Elasticity and thermal expansion anomalies of anisotropic high-T_c superconductors

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The idea of a tensor of elastic moduli with constant superconducting order parameter is used in accordance with Landau's theory of phase transitions. A method is indicated for determining the components of this tensor on the basis of the experimental data. A relation is established between the jump in the heat capacity at the superconducting transition and the experimentally measured jumps of the thermal-expansion coefficients and the temperature derivatives of the elastic moduli. An interpretation is given for the observed changes in the slope angles of the temperature dependence of the thermal-expansion coefficients in a $YBa_2Cu_3O_{7-x}$ single crystal at the superconducting transition and it is predicted that the pressure dependence of the heat-capacity jump in this single crystal is anisotropic. It is predicted that superconductivity has an anisotropic effect on the elasticity and some second derivatives of the superconducting transition temperature with respect to strains in the $YBa_2Cu_2O_{7-x}$ single crystal are determined. Some components of the tensor of coefficients of superconducting elasticity are determined and it is concluded that the pressure derivatives of the parameters in Landau's expansion of the free energy in this single crystal are anisotropic.

1. INTRODUCTION

In any investigation of the properties of a high-T_c superconductor it is often asserted that these properties are anomalous. Such assertions are, in turn, associated with the fact that due to the high superconducting transition temperature T_c superconductivity is found to have an anomalously strong effect on a number of the most important properties of high-T_c superconductors. Thus, in particular, in comparatively recent experimental studies of thermal expansion[1] and of the elastic properties[2,3] of perfect $YBa_2Cu_3O_{7-x}$ and $La_{2-x}Sr_xCuO_4$ single crystals it was concluded that superconductivity significantly influences the properties investigated. This influence on the thermal expansion of $YBa_2Cu_3O_{7-x}$ (Ref. 1) and elasticity of $La_{2-x}Sr_xCuO_4$ (Ref. 3) is characterized by very strongly expressed anisotropy. Analysis of the experimental data of Refs. 1–3 shows that, first, the usual (for the superconducting transition) jumps of the thermal-expansion coefficients and elastic moduli are determined well. Besides this, a significant difference was observed in the slope angles of the temperature dependence of both the thermal-expansion coefficients and the elastic moduli above and below T_c.

In order to analyze the experimental data it is necessary to have a theory that 1) makes it possible to correlate different experimental results, 2) enables quantitative predictions on the basis of existing data, and 3) makes it possible to determine, on the basis of the experimental data, the values of the parameters of interest in order to develop new theoretical models.

In the present paper we show how the latest experimental data for anisotropic crystals of high-T_c superconductors[1–3] can be understood on the basis of Landau's simple theory of second-order phase transitions, taking into account the effect of the superconducting order parameter so as to be able to describe quantitatively the superconductivity-induced changes in the temperature dependence of the thermal expansion and elasticity. In addition, we show how to obtain from the experimental data the numerical values of the components of the tensor of superconducting elasticity coefficients, which characterizes the dependence of the elastic moduli on the superconducting order parameter. Knowledge of this tensor is necessary in order to ascertain the importance of the dependence of elasticity on the superconducting order parameter, which traditional microscopic theories usually neglect (compare Refs. 4 and 5), in order to construct a theory of superconductivity of high-T_c superconductors.

The consequences of Landau's theory of second-order phase transitions, in application to the description of the anomalies of elasticity of a superconductor, are presented in Sec. 2. The idea of the tensor of isothermal elastic moduli with a constant superconducting order parameter is employed. It is this tensor that can be employed directly in a theory of superconductivity that takes into account the dependence of the elastic properties of a superconductor on the order parameter. A relation is established between this tensor and the tensor of the experimentally measured elastic moduli. Relations determining the jump in the temperature derivative of the elastic moduli at the superconducting transition are derived together with the usual jumps of the elastic moduli.

A theory of anomalies of the thermal expansion of anisotropic high-T_c superconductors is developed in Sec. 3. Just as was shown for the elastic moduli, here jumps in the temperature derivatives of the thermal expansion coefficients at a superconducting transition are established together with the usual jumps in these coefficients. Formulas relating the experimental results to the parameters in

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Landau's expansion of the free energy are derived for the jumps of such temperature derivatives. A relation is established between the jump in the heat capacity and the jumps of the thermal-expansion coefficients and of the temperature derivatives of the elastic moduli.

A method made possible by the theory presented in Secs. 2 and 3 for finding the components of the coefficients of the superconducting elasticity tensor, which determine the dependence of the elasticity on the superconducting order parameter, is discussed in Sec. 4. An expression is presented for this tensor in the particular case of the BCS model.

Finally, in order to illustrate the possibilities of the theory, in Sec. 5 the theory is compared with experiments. An interpretation is given for the observed changes in the slope angles of the temperature dependence of the thermal-expansion coefficients in a $YBa_2Cu_3O_{7-x}$ single crystal at the superconducting transition and it is predicted that the pressure dependence of the jump in the heat capacity in this single crystal is anisotropic. It is predicted that the effect of superconductivity on elasticity is anisotropic and some second derivatives of the superconducting transition temperature with respect to strains in the $YBa_2Cu_3O_{7-x}$ single crystal are determined. Some components of the tensor of superconducting elasticity coefficients are determined and it is concluded that in this crystal the pressure derivatives of the parameters in Landau's expansion of the free energy are anisotropic.

2. ANOMALIES OF THE ELASTIC MODULI

We start with the free-energy density $F_\Delta(T,\Delta,\hat{e})$ of a superconductor as a function of the thermodynamic variables: the temperature T, the superconducting order parameter Δ, and the pure-strain tensor $\hat{e}$. In order to simplify the formulas for the tensor expressions, we employ below the following notations for the components of the stress tensor e_i $(i=1\text{–}6)$, where $e_1=e_{11}$, $e_2=e_{22}$, $e_3=e_{33}$, $e_4=e_{23}$, $e_5=e_{13}$, and $e_6=e_{12}$. Then the equation of the superconducting state of a strained superconductor in the absence of a magnetic field will be determined by the relation

$$\left(\frac{\partial F_\Delta}{\partial \Delta}\right)_{T,\hat{e}}=0. \tag{2.1}$$

In accordance with the free energy $F_\Delta(T,\Delta,\hat{e})$, we employ, by analogy to ferromagnetics, the idea of the tensor of isothermal moduli of elasticity with a constant superconducting order parameter (see, for example, Refs. 6 and 7)

$$C^\Delta_{ij}(T,\Delta)=\left(\frac{\partial^2 F_\Delta}{\partial e_i \partial e_j}\right)_{T,\Delta}. \tag{2.2}$$

The usefulness of such a tensor in application to superconductors is due, in part, to the problem of constructing a theory of superconductivity that takes into account the dependence of the phonon spectrum on the superconducting order parameter in a high-T_c superconductor.[4,5] At the same time, as will be demonstrated below, it seems natural to employ such a tensor in the thermodynamics of superconductors.

The equation (2.1) determines the order parameter $\Delta(T,\hat{e})$ as a function of the variables T and $\hat{e}$. This makes it possible to switch from the free-energy density $F_\Delta(T,\Delta,\hat{e})$ to the free-energy density $F_s(T,\hat{e})$ of a superconductor as a function of the variables T and $\hat{e}$ by means of the relation

$$F_s(T,\hat{e})=F_\Delta(T,\Delta(T,\hat{e}),\hat{e}). \tag{2.3}$$

The free energy (2.3) makes it possible to determine the tensor of isothermal elastic moduli of the superconductor

$$C^s_{ij}(T)=\left(\frac{\partial^2 F_s}{\partial e_i \partial e_j}\right)_T, \tag{2.4}$$

which can be measured experimentally. Having in mind, e.g., the problem of finding for a superconductor the tensor $\hat{C}^\Delta(T,\Delta)$ as a function of Δ, we now establish a relation between this tensor and the experimentally measured tensor $\hat{C}^s(T)$ of elastic moduli. Using the relations (2.1)–(2.4) we find

$$C^s_{ij}=C^\Delta_{ij}+\left(\frac{\partial^2 F_\Delta}{\partial \Delta \partial e_j}\right)_T\left(\frac{\partial \Delta}{\partial e_i}\right)_T. \tag{2.5}$$

Differentiating Eq. (2.1) with respect to the tensor $\hat{e}$ with $T=\text{const}$ gives

$$\left(\frac{\partial^2 F_\Delta}{\partial e_j \partial \Delta}\right)_T=-\left(\frac{\partial^2 F_\Delta}{\partial \Delta^2}\right)_{T,\hat{e}}\left(\frac{\partial \Delta}{\partial e_j}\right)_T. \tag{2.6}$$

The relations (2.5) and (2.6) make it possible to write down the desired relation between the tensors in the form

$$C^s_{ij}=C^\Delta_{ij}-\left(\frac{\partial^2 F_\Delta}{\partial \Delta^2}\right)_{T,\hat{e}}\left(\frac{\partial \Delta}{\partial e_i}\right)_T\left(\frac{\partial \Delta}{\partial e_j}\right)_T. \tag{2.7}$$

We now work out in detail the relations discussed here for the neighborhood of a superconducting transition, when Landau's expansion in the parameter Δ can be used for the free energy $F_\Delta(T,\Delta,\hat{e})$:

$$F_\Delta(T,\Delta,\hat{e})=F_0(T,\hat{e})+\tfrac{1}{2}a(T,\hat{e})\Delta^2+\tfrac{1}{4}b(\hat{e})\Delta^4, \tag{2.8}$$

where $F_0(T,\hat{e})$ is the normal-state free-energy density of the metal. Assuming the elastic strains to be small, we expand Eq. (2.8) in powers of the strain tensor up to quadratic terms (compare Ref. 8):

$$F_\Delta(T,\Delta,\hat{e})=F_0(T,0)+\frac{1}{2}a(T,0)\Delta^2+\frac{1}{4}b(0)\Delta^4+\left[\sigma^0_i(T)+\frac{1}{2}\left(\frac{\partial a}{\partial e_i}\right)_T\Delta^2+\frac{1db}{4de_i}\Delta^4\right]e_i+\frac{1}{2}C^\Delta_{ij}(T,\Delta)e_ie_j. \tag{2.9}$$

Here terms $\sim\Delta^4 e_i$ were retained and terms $\Delta^4 e_ie_j$ were neglected. In addition,

$$C^\Delta_{ij}(T,\Delta)=C^0_{ij}(T)+C'_{ij}(T)\Delta^2, \tag{2.10}$$

where we call the tensor

$$C'_{ij}(T)=\frac{1}{2}\left(\frac{\partial^2 a}{\partial e_i \partial e_j}\right)_T \qquad (2.11)$$

the tensor of coefficients of superconducting elasticity by analogy to the tensor of coefficients of magnetoelasticity employed for ferromagnetics (see, for example, Refs. 6 and 7). The tensors $\sigma_i^0(T)=(\partial F_0/\partial e_i)_T$ and $C_{ij}^0(T)=(\partial^2 F_0/\partial e_i \partial e_j)_T$ determine the isothermal internal stresses and the elastic moduli of the metal in the normal state.

Using the expansion (2.9), we write an equation for the superconducting state (2.1) to within terms linear in the strain tensor

$$a+b\Delta^2+\left[\left(\frac{\partial a}{\partial e_i}\right)_T+\frac{db}{de_i}\Delta^2\right]e_i=0 \qquad (2.12)$$

and an expression for the second derivative of the free energy in absence of strain ($\hat{e}=0$)

$$\left(\frac{\partial^2 F_\Delta}{\partial \Delta^2}\right)_{T,\hat{e}=0}=2b\Delta^2. \qquad (2.13)$$

Differentiating Eq. (2.12) with respect to the tensor $\hat{e}$ with $T=\text{const}$ we obtain the derivative

$$\left(\frac{\partial \Delta}{\partial e_i}\right)_T=-\frac{1}{2b\Delta}\left[\left(\frac{\partial a}{\partial e_i}\right)_T+\frac{db}{de_i}\Delta^2\right]. \qquad (2.14)$$

As a result, the relation (2.7), taking into account Eqs. (2.13) and (2.14), near the superconducting transition temperature assumes for temperatures $T<T_c$ the following form (accurate up to quadratic terms in the parameter Δ):

$$C_{ij}^s=C_{ij}^\Delta-\frac{1}{2b}\left\{\left(\frac{\partial a}{\partial e_i}\right)_T\left(\frac{\partial a}{\partial e_j}\right)_T+\left[\left(\frac{\partial a}{\partial e_i}\right)_T\frac{db}{de_j}+\left(\frac{\partial a}{\partial e_j}\right)_T\frac{db}{de_i}\right]\Delta^2\right\}. \qquad (2.15)$$

In order to specify further the expressions (2.11) and (2.15), we must introduce, as is usually done in Landau's theory, the temperature dependence of the parameter a near the superconducting transition temperature

$$a(T)=\alpha(T-T_c). \qquad (2.16)$$

Then for the tensor C'_{ij} (2.11) of the coefficients of superconducting elasticity, which determine the dependence of the tensor $\hat{C}^\Delta(T,\Delta)$ of the elastic moduli on the superconducting order parameter Δ, we obtain

$$C'_{ij}=-\frac{\alpha}{2}\left[\frac{d\ln\alpha}{de_i}\frac{dT_c}{de_j}+\frac{d\ln\alpha}{de_j}\frac{dT_c}{de_i}+\frac{d^2T_c}{de_i de_j}\right], \qquad (2.17)$$

and the expression (2.15), which establishes a relation between the tensor $\hat{C}^\Delta$ and the tensor $\hat{C}^s$ of experimentally measured elastic moduli of the superconductor, assumes the form ($T<T_c$)

$$C_{ij}^s(T)=C_{ij}^\Delta(T,\Delta(T))-\frac{\alpha^2}{2b}\frac{dT_c}{de_i}\frac{dT_c}{de_j}-\frac{\alpha}{2}\left[\frac{d\ln(\alpha/b)}{de_i}\frac{dT_c}{de_j}+\frac{d\ln(\alpha/b)}{de_j}\frac{dT_c}{de_i}\right]\Delta^2(T), \qquad (2.18)$$

where the temperature dependence of the superconducting order parameter, in the absence of strain, is determined from Eqs. (2.12) and (2.16) by the standard expression

$$\Delta^2(T)=\frac{\alpha}{b}(T_c-T). \qquad (2.19)$$

Since $\hat{C}^s$ and $\hat{C}^\Delta$ are tensors of rank 4 and the number of independent and nonzero components of tensors of rank 2, corresponding in our notation to the expressions dT_c/de_i and $d\ln(\alpha/b)/de_i$, can be reduced in our case to three ($i=1, 2, 3$), in accordance with the formula (2.18) the tensors $\hat{C}^s$ and $\hat{C}^\Delta$ differ for the components with $i, j=i$, 2, 3. For the other components, we have from Eq. (2.18) the simple relation

$$C_{ij}^s(T)=C_{ij}^\Delta(T,\Delta(T))=C_{ij}^0(T)+C'_{ij}\Delta^2(T), \qquad (2.20)$$

where

$$C'_{ij}=-\frac{\alpha}{2}\frac{d^2T_c}{de_i de_j}. \qquad (2.21)$$

We also give an expression describing the explicit temperature dependence [corresponding to Eq. (2.16)] of the tensor $\hat{C}^s(T)$ of the elastic moduli. Using Eqs. (2.17)–(2.19) we find ($T<T_c$)

$$C_{ij}^s(T)=C_{ij}^0(T)-\frac{\alpha^2}{2b}\left\{\frac{dT_c}{de_i}\frac{dT_c}{de_j}+\left[\frac{d\ln(\alpha^2/b)}{de_i}\frac{dT_c}{de_j}+\frac{d\ln(\alpha^2/b)}{de_j}\frac{dT_c}{de_i}+\frac{d^2T_c}{de_i de_j}\right](T_c-T)\right\}. \qquad (2.22)$$

The expression (2.22) represents the general thermodynamic relations derived in Ref. 9 for an arbitrary second-order phase transition and worked out in detail for Landau's theory of phase transitions using the expansion (2.9) and (2.16) for the free energy. The expression (2.22) describes, besides the usually discussed jump of the elastic moduli of a superconductor at the phase-transition point (see, for example, Ref. 3)

$$\delta C_{ij}^s(T_c)=-\frac{\alpha^2}{2b}\frac{dT_c}{de_i}\frac{dT_c}{de_j}, \qquad (2.23)$$

the change in the slope angle of the temperature dependence of the elastic moduli, which is proportional to $\Delta^2(T)$, at a transition from the normal state into the superconducting state. As a result, the temperature derivative of the elastic moduli (2.22) taken below and above T_c will undergo a jump by the amount

$$\delta\left(\frac{dC_{ij}^s}{dT}\right)=\frac{\alpha^2}{2b}\left[\frac{d\ln(\alpha^2/b)}{de_i}\frac{dT_c}{de_j}+\frac{d\ln(\alpha^2/b)}{de_j}\frac{dT_c}{de_i}+\frac{d^2T_c}{de_i de_j}\right]. \tag{2.24}$$

We note here that, on the basis of our approximations (2.9) and (2.16), the relation (2.24) can be represented in a form identical to the result obtained with the model approach of Ref. 10 if we set $\alpha^2/2b=N$, where N is the electron density of states per unit volume at the Fermi level. This assumption differs from the BCS model (see Eq. (4.16) below) by the numerical factor $4\pi^2/7\zeta(3)\simeq 4.7$, if the temperature is expressed in energy units. Comparing the relations obtained above to experiment shows that it is convenient to determine the ratio $\alpha^2/2b$ from the jump in the heat capacity at the phase-transition point

$$\frac{\alpha^2}{2b}=\frac{\delta C_v(T_c)}{T_c}, \tag{2.25}$$

where $\delta C_v(T_c)$ is the jump in the heat capacity at constant volume per unit volume. Here no distinction need be made between the jumps of the heat capacity at constant volume $\delta C_v(T_c)$ and at constant pressure $\delta C_p(T_c)$, since this difference is small.[10]

3. THERMAL-EXPANSION ANOMALIES

In the case of free thermal expansion the components of the internal-stress tensor in a superconductor should vanish. Thus if one works with the free energy $F_\Delta(T,\Delta,\hat{e})$, then one should talk about the tensor $\sigma^\Delta(T,\Delta,\hat{e})$ of isothermal internal stresses with constant superconducting order parameter. Under these conditions the thermal expansion of the superconductor will be described by the equation

$$\sigma_i^\Delta(T,\Delta,\hat{e})=\left(\frac{\partial F_\Delta}{\partial e_i}\right)_{T,\Delta}=0, \tag{3.1}$$

which determines the strain tensor $\hat{e}^\Delta(T,\Delta)$ as a function of the variables T and Δ. Here and below, by analogy to the strain tensor $\hat{e}$, the notation σ_i $(i=1$–$6)$, where $\sigma_1=\sigma_{11}$, $\sigma_2=\sigma_{22}$, $\sigma_3=\sigma_{33}$, $\sigma_4=\sigma_{23}$, $\sigma_5=\sigma_{13}$ and $\sigma_6=\sigma_{12}$, is employed for the components of the stress tensor $\hat{\sigma}$.

We now introduce the tensor of linear thermal-expansion coefficients of the superconductor with constant order parameter by the following relation:

$$\beta_j^\Delta(T,\Delta)=\left(\frac{\partial e_j^\Delta}{\partial T}\right)_\Delta. \tag{3.2}$$

Then, differentiating Eq. (3.1) with respect to the temperature with $\Delta=$const and using the definitions (2.2) and (3.2), we obtain

$$\left(\frac{\partial^2 F_\Delta}{\partial T\partial e_i}\right)_\Delta+C_{ij}^\Delta(T,\Delta)\beta_j^\Delta(T,\Delta)=0. \tag{3.3}$$

Switching to the free energy $F_s(T,\hat{e})$ (2.3), we have instead of Eq. (3.1) the following equation, describing the thermal expansion of the superconductor

$$\sigma_i^s(T,\hat{e})=\left(\frac{\partial F_s}{\partial e_i}\right)_T\equiv\left(\frac{\partial F_\Delta}{\partial e_i}\right)_{T,\Delta(T,\hat{e})}=0, \tag{3.4}$$

which determines the strain tensor $\hat{e}^s(T)$ as a function of temperature and makes it possible to introduce the tensor of the linear thermal-expansion coefficients of a superconductor, usually employed in experiments:

$$\beta_j^s(T)=\frac{de_j^s}{dT}. \tag{3.5}$$

We now establish a relation between the tensors $\hat{\beta}^s(T)$ and $\hat{\beta}^\Delta(T,\Delta)$. For this we differentiate Eq. (3.4) with respect to the temperature and use the definitions (2.2) and (3.5). Then we obtain

$$\left(\frac{\partial^2 F_\Delta}{\partial T\partial e_i}\right)_\Delta+\left(\frac{\partial^2 F_\Delta}{\partial\Delta\partial e_i}\right)_T\left[\left(\frac{\partial\Delta}{\partial T}\right)_{\hat{e}}+\left(\frac{\partial\Delta}{\partial e_j}\right)_T\beta_j^s(T)\right]+C_{ij}^\Delta(T,\Delta)\beta_j^s(T)=0. \tag{3.6}$$

Next, using the relations (2.6), (2.7), and (3.3) we obtain the desired relation

$$C_{ij}^s\beta_j^s=C_{ij}^\Delta\beta_j^\Delta+\left(\frac{\partial^2 F_\Delta}{\partial\Delta^2}\right)_{T,\hat{e}}\left(\frac{\partial\Delta}{\partial T}\right)_{\hat{e}}\left(\frac{\partial\Delta}{\partial e_i}\right)_T. \tag{3.7}$$

We now consider the consequences of the relations (3.3) and (3.7) near a superconducting transition, when the free-energy expansion (2.9) can be used. Then we have

$$\left(\frac{\partial^2 F_\Delta}{\partial T\partial e_i}\right)_\Delta=\frac{d\sigma_i^0}{dT}+\frac{1}{2}\left(\frac{\partial^2 a}{\partial T\partial e_i}\right)\Delta^2 \tag{3.8}$$

and the relation, following from Eqs. (2.10) and (3.3),

$$\frac{d\sigma_i^0}{dT}+\frac{1}{2}\left(\frac{\partial^2 a}{\partial T\partial e_i}\right)\Delta^2+C_{ij}^0\beta_j^\Delta(T,\Delta)=0 \tag{3.9}$$

up to terms quadratic in the order parameter Δ. In Eq. (3.9) the corrections of order $C'_{ij}\beta_j^0\Delta^2$ are small because the parameter $(dT_c/de_j)\beta_j^0\ll 1$ is small. We find from Eq. (3.9) the following expression for the tensor of thermal-expansion coefficients of the superconductor with constant order parameter

$$\beta_j^\Delta(T,\Delta)=\beta_j^0(T)+\beta'_j\Delta^2, \tag{3.10}$$

where

$$\beta_j^0(T)=-[C_{ji}^0]^{-1}\frac{d\sigma_i^0}{dT} \tag{3.11}$$

is the tensor of thermal-expansion coefficients of the normal-state metal, and

$$\beta'_j=-\frac{1}{2}[C_{ji}^0]^{-1}\left(\frac{\partial^2 a}{\partial T\partial e_i}\right)=-\frac{\alpha}{2}[C_{ji}^0]^{-1}\frac{d\ln\alpha}{de_i}. \tag{3.12}$$

Next, using the expressions (2.13), (2.14), (2.16), and (2.19), we find from Eq. (3.7) a relation between the tensor $\hat{\beta}^s$ and $\hat{\beta}^\Delta$ of the thermal-expansion coefficients near the superconducting transition temperature $(T<T_c)$

$$C^s_{ij}\beta^s_j=C^\Delta_{ij}\beta^\Delta_j-\frac{\alpha^2}{2b}\frac{dT_c}{de_i}-\frac{\alpha}{2}\frac{d\ln(\alpha/b)}{de_i}\Delta^2. \tag{3.13}$$

In order to obtain an explicit expression for the experimentally measured tensor $\hat{\beta}^s(T)$ of thermal-expansion coefficients of the superconductor, we employ the formulas (2.16), (3.3), (3.8), and (3.11). Then we obtain from Eq. (3.13) $(T<T_c)$

$$C^s_{ij}\beta^s_j=C^0_{ij}\beta^0_j(T)-\frac{\alpha^2}{2b}\frac{dT_c}{de_i}-\frac{\alpha}{2}\frac{d\ln(\alpha^2/b)}{de_i}\Delta^2(T), \tag{3.14}$$

where the temperature dependence $\Delta^2(T)$ is determined by Eq. (2.19).

The expression (3.14) represents the general thermodynamic relations, derived in Ref. 9 for an arbitrary second-order phase transition, applied to Landau's theory of phase transitions using the expansion (2.9) and (2.16) for the free energy.

The main difference between expression (3.14) and the corresponding result of Ref. 10 is that it was written to within terms quadratic in the order parameter Δ, and for this reason it describes not only the usual jumps of the thermal-expansion coefficients at the phase-transition point (see, for example, Ref. 1)

$$\delta\beta^s_j(T_c)=-\frac{\alpha^2}{2b}[C^0_{ji}]^{-1}\frac{dT_c}{de_i}, \tag{3.15}$$

but it also describes the change in the slope angle of the temperature dependence of the components of the tensor $\hat{\beta}^s(T)$ at the transition into the superconducting state. As a result, it is possible to describe the jump in the temperature derivative of the thermal-expansion coefficients, which is taken above and below T_c, by the amount

$$\delta\left(\frac{d\beta^s_j}{dT}\right)=\frac{\alpha^2}{2b}[C^0_{ji}]^{-1}\frac{d\ln(\alpha^2/b)}{de_i}. \tag{3.16}$$

Comparing the expressions (2.24) and (3.16), we can establish with the help of Eq. (2.25) an identity relating the jumps of the temperature derivatives of the elastic moduli and the thermal-expansion coefficients to the jump in the heat capacity

$$\delta\left(\frac{dC^s_{ij}}{dT}\right)-\delta\left(\frac{d\beta^s_k}{dT}\right)\left[C^0_{ik}\frac{dT_c}{de_j}+C^0_{jk}\frac{dT_c}{de_i}\right]=\frac{\delta C_v}{T_c}\frac{d^2T_c}{de_ide_j}. \tag{3.17}$$

Since the quantities appearing on the left-hand side of the formulas (3.16) and (3.17) can be measured experimentally, just like the jump in the heat capacity $\delta C_v(T_c)$ and the tensor of elastic moduli C^0, the relations (3.16) and (3.17) make it possible to determine the parameters of superconductors, such as the derivatives $d\ln(\alpha^2/b)/de_i$ and (d^2T_c/de_ide_j).

4. POSSIBLE EXPERIMENTAL DETERMINATION OF THE TENSOR OF COEFFICIENTS OF SUPERCONDUCTING ELASTICITY

The situation is simplest for the components of the tensor C'_{ij} of superconducting elasticity, which correspond to the components C^s_{ij} of the tensor of elastic moduli, which do not change discontinuously at the superconducting transition (i.e., $i=4, 5, 6$ or $j=4, 5, 6$), and for which, according to Eq. (2.21), it is sufficient to know the coefficient α and the derivatives d^2T_c/de_ide_j. Since for such coefficients the formula (3.17) assumes the form

$$\delta\left(\frac{dC^s_{ij}}{dT}\right)=\frac{\delta C_v}{T_c}\frac{d^2T_c}{de_ide_j}, \tag{4.1}$$

it is obvious that if the jump of the temperature derivative of the elastic moduli and the jump of the heat capacity, which are determined experimentally, are known, then the derivatives d^2T_c/de_ide_j can be found directly.

The coefficient α can be determined by using the experimentally determined temperature dependence of the squared order parameter near T_c, when in the absence of strain we have from Eq. (2.19)

$$\left(\frac{\partial\Delta^2}{\partial T}\right)_{\hat{e}=0}=-\frac{\alpha}{b}. \tag{4.2}$$

Correspondingly, and using the expression (2.25), we find

$$\alpha=-2\frac{\delta C_v}{T_c}\left(\frac{\partial\Delta^2}{\partial T}\right)^{-1}_{\hat{e}=0}. \tag{4.3}$$

Thus we obtain, for the components of the tensor C'_{ij} with $i=4, 5, 6$ or $j=4, 5, 6$, from Eqs. (2.21), (4.1), and (4.3) the following expression:

$$C'_{ij}=\left(\frac{\partial\Delta^2}{\partial T}\right)^{-1}_{\hat{e}=0}\delta\left(\frac{dC^s_{ij}}{dT}\right). \tag{4.4}$$

Somewhat more experimental information is required in order to determine the tensor components C'_{ij} with $i,j=1, 2, 3$, which correspond to the components of the elastic moduli C^s_{ij} which undergo a jump at a phase transition. Here it is sufficient to know in addition the quantities dT_c/de_i and $d\ln\alpha/de_i$.

The pressure derivatives of the superconducting transition temperature are usually determined experimentally. If we now switch to derivatives with respect to the pressure tensor $\hat{p}=-\hat{\sigma}^0$ by means of the relation

$$\frac{d}{de_i}=-C^0_{ij}\frac{d}{dp_j}, \tag{4.5}$$

then, for example, Eqs. (3.15) and (3.16) will assume the form

$$\delta\beta^s_i(T_c)=\frac{\delta C_v}{T_c}\frac{dT_c}{dp_i}, \tag{4.6}$$

$$\delta\left(\frac{d\beta^s_i}{dT}\right)=-\frac{\delta C_v}{T_c}\frac{d\ln(\alpha^2/b)}{dp_i}. \tag{4.7}$$

Here the expression (2.25) was also taken into account. If the pressure (anisotropic) derivatives of T_c are determined experimentally, then the required values of the derivatives dT_c/de_i can be determined with the help of Eq. (4.5) and the known tensor of elastic moduli C_{ij}^0 of the normal-state metal. Similarly, the formula (4.7) or the equivalent formula (3.16) makes it possible to find

$$\frac{d\ln(\alpha^2/b)}{de_i}=C_{ij}^0\delta\left(\frac{d\beta_j^s}{dT}\right)\frac{T_c}{\delta C_v}, \tag{4.8}$$

from the experimentally measured jump of the temperature derivative of the thermal-expansion coefficients. However, this is still not enough in order to find the derivatives $d\ln\alpha/de_i$ experimentally. The desired derivatives can be found by determining experimentally the strain (or pressure) dependence of the slope angle of the temperature dependence of the squared superconducting gap near T_c, since

$$\frac{\partial^2\Delta^2}{\partial e_i\partial T}=\left(\frac{\partial\Delta^2}{\partial T}\right)_{\hat{e}=0}\frac{d\ln(\alpha/b)}{de_i}. \tag{4.9}$$

Then we find from Eqs. (4.8) and (4.9)

$$\frac{d\ln\alpha}{de_i}=C_{ik}^0\delta\left(\frac{d\beta_k^s}{dT}\right)\frac{T_c}{\delta C_v}-\frac{\partial^2\Delta^2}{\partial e_i\partial T}\left(\frac{\partial\Delta^2}{\partial T}\right)_{\hat{e}=0}^{-1}. \tag{4.10}$$

Now, using the relations (2.17), (3.17), (4.3), and (4.10), we can write the tensor C_{ij}' of coefficients of superconducting elasticity for any i and j in the following form:

$$C_{ij}'=\left(\frac{\partial\Delta^2}{\partial T}\right)_{\hat{e}=0}^{-1}\left\{\delta\left(\frac{dC_{ij}^s}{dT}\right)-\delta C_vQ_{ij}\right\}, \tag{4.11}$$

where

$$Q_{ij}=\frac{d\ln T_c}{de_i}\frac{d}{de_j}\ln\frac{\partial\Delta^2}{\partial T}+\frac{d\ln T_c}{de_j}\frac{d}{de_i}\ln\frac{\partial\Delta^2}{\partial T}. \tag{4.12}$$

In order to have some idea of the possible magnitude of Q_{ij} we point out that in the particular case of the BCS model

$$\left(\frac{\partial\Delta^2}{\partial T}\right)_{\hat{e}=0}=-\frac{\alpha}{b}=-\frac{8(\pi k)^2}{7\zeta(3)}T_c, \tag{4.13}$$

$$\frac{\partial^2\Delta^2}{\partial e_i\partial T}=-\frac{8(\pi k)^2}{7\zeta(3)}\frac{dT_c}{de_i}, \tag{4.14}$$

where k is Boltzmann's constant and $\zeta(3)\approx1.20$. In accordance with Eqs. (4.13) and (4.14) we have, according to the BCS model,

$$Q_{ij}^{\rm BCS}=2\frac{d\ln T_c}{de_i}\frac{d\ln T_c}{de_j}. \tag{4.15}$$

Finally, we point out that according to the BCS model

$$\alpha=-\frac{N}{T_c},\quad b=\frac{7\zeta(3)N}{8(\pi kT_c)^2}, \tag{4.16}$$

where N is the electron density of states per unit volume at the Fermi level.

5. DISCUSSION AND RESULTS

We now discuss the consequences of the theory, expounded above, in application to experiments with YBaCuO single crystals, which have orthorhombic structure near the superconducting transition temperature. We begin our discussion with thermal expansion, having in mind the remarkable experimental results of Ref. 1 on thermal-expansion anomalies of YBaCuO-single crystals. The thermal expansion of an orthorhombic lattice is determined by three expansion coefficients $\beta_i^s(T)$ ($i=1, 2, 3$) along the axes a, b, and c, respectively. In Ref. 1 a strong anisotropy was observed in the anomalous behavior of the thermal expansion of a YBaCuO single crystal near the superconducting transition. In particular, it was found that the jumps in the thermal-expansion coefficients $\beta_i^s(T)$ at $T=T_c\simeq91$ K along the a and b axes have different signs $\delta\beta_a^s(T_c)\simeq-\delta\beta_b^s(T_c)$, and no jump was observed in the thermal expansion coefficient along the c axis: $\delta\beta_c^s(T_c)\simeq0$. The experimental results of Ref. 1 were analyzed with the help of Eq. (4.6), on the basis of which it was concluded that the pressure derivatives of the superconducting transition temperature dT_c/dp_i are strongly anisotropic along the a, b, and c axes. The following estimates were given: $dT_c/dp_a\simeq-1.9$ K GPa^{-1}, $dT_c/dp_b\simeq2.2$ K GPa^{-1}, and $dT_c/dp_c\simeq0$. In addition, another thermal-expansion anomaly—for which an interpretation has still not been found—was observed in Ref. 1: The jumps in the temperature derivatives of the thermal-expansion coefficients along the a, b, and c axes at $T=T_c$ are strongly anisotropic. On the basis of Eqs. (3.16) and (3.17) of our paper, this anomaly of the thermal expansion is natural and connected with the analogous anomaly of the temperature derivatives of the elastic moduli (2.24). The experimental data of Ref. 1 can be used, together with Eq. (4.7), to find the derivatives $d\ln(\alpha^2/b)/dp_i$, which determine the jumps in the temperature derivatives of the thermal-expansion coefficients. According to Ref. 1 we have $\delta C_v(T_c)/T_c\simeq0.47$ mJ/cm$^3\cdot$K^2 and for the jumps of the derivatives $\delta(d\beta_a^s/dT)\simeq-\delta(d\beta_b^s/dT)\simeq\delta(d\beta_c^s/dT)\simeq-2.2\cdot10^{-8}$ K^{-2}, measured along the a, b, and c axes, respectively. Then, according to Eq. (4.7), the jumps, measured in Ref. 1, in the temperature derivatives of the thermal-expansion coefficients give the following values for the pressure derivatives parallel to the three axes

$$\frac{d\ln(\alpha^2/b)}{dp_a}\simeq-\frac{d\ln(\alpha^2/b)}{dp_b}\simeq\frac{d\ln(\alpha^2/b)}{dp_c}\simeq4.7\cdot10^{-2}\ \text{GPa}^{-1}, \tag{5.1}$$

which are thus found to be anisotropic, and they are of the same absolute order of magnitude as the pressure derivatives $d\ln T_c/p_a\simeq-2.1\cdot10^{-2}$ GPa^{-1} and $d\ln T_c/p_b\simeq2.4\cdot10^{-2}$ GPa^{-1}. We note here that the BCS model the ratio α^2/b is proportional to the electron density of states N at the Fermi level. In spite of the limited applicability of the BCS model, it can be conjectured that the pressure derivatives of the electron density of states N along different axes are also anisotropic.

The anisotropy of the derivatives dT_c/dp_i and $d\ln(\alpha^2/b)/dp_i$ suggests a corresponding anisotropy of the heat-capacity jump at the phase transition as a function of the pressure applied along different axes a, b, and c in a YBaCuO single crystal. For this, using Eq. (2.25) we calculate the derivative

$$\frac{d\ln\delta C_p}{dp_i}=\frac{d\ln(\alpha^2/b)}{dp_i}+\frac{d\ln T_c}{dp_i}, \tag{5.2}$$

where we have neglected the insignificant difference in the jumps of the heat capacities δC_p and δC_v. On the basis of Eq. (5.2) and the values presented above for dT_c/dp_i from Ref. 1 and using also our data (5.1), we find the following anisotropic values of the pressure derivatives of the logarithm of the jump of the heat capacity:

$$\begin{aligned}\frac{d\ln\delta C_p}{dp_a}&=2.6\cdot10^{-2}\ \mathrm{GPa}^{-1},\\ \frac{d\ln\delta C_p}{dp_b}&=-2.3\cdot10^{-2}\ \mathrm{GPa}^{-1},\\ \frac{d\ln\delta C_p}{dp_c}&=4.7\cdot10^{-2}\ \mathrm{GPa}^{-1}.\end{aligned} \tag{5.3}$$

The predicted values of the pressure derivatives (5.3) are found to be of the same order of magnitude as the pressure derivatives $d\ln T_c/dp_i$ and $d\ln(\alpha^2/b)/dp_i$. The dependence of the heat capacity jump δC_p on the stresses in the superconductor was observed in Ref. 11 experimentally in the compound with the structure of A-15 V_3Si and it was interpreted in Ref. 12, where it was conjectured that the effect under discussion is produced mainly by the stress (or strain) dependence of the superconducting transition temperature. For YBaCuO single crystals we point out that, first, this effect is anisotropic and, second, both terms on the right-hand side of Eq. (5.2) are of the same order of magnitude. The latter fact distinguishes qualitatively the result of our analysis for YBaCuO from the corresponding analysis of Ref. 12 for V_3Si.

On the basis of the experimental data on the anisotropy of thermal expansion of YBaCuO single crystals,[1] we now discuss the consequences for the elastic properties of the orthorhombic crystal YBaCuO, which are determined by nine moduli of elasticity. In connection with the formulas (2.23) and (2.24), it is convenient to use Eq. (4.5) to transform from the pressure derivatives dT_c/dp_i and $d\ln(\alpha^2/b)/dp_i$ to the corresponding derivatives with respect to the components of the strain tensor along the a, b, and c axes. This transformation was made in Ref. 1 for derivatives of the superconducting transition temperature, where the following values are given for the strain derivatives of T_c: $dT_c/de_a\simeq217$ K, $dT_c/de_b\simeq-316$ K, and $dT_c/de_c\simeq-30$ K. The derivative dT_c/de_c was found to be an order of magnitude smaller than the two other derivatives. Here one should note the possible inaccuracy of such a conversion, since the experimentally known set of moduli of elasticity of YBaCuO single crystals[13] $C^0_{11}=230$ GPa, $C^0_{12}=100$ GPa, $C^0_{13}=100$ GPa, and $C^0_{33}=150$ GPa is insufficient for conversion using Eq. (4.5), and in Ref. 1 the approximations $C^0_{12}\approx C^0_{11}$ and $C^0_{23}\approx C^0_{13}$ were employed. As a result of such an approximation, for example, for the derivative $dT_c/de_c\simeq-C^0_{13}(dT_c/dp_a+dT_c/dp_b)$ the terms $dT_c/dp_a=-1.9$ K GPa^{-1} and $dT_c/dp_b=2.2$ K GPa^{-1} cancel significantly, and this could be why dT_c/de_c is so small. Keeping in mind this remark and the pressure derivatives (5.1), we also present the corresponding values of the derivatives with respect to the strains along the a, b, and c axes: $d\ln(\alpha^2/b)/de_a\simeq-10.8$, $d\ln(\alpha^2/b)/de_b\simeq1.4$, $d\ln(\alpha^2/b)/de_c\simeq-7.0$. For comparison we also give the dimensionless derivatives following from Ref. 1: $d\ln T_c/de_a\simeq2.4$, $d\ln T_c/de_b\simeq-3.5$, $d\ln T_c/de_c\simeq-0.3$.

It can be asserted on the basis of Eq. (2.23) that only the elastic moduli C^s_{ij} with $i,j=1, 2, 3$ can undergo jumps at a phase transition. Since the parameter b is positive, the diagonal components of the elastic moduli C^s_{11}, C^s_{22}, and C^s_{33} will become softer at the superconducting transition. The sign of the jump of the off-diagonal ($i\neq j=1, 2, 3$) components of C^s_{ij} depends on the sign of the derivative $(dT_c/de_i)(dT_c/de_j)$, and for this reason these components of the tensor of elastic moduli can become stiffer at $T=T_c$. The results of Ref. 1, indicated above, on the anisotropy of the derivatives dT_c/de_i enable us to predict that the jumps of the elastic moduli of YBaCuO single crystals at $T=T_c$ will exhibit corresponding anisotropy. Keeping in mind the data of Ref. 1 presented above for $\delta C_v/T_c$ and for the derivatives dT_c/de_i, we find the corresponding magnitudes of the predicted jumps for the diagonal components

$$\begin{aligned}&\delta C^s_{11}(T_c)\simeq-22\ \mathrm{MPa},\quad \delta C^s_{22}(T_c)\simeq-47\ \mathrm{MPa},\\ &\delta C^s_{33}(T_c)\simeq-0.4\ \mathrm{MPa}\end{aligned} \tag{5.4}$$

and the off-diagonal components

$$\begin{aligned}&\delta C^s_{12}(T_c)\simeq32\ \mathrm{MPa},\quad \delta C^s_{13}(T_c)\simeq3.0\ \mathrm{MPa},\\ &\delta C^s_{23}(T_c)\simeq-4.4\ \mathrm{MPa}\end{aligned} \tag{5.5}$$

of the elastic moduli of YBaCuO single crystals. We note, first, that the jumps of the off-diagonal components $\delta C^s_{12}(T_c)$ and $\delta C^s_{13}(T_c)$ are positive. This corresponds to stiffening of the corresponding elastic moduli at the superconducting transition temperature. Second, we note the small magnitude of the jumps of the elastic moduli δC^s_{i3} ($i=1, 2, 3$). This is a consequence of the above fact that the derivative dT_c/de_c is small, which could be due to the inaccuracy of the analysis of the experimental data.

We now compare the estimates obtained above to existing experimental data[2,14,15] for the relative jumps of the speeds of longitudinal sound waves $\delta V_1/V_1$ and $\delta V_3/V_3$ at $T=T_c$ in a YBaCuO single crystal, which are determined by the corresponding relative jumps of the elastic moduli $\delta C^s_{11}/2C^0_{11}$ and $\delta C^s_{33}/2C^0_{33}$. This gives the estimate $\delta C^s_{11}/2C^0_{11}\simeq-4.8\cdot10^{-5}$, where we used $C^0_{11}=230$ GPa, according to Ref. 13. This estimate is close to the relative softening of the speed of a sound wave $\delta V_1/V_1\simeq-5\cdot10^{-5}$, measured in Ref. 2. For the elastic modulus we have the estimate $\delta C_{33}^s/2C_{33}^0 \simeq -1.3\cdot 10^{-6}$, where $C_{33}^0 \simeq 150$ GPa,[13] which is significantly less than for the modulus C_{11}^s. On the one hand, this corresponds to the result of Ref. 14, where no anomaly was observed in the speed V_3, determined by the modulus C_{33}^s, of a longitudinal sound wave at the superconducting transition. On the other hand, however, a year after Ref. 15 was published the same group observed the jump $\delta V_3/V_3 \simeq -5\cdot 10^{-5}$, just as for $\delta V_1/V_1$ and significantly larger than the estimate $\delta C_{33}^s/2C_{33}^0 \simeq -1.3\cdot 10^{-6}$. Using now the experimental result of Ref. 15 $\delta V_3/V_3 \simeq -5\cdot 10^{-5}$ and the values $\delta C_v/T_c = 0.47$ mJ/cm$^3\cdot$K^2 (Ref. 1) and $C_{33}^0 = 150$ GPa,[13] we obtain on the basis of Eq. (2.23) the estimate $|dT_c/de_c| \simeq 180$ K of the absolute value of the derivative, which is significantly greater than the value obtained for this derivative in Ref. 1. As we have noted above, this discrepancy could be due to the error introduced by the approximation $C_{23}^0 \simeq C_{13}^0$, employed in Ref. 1. Just as for the off-diagonal ($i\neq j = 1, 2, 3$) components of the tensor C_{ij}^s, we are not aware of any experimental measurements of temperature dependence for the elastic modulus C_{22}^s.

Besides jumps of the elastic moduli C_{11}^s and C_{33}^s of a YBaCuO single crystal at $T=T_c$, in Refs. 2 and 15 jumps were observed in the temperature derivatives of the elastic moduli $\delta(dC_{11}^s/dT) \simeq -(6\cdot 10^{-5}\ \mathrm{K}^{-1})C_{11}^0$ (Ref. 2), $\delta(dC_{33}^s/dT) \simeq -(2.6\cdot 10^{-5}\ \mathrm{K}^{-1})C_{33}^0$ (Ref. 15) and $\delta(dC_{44}^s/dT) \simeq -(1.2\cdot 10^{-4}\ \mathrm{K}^{-1})C_{44}^0$ (Ref. 15). Given the experimental data of Ref. 1 on the jumps of the temperature derivatives of the thermal-expansion coefficients in this crystal, the values of $\delta C_v/T_c$, C_{1j}^0 ($j=1, 2, 3$), and $C_{23}^0 \approx C_{13}^0$ and C_{33}^0, indicated above, and using also $C_{44}^0 = 25$ GPa,[15] we can calculate, on the basis of Eq. (3.17), the corresponding second derivatives of the superconducting transition temperature with respect to the strains. In this manner we obtained the following values:

$$\frac{1}{T_c}\frac{d^2T_c}{de_a^2} \simeq -275, \quad \frac{1}{T_c}\frac{d^2T_c}{de_c^2} \simeq -96, \quad \frac{1}{T_c}\frac{d^2T_c}{de_4^2} \simeq -70, \tag{5.6}$$

which are two orders of magnitude greater than the first derivatives of $\ln T_c$ with respect to the strains. This fact was pointed out in Ref. 10, where an estimate is given for the dimensionless derivative $T_c^{-1}d^2T_c/de^2 \sim -10^3$ with respect to the pure shear strain for YBaCuO ceramic. The question naturally arises of how specific a property of high-T_c superconductors such the large magnitude of the derivatives (5.6) is. In order to answer this question we employed the experimental data of Ref. 16, where a jump was observed in the temperature derivative of the shear modulus C_{44}^s in a cubic single crystal of the low-temperature superconductor vanadium with $T_c = 5.4$ K. From Ref. 16 we found $\delta(dC_{44}^s/dT) \simeq (4.6\cdot 10^{-5}\ \mathrm{K}^{-1})C_{44}^0$, $C_{44}^0 = 46$ GPa. Then, since the formula (3.17) assumes the simpler form (4.1) for the shear modulus, and using the data $\delta C_v/T_c = 1.62$ mJ/cm$^3\cdot$K^2,[17] we obtain $d^2T_c/de_4^2 \simeq 1.3\cdot 10^3$ K. The corresponding dimensionless derivatives $T_c^{-1}d^2T_c/de_4^2 \simeq 241$ for vanadium and $T_c^{-1}d^2T_c/de_4^2 \simeq -70$ for YBaCuO single crystals are of the same order of (absolute) magnitude. The difference in the signs of these derivatives corresponds to softening of the shear modulus C_{44}^s in vanadium and stiffening of the modulus C_{44}^s in YBaCuO at the superconducting transition. We call attention here also to Ref. 18, where anomalously large values of the second derivatives of T_c with respect to both shear and compressive strains were established for superconductors with A-15 structure V_3Si and V_3Ge. Thus the anomalously large relative values of the second derivatives of T_c with respect to the strains are not typical only of b.

We now calculate the components of the tensor C'_{ij} (4.11) of coefficients of superconducting-elasticity for a YBaCuO single crystal. According to the experimental data of Ref. 19, the BCS theory describes well the squared superconducting order parameter $\Delta^2(T)$ at temperatures $T \lesssim T_c$ in a YBaCuO single crystal. Then, according to Eq. (4.13), we obtain near T_c the value of the derivative $(\partial\Delta^2/\partial T)_{\hat{e}=0} \simeq -6.32$ meV^2/K at $T_c = 91$ K. Since for the shear modulus C_{44}^0 the experimental value of the jump in the temperature derivative is known $\delta(dC_{44}^s/dT) \simeq -3\cdot 10^{-3}$ GPa/K,[15] according to Eq. (44) it is easy to calculate the corresponding value of the component of the superconducting-elasticity tensor

$$C'_{44} \simeq 4.7\cdot 10^{-4}\ \mathrm{GPa/meV^2}. \tag{5.7}$$

This component is the only component of the tensor C'_{ij} which can be determined for YBaCuO single crystals directly from experiment. In estimating the other components C'_{ij} we hypothesize that the second term in braces on the right-hand side of Eq. (4.11) can be neglected. In order to elucidate the meaning of this hypothesis for YBaCuO, we indicate first that according to Ref. 1 $\delta C_v = 4.3\cdot 10^{-5}$ GPa/K. This expression should be compared with the experimental values $\delta(dC_{11}^s/dT) \simeq -1.4\cdot 10^{-2}$ (GPa/K) (Ref. 2) and $\delta(dC_{33}^s/dT) \simeq -3.9\cdot 10^{-3}$ (GPa/K) (Ref. 15). It is obvious that this hypothesis can be violated for C'_{11} only if the inequality $Q_{11} \ll 3\cdot 10^2$ is violated and for C'_{33} if the inequality $Q_{33} \ll 1\cdot 10^2$ is violated. Since according to Eq. (4.15) $Q_{11}^{\mathrm{BCS}} \simeq 11$ and $Q_{33}^{\mathrm{BCS}} \simeq 0.2$, the assumption made above can break down only if the values of the derivatives $(\partial^2\Delta^2/\partial e_i\partial T)$ for $i=1, 3$ in YBaCuO exceed their BCS values (4.14) by approximately two orders of magnitude. For this reason, our assumption reduces to the assertion that such a large excess does not occur. Then it is found that the following estimates can be given:

$$C'_{11} \simeq 2.2\cdot 10^{-3}\ \mathrm{GPa/meV^2},$$
$$C'_{33} \simeq 6.2\cdot 10^{-4}\ \mathrm{GPa/meV^2}. \tag{5.8}$$

We now estimate the pressure derivatives along the a, b, and c axes of the parameters α and b in Landau's expansion of the free energy in a YBaCuO single crystal. For this, we use the relations (4.5) and the expression (4.8) to switch in Eq. (4.10) to derivatives with respect to the pressure tensor. Then we obtain

$$\frac{d\ln\alpha}{dp_i} = \frac{d\ln(\alpha^2/b)}{dp_i} - \frac{\partial^2\Delta^2}{\partial p_i\partial T}\left(\frac{\partial\Delta^2}{\partial T}\right)^{-1}_{\hat{e}=0}. \tag{5.9}$$

As noted above, according to the experimental data of Ref. 19 for the derivative $(\partial\Delta^2/\partial T)_{\hat{e}=0}$, the relation (4.13) holds in a YBaCuO single crystal. We now estimate, according to the BCS model, the derivative $(\partial^2\Delta^2/\partial p_i\partial T) = -[8\pi^2/7\zeta(3)]dT_c/dp_i$, which we do not know from experiment. Then the second term on the right-hand side of Eq. (4.9) reduces to the derivative $-d\ln T_c/dp_i$. Since for the c axis the experimental value of the derivative satisfies $d\ln T_c/dp_c \simeq 0$ according to Ref. 1, it can be assumed that the second term on the right-hand side of Eq. (5.9) is small compared to the first term. Thus the following relation can be obtained for the c axis from Eqs. (5.1) and (5.9):

$$\frac{d\ln\alpha}{dp_c} \simeq \frac{d\ln b}{dp_c} \simeq 4.7\cdot 10^{-2}\ \mathrm{GPa}^{-1}. \tag{5.10}$$

For the two other axes a and b the experimental values[1] of the derivative $d\ln T_c/dp_i$ indicated above are only two times smaller than the corresponding experimental values of the derivative $d\ln(\alpha^2/b)/dp_i$ given by (5.1). As a result, the second term on the right-hand side of Eq. (5.9) for the a and b axes cannot be neglected. Nonetheless, because of the strong anisotropy of the derivatives (5.1) and $d\ln T_c/dp_i$, on the basis of this analysis we can conclude that the pressure derivatives along different axes of the parameters in the Landau expansion of the free energy in a YBaCuO single crystal are anisotropic.

Finally, we note that the anisotropic effect of superconductivity on the elastic properties was also observed in single crystals of a different high-T_c superconductor, $La_{2-x}Sr_xCuO_4$.[3] It was observed that the elastic modulus C_{33}^s charges discontinuously at $T=T_c$, while the modulus C_{11}^s does not manifest such an anomaly.[3] On this basis and in accordance with Eq. (2.23), it can be conjectured that $|dT_c/de_a| \ll |dT_c/de_c|$. The change in the slope angle of the temperature dependence at a superconducting transition is demonstrated in Ref. 3 for both the elastic moduli C_{11}^s and C_{33}^s and for the shear elastic modulus C_{66}^s. On the other hand, the other shear modulus C_{44}^s does not exhibit any anomaly near T_c (Ref. 3). This suggests that the second derivative of the superconducting transition temperature with respect to shear strains (2.21) is anisotropic in LaSrCuO. Unfortunately, we do not know of any experimental results on thermal expansion of LaSrCuO single crystal near T_c similar to the data of Ref. 1 for YBaCuO, and this makes it difficult to perform a study for LaSrCuO single crystals similar to the one performed above for YBaCuO single crystals.

Thus our analysis shows how it is possible to determine an entire series of important parameters of superconductors and, in particular, how to establish the relation between the elastic properties and the superconducting order parameter by studying simultaneously anomalies of the thermal expansion and elastic moduli of single crystals of high-T_c superconductors near the superconducting transition.

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