E_\nu m^2}\sum_n\int dk_3\theta(\mu-E_n)\times\int_0^\infty dt\exp(-i\lambda t-\kappa t^2)\times\left[L_n(z)-L_{n-1}(z)+i\frac{H}{H_0}t(L_n+L_{n-1})\right], \tag{33}$$

where we have adopted the notation

$$\varkappa=\frac{H}{B_0}\left(\frac{E_\nu}{m_e}\right)^2,\quad\lambda=\frac{M_0^2+2E_\nu E_n}{m_e^2},$$

$$M_0^2=M_W^2-m_e^2-m_\nu^2. \tag{34}$$

Henceforth we shall confine ourselves to considering a sufficiently wide range of values of the neutrino energy:

$$M_W\ll E_\nu\ll\frac{M_W^2}{\nu}. \tag{35}$$

We first consider the case of relatively strong fields, when the condition (21) is valid. Making use of the fact that $L_0(z)=1$ and performing the integration over the variables t and k_3 in (33), we find

$$\text{Re}(\Delta E_\nu^s)=\frac{g^2 sm_\nu}{4E_\nu M_W^2}\left[n_e-2eHn_e\left(\frac{E_\nu}{M_W^2}\right)^2-2\frac{E_\nu}{M_W^2}w_e\right], \tag{36}$$

where w_e is the (volume) energy density of the electron gas:

$$w_e=\frac{E_e}{V}=\frac{eH}{4\pi^2}m_e^2\left[\frac{\mu}{m_e}\sqrt{\left(\frac{\mu}{m_e}\right)^2-1}+\ln\left(\frac{\mu}{m_e}+\sqrt{\left(\frac{\mu}{m_e}\right)^2-1}\right)\right]. \tag{37}$$

According to this formula, the AMM of the neutrino is equal to

$$\Delta\mu_\nu=\frac{32\pi^2}{3}\frac{n_e}{m_\nu M_W^2}\left(\frac{E_\nu}{M_W}\right)^2\mu_\nu^0. \tag{38}$$

We note that the condition (21) is equivalent to the following restriction on the electron density:[24]

$$n_e < \pi^{-2}(eH)^{3/2}\frac{1}{\sqrt{2}}, \tag{39}$$

which for $H \sim B_0$ is several orders of magnitude smaller than the highest of the available estimates for the density of the electron-positron plasma in the magnetosphere of pulsars.

In the limit under consideration, when the restrictions (32), (35), and (21) are valid, for the part of the energy shift that does not depend on the orientation of the neutrino spin the result (30) follows in the first approximation, and this can confirm the correctness of the calculations performed.

Next we investigate the case of weak fields, when (27) is valid, and confine ourselves to considering the AMM of the neutrino.

In this case, in Eq. (33), in the important region, we have $z \ll 1$. Using the asymptotic form of the Laguerre functions for small values of the argument,[21] for values of the neutrino energy that lie in the region (35) we obtain

$$\Delta\mu_\nu = -\frac{16}{3}\frac{(3\pi^2 n)^{1/3}}{m_\nu}\mu_\nu^0. \tag{40}$$

In this case, as can be seen, $\Delta\mu$ differs only by a kinematic factor from the corresponding value (31) for longitudinal motion.

4. DISCUSSION OF THE RESULTS

First of all we shall perform numerical estimates of the results obtained.

To estimate the AMM of the neutrino in the limiting case of relatively strong fields, we set $n_e \simeq \frac{1}{2}\pi^{-2}(H/B_0)^{3/2}m^3$, which corresponds to a chemical-potential value given by $\sqrt{\mu^2-m^2} = m(H/B_0)^{1/2}$. Then from Eq. (38), setting $m_\nu = 10$ eV, we have

$$\frac{\Delta\mu_\nu}{\mu_\nu^0} \simeq 10^{-5}\left(\frac{H}{B_0}\right)^{3/2}\left(\frac{E_\nu}{M_W}\right)^2. \tag{41}$$

For $H = 10^{-2}B_0$ and $E_\nu \simeq 10^4 M_W$ [the latter agrees with the restriction (35)] it follows from (41) that $\Delta\mu_\nu = \mu_\nu^0$, while for $H = B_0$ and $E_\nu \simeq 10^3 M_W$ we have $\Delta\mu_\nu = 10\mu_\nu^0$.

In the case of weak magnetic fields, substituting $n_e = 10^{24}$ cm^{-3} into Eq. (40), corresponding to the density of the free electron gas in metals, we find $|\Delta\mu_\nu| \approx 50\mu_\nu^0$, while for $n_e = 10^{38}$ cm^{-3} we have $|\Delta\mu_\nu| \simeq 5\times 10^4\mu_\nu^0$. With regard to the latter estimate we note that there are indications that the core of a neutron star can be represented as a lattice of ions immersed in an electron gas that does indeed possess a sufficiently high density $n_e \sim 10^{38}$ cm^{-3} and a moderate temperature $T \sim 10^6$–10^9 K, i.e., is strongly degenerate.

As follows from the estimates given, the AMM of the neutrino in a dense electron medium at zero temperature can be substantially greater than its static value (1) both in the case of a relatively strong field and in the case of weak fields.

Also interesting is the fact that as the magnetic-field strength increases the AMM of the neutrino, which is negative in the case of weak fields, increases, passes through zero, and becomes positive.

The question of the compatibility of the small mass and large magnetic moment of the neutrino in the framework of the problem under consideration receives a positive answer, and for this there is no need to go beyond the standard Weinberg–Salam–Glashow model. This is because the AMM of the neutrino is directly proportional to the density of the medium, and not to the neutrino mass as in the absence of a medium.

In conclusion, we note that the energy shift of the neutrino [cf., e.g., Eqs. (25) and (30)] depends dynamically on the magnetic-field strength and the neutrino energy. This can turn out to be important in the study of the character of the oscillations for a beam of neutrinos in matter in an external magnetic field.

APPENDIX

The integrals F_1 and F_2 are equal to

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \frac{1}{2}B \pm \frac{1}{2}(m_e^2 + 2eHn)A.$$

Here,

$$A = G(x_1) - G(x_2), \quad B = Q(x_1) - Q(x_2),$$

$$x_1 = \mu + \sqrt{\mu^2 - m_e^2 - 2eHn}, \quad x_2 = \mu - \sqrt{\mu^2 - m_e^2 - 2eHn}.$$

The functions $G(x)$ and $Q(x)$ are equal to

$$G(x) = \frac{1}{2c}\ln\frac{x^2}{ax^2+bx+c} - \frac{b}{2c}P(x),$$

$$Q(x) = \frac{1}{2a}\ln(ax^2+bx+c) - \frac{b}{2a}P(x),$$

where

$$P(x) = \begin{cases} \dfrac{1}{\sqrt{-\Delta}}\ln\dfrac{b+2ax-\sqrt{-\Delta}}{b+2ax+\sqrt{-\Delta}}, & \Delta < 0 \\ \dfrac{2}{\sqrt{\Delta}}\operatorname{arctg}\dfrac{b+2ax}{\sqrt{\Delta}}, & \Delta > 0 \end{cases}$$

$$a = E_\nu - q_3, \quad b = M^2, \quad c = (E_\nu + q_3)(m_e^2 + 2eHn),$$

$$\Delta = 4ac - b^2.$$

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