Franson-type interference of entangled multi-photon states

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On the basis of a detailed investigation of the photon correlation of arbitrary order S of two spatially separated modes we elicit characteristic features of their interference, which appears in spite of the absence of mode mixing. Here each mode is exposed to the action of an independent two-ray interferometer. We establish that the maximal contrast of the interference picture is achieved in the framework of the quantum description and the isolated contrast takes place for $2S$ photon states for odd S only. With increasing S the interference maxima become sharper and the minima become smoother. The classical theory predicts, on the other hand, significantly lower visibility and contrast.

INTRODUCTION

A significant role in modern studies in quantum optics is played by a variety of projects on the observation of interference between so-called entanglement states, characterized by hard pairwise correlation of photons belonging to different modes.[1–11] Here, as a rule, the two-channel detection regime is utilized, and only cases of coincident photon appearance in the channels is taken into account. Thus we are in fact dealing with modifications of the interference of the intensities, or with mixed moments of another order.

The most thorough analysis in this sense was given to two-photon states, whose correlated photons are in two modes separated in space or in frequency. Their preparation presents no difficulty: most simply they are obtained in the process of nondegenerate parametric scattering in a piezocrystal, where a pump photon fissions into two photons (signal and idler) with frequency conversion down ($\omega_0 \rightarrow \omega_a + \omega_b$). Therefore the numerous theoretical predictions are already supported by many experiments. According to the classification in Refs. 7 and 8 they can be divided in two large groups. To the first group belong schemes with mixing of the modes produced in the parametric process, which can be achieved with the help of a two-ray interferometer or a simple beam splitter. The second group is interesting in that the prepared modes are not mixed yet interference is present: each of them separately passes its two-ray interferometer (for example, Mach–Zehnder), and the rate of pair photocounts depends on the cosine of the sum of the path differences in these independent interferometers. The reason for this at first sight unexpected behavior lies in the correlation of photons of the entangled state. This last group of experiments has received the name of interference of the Franson type.

Thus, two-photon fields have by now been extensively studied, which cannot be said of multiphoton processes whose study is still in infancy (see, for example, Refs. 7–11, and the literature cited therein). And whereas schemes with mixing have been already subjected to a definite analysis,[7–9] the multiphoton interference of the Franson type is in that sense lagging. The present work is an attempt to fill this gap. In this connection it is necessary to note that the investigation of the correlation of three or more photons displays also a new type of contradiction between quantum theory and the concept of hidden parameters,[10,11] which is sharper and more explicitly pronounced than by means of violation of the Bell inequalities.[12,13]

1. HOW TO OBSERVE FRANSON-TYPE INTERFERENCE

The scheme of a possible experiment for the organization of Franson-type interference is shown in Fig. 1. It is particularized for the four-photon state (two photons in each mode: $S=2$), which we will consider in greatest detail. In the case of an arbitrary $2S$-photon state, small changes not of principal significance have to be introduced into the detection system (increase of the number of photoreceptors).

There are the following main possibilities for the preparation of the four-photon state $|22\rangle$. First, use can be made of the four-cascade transition of an atom from the excited to the ground state, for which each photon pair is degenerate (belongs to the same mode). An alternate procedure consists in the application of parametric scattering either in a medium with nonlinearity $\chi^{(4)}$ (by analogy with Ref. 14), or as a result of a cascade process analogous to that described in Refs. 7 and 15. In the latter case two photons are generated in the first stage of strong monochromatic pumping in the process of nondegenerate parametric scattering in the piezocrystal, for example of the type $\omega_0 \rightarrow \omega_a' + \omega_b'$ (the subscripts a and b correspond to different modes, separated either in space, or in frequency, or in polarization type), and in the second stage these photons split into pairs already in the degenerate regime, i.e., in the generation of subharmonics: $2\omega_{a,b}' \rightarrow \omega_{a,b}$. In the following we suppose for simplicity that $\omega_a=\omega_b=\omega$.

The beams formed in this fashion then enter independent Mach–Zehnder interferometers, in which phase shifts in the arms can be introduced by the displacement of any one of the total-reflection mirrors. The emerging beams are detected by photodetectors whose signals are analyzed for coincidences and as a result one determines the desired

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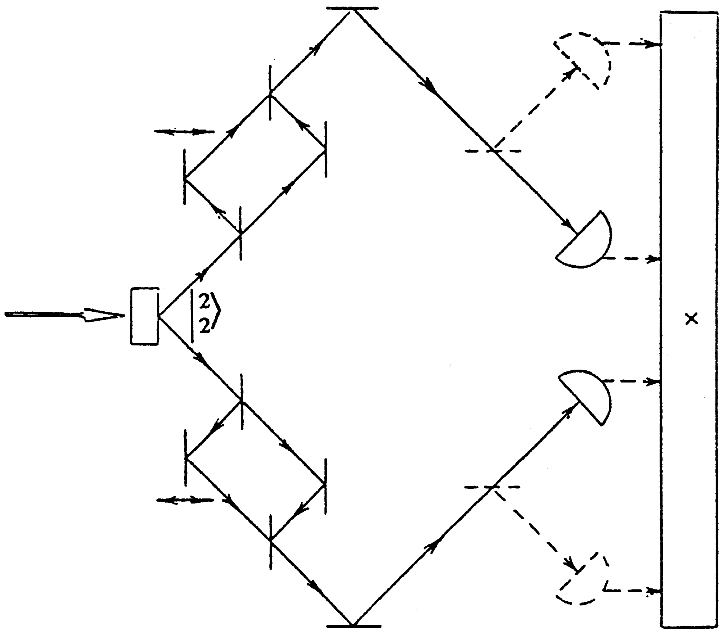

FIG. 1. Possible variant of a scheme for the observation of Franson-type interference in a two-mode four-photon state. All beam splitters have identical (50%) transmission and reflection. Dashed lines indicate additional photo-detectors and beam splitters necessary for the detection of the fourth moment G_{22}. The coincidence scheme registers only cases of simultaneous detection of photons in each of the receptors. The filters in front of the photo-receptors are not shown.

moment $G_{SS}=\langle :n_a^S n_b^S: \rangle$, where $n_a=a^+a$, $n_b=b^+b$, with a^+, a and b^+, b the creation and annihilation operators of each of the modes, and the colons denote normal ordering. To measure G_{11} (intensity interference) two detectors are sufficient, to measure G_{22}-four (additional receptors are shown in Fig. 1 by dashed lines).

2. STARTING RELATIONS

The process of formation of the $2S$-photon state, which consists of the decay of the pump photon into two S photons (one S in each mode) is described by the following Hamiltonian

$$H=i\hbar\chi[(a^+b^+)^S-(ab)^S]/S, \tag{1}$$

where χ is a coefficient that characterizes the nonlinearity of the system (for parametric scattering it is proportional to the nonlinearity $\chi^{(2S)}$ of the medium and the pumping amplitude, which we assume to be classical and inexhaustible).

In the Heisenberg picture the evolution of the operators determines the following equations of motion

$$\frac{da}{d\tau}=a^{+S-1}b^{+S}, \quad \frac{db}{d\tau}=a^{+S}b^{+S-1}. \tag{2}$$

Here $\tau=\chi t$.

3. PERTURBATION THEORY FOR THE FOUR-PHOTON STATES

For $S=2$ the solution of the system of equations (2) to econd order of perturbation theory in τ has the form (4)

$$a(\Omega)=a_0(\Omega)+\tau a_0^+(\Omega)b_0^{+2}(-\Omega)$$
$$+\frac{\tau^2}{2}[2a_0^+(\Omega)a_0(\Omega)b_0^+(-\Omega)b_0(-\Omega)$$
$$+a_0^+(\Omega)a_0(\Omega)+b_0^{+2}(-\Omega)b_0^2(-\Omega)$$
$$+4b_0^+(-\Omega)b_0(-\Omega)+2]a_0(\Omega), \tag{3}$$

where the subscript "0" corresponds to the initial operators for $\tau=0$, and Ω is the detuning frequency from the carrier $\bar{\omega}$, i.e., $\omega=\bar{\omega}+\Omega$. The appearance of negative arguments $(-\Omega)$ in (3) is connected with the condition $\omega_0=2(\omega_a+\omega_b)=4\omega$, which follows from the law of conservation of energy (ω_0 is the frequency of the monochromatic pumping), a consequence of which is $\Omega=\Omega_a=-\Omega_b$. The relation for $b(\Omega)$ is obtained from (3) by exchanging a_0 and b_0, but keeping the original arguments.

It is not hard to verify that the above solutions satisfy the standard commutation relations for bosons:

$$[a(\Omega),a^+(\Omega')]=\delta(\Omega-\Omega'),$$
$$[b(\Omega),b^+(\Omega')]=\delta(\Omega-\Omega'), \tag{4}$$

$$[a(\Omega),a(\Omega')]$$
$$=[a^+(\Omega),a^+(\Omega')]=[b(\Omega),b(\Omega')]$$
$$=[b^+(\Omega)b^+(\Omega')]=[a(\Omega),b(\Omega')] \tag{5}$$
$$=[a(\Omega),b^+(\Omega')]=0$$

accurate to second order in τ inclusive.

At the input of the parametric system we specify thermal noise with an average number of photons $N_0=\langle a_0^+a_0\rangle=\langle b_0^+b_0\rangle$. The nonzero normal-ordered moments of order S are then equal to

$$\langle:(a_0^+a_0)^S:\rangle=\langle:(b_0^+b_0)^S:\rangle=S!N_0^S. \tag{6}$$

The main reason for using initial thermal noise is that it allows the accomplishment of a gradual passage from a quantum to a classical model. Indeed, the case $N_0=0$ corresponds to the vacuum at the input of the parametric scatterer, i.e., a purely quantum situation. In the opposite limit, the noncommutativity of a_0, a_0^+ and b_0, b_0^+ ceases to affect the results and we arrive at the classical description of the system. In this way it will be easy to follow the specifics of each of the characteristic cases.

4. TRANSFORMATION OF THE RADIATION BY THE INTERFEROMETERS AND ITS SUBSEQUENT DETECTION

We pass now to a description of the action of the interferometers (see Fig. 1), which accomplish a unitary transformation of the form

$$a'(\Omega)=\{[1+\exp(-i(\alpha+\Omega\Delta t_a))]a(\Omega) + [1-\exp(-i(\alpha+\Omega\Delta t_a))]c(\Omega)\}/2,$$

$$b'(\Omega)=\{[1+\exp(-i(\beta+\Omega\Delta t_b))]b(\Omega) + [1-\exp(-i(\beta+\Omega\Delta t_b))]d(\Omega)\}/2, \quad (7)$$

where the operators $c(\Omega)$ and $d(\Omega)$, as well as all the others, describe in the Heisenberg picture the vacuum at the "second" inputs of the interferometers (on the other side of the incident beam splitters); $\Delta t_{a,b}$ are the relative time delays of the light by the arms of the interferometers; $\alpha=\bar{\omega}\Delta t_a$, $\beta=\bar{\omega}\Delta t_b$.

Since we are interested in normally-ordered moments G_{SS}, the operators c and d do not contribute upon averaging over the vacuum, and in what follows the corresponding terms will be omitted.

The photocurrents in the detectors are determined by the operators

$$n_a(t_a)=a''^+(t_a)a''(t_a), \quad n_b(t_b)=b''^+(t_b)b''(t_b), \quad (8)$$

with

$$a''(\Omega)=\eta_a^{1/2}(\Omega)a'(\Omega)+...,$$

$$b''(\Omega)=\eta_b^{1/2}(\Omega)b'(\Omega)+..., \quad (9)$$

where $\eta_{a,b}(\Omega)$ are the spectral characteristics of the photoreceptors (in intensity), which can be regulated by the linear filters placed in front of them (which are not shown in Fig. 1). In (9) terms that vanish upon calculation of normal-ordered moments, similarly to c and d, were omitted. Their explicit form is given, for example, in Ref. 7. Losses due to additional light dividers when operating with four detectors can also be included in the coefficient $\eta_{a,b}(\Omega)$.

Operators in the temporal and spectral representation used in (8) and (9) are connected by a Fourier transformation, for example,

$$a''(t_a)=\int a''(\Omega)\exp(-i\Omega t_a)d\Omega. \quad (10)$$

The integration limits can be taken here approximately as infinite.

We first analyze a variant of observation of intensity interference. To this end we need in the experimental scheme (Fig. 1) two photodetectors. The problem reduces to the calculation of the correlation function

$$G_{11}(t_a,t_b)\equiv\langle n_a(t_a)n_b(t_b)\rangle = \frac{1}{16}\int_{-\infty}^{\infty}\cdots\int [\eta_a(\Omega_1)\eta_a(\Omega_3)\eta_b(\Omega_2)\eta_b(\Omega_4)]^{1/2} \times[1+\exp(i(\alpha+\Omega_1\Delta t_a))][1+\exp(i(\beta+\Omega_2\Delta t_b))] \times[1+\exp-(i(\alpha+\Omega_3\Delta t_a))][1+\exp(-i(\beta+\Omega_4\Delta t_b))]\langle a^+(\Omega_1)b^+(\Omega_2)a(\Omega_3)b(\Omega_4)\rangle \times\exp(it_a(\Omega_1-\Omega_3)+it_b(\Omega_2-\Omega_4))d\Omega_1...d\Omega_4. \quad (11)$$

Here we made use of (7)–(10).

The expectation value in the integrand can be determined with the help of Eqs. (3)–(6). Omitting the unwieldy intermediate steps, we obtain in the end

$$\langle...\rangle=[A+B\delta(\Omega_1+\Omega_2)]\delta(\Omega_1-\Omega_3,\Omega_2-\Omega_4), \quad (12)$$

$$A=N_0^2+2\tau^2(8N_0^4+12N_0^3+8N_0^2+N_0), \quad (13)$$

$$B=4\tau^2(12N_0^4+21N_0^3+17N_0^2+7N_0+1). \quad (14)$$

Thus,

$$G_{11}(t_a,t_b) = \frac{A}{4}\int_{-\infty}^{\infty}\eta_a(\Omega)[1+\cos(\alpha+\Omega\Delta t_a)]d\Omega \times\int_{-\infty}^{\infty}\eta_b(\Omega)[1+\cos(\beta+\Omega\Delta t_b)]d\Omega + \frac{B}{4}\int_{-\infty}^{\infty}\eta_a(\Omega)\eta_b(-\Omega)\{1+\cos(\alpha+\Omega\Delta t_a) +\cos(\beta-\Omega\Delta t_b)+\tfrac{1}{2}\cos[\alpha+\beta+\Omega(\Delta t_a-\Delta t_b)] +\tfrac{1}{2}\cos[\alpha-\beta+\Omega(\Delta t_a+\Delta t_b)]\}d\Omega. \quad (15)$$

The first term represents the product of ordinary single-photon interference terms. The first two cosines in the second term of the integrand are also of the single-photon type. We are interested in higher orders, for which the additional single-photon modulation is parasitic and can be eliminated by an appropriate choice of spectral characteristics $\eta_{a,b}(\Omega)$. For simplicity we set $\eta_a(\Omega)=\eta_b(\Omega)=\eta(\Omega)$. Let the bandwidth of the detector (or the filter placed in front of it) be $\Delta\Omega>2\pi/\Delta t_{a,b}$. Then, according to Eq. (15),

$$G_{11}\simeq\frac{A}{4}\left(\int_{-\infty}^{\infty}\eta(\Omega)d\Omega\right)^2+\frac{B}{4}\int_{-\infty}^{\infty}\eta^2(\Omega) \times\{1+\tfrac{1}{2}\cos[\bar{\omega}(\Delta t_a+\Delta t_b)+\Omega(\Delta t_a-\Delta t_b)]\}d\Omega, \quad (16)$$

or for $\Delta t_a=\Delta t_b=\Delta t$

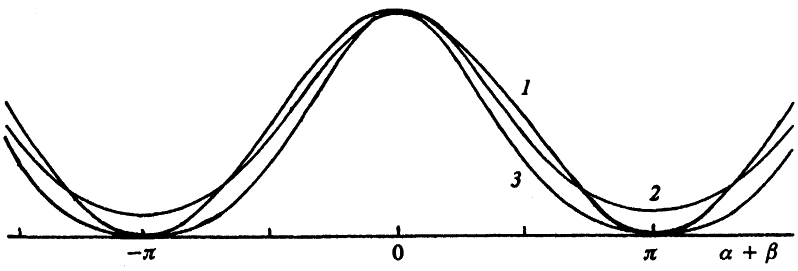

FIG. 2. Graphs of the interference curves of the moments G_{SS}, normalized to their maximum value. The numbers by the curves correspond to $S=1$ to 3. We have set $N_0=0$ everywhere.

$$G_{11}=\frac{A}{4}\left(\int_{-\infty}^{\infty}\eta d\Omega\right)^2+\frac{B}{4}\left[1+\tfrac{1}{2}\cos(\alpha+\beta)\right]\int_{-\infty}^{\infty}\eta^2 d\Omega. \tag{17}$$

Thus the interference is determined by the oscillating factor of the second term. The visibility V of the interference picture does not exceed 1/2 and reaches maximum in the pure quantum case, when $N_0=A=0$, $B=4$ and

$$G_{11}\propto 1+\tfrac{1}{2}\cos(\alpha+\beta). \tag{18}$$

In passage to the classical description, i.e., with increasing N_0, the visibility decreases due to the increase in $A\simeq N_0^2$ and in the constant component in (17), above whose background the oscillations are less noticeable. However the very structure of the interference picture remains unchanged. Only its quantitative parameter V varies. This circumstance relates the case under consideration to interference of intensities of the Franson type of two-photon states, where the visibility also falls on going from the quantum to the classical description.[2,6] However, there are also differences: there the maximal visibility reaches unity (see the graph in Fig. 2 for $S=1$). Further, according to (15)–(17), G_{11} does not depend on t_a and t_b. This means that the pairs of photons can be detected either simultaneously or with an arbitrary relative delay. In the two-photon case, on the other hand, simultaneous detection was required to obtain maximum visibility.

We pass now to the analysis of interference of higher order, i.e., the correlation function $G_{22}(t_a,t_b)$, in the calculation of which the main difficulty lies in the evaluation of the expectation value

$$\begin{aligned}&\langle a^+(\Omega_1)a^+(\Omega_2)b^+(\Omega_3)b^+(\Omega_4)a(\Omega_5)a(\Omega_6)b(\Omega_7)b(\Omega_8)\rangle\\&=N_0^4\{[\delta(\Omega_1-\Omega_5,\Omega_2-\Omega_6)+\delta(\Omega_1-\Omega_6,\Omega_2-\Omega_5)]\\&\quad\times[\delta(\Omega_3-\Omega_7,\Omega_4-\Omega_8)+\delta(\Omega_3-\Omega_8,\Omega_4-\Omega_7)]\\&\quad+4\delta(\Omega_1-\Omega_2,\Omega_1-\Omega_5,\Omega_1-\Omega_6,\Omega_3-\Omega_4,\Omega_3\\&\quad-\Omega_7,\Omega_3-\Omega_8)\}+\tau^2N_0[\ldots]\\&\quad+4\tau^2\delta(\Omega_1-\Omega_2,\Omega_5-\Omega_6,\Omega_3-\Omega_4,\Omega_7-\Omega_8,\Omega_1\\&\quad+\Omega_3,\Omega_5+\Omega_7).\end{aligned} \tag{19}$$

Here we made use of (3)–(6). The term in the square brackets, proportional to τ^2N_0, has been omitted in view of its extreme unwieldiness and since the smallness of τ makes it negligibly small relative to the zeroth order of perturbation theory, i.e., the first term in the braces. The last term in (19) is of purely quantum nature and does not vanish for $N_0=0$.

The suppression of lower interference orders, as in the preceding case, is achieved by the choice of a sufficiently wide detection band $\Delta\Omega>2\pi/\Delta t_{a,b}$. Here the first term in (19) provides only a constant background, proportional to N_0^4. This terminates its role as a "destroyer" of the interference visibility, and in the following we will concentrate only on the investigation of the effects due to the last term in (19), corresponding to the purely quantum situation ($N_0=0$).

With (7)–(10) taken into account and for equal phase shifts in the channels ($\Delta t_a=\Delta t_b=\Delta t$) we have ultimately

$$G_{22}(t_a-t_b)=\left[\frac{\tau}{4}(2+\cos 2\bar{\omega}\Delta t)\left|\times\int_{-\infty}^{\infty}\eta^2(\Omega)\exp(i\Omega(t_a-t_b))d\Omega\right|\right]^2. \tag{20}$$

Here we have set $\eta_a(\Omega)=\eta_b(\Omega)=\eta(\Omega)$.

To clarify the features of the final relation (20) we briefly recall the physical essence of the experiment under discussion (Fig. 1). Into each of the independent channels "a" and "b" enter pairs of simultaneously produced photons which, having traversed the interferometers, are also strictly pairwise detected by four receptors. If both photons of one channel are distributed with 50% likelihood not over two detectors but only appear in one, then the counting scheme will not react to such an event. This is because we only count events corresponding to the simultaneous appearance of photo-counts in all four receptors. It is the rate of such coincidences that is determined by (20).

Expanding the detection band $\Delta\Omega$ without bounds, i.e., letting $\eta\to 1$, we obtain

$$G_{22}(t_a-t_b)\propto\delta^2(t_a-t_b). \tag{21}$$

Consequently we are indeed dealing with a four-photon state, all of whose quanta are produced and detected simultaneously.

The normalized graph of the dependence of the interference cofactor in (20) on $\alpha+\beta=2\bar{\omega}\Delta t$ is shown in Fig. 2 ($S=2$). It is seen that it differs insignificantly from the case of interference of two-photon states ($S=1$), but the contrast is lower since $2+\cos(\alpha+\beta)$ has no zeros.

5. HIGHER ORDERS OF INTERFERENCE

We expand the investigation of the purely quantum interference ($N_0=0$) by generalizing it to the case of an arbitrary number of photons S in each mode. To this end we need the expectation value

$$\langle 0|a^+(\Omega_1)...a^+(\Omega_S)b^+(\Omega_{S+1})...b^+(\Omega_{2S})a(\Omega_{2S+1})$$
$$...a(\Omega_{3S})b(\Omega_{3S+1})...b(\Omega_{4S})|0\rangle, \quad (22)$$

where the operators are written in the Heisenberg picture and the averaging is over the initial vacuum states of the modes: $|0\rangle=|0\rangle_{\Omega_1}|0\rangle_{\Omega_2}...|0\rangle_{4S}$. It is clear that in the approximation of second order in perturbation theory the quadratic in τ terms in the expansion of the operators do not contribute to (22). We can therefore limit ourselves to the linear in τ solutions of the equations of motion (2):

$$a(\Omega)\simeq a_0(\Omega)+\tau a_0^{+S-1}(\Omega)b_0^{+S}(-\Omega),$$
$$b(\Omega)\simeq b_0(\Omega)+\tau a_0^{+S}(-\Omega)b_0^{+S-1}(\Omega), \quad (23)$$

whence the correlator (22) is

$$\langle 0|...|0\rangle \propto \tau^2\delta(\Omega_1-\Omega_2,\Omega_1-\Omega_3,...,\Omega_1-\Omega_{2S},\Omega_{4S}-\Omega_{4S-1},...,\Omega_{4S}-\Omega_{2S+1}). \quad (24)$$

Thus

$$G_{SS}(t_a,t_b)\equiv\langle :n_a^S(t_a)n_b^S(t_b):\rangle \propto \left|\int_{-\infty}^{\infty}\eta^S(\Omega)\exp(i\Omega(t_a-t_b)) \times[1+\exp(i(2+\Omega\Delta t_a))]^S \times[1+\exp(i(\beta-\Omega\Delta t_b))]^S d\Omega\right|^2. \quad (25)$$

Setting $\Delta\Omega\Delta t_{a,b}>2\pi$ and $\Delta t_a=\Delta t_b=\Delta t$ we obtain for the interference factor

$$G_{SS}\propto\left|\sum_{K=0}^{S}(C_S^K)^2\exp(iK(\alpha+\beta))\right|^2 = \sum_{K=0}^{S}(C_S^K)^4+2\sum_{L=1}^{S}\times\cos L(\alpha+\beta)\sum_{K=0}^{S-L}(C_S^KC_S^{K+L})^2, \quad (26)$$

where we have the binomial coefficients $C_S^K=S!/K!(S-K)!$

For the particular case $S=2$ we have the interference factor (20) identically equal to $2[9+8\cos(\alpha+\beta)+\cos 2(\alpha+\beta)]$.

For $S=3$ we have

$$G_{33}\propto 2[82+99\cos(\alpha+\beta)+18\cos 2(\alpha+\beta)+\cos 3(\alpha+\beta)]\equiv 8[16+24\cos(\alpha+\beta)+9\cos^2(\alpha+\beta)+\cos^3(\alpha+\beta)]. \quad (27)$$

Consequently, with increasing S the number of harmonics of the interference term increases being equal to $S+1$.

The normalized graph of G_{33} is also shown in Fig. 2. One notices first the return of 100% contrast in the interference (G_{33} again takes on zero values). This is explained by the following circumstance. For odd S the number of even harmonics (as well as of the even powers of the cosine) is equal to the number of odd harmonics (odd powers), and as a result they can compensate each other and turn the sum into zero. For even values of S, on the other hand, the number of odd harmonics is one less than the number of even ones. But it is the odd cosine powers which can take on negative values, and so their insufficiency results in incomplete compensation and a decrease in the interference contrast.

There is a general tendency towards sharper interference maxima with increasing S and towards smoother minima, which is also connected with the increased role of higher cosine powers. However the distribution and number of the maxima and minima are unchanged, which is in contrast with the interference of multi-photon states when they are mixed with the help of a beam splitter or polarization rotator,[7-9] where an increase in S is accompanied by an increasing number of zeros and local maxima.

CONCLUSION

The variants of Franson-type interference here investigated are endowed with characteristic features that distinguish them from the entire class of multi-photon interferometers, which, it is to be hoped, will raise the interest of the reader and enthusiasm of the experimenter.

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