Parametrization of electro-weak radiative corrections

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Using the $\alpha(m_Z)$, m_Z, G_μ, m_t, m_H parametrization we treat the leading (in m_t) radiative corrections to a large number of electro-weak observables. We indicate a program for the evaluation of the functions $V(t,h)$ for those processes for which they have not been as yet explicitly evaluated.

1. INTRODUCTION

The electro-weak minimal standard model (MSM), sometimes also called Quantum Flavor Dynamics (QFD), is along with QED and QCD one of the greatest accomplishments in physics. Its creation was a confluent effort of experimentalists and theorists. Just like QED, which is contained in the MSM, the MSM is a renormalizable theory and permits the calculation of radiative corrections. However there are differences between radiative corrections in the MSM and in QED. The most significant is that two important elements of the MSM, the t quark and the Higgs boson, have not been yet discovered and the values m_t and m_H of their masses, on which the radiative corrections depend significantly, are as yet unknown.

Another difference consists of the fact that both fundamental constants of QED, e and m—the charge and mass of the electron, are known to very high precision:

$$\alpha = e^2/4\pi = 1/137{,}0359895(61) \quad [1]\,, \tag{1}$$

$$m = 0{,}51099906(15) \text{ MeV} \quad [1]\,. \tag{2}$$

It is therefore natural in QED to express the results of loop calculations in terms of α and m and not, say, in terms of α and the anomalous magnetic moment of the electron, which is a secondary quantity, although it is measured to exceedingly high precision:

$$\mu_e = 1{,}001159652193(10) \quad [1]\,. \tag{3}$$

In the case of QFD there is some arbitrariness in the choice of the three fundamental constants, and of the three best measured electro-weak observables

$$\bar{\alpha} \equiv \alpha(m_Z) = 1/128{,}87(12) \quad [2]^{1)}\,, \tag{4}$$

$$m_Z = 91{,}187(7) \quad [4]\,, \tag{5}$$

$$G_\mu = 1{,}16639(2)\cdot 10^{-5}\ \text{GeV}^{-2} \quad [1]^{2)} \tag{6}$$

one, G_μ, is secondary.

It is understood that the expressions for the radiative corrections would be more natural and elegant if one were to use instead of G_μ, say, the mass of the W boson, but it is known much more poorly:

$$m_W = 80{,}22 \pm 0{,}26 \quad [1]^{3)}\,. \tag{7}$$

The same applies too the coupling constants of the W and Z bosons to the weak currents:

$$\alpha_W = g^2/4\pi\,, \qquad \alpha_Z = f^2/4\pi \tag{8}$$

and, in particular, to their values

$$\bar{\alpha}_W = \alpha_W(m_W^2) \quad \text{and} \quad \bar{\alpha}_Z = \alpha_Z(m_Z^2)\,. \tag{9}$$

Historically in the literature on electro-weak radiative corrections the quantities most commonly used as the three fundamental constants are α, G_μ and the so-called Sirlin angle θ_W:[7]

$$\sin^2\theta_W = 1 - m_W^2/m_Z^2\,, \tag{10}$$

i.e., in essence, the ratio of the W- and Z-boson masses. This angle was introduced long before the discovery of the W and Z bosons themselves, when the masses of both were unknown to the same extent. It was then that radiative corrections were calculated to such low-energy processes as μ decay, β decay of the neutron and the interaction of the neutrino with electrons and nucleons. In preparation for the construction of the electron-positron colliders SLC and LEP the same parametrization was used for radiative corrections to the decays of Z bosons.[8] It is also used in the most recent review of the properties of elementary particles,[1] even though the precision of m_Z^2 is almost two orders of magnitude better than the precision of $\sin^2\theta_W$ as a result of the experiments at LEP.

The parametrization of electro-weak radiative corrections in terms of $\bar{\alpha}$, m_Z, G_μ, m_t, m_H has unquestionable advantages compared to parametrization in terms of α, $\sin^2\theta_W$, G_μ, m_t, m_H. In the first place, using $\bar{\alpha}$ in place of α automatically takes into account the very large ($\sim$6%) purely electromagnetic effect of the change in the electric charge due to the photon polarization of the vacuum, so that only intrinsically electro-weak corrections remain. (This circumstance, that $\bar{\alpha}$ and not α is the natural quantity for all electro-weak processes, is connected with the fact that even low-energy weak processes, such as, for example, μ decay or $\nu_\mu e$ scattering, are determined by the values of $\bar{\alpha}_W$ and $\bar{\alpha}_Z$ and, consequently, by $\bar{\alpha}$; see Appendix C. This applies even more to the decays of the Z bosons.) In the second place, using m_Z in place of $\sin^2\theta_W$ permits the prediction of the values of electro-weak observables in the tree approximation with minimal errors.

It should be noted that making use of the secondary quantity G_μ as one of the basic parameters implies that the expression for the radiative correction to some electro-weak observable does not correspond literally to the Feynman diagrams for that observable, but also contains radiative corrections to G_μ. In the parametrization in terms of $\sin^2\theta_W$ this inconsistency acquires grotesque features. Thus, for example, the vertex parts in the purely leptonic decays of Z bosons

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due to triangle diagrams containing only leptons and W and Z bosons, turn out to depend on m_t (see, for example, Ref. 8).

As was already mentioned, from the direct experiment on the measurement of the mass m_W the quantity $\sin^2\theta_W$ is known only poorly. On the other hand, extraction of this quantity from other electro-weak data requires knowledge of m_t and m_H. Thus, for example, the experimental data on deep inelastic scattering were represented in terms of $\sin^2\theta_W$ for a set of fixed values of m_t and m_h and the reader is unable to deduce for himself from these data the restrictions on m_t and m_H (see below).

The representation of the radiative corrections in terms of functions of the parameters m_t and m_H specified "once and for all" is found to be impossible if $\sin^2\theta_W$ is used as one of the basic quantities.

Such a representation turns out to be possible if parametrization in terms of $\bar{\alpha}$, m_Z, G_μ, m_t, and m_H is used. The mass of the Z boson is known so well that the main imprecision in the theoretical predictions is due to $\bar{\alpha}$. Further, in the one-loop approximation in the electromagnetic interaction the answer for each observable can be put in the form of the sum of functions, one of which depends on $t=(m_t/m_Z)^2$ and the other on $h=(m_H/m_Z)^2$. Both functions, as well as the value of the observables in the tree approximation, depend on the parameter θ introduced by Peskin[9] and defined by the relation

$$\sin^2 2\theta = 4\sin^2\theta\cos^2\theta = 4s^2c^2 = \frac{4\pi\bar{\alpha}}{\sqrt{2}G_\mu m_Z^2} = 0{,}71094(66)\,, \tag{11}$$

from which

$$\begin{aligned} 2\theta &= 1{,}00316(73)\,, \quad \theta = 0{,}50158(37)\,,\\ s^2 &= 0{,}23118(33)\,, \quad c^2 = 0{,}76881(33)\,,\\ s &= 0{,}48081(33)\,, \quad c = 0{,}87682(19)\,,\\ c^2 - s^2 &= 0{,}53763(62)\,, \quad (1-4s^2)/2 = 0{,}03763(62)\,,\\ 1 - 4s^2 &= 0{,}07526(124)\,. \end{aligned} \tag{12}$$

In Ref. 6 we have calculated explicitly the radiative corrections in terms of θ, t, h, for m_W/m_Z and the axial and vector constants for the leptonic decays of Z bosons, g_A^l and g_V^l:

$$\frac{m_W}{m_Z} = c + \frac{3\bar{\alpha}cV_m(t,h)}{32\pi s^2(c^2-s^2)}\,, \tag{13}$$

$$g_A^l = -\frac{1}{2} - \frac{3\bar{\alpha}V_A(t,h)}{64\pi s^2c^2}\,, \tag{14}$$

$$R = \frac{g_V^l}{g_A^l} \equiv 1 - 4\sin^2\theta_{eff} = (1-4s^2) + \frac{3\bar{\alpha}V_R(t,h)}{4\pi(c^2-s^2)}\,. \tag{15}$$

Each of the V_i ($i=m,A,R$) was represented in Ref. 6 in the form of a sum of four terms:

$$V_i(t,h) = t + T_i(t) + H_i(h) + C_i\,. \tag{16}$$

The coefficients of V_i in Eqs. (13)–(15) are chosen to produce the same asymptotic behavior $V_i \simeq t$ for $t \gg 1$. [For $t\gg 1$ the functions $T_i(t)$ and $H_i(h)$ are proportional to $\ln t$ and $\ln h$ respectively.] This kind of asymptotic normalization facilitates comparison between experimental results referring to different observables.

Our aim in this article is to write down and discuss formulas similar to (13)–(15) for other electro-weak observables. We consider here $\nu_\mu e$ scattering, hadronic decays of Z bosons, including in that number $Z\to b\bar{b}$, deep inelastic neutrino scattering by nucleons, and parity violation in atomic transitions. With the exception of atomic transitions and $Z\to b\bar{b}$ we considered all these processes in Ref. 10, which remains unpublished in the archival literature. As in Ref. 10 we confine ourselves to the asymptotic region $t\gg 1$ only, considering this work as a small step in the direction of bringing radiative electro-weak corrections to the form convenient for understanding, comparing and including new experimental data in the discussion as they appear in the future.

2. RELATION BETWEEN THE BARE QUANTITIES AND $\bar{\alpha}$, G_μ, m_Z

The electro-weak interaction Lagrangian contains the so-called "bare" quantities: the charges e_0, f_0, g_0 characterizing the γ, Z, and W vertices respectively, and the masses

$$m_{Z0} = f_0\eta_0/2\,, \qquad m_{W0} = g_0\eta_0/2\,, \tag{17}$$

where η_0 is the vacuum expectation value of the Higgs condensate. The three charges are related by

$$(e_0/g_0)^2 + (g_0/f_0)^2 = 1\,. \tag{18}$$

It is therefore convenient to introduce the notation:

$$s_0 \equiv \sin\theta_0 \equiv e_0/g_0\,, \qquad c_0 \equiv \cos\theta_0 \equiv g_0/f_0 = m_{W0}/m_{Z0}\,. \tag{19}$$

At times it is convenient to use the notation:

$$\alpha_0 \equiv e_0^2/4\pi\,, \qquad \alpha_{Z0} \equiv f_0^2/4\pi\,, \qquad \alpha_{W0} \equiv g_0^2/4\pi\,. \tag{20}$$

The relations between the observable and bare quantities contain infinities ($\propto \Lambda^2$ and $\ln\Lambda$, where Λ is the cut-off energy or, in dimensional regularization, $\propto 1/\varepsilon$ where 2ε is the deviation of the dimension from four). In expression one set of observables in terms of another these infinities, naturally, cancel out. In Ref. 6 we have followed in detail this cancellation of infinities on the example of m_W/m_Z, g_A^l and g_V^l. In this article we leave them out from the very beginning, since we know that they will not enter the final formulas. Moreover we do not include Feynman diagrams that contain no t quarks since such diagrams cannot yield terms proportional to m_t^2. Lastly we shall consistently neglect the masses of the W and Z bosons compared to m_t. Making use of the formulas from section 6 of Ref. 6 we obtain (the sign $\simeq$ indicates that all three approximations were used)

$$\bar{\alpha} \simeq \alpha_0\,, \tag{21}$$

$$m_Z^2 \simeq m_{Z0}^2[1 - \Pi_Z(0)]\,, \tag{22}$$

$$m_W^2 \simeq m_{W0}^2[1 - \Pi_W(0)]\,, \tag{23}$$

$$G_\mu \simeq \frac{g_0^2}{4\sqrt{2}m_{W0}^2}[1+\Pi_W(0)]$$

$$= \frac{f_0^2}{4\sqrt{2}m_Z^2}[1+\Pi_W(0)-\Pi_Z(0)]\,. \qquad (24)$$

Dividing (21) by (24) we readily conclude

$$s_0^2c_0^2 \simeq s^2c^2[1-\rho_t]\,, \qquad (25)$$

where

$$\rho_t \equiv \Pi_Z(0)-\Pi_W(0)\,. \qquad (26)$$

Ignoring terms of order ρ_t^2 we find from (25)

$$c_0^2 \simeq c^2+\frac{c^2s^2}{c^2-s^2}\rho_t\,, \qquad (27)$$

$$s_0^2 \simeq s^2-\frac{c^2s^2}{c^2-s^2}\rho_t\,. \qquad (28)$$

From here it immediately follows that

$$m_W^2/m_Z^2 \simeq c_0^2(1+\rho_t) \simeq c_0^2+c^2\rho_t$$

$$\simeq c^2+\left(\frac{c^2s^2}{c^2-s^2}+c^2\right)\rho_t = c^2+\frac{c^4\rho_t}{c^2-s^2}\,. \qquad (29)$$

If use is made of the common value of $\sin^2\theta_W$, see Ref. 10, it then follows from (29) that

$$\sin^2\theta_W = s^2-\frac{c^4}{c^2-s^2}\rho_t\,. \qquad (30)$$

As is shown in Appendix (1)

$$\rho_t \simeq \frac{3\bar{\alpha}}{16\pi s^2c^2}t\,. \qquad (31)$$

From (29) and (31) one reproduces trivially the coefficient of V_m in Eq. (13). From (29) and (31) follows that

$$\frac{m_W}{m_Z} \simeq c+\frac{c^3\rho_t}{2(c^2-s^2)}$$

$$\simeq c+\frac{3\bar{\alpha}ct}{32\pi(c^2-s^2)s^2} = 0{,}8768(2)+0{,}00163t\,. \qquad (32)$$

As it should, the coefficient of t in (32) coincides with the coefficient of V_m in (13).

If we compare (32) and (13) with the experimental value $m_W/m_Z = 0.8797(29)$, see (7) and footnote 3, then we obtain

$$V_m^{exp} = 1{,}78 \pm 1{,}78\,.$$

3. PURELY LEPTONIC PROCESSES

We consider now matrix elements for leptonic decays of the Z boson and $\nu_\mu e$ scattering. We write the amplitude for the decay $Z\to l\bar{l}$ in the form:

$$M_l = \frac{1}{2}f(g_A^l\bar{l}\gamma_\alpha\gamma_5 l + g_V^l\bar{l}\gamma_\alpha l)Z_\alpha\,, \qquad (33)$$

where

$$f = (4\sqrt{2}G_\mu m_Z^2)^{1/2}\,. \qquad (34)$$

Since the Z-boson wavefunction renormalization contains no terms proportional to m_t^2 (this is obvious from dimensional considerations) the Born approximation is valid for M_l:

$$M_l \simeq \frac{1}{2}f_0\left[-\frac{1}{2}\bar{l}\gamma_\alpha\gamma_5 l-\left(\frac{1}{2}-2s_0^2\right)\bar{l}\gamma_\alpha l\right]Z_\alpha\,. \qquad (35)$$

Taking into account that

$$f_0 \simeq f\left(1+\frac{1}{2}\rho_t\right)\,, \qquad (36)$$

and

$$1-4s_0^2 \simeq 1-4s^2+\frac{4c^2s^2}{c^2-s^2}\rho_t\,, \qquad (37)$$

we find that

$$g_A^l \simeq -\frac{1}{2}\left(1+\frac{1}{2}\rho_t\right) = -\frac{1}{2}-\frac{3\bar{\alpha}t}{64\pi s^2c^2}$$

$$= -0{,}5000-0{,}00065t\,, \qquad (38)$$

$$R = \frac{g_V^l}{g_A^l} \simeq 1-4s^2+\frac{4c^2s^2}{c^2-s^2}\rho_t$$

$$= 1-4s^2+\frac{3\bar{\alpha}}{4\pi(c^2-s^2)}t = 0{,}0753(12)+0{,}00345t\,. \qquad (39)$$

The following notation is often used in the literature:

$$\sin^2\theta^{eff} = \frac{1}{4}\left(1-\frac{g_V^l}{g_A^l}\right)\,. \qquad (40)$$

It follows from (39) and (40) that

$$\sin^2\theta^{eff} = s^2-\frac{c^2s^2}{c^2-s^2}\rho_t\,. \qquad (41)$$

Comparing (41) with (30) we see that $\sin^2\theta^{eff}$ depends on t more weakly than $\sin^2\theta_W$. Experimentally

$$g_A^{exp} = -0{,}5000(10) \quad [11]\,, \qquad (42)$$

$$(\sin^2\theta^{eff})^{exp} = 0{,}2324(11) \quad [4]\,, \qquad (43)$$

which corresponds to

$$R^{exp} = 0{,}0704(44)\,. \qquad (44)$$

Comparing (38) and (42) we obtain

$$V_A^{exp} = 0 \pm 1{,}54\,. \qquad (45)$$

Comparing (39) and (44) we obtain

$$V_R^{exp} = -1{,}42 \pm 1{,}32\,. \qquad (46)$$

As was shown in Ref. 6, treating the one-loop corrections exactly changes the average value of t_m slightly but significantly increases the average value of t_A and especially t_R, so that the latter becomes positive. Here t_m, t_A, and t_R denote the values of t extracted from (7), (42), and (44) respectively.

We now turn to the matrix element for $\nu_\mu e$ scattering:

$$M_{\nu e} = \frac{G_\mu}{\sqrt{2}}\bar{\nu}_\mu\gamma_\alpha(1+\gamma_5)\nu_\mu(g_A^{\nu e}\,\bar{e}\gamma_\alpha\gamma_5 e + g_V^{\nu e}\,\bar{e}\gamma_\alpha e)\,. \qquad (47)$$

In the approximation that we are using it is equal to

$$M_{\nu e} \simeq \frac{f_0^2}{m_Z^2}\frac{1}{2}\bar{\nu}_\mu\gamma_\alpha\frac{1+\gamma_5}{2}\nu_\mu\left(-\frac{1}{2}\bar{e}\gamma_\alpha\frac{1+\gamma_5}{2}e + s_0^2\bar{e}\gamma_\alpha e\right). \qquad (48)$$

It follows from (47), (48), (36), (34), and (28) that

$$g_A^{\nu e} = -\frac{1}{2}\frac{f_0^2}{f^2} = -\frac{1}{2}(1+\rho_t) = -\frac{1}{2} - \frac{3\bar{\alpha}}{32\pi s^2c^2}t$$
$$= -0{,}5000 - 0{,}0013t\,, \qquad (49)$$

$$g_V^{\nu e} = \left(-\frac{1}{2} + 2s_0^2\right)\frac{f_0^2}{f^2} = -\frac{1}{2} + 2s^2 - \frac{1-2s^2+4s^4}{2(c^2-s^2)}\rho_t$$
$$= -\frac{1}{2} + 2s^2 - \frac{1-2s^2+4s^4}{(c^2-s^2)c^2s^2}\frac{3\bar{\alpha}}{32\pi}t$$
$$= -0{,}0376(6) - 0{,}00182t\,. \qquad (50)$$

Taking the one-loop corrections exactly into account changes the factor t in (49) and (50) into the functions $V_A^{\nu e}(t,h)$ and $V_V^{\nu e}(t,h)$ respectively. The asymptotic values for $t \gg 1$ of these functions are equal to t.

Measurements of the ν_μ and $\bar{\nu}_\mu$ cross sections for scattering on electrons gave:

$$g_A^{\nu e} = -0{,}503 \pm 0{,}018 \quad [12]\,, \qquad (51)$$

$$g_V^{\nu e} = -0{,}025 \pm 0{,}019 \quad [12]\,. \qquad (52)$$

Comparing these values with Eqs. (49) and (50) we obtain

$$V_A^{\nu e} = 2{,}3 \pm 13{,}8\,, \qquad (53)$$

$$V_V^{\nu e} = -6{,}9 \pm 10{,}4\,. \qquad (54)$$

Although the quantities $g_{A,V}^{\nu e}$ and $g_{A,V}^{e}$ differ from each other only in the upper indices, they are very different entities. This can already be seen from a comparison of the radiative corrections for g_A^e and $g_A^{\nu e}$ in Eqs. (38) and (49). If box-type Feynman diagrams are ignored it is easily seen that

$$g_A^{e\nu} \simeq 2g^\nu g_A^e\,, \qquad (55)$$

$$g_V^{e\nu} \simeq 2g^\nu g_V^e\,. \qquad (56)$$

(We note that $g^\nu = g_A^\nu = g_V^\nu$). To leading order in t we have $g_A^{e\nu} = -2(g_A^e)^2$, and for equal experimental accuracy it is twice as sensitive to the value of t. For this reason direct comparison of the four quantities, $g_{A,V}^{e\nu}$ and $g_{A,V}^{e}$, in one figure [Fig. 2 of Ref. 12] makes no sense physically. On the other hand, as is clear from (55) and (56), in the "no box" approximation

$$g_V^{e\nu}/g_A^{e\nu} = g_V^e/g_A^e\,. \qquad (57)$$

and one may compare the ratio extracted from the data on $\nu_\mu e$ scattering with those obtained in LEP experiments.

4. DECAY OF THE Z BOSON INTO QUARKS

Two specific features distinguish the decay of Z bosons into quarks from the decay into leptons. Firstly, this is nonperturbative fragmentation (hadronization) of quarks, as a result of which the extraction of the vector g_V^q and axial g_A^q constants in the decay into a given $q\bar{q}$ pair has larger systematic errors than in the leptonic case.

Secondly, in the decay into quarks a larger role is played by hard perturbative virtual gluons, characterized by the constant $\bar{\alpha}_s = \alpha_s(m_Z)$.

The corrections due to hard virtual gluons divide naturally into three classes:

1. Corrections due to gluons connecting external quarks (corrections of order $\bar{\alpha}_s$, $\bar{\alpha}_s^2$, $\bar{\alpha}_s^3$).

2. Corrections due to gluons connecting external quarks with internal ones, "sitting' inside the electro-weak loops (corrections of order $\bar{\alpha}\bar{\alpha}_s$).

3. Corrections due to gluons connecting quarks inside the electro-weak loops (corrections of order $\bar{\alpha}\bar{\alpha}_s$).

In the case of leptons we have corrections of only the last type. Naturally in that case the electro-weak quark loops describe vacuum polarization. In quark decays corrections of the third and second class arise also from electro-weak loops describing vertices ($q\bar{q}W$ and qWW triangles and qW bubbles).

The largest corrections are those of the first class in which the electro-weak loops are either absent or factorize. They are best known theoretically. For light quarks, whose masses can be neglected, these corrections appear in the corresponding widths of the Z boson in the form of a factor:[13,14]

$$1 + \delta^{QCD} = 1 + \frac{\bar{\alpha}_s}{\pi} + 1{,}411\left(\frac{\bar{\alpha}_s}{\pi}\right)^2 - 13\left(\frac{\bar{\alpha}_s}{\pi}\right)^3. \qquad (58)$$

We note the similar factorization in the partial width of the Z bosons of the corrections due to "external" hard photons—δ^{QED} is added to δ^{QCD}:

$$\delta^{QED} = \frac{3\bar{\alpha}Q_f^2}{4\pi} - \frac{\bar{\alpha}}{\pi}\frac{\bar{\alpha}_s}{\pi}Q_f^2\,, \qquad (59)$$

where $Q_f^2 = 1$ for leptons, 1/9 for down quarks and 4/9 for up quarks. The term $\propto \bar{\alpha}\bar{\alpha}_s$ in (59) originates from Feynman diagrams containing both a hard gluon and a hard photon. Expression (59) was obtained from Eq. (12) of Ref. 15, in which we have gone over from α to $\bar{\alpha}$ and neglected the term $\propto \bar{\alpha}^2$.

As far as the quantity $\bar{\alpha}_s$ that enters Eqs. (58) and (59) is concerned, it is extracted independently from the data on the energy and angular distributions of hadrons in the decays of the Z boson and the hadronic decay width of the τ

lepton. The existing data are characterized by substantial scatter:

$$\bar{\alpha}_s = 0{,}125 \pm 0{,}005 \quad [16],$$
$$\bar{\alpha}_s = 0{,}125 \pm 0{,}003(\exp) \pm 0{,}008(th) \quad [17],$$
$$\bar{\alpha}_s = 0{,}118 \pm 0{,}001(stat) \pm 0{,}003(syst)^{+0{,}009}_{-0{,}004}(th) \quad [18], \tag{60}$$
$$\bar{\alpha}_s = 0{,}117 \pm 0{,}004 \quad [19],$$
$$\bar{\alpha}_s = 0{,}115 \pm 0{,}008 \quad [1].$$

We note that an error in $\bar{\alpha}_s$ of the order of ± 0.003 would permit the extraction of truly electro-weak corrections from the full hadronic width with an accuracy of ± 0.001. At the present time the accuracy to which truly electro-weak corrections are known can be judged from

$$(2g_A^l)^2 = |1 \pm 0{,}002|^2 = 1 \pm 0{,}004. \tag{61}$$

[The quantity $(2g_A^l)^2$ given here was determined in Ref. 11 from the leptonic widths, including the correction given by Eq. (59), which contains α in place of $\bar{\alpha}$. Utilization of $\bar{\alpha}$ would reduce the average value of $(2g_A^l)^2$ by approximately 0.0001.]

The amplitude for the decay $Z \to \bar{q}q$, without taking into account external photons and gluons, can be written in the form

$$A_{q\bar{q}} = \frac{f}{2} Z_\alpha \bar{q}(g_A^q \gamma_\alpha \gamma_5 + g_V^q \gamma_\alpha) q. \tag{62}$$

In the Born approximation we have

$$A_{q\bar{q}} = \frac{f_0}{2} Z_\alpha \bar{q}[T_3 \gamma_\alpha \gamma_5 + (T_3 - 2Q_q s_0^2)\gamma_\alpha] q, \tag{63}$$

where $T_3 = +1/2$, $Q = +2/3$ for the u and c quarks, and $T_3 = -1/2$, $Q = -1/3$ for the d, s, and b quarks.

For light quarks, as well as for leptons, taking radiative corrections into account in leading t approximation reduces to the replacement [see Eqs. (31), (36), and (37)]:

$$f_0 \simeq f\left(1 + \frac{\rho_t}{2}\right) = f\left(1 + \frac{3\bar{\alpha} t}{32\pi s^2 c^2}\right), \tag{64}$$

$$s_0^2 \simeq s^2 - \frac{c^2 s^2}{c^2 - s^2}\rho_t = s^2 - \frac{3\bar{\alpha} t}{16\pi(c^2 - s^2)}. \tag{65}$$

Therefore

$$g_A^q = T_3\left(1 + \frac{3\bar{\alpha} t}{32\pi s^2 c^2}\right), \tag{66}$$

$$g_V^q = \left[T_3 - 2Q_q s^2 + \frac{Q_q 3\bar{\alpha} t}{8\pi(c^2 - s^2)}\right]\left[1 + \frac{3\bar{\alpha} t}{32\pi s^2 c^2}\right]. \tag{67}$$

In going over to the full one-loop corrections the factor t in g_A^q should be replaced by the function $V_A^q(t,h)$, while $R^q = g_V^q/g_A^q$—by the function $V_R^q(t,h)$. Expressions for these functions will be obtained in the future.

In the case of the decay $Z \to b\bar{b}$ the Eqs. (66) and (67) need to be modified to take into account the virtual $b \to tW \to b$ transitions:

$$g_A^b = g_A^q - \frac{\tau}{2} = -\frac{1}{2} + \frac{\bar{\alpha} t}{64\pi c^2 s^2},$$
$$g_V^b = g_V^q - \frac{\tau}{2} = \frac{2}{3}s^2 - \frac{1}{2} + \frac{1 - 6s^2}{s^2 c^2 (c^2 - s^2)} \frac{\bar{\alpha} t}{64\pi}. \tag{68}$$

It is shown in Appendix B that

$$\tau = -\frac{\bar{\alpha} t}{8\pi c^2 s^2}. \tag{69}$$

The exact value of this correction was calculated in several papers,[20,21,22] and its contribution to $V_A^b(t,h)$ and $V_R^b(t,h)$ will be calculated in the future.

The decay $Z \to b\bar{b}$ differs from other quark decays further by the fact that in it terms proportional to m_b^2/m_Z^2 are not negligibly small. Firstly, they appear purely kinematically and reduce the width:

$$\Gamma(Z \to b\bar{b}) = 3\frac{G_\mu M_Z^3}{6\pi\sqrt{2}} \times \sqrt{1 - \frac{4m_b^2}{m_Z^2}}\left\{[(g_A^b)^2 + (g_V^b)^2]\left(1 + \frac{2m_b^2}{m_Z^2}\right) - 6(g_A^b)^2\frac{m_b^2}{m_Z^2}\right\}$$
$$\simeq \frac{G_\mu M_Z^3}{2\pi\sqrt{2}}\left\{(g_A^b)^2 + (g_V^b)^2 - 6(g_A^b)^2\frac{m_b^2}{m_Z^2}\right\}. \tag{70}$$

Secondly, terms, proportional to m_b^2/m_Z^2 appear when external gluon corrections are taken into account. As a result the width $\Gamma(Z \to b\bar{b})$ can be written in the following form:[15]

$$\Gamma(Z \to b\bar{b}) = \frac{G_\mu M_Z^3}{2\pi\sqrt{2}}\{[(g_A^b)^2 + (g_V^b)^2](1 + \delta^{QCD} + \delta^{QED}) + (g_V^b)^2\delta_2^{QCD} + (g_A^b)^2\delta_3^{QCD}\}, \tag{71}$$

where $\delta^{\rm QCD}$ and $\delta^{\rm QED}$ are given by Eqs. (58) and (59), and

$$\delta_2^{QCD} = 12\frac{\bar{m}_b^2}{m_Z^2}\frac{\bar{\alpha}_s}{\pi}\left[1 + 8{,}7\frac{\alpha_s}{\pi} + 45{,}3\left(\frac{\bar{\alpha}_s}{\pi}\right)^2\right], \tag{72}$$

$$\delta_3^{QCD} = -\frac{6\bar{m}_b^2}{m_Z^2}\left[1 + 3{,}67\frac{\bar{\alpha}_s}{\pi} + 14{,}3\left(\frac{\bar{\alpha}_s}{\pi}\right)^3\right], \tag{73}$$

$$\bar{m}_b = m_b(m_Z) = 2{,}7\ \text{GeV}. \tag{74}$$

Taking $\alpha_s = 0.12 \pm 0.01$ we obtain

$$\delta^{QCD} = 0{,}039 \pm 0{,}003, \tag{75}$$

$$\delta_2^{QCD} = 0{,}0006, \tag{76}$$

$$\delta_3^{QCD} = -0{,}006. \tag{77}$$

If we ignore small corrections and confine ourselves to the roughest approximation then the full hadronic decay width of the Z boson is

$$\Gamma_h = \frac{\sqrt{2}G_\mu m_Z^3}{8\pi}\left(1+\frac{\bar{\alpha}_s}{\pi}\right)(1+\rho_t)$$

$$\times\left[5-\frac{28}{3}s^2+\frac{88}{9}s^4+\left(\frac{28}{3}-\frac{176}{9}\right)\right.$$

$$\left.\times\frac{c^2s^2}{c^2-s^2}\rho_t+\left(2-\frac{4}{3}s^2\right)\tau\right]$$

$$=\frac{\sqrt{2}G_\mu m_Z^3}{8\pi}\left(1+\frac{\bar{\alpha}_s}{\pi}\right)(1+\rho_t)\cdot 3{,}36\left(1+0{,}59\frac{\bar{\alpha}t}{4\pi}\right). \quad (78)$$

Experimenters usually quote the value of the ratio

$$R_{hl} = \Gamma_h/\Gamma_l. \quad (79)$$

Taking into account that

$$\Gamma_l = \frac{\sqrt{2}G_\mu m_Z^3}{12\pi}[(g_A^l)^2+(g_V^l)^2]$$

$$=\frac{\sqrt{2}G_\mu m_Z^3}{48\pi}(1+\rho_t)\left\{1+\left[1-4s^2+\frac{3\bar{\alpha}t}{4\pi(c^2-s^2)}t\right]^2\right\}, \quad (80)$$

we have

$$R_{hl} = 6\cdot 3{,}36\left(1+\frac{\bar{\alpha}_s}{\pi}\right)[1+(1-4s^2)^2]^{-1}$$

$$\times\left(1+0{,}59\frac{\alpha t}{4\pi}\right)\left(1-0{,}82\frac{\alpha t}{4\pi}\right)$$

$$=20{,}07\left(1+\frac{\bar{\alpha}_s}{\pi}\right)\left(1-0{,}23\frac{\bar{\alpha}t}{4\pi}\right). \quad (81)$$

It thus turns out that R_h depend very weakly on m_t, hence within the accepted accuracy (neglect of terms of order $\bar{\alpha}/\pi$) the term $0.23\bar{\alpha}t/4\pi$ should also be neglected.

Comparing (81) with the experimental value[4]

$$R_{hl} = 20{,}85 \pm 0{,}07, \quad (82)$$

we find

$$\bar{\alpha}_s \simeq 0{,}122 \pm 0{,}011. \quad (83)$$

This value should be compared with the quantity

$$\bar{\alpha}_s \simeq 0{,}135 \pm 0{,}011, \quad (84)$$

obtained from R_{hl} with all the corrections which we ignored included.[4] We note that inclusion of the second term in Eq. (58) would decrease $\bar{\alpha}_s$ from 0.122 to 0.117 and the disagreement with (84) would increase, so that the corrections of order $\bar{\alpha}/\pi$ which we omitted are significant.

We return now to the decays $Z\to b\bar{b}$. Recently the charge asymmetry A_{FB}^b in these decays was measured:[4]

$$(A_{FB}^b)_{exp}^0 = 0{,}098 \pm 0{,}012, \quad (85)$$

where the superscript 0 indicates that all "extraneous" effects—$B^0\leftrightarrow B^0$ transitions, interference with the photon, etc.—have been taken into account and the result (85) should be compared with the bare theoretical formula:

$$(A_{FB}^b)^0 = \frac{3}{4}A_e^0A_b^0, \quad (86)$$

where

$$A_e^0 = \frac{2g_V^eg_A^e}{(g_V^e)^2+(g_A^e)^2} = \frac{2g_V^e/g_A^e}{1+(g_V^e/g_A^e)^2}, \quad (87)$$

and

$$A_b^0 = \frac{2g_V^bg_A^b}{(g_V^b)^2+(g_A^b)^2}. \quad (88)$$

The relative error in the measurement of $(A_{FB}^b)^0$ is of order 10%. At the same time $(g_V^e/g_A^e)^2\simeq 5\cdot 10^{-3}$; therefore the term $(g_V^e/g_A^e)^2$ in the denominator of (87) can be neglected. Substituting (87) and (88) into (86) and taking into account the above comment we obtain

$$(A_{FB}^b)^0 = 3\frac{g_V^e}{g_A^e}\frac{g_V^bg_A^b}{(g_V^b)^2+(g_A^b)^2}. \quad (89)$$

By substituting the theoretical formulas (39) and (11) into (89) we can obtain a bound on the value of t by comparison with the experimental result (85). A cleverer approach, used in the literature, consists of the following. Since g_V^e is numerically close to zero the ambiguity in g_V^b and g_A^b due to a poor knowledge of m_t is unimportant, and measurement of $(A_{FB}^b)^0$ permits a direct determination of g_V^e/g_A^e or $\sin^2\theta^{\text{eff}}$ (see Ref. 4). Setting $t=0$, in (68) from (89) and (85) we obtain

$$R \equiv g_V^e/g_A^e = 0{,}070 \pm 0{,}009. \quad (90)$$

In the experimental result (44) the data on the lepton asymmetry A_{FB}^e, on the polarization of the τ leptons produced in the decay of the Z, and on A_{FB}^b were included. They all have approximately the same experimental errors and determine one and the same quantity—the ratio g_V^e/g_A^e.

5. DEEP INELASTIC νN SCATTERING

The first determination of $\sin^2\theta_W$ and, consequently, prediction of the ratio of the W- and Z-boson masses, resulted from a measurement of the ratio of the cross sections due to neutral neutrino and quark currents NC and the corresponding charged currents CC.[23] Since then it has become customary to express the results of similar experiments in terms of $\sin^2\theta_W$. The most precise results were obtained by the CHARM,[24] CDHS[25] and CCFR[26] groups and were utilize to determine $\sin^2\theta_W$ and, consequently, the experimental values of the electro-weak radiative corrections to $\sin^2\theta_W$. In this procedure use is made of the circumstance[27] that the leading t corrections make their main contribution precisely through $\sin^2\theta_W$ and not indirectly:

$$R_\nu = \frac{1}{2}-\sin^2\theta_W+\frac{5}{9}\sin^4\theta_W(1+r)+\frac{1+10r}{9\cos^2\theta_W}\frac{3\bar{\alpha}}{16\pi}t, \quad (91)$$

where

$$R_\nu \equiv \sigma_{\nu N}^{NC}/\sigma_{\nu N}^{CC}\,, \tag{92}$$

and

$$r \equiv \sigma_{\bar\nu N}^{CC}/\sigma_{\nu N}^{CC}\,. \tag{93}$$

Nevertheless $\sin^2\theta_W$ cannot be extracted from the experimental data without making assumptions about the values of m_t and m_H. And although the dependence on these values is not strong it seems more rational to us at the present stage to give up this two-step procedure and directly analyze R_ν in terms of $\bar\alpha$, m_Z, G_μ, m_t, and m_H. In the parton approximation for the isoscalar quark target to leading order in t and not including the second generation of quarks or nuclear effects we have

$$\begin{aligned}R_\nu &= \left[\frac{f_0^2}{m_Z^2}\frac{m_W^2}{g_0^2}\right]^2\left[\frac{1}{2}-s_0^2+\frac{5}{9}s_0^4(1+r)\right]\\ &= (1+2\rho_t)\left[\frac{1}{2}-s^2+\frac{5}{9}s^4(1+r)+\frac{c^2s^2}{c^2-s^2}\rho_t\right.\\ &\quad\left.-\frac{10}{9}(1+r)\frac{c^2s^4}{c^2-s^2}\rho_t\right]\\ &= \frac{1}{2}-s^2+\frac{5}{9}s^4(1+r)+\rho_t\left[1-2s^2+\frac{80}{9}s^4(1+r)\right.\\ &\quad\left.+\frac{c^2s^2}{c^2-s^2}-\frac{10}{9}(1+r)\frac{c^2s^4}{c^2-s^2}\right]\\ &= \frac{1}{2}-s^2+\frac{5}{9}s^4(1+r)+\frac{3\bar\alpha t}{16\pi c^2s^2(c^2-s^2)}[(c^2-s^2)^2\\ &\quad+c^2s^2-\frac{10}{9}(1+r)s^6]\,.\end{aligned} \tag{94}$$

When the remaining so-called small terms in the radiative corrections are taken into account the quantity t in (94) should be replaced by $V_{R_{\nu N}}(t,h)$. We are in the process of calculating the explicit expression for this function. For $r=0.38(1)$ (see Ref. 4) Eq. (94) gives

$$R_\nu = 0{,}3098(4)+0{,}00217t \to 0{,}3098+0{,}00217V_{R_{\nu N}}(t,h)\,. \tag{95}$$

Here the ± 4 error is due in equal measure to errors in s^2 and in r. We turn to the collaborations CHARM, CDHS, and CCFR with the request that they publish the experimental values of $R_\nu^{\rm exp}$ *without* any radiative corrections. The value of $R_\nu^{\rm exp}$ immediately gives $V_{R_{\nu N}}(t,h)^{\rm exp}$. Knowing the latter one can draw direct conclusions about the bounds on m_t and m_H, similarly to what was done for leptonic decays of Z bosons in Ref. 6.

6. PARITY VIOLATION IN ATOMS

The effects of parity violation in atoms observed so far are due to the interaction of the neutral axial electron current $(\bar e\gamma_\alpha\gamma_5 e)$ with the neutral vector nucleon current, which in the static limit reduces to the so-called weak charge of the nucleus, Q_W. The amplitude for this interaction is usually written in the form

$$A = \frac{G_\mu Q_W}{2\sqrt{2}}\bar e\gamma_0\gamma_5 e\,. \tag{96}$$

In the Born approximation the amplitude of interest for the scattering of the electron e on the nucleon N equals:

$$A_N = \frac{f_0^2}{8m_Z^2}\bar N(T_3-2Qs_0^2)N\bar e\gamma_0\gamma_5 e\,, \tag{97}$$

where $T_3=\frac{1}{2}(-\frac{1}{2})$, $Q=+1(0)$ for the proton (neutron).

Using

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8m_W^2}\,, \tag{98}$$

we obtain for the scattering amplitude on the nucleus containing Z protons and N neutrons:

$$A = \frac{G_\mu}{2\sqrt{2}}\frac{8m_W^2}{g_0^2}\frac{f_0^2}{8m_Z^2}[(1-4s_0^2)Z-N]\bar e\gamma_0\gamma_5 e\,. \tag{99}$$

Taking into account that

$$\frac{m_W^2}{m_Z^2}\frac{f_0^2}{g_0^2} = 1+\rho_t\,,$$

$$1-4s_0^2 = 1-4s^2+\frac{4c^2s^2\rho_t}{c^2-s^2}\,,$$

we obtain

$$\begin{aligned}Q_W &= (1+\rho_t)\left[(1-4s^2)Z-N+Z\frac{4c^2s^2}{c^2-s^2}\rho_t\right]\\ &= (1-4s^2)Z-N+\left\{[(1-4s^2)-N]+Z\frac{4c^2s^2}{c^2-s^2}\right\}\rho_t\\ &= (1-4s^2)Z-N+\left[\frac{Z}{c^2-s^2}+\frac{(1-4s^2)Z-N}{4c^2s^2}\right]\frac{3\bar\alpha t}{4\pi}\,.\end{aligned} \tag{100}$$

For the isotope of cesium, for which the most accurate experiments were performed,[28] we have $Z=55$, $N=78$. Introducing these numbers into (100) we have

$$Q_W(^{133}Cs_{55}) = -73{,}9-1{,}64\frac{3\bar\alpha}{4\pi}t = -73{,}9-0{,}0030t\,. \tag{101}$$

From the experiment, Ref. 28,

$$Q_W(^{133}Cs_{55}) = -71{,}04\pm1{,}58\pm0{,}88\,. \tag{102}$$

It follows that the quantity $Q_W(^{133}Cs_{55})$ is practically insensitive to the value of the mass of the t quark. This result is well known in the literature[28,29] and permits one to hope that with improved experimental accuracy there might appear here signs of new physics, most importantly the Z' boson.

APPENDIX A

CALCULATION OF THE PARAMETER ρ_t

In this Appendix we obtain an expression for the parameter ρ_t, using dimensional regularization for the integrals appearing in it. This same expression can be obtained without a specific regularization procedure, as was shown in Appendix B of our paper, Ref. 6. The point is that the difference of the correlators of chiral currents that enters ρ_t is finite, because in the integral over momentum the integrand falls rapidly with increasing momentum. On the other hand the correlator of the electromagnetic currents in the polarization operator for the Z boson, proportional to Q^2, was set equal to zero in Ref. 6 because otherwise the phonon acquires mass. Similarly one sets equal to zero the vector correlator proportional to QT_3. The advantage of dimensional regularization is that the correlator of the vector currents in that procedure is automatically equal to zero for $q^2=0$.

Although the expression for ρ_t is well known, we consider it necessary that all the principal quantities determining electro-weak corrections be derived systematically and discussed within the framework of the parametrization procedure for electro-weak corrections developed here. The derivation given here has a definite methodological improvement: consistent utilization of vector current conservation significantly simplifies the calculations (this applies particularly to the parameter τ; see Appendix B).

Let us pass to the calculations. By definition,

$$\rho_t = \Pi_Z(0) - \Pi_W(0), \tag{A1}$$

where $\Pi_Z(0)$ is described by the $t\bar{t}$ and $b\bar{b}$ loops, and $\Pi_W(0)$ by the tb loop. Since the diagonal vector current is conserved we keep in $\Pi_Z(0)$ only the contribution of the axial current. Ignoring the mass of the b quark we obtain

$$\begin{aligned}\rho_t &= (-)i\cdot 3\int \frac{d^nk}{(2\pi)^n}\,\mathrm{Sp}\left(\gamma_\alpha\cdot\frac{1}{2}\frac{\gamma_5}{2}\frac{1}{\hat{k}-m_t}\gamma_\alpha \cdot\frac{1}{2}\frac{\gamma_5}{2}\frac{1}{\hat{k}-m_t}\right)\frac{f_0^2}{4m_Z^2}\\
&+(-)i\cdot 3\int\frac{d^nk}{(2\pi)^n}\,\mathrm{Sp}\left(\gamma_\alpha\cdot\frac{1}{2}\frac{\gamma_5}{2}\frac{1}{\hat{k}}\gamma_\alpha\cdot\frac{1}{2}\frac{\gamma_5}{2}\frac{1}{\hat{k}}\right)\cdot\frac{1}{4}\frac{f_0^2}{m_z^2}\\
&-(-)i\cdot 3\int\frac{d^nk}{(2\pi)^n}\,\mathrm{Sp}\left(\gamma_\alpha\frac{1}{\hat{k}-m_t}\gamma_\alpha\frac{1+\gamma_5}{2}\frac{1}{\hat{k}}\right)\cdot\frac{1}{4}\frac{g_0^2}{2m_W^2}\\
&=\frac{-3\cdot if_0^2}{64(2\pi)^n m_Z^2}\left\{\int\frac{d^nk}{(k^2-m_Z^2)^2}\mathrm{Sp}[\gamma_\alpha(\hat{k}-m_t)\gamma_\alpha(\hat{k}+m_t)] + \int\frac{d^nk}{k^4}\mathrm{Sp}[\gamma_\alpha\hat{k}\gamma_\alpha\hat{k}]\right\}+\frac{3\cdot ig_0^2}{16(2\pi)^n m_W^2}\\
&\times\int\frac{d^nk}{k^2(k^2-m_Z^2)}\mathrm{Sp}(\gamma_\alpha\hat{k}\gamma_\alpha\hat{k}).\end{aligned}\tag{A2}$$

Using the following relation

$$\mathrm{Sp}(\gamma_\alpha\gamma_\alpha)=4n,\quad \gamma_\alpha\hat{k}\gamma_\alpha = 2k_\alpha\gamma_\alpha - n\hat{k} = (2-n)\hat{k},$$

$$\mathrm{Sp}(\hat{k}\hat{k}) = 4k^2,$$

we obtain

$$\begin{aligned}\rho_t &= -i\frac{3f_0^2}{64(2\pi)^n m_Z^2}\left\{\int\frac{d^nk}{(k^2-m_t^2)^2}[4(2-n)k^2-4nm_t^2] + \int\frac{d^nk}{k^4}\cdot 4(2-n)k^2 - \int\frac{d^4k}{k^2(k^2-m_t^2)}\cdot 16(2-n)k^2\right\}\\
&=\frac{3f_0^2}{64(2\pi)^n m_Z^2}\left\{-\int\frac{d^nk}{(k^2+m_t^2)^2}[4(2-n)k^2+4nm_t^2] - 4(2-n)\int\frac{d^nk}{k^2}+16(2-n)\int\frac{d^nk}{(k^2+m_t^2)}\right\}.\end{aligned}\tag{A3}$$

To evaluate the integrals we make use of the formula[30]

$$\begin{aligned}&\int\frac{\rho^{2s}d^n\rho}{(\rho^2+m^2)^\alpha} = \frac{\pi^{n/2}}{\Gamma(n/2)}\times\frac{\Gamma(n/2+s)\Gamma(\alpha-n/2-s)}{\Gamma(\alpha)}(m^2)^{n/2-\alpha+s},\\
&\Gamma(x+1) = x\Gamma(x),\end{aligned}\tag{A4}$$

after which we obtain

$$\begin{aligned}\rho_t &= \frac{-3f_0^2(m_t^2)^{n/2-1}}{16(4\pi)^{n/2}m_Z^2}\left\{n\Gamma\left(2-\frac{n}{2}\right)+\frac{n}{2}(2-n)\Gamma\left(1-\frac{n}{2}\right) + \frac{2-n}{n/2-1}\Gamma\left(2-\frac{n}{2}\right) - 3(2-n)\Gamma\left(1-\frac{n}{2}\right)\right\}\\
&=\frac{3f_0^2(m_t^2)^{n/2-1}}{16(4\pi)^{n/2}m_Z^2}\left\{6\Gamma\left(2-\frac{n}{2}\right)-2n\Gamma\left(2-\frac{n}{2}\right) + 2\Gamma\left(2-\frac{n}{2}\right)\right\} = \frac{3f_0^2(m_t^2)^{n/2-1}}{4(4\pi)^{n/2}m_Z^2}\Gamma\left(3-\frac{n}{2}\right).\end{aligned}\tag{A5}$$

Lastly we can set $n=4$ in the last expression and, replacing f_0^2 by $4\pi\bar{\alpha}_Z$, obtain

$$\rho_t = \frac{3\bar{\alpha}_Z}{16\pi}\left(\frac{m_t}{m_Z}\right)^2 = \frac{3\bar{\alpha}}{16\pi c^2 s^2}t.\tag{A6}$$

APPENDIX B

CALCULATION OF THE PARAMETER τ

In the tree approximation the amplitude for the decay $Z\to b\bar{b}$ equals:

$$A_{b\bar{b}} = \frac{f_0}{2}Z_\alpha\left[\left(\frac{2}{3}s_0^2-\frac{1}{2}\right)\bar{b}\gamma_\alpha b - \frac{1}{2}\bar{b}\gamma_\alpha\gamma_5 b\right].\tag{B1}$$

When one-loop corrections are taken into account terms $\sim\alpha t$ appear in the corrections to f_0 and s_0^2 in the basic equations for other amplitudes for $Z\to f\bar{f}$ transitions as well. The specifics of the amplitude $Z\to b\bar{b}$ have to do with the fact that it has an additional source of corrections proportional to αt: the diagrams containing the transition of the b quark into a virtual t quark. These corrections give rise to the following change in the amplitude (B1):

$$A_{b\bar{b}} = \frac{f_0}{2} Z_\alpha \left[\left(\frac{2}{3}s_0^2 - \frac{1+\tau}{2}\right)\bar{b}\gamma_\alpha b - \frac{1+\tau}{2}\bar{b}\gamma_\alpha\gamma_5 b\right]. \quad \text{(B2)}$$

It is to the calculation of τ that this appendix is devoted. We set $m_b, m_Z, m_W = 0$ and carry out the calculation in the renormalized R_ξ gauge. We have 10 diagrams: two vertex diagrams with W-boson exchange (WWt and Wtt) and the exchange of the isotropic partner of the Higgs boson Φ^+ ($\Phi^+\Phi^+t$ and Φ^+tt), two vertex diagrams with the exchange of W^+ and Φ^+ ($W^+\Phi^+t$ and Φ^+W^+t), as well as two insertions with the exchanges of W^+ and Φ^+ in each of the b-quark propagators (W^+t and Φ^+t). It is easy to see that diagrams with the exchange of W^+ do not produce terms proportional to m_t^2. The point is that in the limit $m_t^2 \gg m_W^2$ we can set the mass of the W boson in its propagator equal to zero—this results in no power-law infrared divergences. The integral for the vertex part is dimensionless, so no term proportional to m_t^2 can arise. This argument is also valid for the proper energy of the b quark; the necessary first power of momentum is supplied by the incident momentum of the b quark.

At first sight these arguments are also valid for the diagrams with the exchange of the Φ^+; in them we can also neglect the "mass" of this particle in its propagators, and the integrals over momentum do not give rise to terms proportional to m_t^2. However in this case the factor proportional to m_t^2 arises from the vertices $\bar{t}_R b_L \Phi^+$ and $\bar{b}_L t_R \Phi^+$, proportional to m_t/v, where v is the vacuum expectation value of the Higgs field.

It is thus seen that the contribution to τ comes not from the gauge but from the Higgs sector of the electro-weak theory. This circumstance was recently used in the remarkable paper, Ref. 31.

The interaction of the Z boson with the quarks is described by the sum of the vector and axial currents. In view of conservation of the vector current there are no corrections due to it in the amplitude $Z\to b\bar{b}$, and it is sufficient to consider the bare axial coupling. Since the coupling $Z\Phi + \partial_\mu\Phi$ is vector-like, the corresponding vertex diagram $\Phi^+\Phi^+t$ can be omitted.

We start the calculations with the vertex diagram $tt\Phi^+$. Call its contribution A_a. The axial vertex $Z\bar{t}t$ has the form

$$\frac{1}{2}\frac{f_0}{2} Z_\alpha \bar{t}\gamma_\alpha\gamma_5 t. \quad \text{(B3)}$$

The Yukawa coupling $\Phi^+\bar{t}b$ is

$$\sqrt{2}\frac{m_t}{v}\bar{t}\left(\frac{1+\gamma_5}{2}\right)b\Phi^+. \quad \text{(B4)}$$

Hence

$$A_a = 2i\left(\frac{m_t}{v}\right)^2 \frac{f_0}{4}\int \frac{d^nk}{(2\pi)^n k^2}\bar{b}\frac{1-\gamma_5}{2} \times \frac{1}{\hat{k}-m_t}\gamma_\alpha\gamma_5\frac{1}{\hat{k}-m_t}\frac{1+\gamma_5}{2}b$$

$$= i\frac{f_0}{2}\left(\frac{m_t}{v}\right)^2\int\frac{d^nk}{(2\pi)^n k^2(k^2-m_t^2)^2}\bar{b}(-\hat{k}\gamma_\alpha\hat{k} + m_t^2\gamma_\alpha)\frac{1+\gamma_5}{2}b$$

$$= i\frac{f_0}{2}\left(\frac{m_t}{v}\right)^2\int\frac{d^nk[(1-2/n)k^2+m_t^2]}{(2\pi)^n k^2(k^2-m_t^2)^2}\bar{b}\gamma_\alpha\frac{1+\gamma_5}{2}b$$

$$= i\frac{f_0}{2}\left(\frac{m_t}{v}\right)^2\frac{1}{(2\pi)^n}\bar{b}\gamma_\alpha\frac{1+\gamma_5}{2}b\left[\left(1-\frac{2}{n}\right)I + m_t^2 J\right], \quad \text{(B5)}$$

where

$$I = \int\frac{d^nk}{(k^2-m_t^2)^2} = \int\frac{id^nk}{(k^2+m_t^2)^2} = i\frac{\pi^{n/2}}{\Gamma(n/2)}\frac{\Gamma(n/2)}{\Gamma(2)}\Gamma\left(2-\frac{n}{2}\right)(m_t^2)^{n/2-2} = i\pi^{n/2}\Gamma(2-n/2)(m_t^2)^{n/2-2}, \quad \text{(B6)}$$

$$J = \int\frac{d^nk}{k^2(k^2-m_t^2)^2} = -i\int\frac{d^nk}{k^2(k^2+m_t^2)^2} = -i\frac{\pi^{n/2}}{\Gamma(n/2)}\Gamma\left(\frac{n}{2}-1\right)\Gamma\left(3-\frac{n}{2}\right)(m_t^2)^{n/2-3} = -i\pi^{n/2}(m_t^2)^{-1}. \quad \text{(B7)}$$

For the amplitude A_a we finally obtain

$$A_a = i\frac{f_0}{2}\left(\frac{m_t}{v}\right)^2\frac{i}{(4\pi)^{n/2}}\bar{b}\gamma_\alpha\frac{1+\gamma_5}{2}b \times\left[\frac{n-2}{n}\Gamma\left(2-\frac{n}{2}\right)(m_t^2)^{n/2-2} - 1\right]. \quad \text{(B8)}$$

We pass now to the calculation of the amplitude A_b, which takes into account the contribution of the tW diagrams to the b-quark propagators. The axial vertex $Z\bar{b}b$ has the form $(-\frac{1}{2})(f_0/2)Z_\alpha\bar{b}\gamma_\alpha\gamma_5 b$, and a factor $\frac{1}{2}$ should be included in the insertion in the external line, which is balanced by the fact that there are two external propagators:

$$A_b = 2\cdot\frac{1}{2}\cdot 2\left(\frac{m_t}{v}\right)^2 i\left(-\frac{f_0}{4}\right)\bar{b}\frac{1-\gamma_5}{2}\int\frac{d^nk}{(2\pi)^n(k-p)^2}\frac{1}{\hat{k}-m_t} \times\frac{1+\gamma_5}{2}\frac{1}{\hat{p}}\gamma_\alpha b$$

$$= -i\frac{f_0}{4}\left(\frac{m_t}{v}\right)^2\bar{b}\int\frac{d^nk k_\mu}{(2\pi)^n(k-p)^2(k^2-m_t^2)}\gamma_\mu\frac{1}{\hat{p}}\gamma_\alpha\frac{1+\gamma_5}{2}b. \quad \text{(B9)}$$

Combining the denominators,

$$\frac{1}{(k-p)^2(k^2-m_t^2)} = \int_0^1 \frac{dx}{[k^2-2xkp-m_t^2(1-x)]^2}$$
$$= \int_0^1 \frac{dx}{[(k-px)^2-m_t^2(1-x)]^2}, \qquad \text{(B10)}$$

and shifting the variable of integration $k \to k + px$, we obtain

$$\int \frac{d^n k k_\mu}{(k-p)^2(k^2-m_t^2)} = p_\mu \int \frac{x dx d^n k}{[k^2-m_t^2(1-x)]^2}$$
$$= p_\mu \int_0^1 x dx i\pi^{n/2} \Gamma\left(2-\frac{n}{2}\right)[m_t^2(1-x)]^{n/2-2}. \qquad \text{(B11)}$$

Introducing the last formula into A_b we obtain

$$A_b = 2\frac{f_0}{4}\left(\frac{m_t}{v}\right)^2 \bar{b}\gamma_\alpha \frac{1+\gamma_5}{2} b \frac{1}{(4\pi)^{n/2}}$$
$$\times \int_0^1 x dx \Gamma\left(2-\frac{n}{2}\right)[m_t^2(1-x)]^{n/2-2}. \qquad \text{(B12)}$$

We expand the expression in the square brackets in $n/2-2$ and keep just the linear term: $[m_t^2(1-x)]^{n/2-2} = 1 + (n/2-2)\ln[(1-x)m_t^2]$. Carrying out the integration over x we obtain

$$A_b = 2\frac{f_0}{4}\left(\frac{m_t}{v}\right)^2 \frac{1}{(4\pi)^{n/2}} \bar{b}\gamma_\alpha \frac{1+\gamma_5}{2} b\Gamma\left(2-\frac{n}{2}\right)$$
$$\times\left[\frac{1}{2} - \frac{3}{4}\left(\frac{n}{2}-2\right) + \frac{1}{2}\left(\frac{n}{2}-2\right)\ln m_t^2\right]. \qquad \text{(B13)}$$

Collecting all terms we obtain

$$A_a + A_b = \frac{f_0}{2}\left(\frac{m_t}{v}\right)^2 \frac{1}{(4\pi)^{n/2}} \bar{b}\gamma_\alpha \frac{1+\gamma_5}{2}$$
$$\times b\left\{1 - \frac{n-2}{n}\Gamma\left(2-\frac{n}{2}\right)\right.$$
$$- \frac{n-2}{n}\left(\frac{n}{2}-2\right)\Gamma\left(2-\frac{n}{2}\right)\ln m_t^2$$
$$+ \frac{1}{2}\Gamma\left(2-\frac{n}{2}\right) - \frac{3}{4}\left(\frac{n}{2}-2\right)$$
$$\left.\times\Gamma\left(2-\frac{n}{2}\right) + \frac{1}{2}\left(\frac{n}{2}-2\right)\Gamma\left(2-\frac{n}{2}\right)\ln m_t^2\right\}$$
$$= \frac{f_0}{2}\left(\frac{m_t}{v}\right)^2 \frac{1}{(4\pi)^{n/2}} \bar{b}\gamma_\alpha \frac{1+\gamma_5}{2} b\left\{1 + \left(2-\frac{n}{2}\right)\Gamma\left(2-\frac{n}{2}\right)\right.$$
$$\left.\times\left[\frac{3}{4} + \frac{4-2n}{n(4-n)} + \frac{n}{n(4-n)} + \left(\frac{n-2}{n}-\frac{1}{2}\right)\ln m_t^2\right]\right\}$$
$$= \frac{f_0}{2}\left(\frac{m_t}{v}\right)^2 \frac{1}{(4\pi)^{n/2}} \bar{b}\gamma_\alpha \frac{1+\gamma_5}{2} b$$
$$\times\left\{1 + \Gamma\left(3-\frac{n}{2}\right)\left[\frac{3}{4} + \frac{1}{n} + \frac{n-4}{2n}\ln m_t^2\right]\right\}. \qquad \text{(B14)}$$

In the last expression we can set $n = 4$:

$$A_a + A_b = \frac{f_0}{16\pi^2}\left(\frac{m_t}{v}\right)^2 \bar{b}\gamma_\alpha \frac{1+\gamma_5}{2} b = \frac{f_0 \bar{\alpha} t}{16\pi c^2 s^2} \bar{b}\gamma_\alpha \frac{1+\gamma_5}{2} b. \qquad \text{(B15)}$$

Returning to formula (B2) we finally obtain

$$\tau = -\frac{\bar{\alpha} t}{8\pi c^2 s^2}. \qquad \text{(B16)}$$

APPENDIX C

DISTANCES SELECTED BY THE WEAK INTERACTIONS

In calculating radiative corrections one often encounters in finite formulas logarithms of ratios of the characteristic momenta at which the processes in question take place. A convenient method for taking into account such large corrections is to already normalize the charges in the tree formula at the correct point, namely, the point at which the interaction takes place. Some of the parameters of the electro-weak theory (m_W, m_Z) are evidently defined at momenta $q^2 \sim m_{W,Z}^2$ (or at distances $r \sim 1/m_{W,Z}$), while the fine structure constant α is defined at larger distances $\sim 1/m_e$. To avoid large corrections $\sim \alpha \ln(m_{W,Z}/m_e)$ we go over to $\bar{\alpha} = \alpha(m_Z^2)$. Then in radiative corrections to m_W and in parameters characterizing the Z decay we do not encounter large logarithms $\sim \alpha \ln(m_{W,Z}/m_e)$. But what is the situation with corrections to weak processes at low energies—muon decay, $\nu_\mu e$ and $\nu_\mu N$ scattering, etc.? At first glance these processes are characterized by distances $\sim 1/m_\mu^2$, $1/q^2$ and large factors $\ln(m_{W,Z}^2/m_\mu^2, q^2)$ should arise in the radiative corrections. If indeed such a logarithm were present in the muon decay, then in our procedure involving the constant G_μ it would "leak through" into the expressions for m_W and the vector and axial constants of the Z-boson decay. But in fact there are no such logarithms—the weak interactions at low (compared to $m_{W,Z}$) energies proceed at short distances $\sim 1/m_{W,Z}$. We illustrate the latter assertion for the case of muon decay. The source of a large logarithm could be the insertions in the W-boson propagator [insertions into the vertices and propagators of the external fermions cancel out to within algebraically suppressed terms $\sim \alpha(m_\mu^2/m_W^2)\ln(m_W^2/m_\mu^2)$]. For example, the loop $(e\nu_e)$ indeed contains the infrared logarithm $\sim \alpha \ln(\Lambda^2/m_\mu^2)$ (it does not "reach" m_e, but is cut off at $q^2 \sim m_\mu^2$), which modifies the W-boson propagator in the following manner: $(G_W)^{-1} = q^2 - m_W^2 - \alpha q^2 \ln(\Lambda^2/m_\mu^2)$. But since $m_W^2 \gg q^2 \sim m_\mu^2$, this logarithm gives rise to no noticeable effects. The corrections to the W propagator (and consequently to the effective coupling constant g) are negligibly small. This example illustrates the "freezing" of the effective coupling constants of the heavy vector bosons at their masses—the light fermions do not give rise to logarithmic evolution at momentum transfers less than m_W. In the case of a massless vector particle—the photon—the evolution proceeds all the way down to the mass of the lightest charged particle—the electron.

[1)] In an earlier paper[3] a somewhat different value of $\overline{\alpha}$ was used: $\overline{\alpha} = 1/128.78(12)$.

[2)] In a previous edition of the Tables[5] a slightly different value of G_μ was used: $G_\mu = 1.16637(2)$. This change is too small to be significant.

[3)] This value of m_W gives $m_W/m_Z = 0.8797(29)$, which differs slightly from the value $M_W/m_Z = 0.8790(30)$ which we used in Ref. 6.

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