Stability of a flame in a gravitational field

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The problem of the hydrodynamic stability of a planar flame front is solved for the complete system of equations describing a subsonically propagating flame, including thermal conduction, energy release due to chemical reactions and viscosity, for the case of the flame propagation in a gravitation field. It is shown that the supplementary condition that the velocity of the flame normal to the front be constant, which is needed to solve the problem in ideal hydrodynamics, is valid in the presence of all of these processes for perturbations with wavelength very long compared with the thickness of the combustion zone. For such perturbations the growth rate of the instability is given asymptotically by the Landau formula for the hydrodynamic instability of the flame front, independently of the activation energy and the order of the chemical reaction. It is shown that in practice the viscosity does not influence the growth rate of the instability and the instability growth rate becomes considerably less than the value given by the Landau formula for wavelengths large compared with the thickness of the flame, $\lambda \geqslant 20\Delta$. The Rayleigh–Taylor instability of a flame propagating in a strong gravitational field becomes dominant for the long-wavelength perturbations. Suppression of the growth rate of Rayleigh–Taylor instability due to convection (mass flow across the flame front) becomes particularly pronounced when the finite thickness of the combustion zone is treated.

1. INTRODUCTION

As is well known from fluid dynamics,[1] all surfaces of discontinuity in ideal dissipationless fluid motion can be divided into two classes: tangential discontinuities, in which there is no material flow across the surface of discontinuity, and those in which material does flow across the surface of discontinuity. The latter are divided into shock waves (which move with supersonic velocity relative to the original material) and deflagration waves. In the system of coordinates attached to a deflagration wave front both the incoming flux (the flux of the unburned material) and the outgoing flux are subsonic. In order that a flow with a surface of discontinuity exist it must satisfy the condition of evolutionarity, i.e., it must respond uniquely to small perturbations. In treatments of the ideal hydrodynamics equations this means that the number of conditions at the discontinuity must be one more than the number of simple waves that can move away from the discontinuity along characteristics. For tangential discontinuities and shock waves the evolutionarity condition is satisfied, while for a slow combustion wave (plane) evolutionarity requires that an additional condition be imposed in order to determine the velocity of the surface of discontinuity.[1,2] Slow combustion is a flow in which the region of exothermic reactions propagates relative to the unburned gas due to thermal conductivity. This means that heat from the hot burning gas is transmitted by thermal conductivity to the adjacent cold layers of unburned gas, causing the reaction rate there to rise sharply and initiating motion. The velocity with which a flame propagates in the slow combustion regime is substantially less than the speed of sound due to the high activation barrier; this speed is determined by the chemical reaction time and the rate of heat release from the burning zone into the cold unburned gaseous mixture. It can be found as an eigenvalue by simultaneously solving the chemical kinetics equations and the hydrodynamics equations including heat conduction.[1,3] Then the thickness of the combustion region is found to be of order $\Delta = \chi_1/u_1$, where u_1 is the velocity with which the flame propagates relative to the unburned gas and χ_1 is the thermal conductivity.

In most cases of practical interest the thickness of the combustion zone is negligible in comparison with the typical length scales of the problem, and hence the flame can be regarded as a discontinuous surface of zero thickness, i.e., a flame front, which separates the combustion products and the unburned gaseous mixture. This assumption, however, prevents us from treating the processes that determine the velocity with which the front moves; since this velocity is indeterminate, it must be prescribed *a priori*. Accordingly, in order to treat the stability of this motion some additional condition must be imposed on the perturbed front velocity, which is not a consequence of the equations of hydrodynamics. The question of how to choose the correct auxiliary condition for this problem has been posed in many papers, including those aimed at the stability of slow combustion.

The first to treat the hydrodynamic stability of a planar slow-combustion front using the model of discontinuous flow were Landau[4] and Darrieus.[5] As the additional condition they assumed that the normal component of the flame propagation speed relative to the unburned gas remains unchanged as a result of perturbations. With this assumption it was found that a flame is absolutely unstable against arbitrary perturbations that bend the flame front,

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for perturbations with wavelength down to the front thickness. The rate at which the instability grows as a function of perturbation wave number $k=2\pi/\lambda$ is given by

$$\sigma=ku_1\frac{\Theta_2}{1+\Theta_2}\left(\sqrt{1+\Theta_2-(1/\Theta_2)}-1\right). \quad (1)$$

Here $\Theta_2=T_2/T_1=\rho_1/\rho_2$ is the amount of thermal expansion the gas undergoes in the flame, i.e., the ratio of temperatures (densities) in the gas behind the front (subscript 2) and ahead of it (subscript 1).

From (1) we would expect that the flow behind a flame front would become turbulent even at values of the Reynolds number $\mathrm{Re}\gg\Delta u_2/\nu\sim 1$, with typical density gradient lengths comparable with the thickness $\sim\Delta$ of the combustion zone. But experiments show that spontaneous turbulence in the flow behind a flame front only sets in for values $\mathrm{Re}\sim 10^3$ of the Reynolds number and above, and that the characteristic length scale of the gradients is two orders of magnitude greater than the flame thickness.[3,6]

Thus far many papers have been published (see Refs. 3, 6, and 7 and the citations they contain, Refs. 8–17), in which the authors try to resolve this contradiction by taking into account various factors which might stabilize a flame front due to thermal diffusive and viscous effects. Among the latter we note the phenomenological approach of Markstein,[7] who assumed that the velocity of a flame depends on the curvature of the front. This dependence is associated with transverse diffusion and thermal conduction, but just as in the case of the Landau condition, was not derived from the equations of hydrodynamics. Another phenomenological approach was used in Refs. 13–18, where transport processes (thermal conduction, diffusion, viscosity) were included in the solution of the stability problem, while at the chemical reaction front, regarded as a surface of discontinuity, an additional condition was introduced which determines the dependence of the reaction rate on the temperature ahead of the front. It is evident that dissipative effects stabilize the instability, as a result of which these treatments yield reasonable estimates for the stabilization scale length. However, an exact solution of the spectral problem requires rigorous justification and formulation; all attempts made thus far to prove the conditions at the reaction front[18,19] contain some assumptions, equivalent to the condition being derived.

A similar problem, related to the insufficiency of the boundary conditions at a surface of discontinuity, has been intensively investigated in recent years in connection with the problem of the stability of an ablation front. An ablation wave is formed when a target is irradiated by laser radiation at high intensity. This front is a thermal conduction wave which transports the energy absorbed from the laser radiation in the direction of the cold initial material. The instability of an ablation front resembles the Rayleigh–Taylor (R–T) instability, but the presence of a flux of material and other stabilizing factors substantially changes the appearance of the instability spectrum. In order to describe convective stabilization of the R–T instability many workers[20–23] have used the model of discontinuous flow, which necessitates the introduction of additional conditions on the surface of discontinuity. Just as in the case of the stability of a flame front, it is impossible in principle to derive these conditions from this model.

The only way to get an exact solution to the problem of the stability of the deflagration wave is to include all processes that affect wave propagation. For a slow combustion wave this means solving the full system of hydrodynamic equations, including thermal conduction and chemical kinetics. Here the unperturbed flow is found from the continuous solution determined by these physical processes. This approach enables us to derive a solution to the flame-front stability problem in the discontinuous limit as the first approximation in an expansion in powers of $\Delta/\lambda\to 0$ and to find the exact boundary condition at the discontinuity. Liberman *et al.*[24] have derived a complete solution to the problem of the hydrodynamic stability of a slow combustion wave for an inviscid fluid including chemical reaction kinetics and thermal conduction. It was shown there that for long-wavelength perturbations $(\Delta/\lambda\to 0)$ the condition that the propagation speed relative to the unburned gas be constant follows from the complete system of equations for small perturbations including thermal conduction, and the instability growth rate is given by Eq. (1). Furthermore, the growth rate is substantially reduced in comparison with the Landau formula (1) even at long wavelengths $\lambda\sim 10^2\Delta\gg\Delta$. The growth rate has a maximum at $\lambda\sim 40\Delta$ and vanishes at $\lambda\sim 20\Delta$.

In the present work we derive a solution to the flame-front stability problem including viscosity, thermal conduction, and chemical reaction kinetics. We show that for all physically reasonable values of the viscosity the latter has no effect on the rate at which the flame instability grows. The solution of the problem of stability of a slow combustion front in a gravitational field shows that even in the presence of acceleration the correct auxiliary condition in the discontinuous model is the Landau condition that the normal velocity of the front relative to the unburned gas remain unchanged. We show that convective stabilization of the Rayleigh–Taylor instability is associated with the finite thickness of the transition region, and contains the natural time scale of the unperturbed flow as a parameter.

2. STEADY PROPAGATION OF A SLOW COMBUSTION WAVE

We consider the problem of a slow combustion wave, including the structure of the transition region where combustion takes place for the case in which the Lewis number satisfies $\mathrm{Le}=\rho c_P D/\varkappa=1$, where D and $\varkappa$ are the diffusivity and thermal conductivity of the gas, ρ is the density, and c_P is the specific heat at constant pressure. This assumption imposes no fundamental restriction from the standpoint of the hydrodynamic processes, but enables us to simplify the problem by avoiding treatment of the purely chemical instability. Formally this assumption corresponds to combustion of a mixture of gases with similar molecular weights. The velocity with which the flame propagates in all cases of practical interest is small compared with the speed of sound, so we take $M = u\sqrt{\rho/\gamma P}\ll 1$. It can be shown[3] that for such an isobaric flow when the diffusivity and thermal conductivity are equal the profiles of fuel concentration and temperature are similar, so that the diffusion and thermal conduction equations reduce to the same equation. To be specific we will assume that the combustion process is described by a single equivalent first-order reaction. The reaction rate is assumed to depend on temperature according to the Arrhenius law. We will also assume that the flame velocity is sufficiently small that the gravitational acceleration and viscosity have little effect on the structure of the unperturbed flow. These conditions can be expressed in terms of the smallness of the corresponding dimensionless parameters:

$$M^2 \ll 1, \quad \frac{M^2}{\mathrm{Fr}} \ll 1, \quad \mathrm{Pr}_{1,2} M^2 \ll 1, \tag{2}$$

where $M = u\sqrt{\rho/\gamma P}$ is the Mach number, $\mathrm{Fr}=u^2/g\Delta$ is the Froude number, $\mathrm{Pr}_{1,2}=\nu_{1,2}c_P/\varkappa$ is the Prandtl number with respect to the bulk and shear viscosities respectively, u is the flame speed, P is the pressure, γ is the adiabatic index, g is the gravitational acceleration, and $\nu_{1,2}$ are the bulk and shear viscosities. Under these conditions the energy transport equation contains no terms with viscosity or gravitational acceleration. To simplify the analysis we will assume that the thermal conductivity and viscosity are independent of temperature. Then the system of equations describing flame propagation takes the form

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{u}) = 0, \tag{3}$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_k \frac{\partial u_i}{\partial x_k} + \frac{\partial P}{\partial x_i} = \frac{\partial}{\partial x_l}\left[\nu_1\left(\frac{\partial u_i}{\partial x_l} + \frac{\partial u_l}{\partial x_i} - \frac{2}{3}\delta_{il}\nabla \mathbf{u}\right)\right] + \frac{\partial}{\partial x_i}(\nu_2 \nabla \mathbf{u}) + \rho g_i, \tag{4}$$

$$\rho c_P\left(\frac{\partial T}{\partial t} + \mathbf{u}\nabla T\right) = \nabla(\varkappa \nabla T) + QW. \tag{5}$$

Here Q is the heat of the thermal reaction and $W(\rho,T)$ is its rate. We assume that the unburned and burned gases are described by the ideal-gas equation of state,

$$P = \frac{\gamma - 1}{\gamma} c_P \rho T.$$

Assuming the Arrhenius law for the reaction rate and taking into account the identity of the temperature and concentration profiles,[3,24] we find

$$QW = \frac{\rho c_P}{\tau}(T_2 - T)\exp\left(-\frac{E}{T}\right), \tag{6}$$

where E is the chemical reaction activation energy, τ is the induction time, and $T_2 = T_1 + a_1 Q/c_P$ is the final temperature of the combustion products, determined by the fuel concentration a_1 in the original mixture.

We consider a one-dimensional plane flow. We assume that the z axis is coincident with the material flow velocity. In the comoving coordinate system the unburned gas with density ρ_1 (at $z=-\infty$) flows into the combustion zone with velocity u_1; the reaction products flow out of the combustion zone with density ρ_2 (at $z=\infty$). For very subsonic flow, to within terms of order $M^2 \ll 1$, $M^2/\mathrm{Fr} \ll 1$, $\mathrm{Pr}_{1,2}M^2 \ll 1$, if we take into account Eq. (2), then Eqs. (3) and (4) have the following constants of motion:

$$\rho u_z = \mathrm{const}, \tag{7}$$

$$P = \mathrm{const}. \tag{8}$$

To solve Eq. (5) we find it convenient to go over to dimensionless variables, scaling all quantities by their initial values at $z=-\infty$. Then if we take into account the constants of motion (7) and (8), all variables associated with the unperturbed flow can be expressed in terms of a single quantity, the dimensionless temperature $\Theta = T/T_1$:

$$\frac{\rho_1}{\rho} = \frac{u_z}{u_{z1}} = \frac{T}{T_1} = \Theta. \tag{9}$$

In the flame propagation problem the temperature profile is the eigenfunction of an eigenvalue problem determined by the solution of Eq. (5) with the corresponding boundary conditions. The flame propagation speed is determined from the eigenvalue of the problem.

To solve Eq. (5) we introduce the dimensionless coordinate $\xi = z/\Delta$, where $\Delta = \varkappa/(\rho c_P u_{z1})$ is the thickness of the combustion zone and the dimensionless activation energy is $\mathscr{E} = E/T_1$. In these variables Eq. (5) assumes the form

$$\frac{d^2\Theta}{d\xi^2} - \frac{d\Theta}{d\xi} - \Lambda\left(1 - \frac{\Theta_2}{\Theta}\right)\exp\left(-\frac{\mathscr{E}}{\Theta}\right) = 0, \tag{10}$$

with the boundary conditions

$$\Theta = 1, \quad \xi \to -\infty, \qquad \Theta = \Theta_2, \quad \xi \to \infty. \tag{11}$$

Here $\Lambda = \Delta/(u_{z1}\tau)$ is the eigenvalue of the problem.

For estimates we can conveniently use a simple analytical solution to the problem (10)–(11) for the temperature and for the normal flame propagation speed, derived by Zel'dovich and Frank-Kamenetskii in the limit of large activation energy.[3] In the variables used here the temperature profile assumes the form

$$\Theta(\xi) = \begin{cases} 1 + (\Theta_2 - 1)\exp(\xi), & \xi < 0, \\ \Theta_2, & \xi > 0, \end{cases} \tag{12}$$

and the corresponding eigenvalue is

$$\Lambda_{\mathrm{ZF}} = \frac{\mathscr{E}^2(\Theta_2 - 1)^2}{2\Theta_2^3}\exp\left(\frac{\mathscr{E}}{\Theta_2}\right). \tag{13}$$

Typical scaled profiles $(\Theta-1)/(\Theta_2-1)$ for a slow combustion wave with various values of the activation energy and thermal thickness are shown in Fig. 1, where trace 4 corresponds to the solution (12), which is asymptotically correct for large activation energy.

As can be seen from Fig. 1, the thickness of the heating region is of order unity in the dimensionless variables and is essentially independent of both the activation energy and

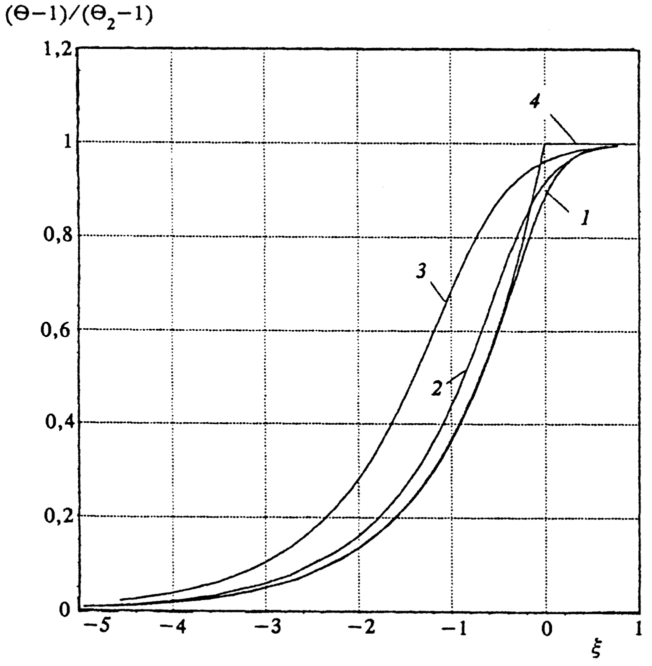

FIG. 1. Scaled temperature profiles in a combustion wave: *1*) $\mathscr{E}=70$, $\Theta_2=8$, $\ln\Lambda=14.3$; *2*) $\mathscr{E}=42$, $\Theta_2=6$, $\ln\Lambda=11.7$; *3*) $\mathscr{E}=21$, $\Theta_2=3$, $\ln\Lambda=10.4$; trace *4* corresponds to Eq. (12).

the amount of thermal spreading. The region where the energy is released in the chemical reaction is considerably narrower.

3. SPECTRAL PROBLEM FOR SMALL PERTURBATIONS

Let us consider the spectral problem for the stability of a steady planar flame against small perturbations which bend the front. By virtue of the symmetry of the problem the solution can be found in the form

$$\tilde{\varphi}=\tilde{\varphi}(z)\exp(\sigma t+ikx), \tag{14}$$

where x is the transverse position variable, k is the perturbation wave number, σ is the growth rate, and $\tilde{\varphi}(z)$ is the amplitude vector of the perturbations. Since we are interested only in perturbations that grow in time, in the future we will assume $\mathrm{Re}(\sigma)>0$.

It can be shown[24] that for $\mathrm{Le}=1$ the concentration and temperature perturbations remain similar, and in the limit $M^2\ll 1$ the density perturbations are related to the temperature perturbations by

$$\frac{\tilde{\rho}}{\rho}=-\frac{\tilde{T}}{T}. \tag{15}$$

Taking into account (9) and (15) we find equations for the small perturbations (14) in the form

$$\frac{d\tilde{j}}{d\xi}=KS\frac{\tilde{\Theta}}{\Theta^2}-K\frac{\tilde{u}}{\Theta}-K\frac{\tilde{w}}{\Theta}, \tag{16}$$

$$\frac{d\tilde{u}}{d\xi}=K\tilde{\mathscr{P}}-2K\Theta\tilde{j}-K\tilde{\Theta}-\left(\frac{KS}{\Theta}+\mathrm{Pr}_1K^2\right)(\tilde{u}+\tilde{w}), \tag{17}$$

$$\frac{d\tilde{\mathscr{P}}}{d\xi}=-K\tilde{u}-K\tilde{w}-\mathrm{Pr}_1K^2\tilde{\Theta}-\left(\frac{KS}{\Theta}+\mathrm{Pr}_1K^2\right)\Theta\tilde{j}-\frac{\tilde{\Theta}}{\mathrm{Fr}\,\Theta^2}, \tag{18}$$

$$\frac{d^2\tilde{\Theta}}{d\xi^2}-\frac{d\tilde{\Theta}}{d\xi}-\Lambda\frac{\tilde{\Theta}}{\Theta^2}\left[\Theta_2-\mathscr{E}\frac{\Theta_2-\Theta}{\Theta}\right]\exp\left(-\frac{\mathscr{E}}{\Theta}\right)$$
$$=\tilde{j}\frac{d\Theta}{d\xi}+KS\frac{\tilde{\Theta}}{\Theta}+K^2\tilde{\Theta}, \tag{19}$$

$$\mathrm{Pr}_1\frac{d\tilde{w}}{d\xi}=\left(1+\mathrm{Pr}_1\frac{KS}{\Theta}+\mathrm{Pr}_1^2K^2\right)\tilde{w}-\mathrm{Pr}_1K\tilde{\mathscr{P}}+\mathrm{Pr}_1K\Theta\tilde{j}$$
$$+\mathrm{Pr}_1\left(\frac{KS}{\Theta}+\mathrm{Pr}_1K^2\right)\tilde{u}, \tag{20}$$

where we have introduced the following notation for the dimensionless temperature perturbations, material flux, transverse and longitudinal momentum fluxes, and the viscous stress:

$$\tilde{\Theta}=\frac{\tilde{T}}{T_1},\quad \tilde{j}=\frac{\tilde{\rho}u_z+\rho\tilde{u}_z}{\rho_1u_{z1}},$$

$$\tilde{w}=\left(\nu_1\frac{di\tilde{u}_x}{dz}-\nu_1k\tilde{u}_z\right)(\rho_1u_{z1}^2)^{-1},$$

$$\tilde{u}=\left(\rho u_zi\tilde{u}_x-\nu_1\frac{di\tilde{u}_x}{dz}-\nu_1k\tilde{u}_z\right)(\rho_1u_{z1}^2)^{-1}, \tag{21}$$

$$\tilde{\mathscr{P}}=\left[\tilde{P}+2\rho u_z\tilde{u}_z+\tilde{\rho}u_z^2-\left(\frac{4}{3}\nu_1+\nu_2\right)\frac{d\tilde{u}_z}{dz}\right.$$
$$\left.-\left(\frac{\nu_1}{3}+\nu_2\right)ki\tilde{u}_x\right](\rho_1u_{z1}^2)^{-1}.$$

The dimensionless wave number and growth rate, respectively, are

$$K=k\Delta,\quad S=\sigma/ku_{z1}.$$

Note that only the bulk viscosity enters explicitly into the equations.

The boundary conditions for Eqs. (16)–(20) are the vanishing of all perturbations in the limit $\xi\to\pm\infty$. The boundary conditions at $\pm\infty$ can be imposed for finite values of ξ_1 and ξ_2 far enough from the transition region so that for $\xi<\xi_1$ and $\xi>\xi_2$ the flow may be treated as uniform. The conditions that the flow be uniform are given by the inequality

$$\left|\frac{d(\ln\Theta)}{d\xi}\right|\ll\min\{1,K\}\quad\text{for } \xi=\xi_1,\xi_2. \tag{22}$$

In the uniform-flow region for $\xi<\xi_1$ and $\xi>\xi_2$ the solution of the equations for the perturbations are found in the form of a superposition of solutions of the form

$$\tilde{\varphi}(\xi)=\tilde{\varphi}\exp(\mu\xi), \tag{23}$$

where the argument of the exponential in (23) is chosen so that $\mu>0$ for $\xi<\xi_1$ and $\mu<0$ for $\xi>\xi_2$. Substituting the perturbations in the form (23) into Eqs. (16)–(20), we find the characteristic equation for the index μ. Taking into account the vanishing of the perturbations at $\xi\to\pm\infty$ we find that in the incident flow three modes are possible: sound

$$\mu_S=K,$$

thermal

$$\mu_T=\tfrac{1}{2}+\sqrt{\tfrac{1}{4}+SK+K^2}$$

and viscous

$$\mu_\eta=\frac{1}{2\,\mathrm{Pr}_1}+\sqrt{\frac{1}{4\,\mathrm{Pr}_1^2}+\frac{SK}{\mathrm{Pr}_1}+K^2}.$$

Three modes are also possible in the outgoing flow: acoustic

$$\mu_A=-K,$$

rotational

$$\mu_V=\frac{1}{2\,\mathrm{Pr}_1}-\sqrt{\frac{1}{4\,\mathrm{Pr}_1^2}+\frac{SK}{\Theta_2\,\mathrm{Pr}_1}+K^2},$$

and chemical

$$\mu_C=\frac{1}{2}-\sqrt{\frac{1}{4}+\frac{SK}{\Theta_2}+K^2+\frac{\Lambda}{\Theta_2}\exp\left(-\frac{\mathscr{E}}{\Theta_2}\right)}.$$

The structure of these modes is similar to that found in Refs. 12 and 24. We note only that the temperature perturbation is nonzero only for the thermal and chemical modes. The condition that the solution of Eqs. (16)–(20) vanish at infinity can be represented in the form of boundary conditions at $\xi=\xi_1$ and $\xi=\xi_2$:

$$\tilde{\varphi}(\xi_1)=\tilde{\varphi}_S+\tilde{\varphi}_T+\tilde{\varphi}_\eta, \qquad (24)$$

$$\tilde{\varphi}(\xi_2)=\tilde{\varphi}_A+\tilde{\varphi}_V+\tilde{\varphi}_C. \qquad (25)$$

Conditions (24) and (25) are relations involving the components of the desired functions $\tilde{\varphi}(\xi)$, which reduce to three algebraic equations for components of the function $\tilde{\varphi}(\xi_1)$ and three equations for the components of $\tilde{\varphi}(\xi_2)$. The system of equations (16)–(20) together with the boundary conditions (24) and (25) completely determine the spectral problem of the stability of a steady combustion wave.

4. STABILITY OF THE FLAME FRONT; ZERO-VISCOSITY CASE

The solution of the spectral problem, the determination of the instability growth rate for arbitrary values of the wave number K, can in general be done only numerically. The numerical solution technique was presented in Ref. 24. The results of the numerical solution of the spectral problem for the case of an inviscid fluid $\mathrm{Pr}=0$ in the absence of a gravitational field ($1/\mathrm{Fr}=0$) is shown in Fig. 2, where the dimensionless growth rate $SK=\sigma\Delta/u_1$ is plotted as a function of the perturbation wave number $K=k\Delta$ for combustion waves with different values of the heat release in the reaction.

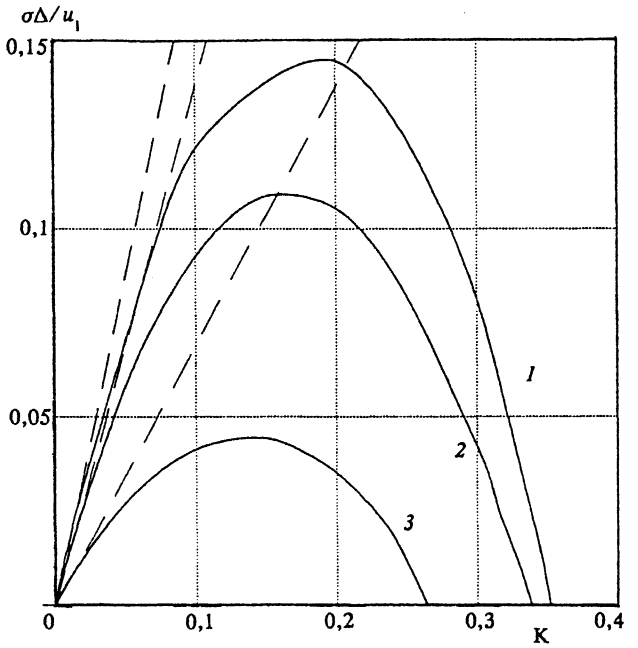

FIG. 2. Instability growth rate $\sigma\Delta/u_{z1}=SK$ versus the dimensionless wave number $K=k\Delta$. The numbers labeling the traces correspond to the temperature profiles in the combustion waves shown in Fig. 1. The dashed straight lines correspond to solutions in the model of a discontinuous flame front.

From Fig. 2 we see that the instability growth rate for small values of the wave number agrees well with the Landau solution (1), regardless of the activation energy and the extent of thermal spreading. However, it should be noted that a significant deviation in the growth rate from values given by Eq. (1) occurs even for wavelengths $\lambda\geqslant 100\Delta$, and for $\lambda\approx 20\Delta$ the growth rate vanishes. Thus, the hydrodynamic instability of a flame occurs only for perturbations with long wavelengths such that $k\Delta\leqslant 0.3$ holds. The calculations show that for fixed Θ_2 the instability growth rate is essentially independent of the activation energy.

An analytical solution of the flame stability problem can be found in the long-wavelength limit for perturbations with wavelengths large in comparison with the thickness of the transition region (the combustion zone). As was noted in the previous section, this implies

$$\varepsilon\equiv K(\xi_2-\xi_1)\ll 1, \qquad (26)$$

where the boundaries of the combustion zone are determined by the inequality (22). Taking into account the exponential decay of $d\Theta/d\xi$ in the uniform-flow region, we can represent (22) in the form

$$\exp(\xi_2-\xi_1)\gg 1/K. \qquad (27)$$

Thus, taking into account (26) and (27) we can represent the condition for the validity of the long-wavelength approximation in the form

$$K\ln(1/K)\ll K(\xi_2-\xi_1)\ll 1. \qquad (28)$$

Note that this condition is more stringent than the usual condition $K=k\Delta\ll 1$ for the applicability of the model of an infinitesimal front thickness.

In view of the very different scales on which the hydrodynamic modes $\tilde{\varphi}_S(\xi),\tilde{\varphi}_V(\xi),\tilde{\varphi}_A(\xi)$ and the thermal modes $\tilde{\varphi}_T(\xi),\tilde{\varphi}_C(\xi)$ fall off, the boundary conditions (24) and (25) for the perturbations of the hydrodynamic variables $\tilde{j}$, $\tilde{u}$, $\tilde{\mathscr{P}}$ can be represented in the form

$$\tilde{u}_1=-\tilde{j}_1,\quad \tilde{\mathscr{P}}_1=-(S-1)\tilde{j}_1 \quad \text{for } \xi=\xi_1, \tag{29}$$

$$\tilde{u}_2+\tilde{\mathscr{P}}_2-(S+2\Theta_2)\tilde{j}_2=0 \quad \text{for } \xi=\xi_2. \tag{30}$$

The boundary conditions for $\tilde{\Theta}$ and $d\tilde{\Theta}/d\xi$ are determined by the behavior of the thermal modes and reduce to exponential decay on scales smaller than or of order the thickness of the combustion zone.

Let us consider the solution of Eqs. (16)–(19) for long-wavelength perturbations. Equation (19) for the temperature perturbations can be represented in the form

$$\hat{F}[\tilde{\Theta}]=\tilde{j}\frac{d\Theta}{d\xi}+KS\frac{\tilde{\Theta}}{\Theta}+K^2\tilde{\Theta}, \tag{31}$$

where

$$\hat{F}\equiv\frac{d^2}{d\xi^2}-\frac{d}{d\xi}-\frac{\Lambda}{\Theta^2}\left[\Theta_2-\frac{\mathscr{E}(\Theta_2-\Theta)}{\Theta}\right]\exp\left(-\frac{\mathscr{E}}{\Theta}\right). \tag{32}$$

Note that

$$\hat{F}[d\Theta/d\xi]=0. \tag{33}$$

It can be shown[24] that the first and second terms on the right-hand side of (31) are of the same order in K, and to lowest order in $\varepsilon\ll 1$ the solution of (31) is

$$\tilde{\Theta}=\zeta_T\frac{d\Theta}{d\xi}, \tag{34}$$

which obviously satisfies the required boundary conditions on $\tilde{\Theta}$ and $d\tilde{\Theta}/d\xi$. The expression for $\tilde{\Theta}$, determined by Eq. (34), signifies a displacement of the flame parallel to the ξ axis by an amount $\delta\xi=\zeta_T$.

Integrating Eqs. (16)–(18), we find to first order in ε

$$\tilde{j}-\tilde{j}_1=KS\zeta_T\frac{\Theta-1}{\Theta}, \tag{35}$$

$$\tilde{u}-\tilde{u}_1=-K\zeta_T(\Theta-1), \tag{36}$$

$$\tilde{\mathscr{P}}=\tilde{\mathscr{P}}_1. \tag{37}$$

When we take into account the expression (35) derived above for $\tilde{j}$, Eq. (31) for the temperature perturbation assumes the form

$$\hat{F}[\tilde{\Theta}]=(KS+\tilde{j}_1/\zeta_T)\tilde{\Theta} \tag{38}$$

to first order in ε. Note that the function $\tilde{\Theta}=\zeta_T d\Theta/d\xi$ is the solution of Eq. (38) corresponding to the zero eigenvalue of the operator $\hat{F}$. This is the only eigenvalue of Eq. (38) for which the perturbed solution can correspond to unstable growth ($\mathrm{Re}\,\sigma>0$). Thus, we obtain the supplementary condition for the perturbation amplitudes

$$KS+\tilde{j}_1/\zeta_T=0. \tag{39}$$

Equation (39) together with the boundary conditions (29), (30) and the constants (35)–(37) at $\xi=\xi_2$ constitute a closed system of equations for the perturbation amplitudes, the condition for whose consistency yields the solution of the problem of the perturbation growth rates:

$$S_0\equiv\frac{\sigma}{ku_{z1}}=\frac{\Theta_2}{\Theta_2+1}\left(\sqrt{\Theta_2+1-\frac{1}{\Theta_2}}-1\right). \tag{40}$$

To elucidate the physical meaning of this solution it is convenient to return to dimensional variables. It is obvious that the constants (35)–(37), applied to the point $\xi=\xi_2$, are nothing but the conditions for the continuity of the mass and momentum flows at the surface of discontinuity, which corresponds to a combustion zone of finite thickness in the long-wavelength approximation. In the case of an incompressible gas $\rho=\rho_1=\text{const}$ the quantity $\zeta=\zeta_T\Delta$ is a small displacement of the flame front, so that Eq. (40) written in dimensional variables as

$$\tilde{u}_{z1}-\frac{\partial\zeta}{\partial t}=0 \tag{41}$$

is nothing but the condition for the constancy of the normal velocity of the flame front, used in Refs. 1, 4, and 5.

5. STABILIZATION OF THE INSTABILITY FOR A COMBUSTION ZONE OF FINITE THICKNESS

The solution obtained above to the problem of flame stability is the first term in an expansion in powers of ε. The numerical solution of the spectral problem shows that considerable stabilization occurs even for wavelengths much longer than the thickness of the combustion zone (see Fig. 2). Consequently, we can expect that the solution of the problem is a good approximation to within terms of order $\varepsilon\ll 1$.

If we include relation (39) the function (34) is the solution of the thermal conduction equation (31) to within terms of order ε^2. Then the constants of Eqs. (16)–(18) to within terms of order ε^2 are

$$\tilde{j}=\tilde{j}_1+KS\zeta_T\frac{\Theta-1}{\Theta}-K\int_{\xi_1}^{\xi}\frac{\tilde{u}_0}{\Theta}d\eta, \tag{42}$$

$$\tilde{u}=-\tilde{j}_1-K\zeta_T(\Theta-1)-KS\int_{\xi_1}^{\xi}\frac{\tilde{u}_0}{\Theta}d\eta+K\int_{\xi_1}^{\xi}\tilde{\mathscr{P}}_0 d\eta$$
$$-2K\int_{\xi_1}^{\xi}\Theta\tilde{j}_0 d\eta, \tag{43}$$

$$\tilde{\mathscr{P}}=-(S-1)\tilde{j}_1-KS\int_{\xi_1}^{\xi}\tilde{j}_0 d\eta-K\int_{\xi_1}^{\xi}\tilde{u}_0 d\eta, \tag{44}$$

where $\tilde{j}_0$, $\tilde{u}_0$, $\tilde{\mathscr{P}}_0$ are first-order terms in the expansion in ε for $\tilde{j}$, $\tilde{u}$, $\tilde{\mathscr{P}}$, determined by the constants (35)–(37) including the boundary conditions (29). Substituting $\tilde{j}$ from (42) in the right-hand side of (31) we find

$$\hat{F}[\tilde{\Theta}]=[(1-K\xi_1)\tilde{j}_1/\zeta_T+SK]\tilde{\Theta}+K^2 f(\xi)\tilde{\Theta},$$

$$f(\xi)\equiv 1-S\frac{\xi}{\Theta}+\frac{\Theta-1}{\Theta}\xi-(S+1)J(\xi) \qquad (45)$$

and

$$J(\xi)=\int_{\xi_1}^{\xi}\frac{\eta}{\Theta^2}\frac{d\Theta}{d\eta}d\eta. \qquad (46)$$

By means of the substitution $\tilde{\Theta}=\tilde{\psi}\exp(\xi/2)$ we can convert Eq. (45) to an equation with a self-adjoint operator,[3] in which the last term on the right-hand side of (45) is a small perturbation in the long-wavelength approximation. The corresponding corrections to the eigenvalue of the equation can be obtained from the standard perturbation theory for a self-adjoint operator.[25] To within terms of order $\varepsilon^2\ll 1$ we have

$$(1-K\xi_1)\tilde{j}_1+[SK+K^2\langle f\rangle]\zeta_T=0, \qquad (47)$$

where

$$\langle f\rangle=\frac{\int_{-\infty}^{\infty}f(\xi)(d\Theta/d\xi)^2\exp(-\xi)d\xi}{\int_{-\infty}^{\infty}(d\Theta/d\xi)^2\exp(-\xi)d\xi}. \qquad (48)$$

In the "ideal hydrodynamic" treatment of the flame stability problem relation (47) can be regarded as a supplementary condition analogous to the Markstein condition,[6] describing the change in the flame velocity associated with curvature of the front. But this condition does not exhaust all the effects associated with the finite thickness of the combustion zone unless the integrals (42)–(44) are taken into account.

The conservation laws (42)–(44), evaluated at $\xi=\xi_2$, together with (29), (30), and (47), yield a closed system of equations for the perturbation amplitudes for which the compatibility condition leads to the following value for the growth rate to within $\varepsilon^2\ll 1$:

$$S=S_0(1-K/K_c), \qquad (49)$$

where S_0 is determined by Eq. (40) and

$$K_c^{-1}=\frac{1}{2S_0\sqrt{\Theta_2+1-1/\Theta_2}}\Bigg\{2(S_0+\Theta_2)\Bigg[1+\langle\xi\rangle-(1+S_0)\left\langle\frac{\xi}{\Theta}+J(\xi)\right\rangle\Bigg]+(S_0^2+2S_0\Theta_2+2\Theta_2)J(\xi_2)-\int_{\xi_1}^{\xi_2}\eta\frac{d\Theta}{d\eta}d\eta\Bigg\}. \qquad (50)$$

Equation (49) predicts stabilization of the unstable flame front for any value of the activation energy. The condition that the fluxes at the boundary points ξ_1, ξ_2 are uniform enables us to replace the limits of integration in (50) by $\pm\infty$. For an estimate we use the value K_c^{-1} which follows from (50) for a combustion wave with a large activation energy in the model (12) of Zel'dovich and Frank–Kamenetskii:

$$K_c^{-1}=\frac{(\Theta_2-1)}{2S_0\sqrt{\Theta_2+1-1/\Theta_2}}\times\left\{1+\frac{S_0^2(\Theta_2+1)+4S_0\Theta_2+2\Theta_2}{(\Theta_2-1)^2}\ln(\Theta_2)\right\}. \qquad (51)$$

Note that since thermal conduction stabilizes the mode even for $K\ll 1$, the estimate (50), (51) for the magnitude of the wave number at which the growth rate vanishes is quite accurate. For example, $K_c=0.3$ at $\Theta_2=8$; $K_c=0.31$ at $\Theta_2=6$; and $K_c=0.28$ at $\Theta_2=4$, which agrees well with the numerical results (Fig. 2).

6. FLAME STABILITY INCLUDING VISCOSITY

The formulation of the flame stability problem in a viscous gas flow is given in Sec. 3. The numerical solution of the spectral problem (16)–(20) in the absence of a gravitational field ($1/\mathrm{Fr}=0$) with the boundary conditions (24), (25) has been obtained by iteration.

For a given approximate value $S=S_n$ of the eigenvalue the system of differential equations (16)–(20) was integrated three times over the interval $[\xi_1,\xi^*]$ with the boundary conditions $[\tilde{\varphi}_1(\xi_1)=\tilde{\varphi}_S,\ \tilde{\varphi}_2(\xi_1)=\tilde{\varphi}_T$ and $\tilde{\varphi}_3(\xi_1)=\tilde{\varphi}_\eta]$ and three times over the interval $[\xi^*,\xi_2]$ with the boundary conditions $\tilde{\varphi}_4(\xi_2)=\tilde{\varphi}_A$, $\tilde{\varphi}_5(\xi_2)=\tilde{\varphi}_V$, $\tilde{\varphi}_6(\xi_2)=\tilde{\varphi}_C$. After this we found a new approximate eigenvalue $S=S_{n+1}$ by Newton's method:

$$S_{n+1}=S_n-D(S_n,\xi^*)/(\partial D/\partial S), \qquad (52)$$

where

$$D(\xi,S)=\det[\tilde{\varphi}_1,\tilde{\varphi}_2,\tilde{\varphi}_3,\tilde{\varphi}_4,\tilde{\varphi}_5,\tilde{\varphi}_6]. \qquad (53)$$

This method of determining the eigenvalues is based on the fact that the desired eigenmode must be expandable in the eigenfunctions $\tilde{\varphi}_S$, $\tilde{\varphi}_T$, and $\tilde{\varphi}_\eta$ at $\xi=\xi_1$ and must go over into a solution that can be represented as a superposition of the functions $\tilde{\varphi}_A$, $\tilde{\varphi}_V$, $\tilde{\varphi}_C$ at $\xi=\xi_2$. This requirement reduces to the condition that the determinant (53) vanish at any point in the interval $[\xi_1,\xi_2]$.

As the "matching" point ξ^* for the solutions from the right and the left we use the point corresponding to the maximum rate of energy release in the thermal conduction equation. This choice of the matching point provides for the greatest accuracy in searching for eigenvalues, since in both cases the trajectories away from the points $\xi=\xi_1$ and $\xi=\xi_2$ are attractive.

We also looked for eigenvalues of the system (16)–(20) in the complex plane. The calculations showed that there are no complex eigenvalues in the problem.

The numerical solutions of the eigenvalue problem reveal that all values of the viscosity of practical interest have no effect on the stability of the flame, and the calculated values of the instability growth rates agree with those obtained for zero viscosity. In the numerical calculations we assumed values of the Prandtl number in the region $0.2<\mathrm{Pr}<1$.

We can arrive at the same conclusion based on the analytical solution if we note that in Eqs. (16)–(20) in the region of heating and burning we have

$$\tilde{w} \sim \mathrm{Pr}_1 K \tilde{j} \ll \tilde{j}. \tag{54}$$

Then repeating calculations analogous to those in Sec. 5, we can show that to within terms of order $\mathrm{Pr}^2\varepsilon^2$ the expression for the growth rate is the same as (49).

7. FLAME STABILITY IN A GRAVITATIONAL FIELD

Let us consider the effect of a gravitational field on the stability of a propagating flame. The combustion wave is assumed to be propagating parallel to the gravitational field. If the acceleration is in the direction of the flame front velocity, then the gravitational field will tend to stabilize the instability; but if the acceleration is directed opposite to the flame velocity, then the cold dense fuel is supported by the light combustion products, and in this flow it is possible for the Rayleigh–Taylor (R–T) instability to develop.[1] The importance of the gravitational field in relation to the actual instability of the flame is determined by the Froude number $\mathrm{Fr}=u_{z1}^2/g\Delta$. Since, as shown in Sec. 6, the viscosity has little effect on the growth rate, we will set $\mathrm{Pr}_1=0$ in what follows.

Let us consider the solution of Eqs. (16)–(19) in the long-wavelength limit. Since the gravitational field does not effect the structure of the hydrodynamic modes $\tilde{\varphi}_S(\xi)$, $\tilde{\varphi}_V(\xi)$, $\tilde{\varphi}_A(\xi)$, the boundary conditions in the incident and outgoing flows for the hydrodynamic variables $\tilde{j}$, $\tilde{u}$, $\tilde{\mathscr{P}}$ remain the same as (29), (30). The gravitational field also does not affect the solution of the perturbed thermal conduction equation, and hence the dimensionless shift ζ_T of the flame front and the perturbation $\tilde{j}_1$ of the mass flux (40). Compared with the previous calculation, a difference arises in the integration of the Euler equation, where an additional term appears:

$$\tilde{\mathscr{P}}-\tilde{\mathscr{P}}_1=\frac{\zeta_T}{\mathrm{Fr}}\frac{\Theta-1}{\Theta}. \tag{55}$$

In zeroth order in ε, corresponding to the model of a flame as a discontinuous front, the expression for the growth rate takes the form

$$S\equiv\frac{\sigma}{u_{z1}k}=\frac{\Theta_2}{\Theta_2+1}\left(\sqrt{\Theta_2+1-\frac{1}{\Theta_2}+\frac{\Theta_2^2-1}{\Theta_2^2}\frac{1}{\mathrm{Fr}K}}-1\right). \tag{56}$$

In the case of a large gravitational field for sufficiently long-wavelength perturbations, Eq. (56) goes over to the usual expression for the Rayleigh–Taylor instability growth rate:

$$\sigma=\sqrt{\frac{\Theta_2-1}{\Theta_2+1}gk}. \tag{57}$$

Stabilization of the flame instability, both the instability of the combustion front and the Rayleigh–Taylor instability, is due to dissipation caused by the finite thickness of the flame front. The corresponding stabilizing terms in the expression for the growth rate appear in higher order in the expansion in $\varepsilon\ll 1$. Calculations similar to those carried out in Sec. 5 yield the following expression for the dimensionless perturbation growth rate S to within terms of order ε^2:

$$S^2\left[1-K\frac{\Theta_2}{\Theta_2+1}\left(2\langle J(\xi)+\xi/\Theta\rangle-J(\xi_2)\right)\right]$$
$$+2\frac{\Theta_2}{\Theta_2+1}S[1+K(1+\langle\xi\rangle-(1+\Theta_2)\langle J(\xi)+\xi/\Theta\rangle$$
$$+\Theta_2 J(\xi_2))]-\Theta_2\frac{\Theta_2-1}{\Theta_2+1}-\frac{\Theta_2-1}{\Theta_2+1}\frac{1}{\mathrm{Fr}K}$$
$$\times\left[1+K\frac{\Theta_2}{\Theta_2-1}J(\xi_2)\right]+\frac{\Theta_2^2}{\Theta_2+1}K\left[1-\frac{1}{\Theta_2}\right.$$
$$\left.+2\left(1+\left\langle\xi-J(\xi)-\frac{\xi}{\Theta}\right\rangle+J(\xi_2)\right)\right]=0. \tag{58}$$

Consider the case in which the main contribution to the instability comes from the gravitational field, $\Theta_2 K\ll \mathrm{Fr}^{-1}$. To first order in ε, i.e., representing the flame as a surface of discontinuity with zero thickness, we have expression (57) for the growth rate.

As is well known,[26–28] when the finite thickness of the front (finite density gradient) is taken into account, the instability growth rate decreases relative to that in expression (57) even in the case of a fluid at rest:

$$\sigma_{\mathrm{RT}}=\sqrt{\frac{\Theta_2-1}{\Theta_2+1}\frac{gk}{1+kL}}, \tag{59}$$

where $L\sim\rho(d\rho/dz)^{-1}$ is the characteristic thickness of the transition region.

It is evident that treating the finite thickness of the flame front for the R–T instability must lead to a similar effect in the terms in expression (58) which are linear in ε. Since they are independent of the mass flow, these terms cannot depend on the Froude number, which determines the relationship between the gravitational field and the mass flow. In view of this we separate those terms, representing them in the form (59). Then in the case of a large gravitational acceleration, neglecting the effect of the flame instability itself, from (58) we find for the dimensionless growth rate of the instability

$$\frac{\sigma\Delta}{u_{z1}}\equiv SK=\sqrt{\frac{\Theta_2-1}{\Theta_2+1}\frac{K}{\mathrm{Fr}(1+Kl)}}\left[1-\left(\frac{K}{K_g}\right)^2\right], \tag{60}$$

where

$$l=\frac{2\Theta_2}{\Theta_2^2-1}\left[J(\xi_2)+(\Theta_2-1)\left\langle\frac{\xi}{\Theta}+J(\xi)\right\rangle\right] \tag{61}$$

and

$$K_g^{-2}=\frac{\Theta_2^2\mathrm{Fr}}{2(\Theta_2+1)}\left[1-\frac{1}{\Theta_2}+2\left(1+\left\langle\xi-J(\xi)\right.\right.\right.$$
$$\left.\left.\left.-\frac{\xi}{\Theta}\right\rangle+J(\xi_2)\right)\right]. \tag{62}$$

Here $l=L/\Delta$ is the effective thickness of the transition region (the flame), and the perturbation wavelength at

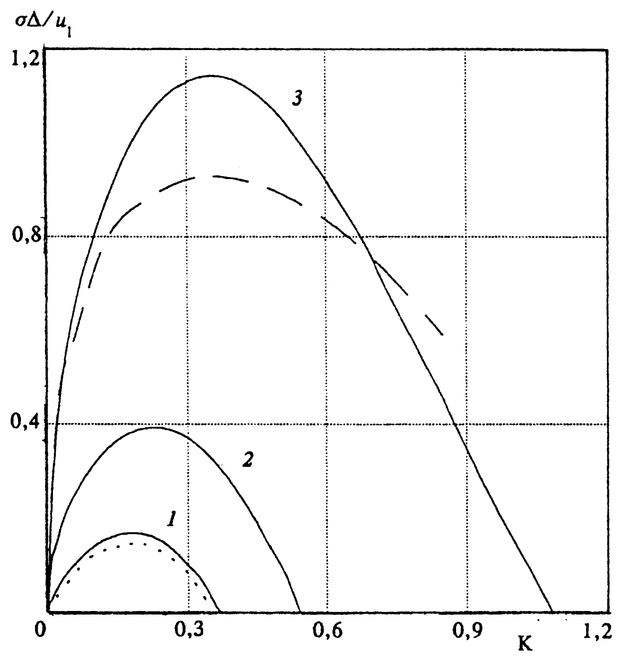

FIG. 3. Dimensionless growth rate as a function of wave number for a flame propagating in a gravitational field with different values of the Froude number: *1*) Fr=10; *2*) Fr=1; *3*) Fr=0.1 for $\Theta_2=\delta$. The dots correspond to the case of zero gravitational field and the dashed curve corresponds to the analytical result (65).

which the growth rate vanishes is $\lambda_g=2\pi\Delta/K_g$. For the case of a flame with a high activation energy the model of Zel'dovich and Frank–Kamenetskii yields for l and K_g

$$l=2\frac{\Theta_2\ln\Theta_2}{\Theta_2-1}, \qquad (63)$$

$$K_g=\frac{\Theta_2-1}{\Theta_2\sqrt{\mathrm{Fr}}}[\ln\Theta_2+(\Theta_2-1)^2 2\Theta_2^{-1/2}. \qquad (64)$$

Returning to dimensional variables, we find for the instability growth rate of a flame with large activation energy the expression

$$\sigma=\sqrt{\frac{\Theta_2-1}{\Theta_2+1}\frac{gk}{1+l\Delta k}}\left[1-\beta\left(\frac{\Delta}{g}\right)u_{z1}^2k^2\right], \qquad (65)$$

where l is determined by (63) and the dimensionless coefficient β is equal to

$$\beta=\frac{\Theta_2^2\ln(\Theta_2)}{(\Theta_2-1)^2}+\frac{\Theta_2}{2}. \qquad (66)$$

Thus, the stabilization of the Rayleigh–Taylor instability is largely due to the finite thickness of the flame and, strictly speaking, cannot be described using the model of an infinitely thin front.[20–23,29–31]

Figure 3 shows the results of solving the flame stability problem numerically in a gravitational field for different values of the Froude number Fr=10, 1, and 0.1 for traces 1, 2, and 3 respectively. For large values $\mathrm{Fr}\ll 1$ of the gravitational acceleration the magnitude of the instability growth rate is mainly determined by the Rayleigh–Taylor instability and agrees satisfactorily with the approximate expression (65).

It is interesting to note that if we consider the motion associated with a combustion wave in a gravitational field for $\mathrm{Fr}\ll 1$ as a model of an ablation wave for a laser target, Eq. (65) yields a satisfactory approximation to the results of numerical modeling,[29–31] and explains the convective stabilization of the Rayleigh–Taylor instability in an ablation flow.

8. CONCLUSION

The solution of the stability problem for a flame front with viscosity, thermal conduction, and the kinetics of the chemical reaction taken into account, in particular for a flame propagating in a gravitational field, reveals that the condition that the velocity with which the flame propagates remain unchanged is valid for perturbations with long wavelengths, considerably greater than the thickness of the combustion zone, $\lambda>100\Delta$. For such long-wavelength perturbations the asymptotic form of the growth rate is determined by the Landau formula (1) in the absence of a gravitational field and by (56) in the presence of such a field. We have shown that the hydrodynamic instability of a flame front is stabilized even for perturbations with wavelength much greater than the flame thickness, where the instability growth rate is independent of viscosity. For the case of a flame propagating in a gravitational field we have shown that convective stabilization of the Rayleigh–Taylor instability can in principle not be described using a flow with a discontinuity, and is related to transport processes responsible for creating the flow and determining the finite thickness of the combustion region. We have obtained an estimate for the characteristic perturbation wave number (64) at which the R–T instability is stabilized by mass flow.

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