Optical hysteresis, switchover, and self-pulsation of excitons and biexcitons in condensed media

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(Submitted 27 July 1992)
Zh. Eksp. Teor. Fiz. **103**, 994–1009 (March 1993)

A theory is developed of stationary and nonstationary optical bi- and multistability, optical switchovers, and self-pulsations in resonant excitation of coherent excitons and biexcitons in condensed media.

Much attention is being paid of late to the study of cooperative processes in the exciton region of the spectrum. Ivanov, Keldysh, and Tikhodeev[1-3] were the first to treat self-consistently condensed modes of excitons and photons, jointly with kinetic equations describing scattering of excitons by phonons. The onset of an energy spectrum of the phonoriton type and of stimulated Brillouin scattering was predicted. Mis'ko *et al.*[4] studied the statistical properties and the stimulated Bose–Einstein condensation of coherently excite polaritons in crystallites of varying dimensionalities. Self-consistent stationary solutions were obtained of the Fokker–Planck equation for coherent polaritons and of the kinetic equations for the scattered quasiparticles. It was shown that the non-equilibrium distribution function of the latter has a real threshold of stimulated Raman scattering, and coherent polaritons possess bunching, antibunching, and squeezed-state properties.

In Refs. 5–10, using generalized Keldysh equations that describe coherent excitons and photons that are weakly inhomogeneous in space and in time, a theory was constructed of optical bistability in the exciton region of the spectrum, and optical switchover between optical-bistability modes was investigated. It was shown, for the first time ever, that regular and stochastic self-pulsations can appear on the long-wave edge of the proper absorption of a crystal, the possibility was predicted of observing spatial turbulence in a system of coherent excitons and photons. Note that optical bistability (OB) in resonant excitation of excitons was first observed in experiment by Anderson and coworkers.[11]

OB has by now become the subject of many theoretical and experimental investigations and is in fact an independent branch of nonlinear optics. It attracts interest because it is one of the most striking examples of optical self-organization of a system, far from thermodynamic equilibrium, and offers tremendous prospects of practical application, primarily for optical information processing and for the development of ne computers with optical logic.

Since optical nonlinearities in semiconductors are particularly large in the exciton region of the spectrum, nonlinear interaction of light with matter is most strongly pronounced just in this frequency region. In addition, exciton-exciton quantum transitions are known to be characterized by giant oscillator strengths.[12-13] Recognizing furthermore that the characteristic relaxation times of excitons and biexcitons are very short ($\tau = 10^{-10}$–10^{-12} s), it becomes obvious that they will play a decisive role in the design of optoelectronic instruments, where ultrafast switchover is necessary. Owing to these properties, optical self-organization of excitons and biexcitons is extensively investigated. A theory of stationary and nonstationary lasing and of self-organization of regular and random time structures was developed in Refs. 14–22 for exciton–biexciton transitions with one- and two-photon quantum transitions. The onset of ultrashort dynamic chaos of coherent excitons, photons, and biexcitons in solids was predicted in Ref. 23.

The present paper is devoted to a theoretical study of stationary and nonstationary optical bi- and multistability, and also optical switchover and self-pulsation in resonant excitation of excitons and biexcitons in condensed media. We consider the case when photons from one and the same pulse can excite excitons from the ground state of a crystal and convert them into biexcitons. The giant oscillator strength inherent in optical generation of excitons and biexcitons contributes to the onset of nonlinear effects in this frequency region even at modern levels of optical excitation.

HAMILTONIAN OF THE PROBLEM AND DYNAMIC EQUATIONS

We use for simplicity a three-level model in which the energies of the ground state transitions of a crystal to an exciton and of an exciton to a biexciton differ by an amount equal to the biexciton binding energy. We consider only one macro-filled mode of coherent excitons, photons, and biexcitons, each of which is characterized by a definite wave vector.

The Hamiltonian of the problem is the sum of the Hamiltonians of the free excitons, biexcitons, and the field and of the Hamiltonian of the interaction between the field and a system of coherent excitons and biexcitons, which takes in the chosen model the form

$$H_{int} = -\hbar g(E^- a + a^+ E^+) - \hbar G g(a^+ b E^- + E^+ b^+ a), \quad (1)$$

where a^+ (b^+) is the operator of exciton (biexciton) creation, g is the exciton-biexciton interaction constant, G is the constant of optical conversion of an exciton into a biexciton, and E^+ (E^-) is the positive- (negative-) frequency component of the electric field of the electromagnetic wave. The energy scheme of the investigated process is shown in Fig. 1.

The Heisenberg equations of motion for the amplitudes of the excitons and biexcitons are of the form

$$i\dot{a} = \omega_{ex} a - i\gamma_{ex} a - gE^+ - GgbE^-, \quad (2)$$

$$i\dot{b} = \omega_{biex} b - i\gamma_{biex} b - GgE^+ a, \quad (3)$$

where γ_{ex} and γ_{biex} are the damping constants of the excitons and biexcitons, respectively and determine the rate of depar-

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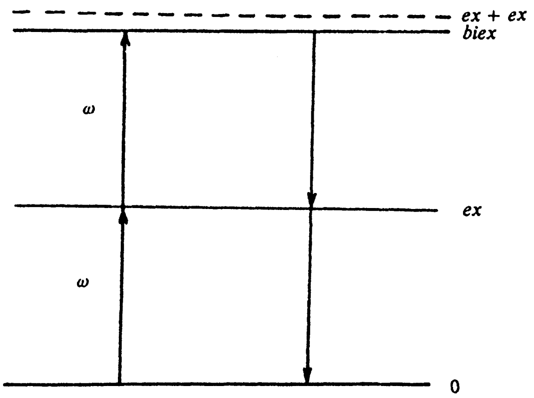

FIG. 1. Energy and quantum-transition schemes of OB theory with allowance for exciton–photon interaction and for optical conversion of excitons into biexcitons: O—ground state of the crystal, ex and biex—energy levels of exciton and biexciton states, ω—electromagnetic-field frequency.

ture of the quasiparticles from coherent into incoherent modes; they were introduced into the equations of motion phenomenologically. Note that these equations can be derived rigorously in the framework of quantum theory of fluctuations and dampings from the flux part of the corresponding Fokker–Planck equation.[20]

The equation of motion for the positive-frequency component of the electromagnetic field E^+ is equivalent to the wave equation

$$c^2\frac{\partial^2 E^+}{\partial z^2}-\frac{\partial^2 E^+}{\partial t^2}=4\pi\hbar g\frac{\partial^2}{\partial t^2}(a+Ga^+b). \tag{4}$$

We write the solution of Eqs. (2)–(4) in the form of a product of slowly varying envelopes and rapidly oscillating components:

$$\begin{aligned} a&=\tilde{A}(z,t)\exp(ikz-i\omega t),\\ b&=\tilde{B}(z,t)\exp(2ikz-2i\omega t),\\ E^+&=\tilde{e}^+(z,t)\exp(ikz-i\omega t), \end{aligned} \tag{5}$$

where ω is the frequency of the electromagnetic wave. We continue the analysis in the approximation of slowly varying envelopes, which are valid under the condition

$$\left|\frac{\partial\tilde{e}^+}{\partial t}\right|\ll\omega|\tilde{e}^+|,\qquad\left|\frac{\partial\tilde{e}^+}{\partial z}\right|\ll k|\tilde{e}^+|. \tag{6}$$

This means that the $A(z,t)$, $B(z,t)$, and $\tilde{e}^+(z,t)$ envelopes are sufficiently smooth functions compared with the rapidly oscillating part. They vary little over the wavelength and during one period of the light incident on the crystal. Substituting (5) in (2)–(4) in the approximation of slowly varying amplitudes, and neglecting effects of spatial dispersion of excitons and biexcitons, which are negligible in the vital region of the spectrum, we obtain the following truncated equations:

$$\frac{\partial\tilde{A}}{\partial t}=i(\omega-\omega_{ex})\tilde{A}-\gamma_{ex}A+ig\tilde{e}^++iGg\tilde{B}\tilde{e}^-, \tag{7}$$

$$\frac{\partial\tilde{B}}{\partial t}=i(2\omega-\omega_{biex})-\gamma_{biex}B+iGg\tilde{e}^+\tilde{A}, \tag{8}$$

$$2ikc^2\frac{\partial\tilde{e}^+}{\partial z}+(\omega^2-c^2k^2)\tilde{e}^++2i\omega\frac{\partial\tilde{e}^+}{\partial t}=4\pi\hbar\omega^2g(A+GA^+B). \tag{9}$$

It is convenient next to change to dimensionless quantities. We introduce

$$\begin{aligned} &X^+=\frac{\tilde{e}^+\sqrt{\gamma_{ex}\gamma_{biex}}}{gG},\quad A=\sqrt{\frac{\gamma_{biex}}{\gamma_{ex}}}\frac{\tilde{A}}{G},\quad B=\tilde{B}\frac{1}{G},\\ &C=\frac{\alpha L}{4T},\quad \alpha=\frac{4\pi\hbar g^2\omega^2}{c^2k\gamma_{ex}},\quad d=\frac{\gamma_{ex}}{\gamma_{biex}},\\ &\delta_0=\frac{2\omega_{ex}-\omega_{biex}}{\gamma_{biex}},\quad \Delta=\frac{\omega^2-c^2k^2}{2\omega\gamma_{biex}},\\ &\sigma=\frac{c^2kT}{L\gamma_{biex}\omega},\quad \delta=\frac{\omega-\omega_{ex}}{\gamma_{biex}},\quad \tau=\gamma_{biex}t, \end{aligned} \tag{10}$$

where T and L are the cavity transmission coefficients and length, and will be introduced below. With (10) taken into account, the system (7)–(9) takes the form

$$\frac{\partial X^+}{\partial\tau}=i\Delta X^+-\frac{\sigma L}{T}\frac{\partial X^+}{\partial z}+2C\sigma i(A+A^+B), \tag{11}$$

$$\frac{\partial A}{\partial\tau}=i\delta A-dA+idX^++idBX^-, \tag{12}$$

$$\frac{\partial B}{\partial\tau}=i(2\delta+\delta_0)B-B+iAX^+. \tag{13}$$

In the general case X^+, A and B are complex. Introducing

$$A_1=\mathrm{Re}\,A,\ A_2=\mathrm{Im}\,A,\ X_1=\mathrm{Re}\,X^+,$$
$$X_2=\mathrm{Im}\,X^+,\ B_1=\mathrm{Re}\,B,\ B_2=\mathrm{Im}\,B$$

we obtain from (11)–(13)

$$\frac{\partial X_1}{\partial\tau}=-\Delta X_2-\frac{\sigma L}{T}\frac{\partial X_1}{\partial z}-2C\sigma(A_2+A_1B_2-A_2B_1), \tag{14}$$

$$\frac{\partial X_2}{\partial\tau}=\Delta X_1-\frac{\sigma L}{T}\frac{\partial X_2}{\partial z}+2C\sigma(A_1+A_1B_1+A_2B_2), \tag{15}$$

$$\frac{\partial A_1}{\partial\tau}=-\delta A_2+\alpha(A_1+X_2B_1-X_1B_2-X_2), \tag{16}$$

$$\frac{\partial A_2}{\partial\tau}=\delta A_1+\alpha(-A_2+X_1B_1+X_2B_2+X_1), \tag{17}$$

$$\frac{\partial B_1}{\partial\tau}=-(2\delta+\delta_0)B_2-B_1-X_2A_1-X_1A_2, \tag{18}$$

$$\frac{\partial B_2}{\partial\tau}=(2\delta+\delta_0)B_1-B_2+X_1A_1-X_2A_2. \tag{19}$$

The system (14)–(19) of nonlinear differential equations describes the evolution of the coherent excitons, biexcitons, and photons in condensed media in the approximation of smooth envelopes, and is the basis of the analysis that follows.

It is impossible to obtain exact analytic solutions of a system of nonlinear partial differential equations. The main feature of the nonlinear passage of light can be revealed in the mean-field model which is widely used in the theory of optical bistability. It corresponds mathematically to replacement of $\int E(z)dz$ by $(E(L)-E(0))L$. In this approximation Eqs. (14) and (15) can be integrated over the coordinate:

$$\frac{\partial X_1}{\partial \tau} = -\Delta X_2 - \frac{\sigma}{T}[X_1 - X_1(0)]$$
$$- 2C\sigma(A_2 + A_1B_2 - A_2B_1), \tag{20}$$
$$\frac{\partial X_2}{\partial \tau} = \Delta X_1 - \frac{\sigma}{T}[X_2 - X_2(0)]$$
$$+ 2C\sigma(A_1 + A_1B_1 - A_2B_2). \tag{21}$$

They describe, together with Eqs. (16)–(19), the dynamic evolution of coherent excitons, photons, and biexcitons in the mean-field approximation.

Since a feature of the present state of research into OB, optical switchover, and optical pulsations is that it is restricted to specific optical instruments with specific experimental geometry, we study here these phenomena in a system of coherent excitons and biexcitons with ring-cavity geometry.

OPTICAL HYSTERESIS AND SWITCHOVER

Let a sample of length L be placed between the entry and exit cavity mirrors having a transmission coefficient T. The two other mirrors are assumed to be ideally reflecting (see Fig. 2). The boundary conditions for the ring cavity are

$$(1 - R)^{1/2}E_I + RE(L, t - \Delta t) = E(0, t),$$
$$E_T = (1 - R)^{1/2}E(L, t), \tag{22}$$

where E_I is the pump amplitude, E_T the field amplitude at the exit from the cavity, $R = 1 - T$ the reflection coefficient of mirrors 1 and 2, $\Delta t = (2l + L)/c_0$, and c_0 the speed of light in vacuum. Introducing dimensionless input and output field amplitudes, we obtain for the normalized amplitudes the boundary conditions

$$TY + R[X_1(L, t - \Delta t)\cos F$$
$$- X_2(L, t - \Delta t)\sin F] = X_1(0, t), \tag{23}$$
$$R[X_1(L, t - \Delta t)\sin F + X_2(L, t - \Delta t)\cos F] = X_2(0, t),$$

where

$$E_I = Y\frac{(\gamma_{ex}\gamma_{biex})^{1/2}}{Gg}T^{1/2}, \quad E_T = X\frac{(\gamma_{ex}\gamma_{biex})^{1/2}}{Gg}T^{1/2},$$

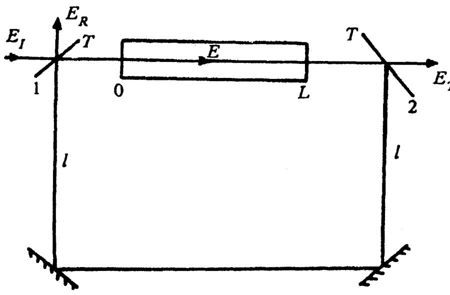

FIG. 2. Diagram of ring cavity E_I, E_R and E_T—amplitudes of incident, reflected, and transmitted fields, respectively.

$F = kL + k_0(2l + L)$ is the field phase advance in a ring cavity, and k_0 is the wave vector in vacuum.

Equations (16)–(21) are nonlinear differential equations describing open resonant systems. The equation of state of the theory of stationary OB in a system of coherent excitons and biexcitons can be easily obtained from (16)–(21) by equating the corresponding derivatives to zero.

Taking the boundary conditions (23) into account, the intensity of the incident field on the intensity of the field emerging from the cavity is given by

$$Y = X(P_1^2 + P_2^2)^{1/2}, \tag{24}$$

where

$$P_1 = \frac{1 - R\cos F}{T} + \frac{2C(1 + 2X^2 + (2\delta + \delta_0)^2)}{(1 + (\delta/d)(2\delta + \delta_0) + X^2)^2 + (\delta/d + 2\delta + \delta_0)^2},$$

$$P_2 = \frac{\Delta}{\sigma} + \frac{R}{T}\sin F + \frac{2C[(2\delta + \delta_0)X^2 - \delta/d - (\delta/d)(2\delta + \delta_0)^2]}{(1 + (\delta/d)(2\delta + \delta_0) + X^2)^2 + (\delta/d + 2\delta + \delta_0)^2}$$
$$- 2CX^2\{[(1 + X^2 + (2\delta + \delta_0)^2)\left(2\delta + \delta_0 + \frac{\delta}{d}\right)$$
$$+ \left(X^2 + \frac{1}{d} - \frac{\delta(2\delta + \delta_0)}{d}\right)(X^2(2\delta + \delta_0)$$
$$- \frac{\delta}{d} - \frac{\delta}{d}(2\delta + \delta_0)^2)]$$
$$\times [(1 - \frac{\delta}{d}(2\delta + \delta_0) + X^2)^2 + \frac{\delta}{d} + 2\delta + \delta_0)^2]^{-2}\}.$$

If the phase advance in the cavity is $F = 2\pi n$, where n is an integer, under exact-resonance conditions ($\omega = \omega_{ex}$, $\delta_0 = 0$, $\Delta = 0$, $ck = \omega$, $\sigma \neq 0$) the equation of state of the OB theory in a system of excitons and biexcitons is

$$Y = X\left[1 + 2C\frac{1 + 2X^2}{(1 + X^2)^2}\right] \tag{25}$$

and differs from the analogous OB equation of state on the model of two-level atoms:

$$Y = X\left[1 + \frac{2C}{1 + X^2}\right]. \tag{26}$$

The criterion for the existence of OB in the mean-field approximation and under conditions of exact resonance is the inequality $C > C_{cr} = 54/17$, whereas the criterion for the existence of OB in a system of two-level atoms is the inequality $C > 4$.

One of the important and fundamental questions of OB is an investigation of the stability of the OB curve. Note that in the case of exact resonance the stationary states of the OB curve are stable and under these conditions no self-oscillations can arise in the system. This fact can ensure reliable

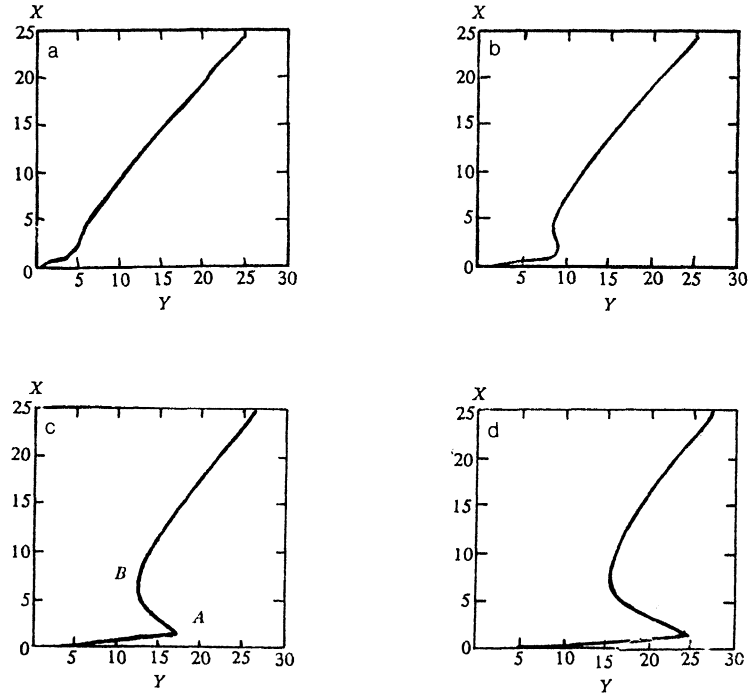

FIG. 3. Stationary OB $X(Y)$ in a ring cavity at various values of the parameters $\delta = \delta_0 = 0$, $F = 2\pi n$, $C = 2$(a), 5(b), 10(c), 5(d).

reconnections of OB modes and can be used to construct instruments for optical reduction of information and computers with optical logic.

Figure 3 shows the dependences of the amplitudes X of the emerging radiation on the amplitudes Y of the incoming radiation, for different values of the bistability parameter. Inasmuch as under exact resonance conditions both the upper and the lower parts of the OB curve are stable, it is of interest to study their switchover times. The investigation of the switchover times is the set of equations (16)–(21). At present there is no standard algorithm for the solution of general-form nonlinear differential equation, and obtaining analytic solutions of the system (16)–(21) is a difficult task. We have therefore performed a computer experiment. The initial conditions are chosen such that they correspond to the value of Y near the threshold of downward switchover. At the instant $t = 0$ we specify a jumplike change of the pump Y, such that $Y \pm \Delta Y$ are on opposite sides of the corresponding switchover threshold. Figure 4 shows switchovers from the lower OB branch to the upper, of the exciton and biexciton amplitudes, and also the projection of the trajectories in phase space on a plane with the exciton and biexciton amplitudes as the coordinate axes, for the parameter values $C = 10$, $d = 0.1$, and $\sigma = 0.1$. It is seen from the figure

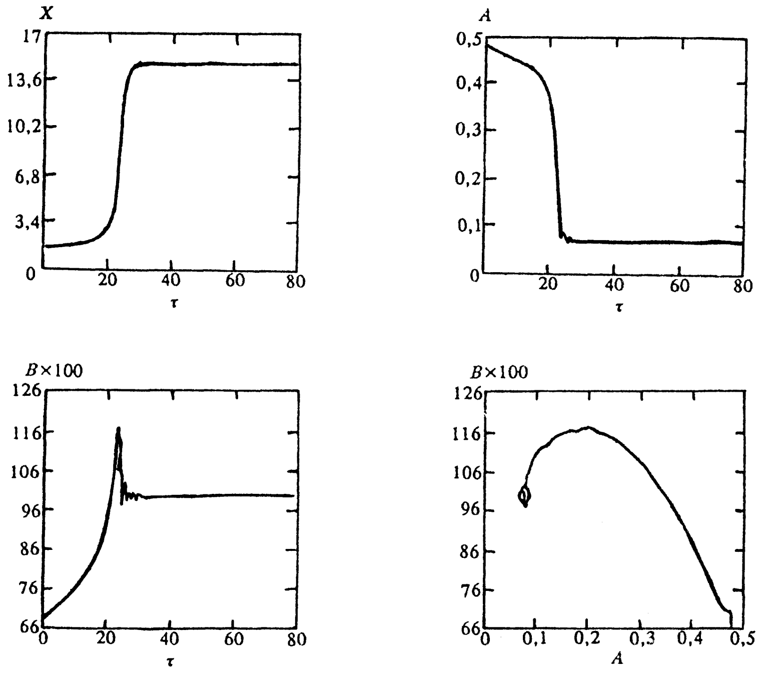

FIG. 4. Optical switchover from lower to upper branch, dynamic evolution of exciton (A) and biexciton (B) amplitudes, and phase portrait of the trajectory at $\sigma = 0.1$, $d = 0.1$, $C = 10$.

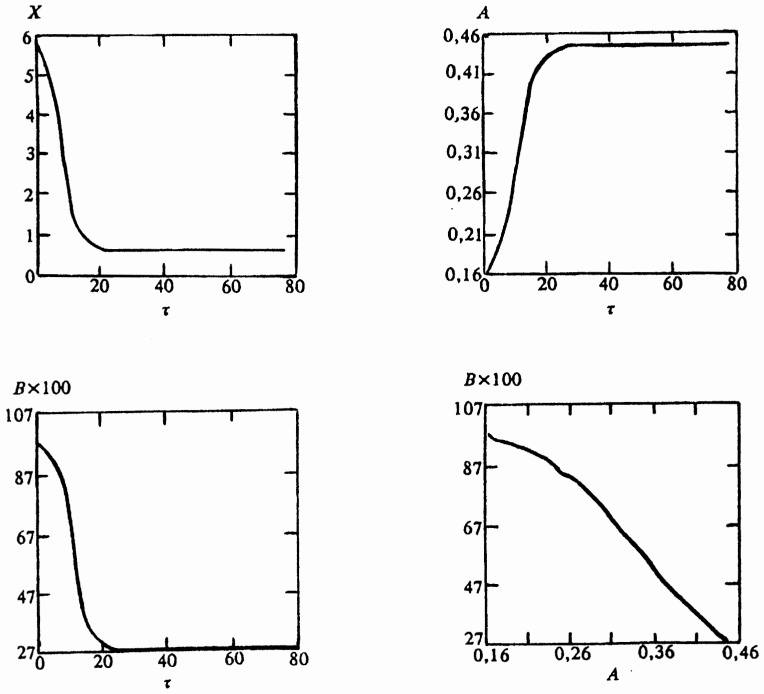

FIG. 5. Optical switchovers from a lower to an upper branch at $\sigma = 0.1$, $d = 0.1$, $C = 10$.

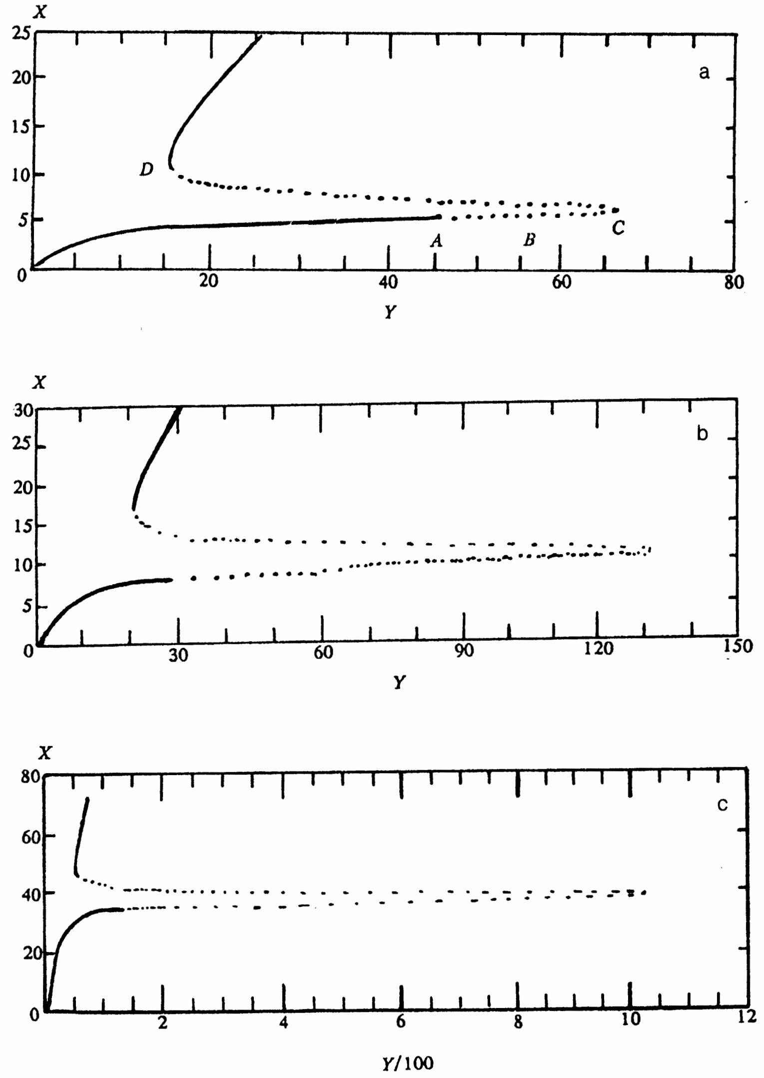

FIG. 6. Stationary OB $X(Y)$ in a ring cavity at different values of the parameters $C = 5$, $\delta_0 = 2$, $F = 2\pi n$, $\delta = 2$(a), 5(b), 10(c).

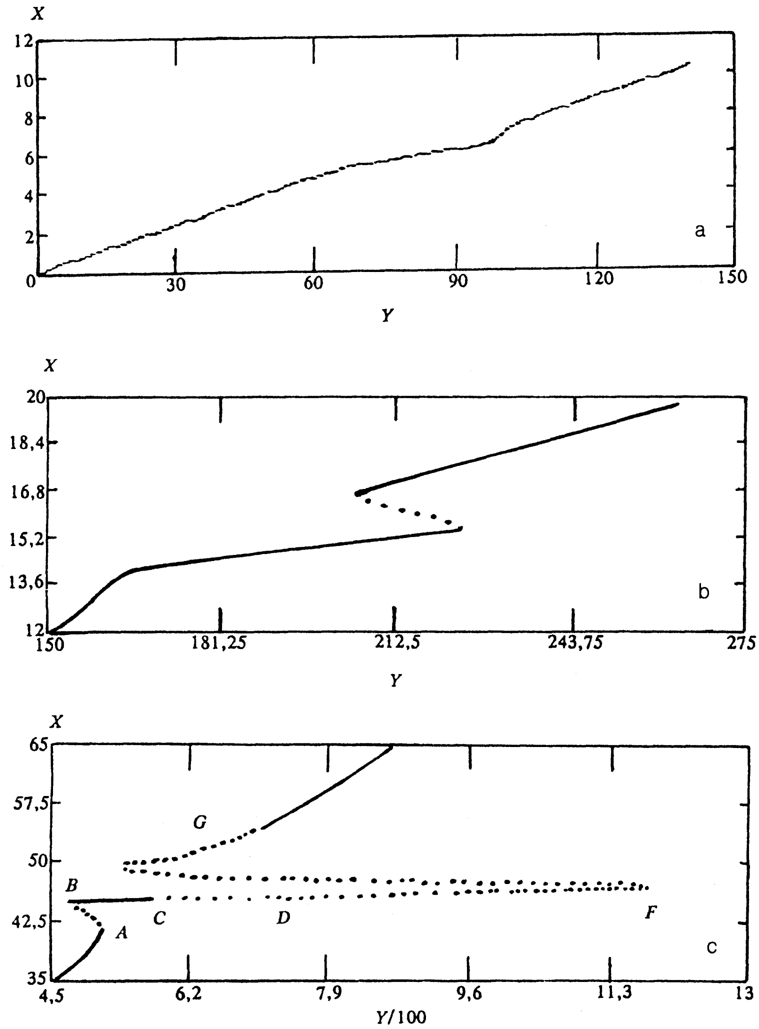

FIG. 7. Stationary OB $X(Y)$ in a ring cavity at different values of the parameters $C=5$, $\delta_0=2$, $F=\pi/2+2\pi n$, $\delta=2$(a), 5(b), 10(c).

that the system goes over from the lower branch of curve B to the upper within a time on the order of $t=(20\text{–}30)\gamma_{\text{biex}}^{-1}$. The worsening of the cavity Q shortens the switchover time, but self-oscillations with finite lifetimes appear in this case. Figure 5 shows the optical switchovers and the corresponding phase portrait in the case of a transition from an upper OB mode to a lower at the same parameter values. In contrast to two-level systems, where the downward and upward switchover times differ substantially in the case of OB in a system of coherent excitons and biexcitons, as seen from the figure, they turn out to be of the same order. Since the exciton and biexciton relaxation times in semiconductors are $(\tau_{\text{ex}}\sim\tau_{\text{biex}}\sim 10^{-11}-10^{-12}\,\text{s})$, the optical switchovers in a system of coherent excitons and biexcitons are of the order of picoseconds.

When the electromagnetic field frequency is not equal to the exciton and biexciton transition frequencies, the equation of state of the OB theory is described by Eq. (24), which differs substantially from the equation of state of two-level atoms. Note that the nonzero deviation from resonance alters in principle the character of the nonlinear passage of the light through the cavity in the case of coherent excitation of excitons and biexcitons. Thus, for example, increasing the detuning from resonance leads to a more pronounced onset of OB, which differs from the OB in a system of two-level atoms, where the increase of δ contributes to poorer observation of the OB. Figure 6 shows plots of the dependence for $d=0.1$, $\delta_0=2$, $C=5$, and different values of the detuning from resonance. It is evident that the OB becomes more and more pronounced with increase of δ_0. Moreover, for $F=\pi/2+2\pi n$, $C=5$ and certain values of δ optical instability can be observed in a system of coherent excitons and biexcitons (Fig. 7). An investigation of the stability of the stationary states of the optical hysteresis to small perturbations is determined by the characteristic equation for the Jacobian of the system

$$|J-\lambda E| = \begin{vmatrix} -\lambda-Q_1, & -Q_2, & -2C\sigma B_2, & 2C\sigma(B_1-1), & 2C\sigma B_2, & -2C\sigma A_1, \\ Q_2, & -\lambda-Q_1, & 2C\sigma(A_1+1), & 2C\sigma B_2, & 2C\sigma A_1, & 2C\sigma A_2, \\ -dB_2, & d(B_2-1), & -\lambda-d, & \delta, & dX_2, & -dX_1, \\ d(1+B_1), & dB_2, & \delta, & -\lambda-d, & dX_1, & dX_2, \\ A_2, & A_1, & X_2, & X_1, & \lambda-d, & -2\delta-\delta_0, \\ A_1, & -A_2, & X_1, & X_2, & 2\delta+\delta_0, & -\lambda-1 \end{vmatrix} \quad (27)$$

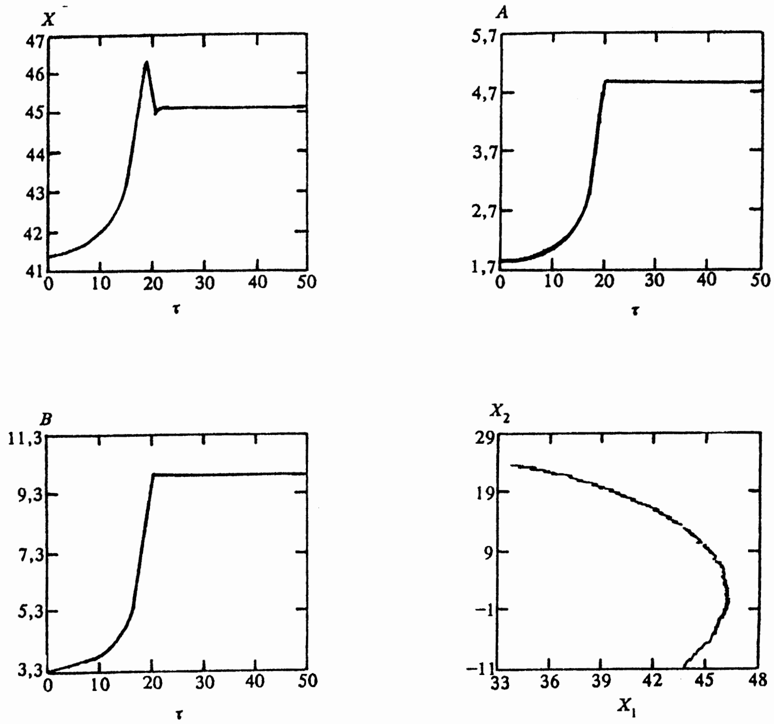

FIG. 8. Optical switchovers from a lower to an upper first loop of optical hysteresis at points A and B of Fig. 7.

where $Q_1 = \sigma/T - (\sigma R/T)\cos F, Q_2 = \Delta + (\sigma R/T)\sin F$, and E is a unit matrix. If the real parts of all the roots of the characteristic equation are negative, the corresponding stationary states are stable to small perturbations. Using the Routh–Hurwitz criterion, we investigated the stability of stationary states for various values of the parameters. The result was that a nonzero detuning from resonance not only contributes to a better manifestation of OB and to the onset of optical multistability, but also causes parts of the stationary optical hysteresis curve to become unstable and produces nonlinear self-pulsations in the system. At the parameter values $C=5$, $\delta_0=2$, $\delta=2$, and $\sigma=0.1$ the upper branch of the OB curve is stable, whereas part of the lower branch becomes unstable, the instability section increasing with δ (Fig. 6). As the representative points moves to the right side of the instability window, metastable nonlinear oscillations characterized by a definite lifetime are observed in the system, after which the system jumps over to a stable

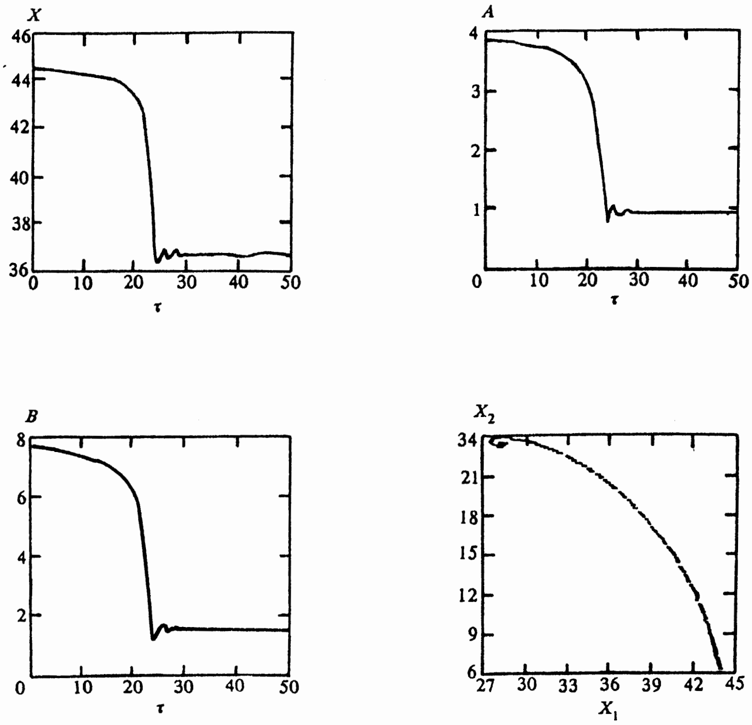

FIG. 9. Optical switchovers from an upper to a lower first loop of optical hysteresis at points A and B of Fig. 7.

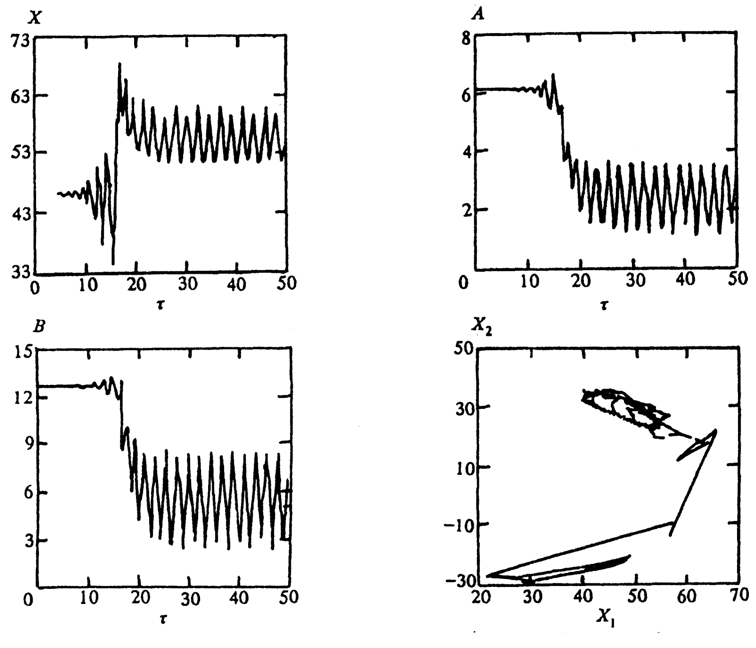

FIG. 10. Evolution of system with time at $F = \pi/2 + 2\pi n$ (interval CD of Fig. 7).

branch of the OB curve (point B). At the point C the system switches over to the upper branch of the OB curve with practically no onset of self-oscillations, and furthermore after time shorter by an order of magnitude than the transitions from the point B. The time of switchover from the upper OB branch to the lower stable part can be seen to be in this case of the same order as the time of switchover from the point C.

Figures 8–10 show the switchover dynamics if the two-loop optical hysteresis shown in Fig. 7 is realized. Figures 8 and 9 show switchovers from the lower to the upper and from the upper to the lower branches of the first optical hysteresis loop, at the points A and B, respectively, as well as the corresponding projections of the phase trajectories. It is seen from the figure that the times of the "up" and "down" transitions are of the same order, 20 τ. The CF section of the second optical-hysteresis loop is unstable, and self-oscillations appear as a result. Figure 10 shows self-pulsations in a system of coherent excitons, photons, and biexcitons; they appear on the nonstable section CD of the lower branch of the second OB branch. Evidently, self pulsations comprising complicated nonlinear self-oscillations appear in the system.

The onset of nonlinear oscillations in a system of coherent excitons, photons, and biexcitons can be used to convert

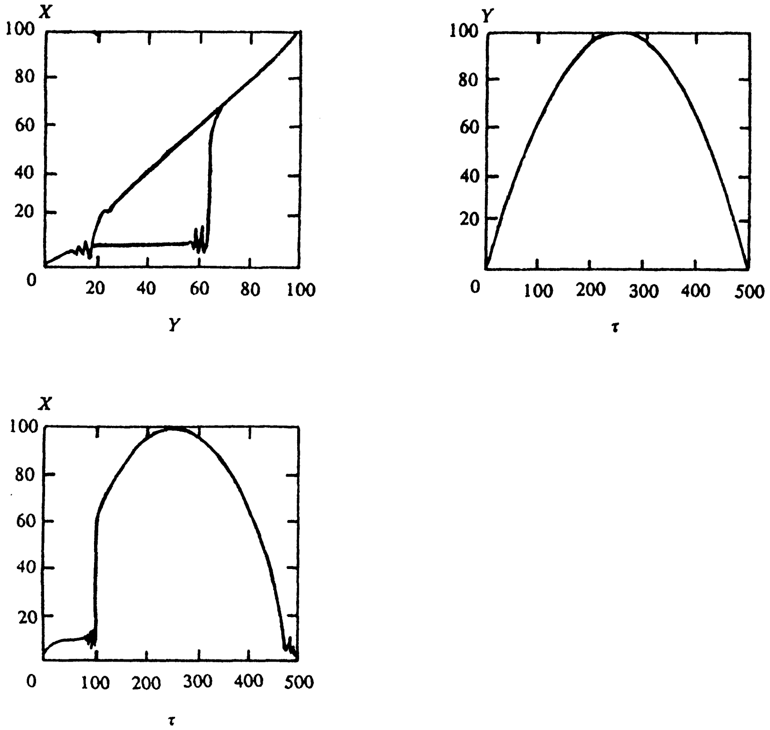

FIG. 11. Dynamic bistability from external parabolic pulse of duration $\tau = 500$, $F = 2\pi n$.

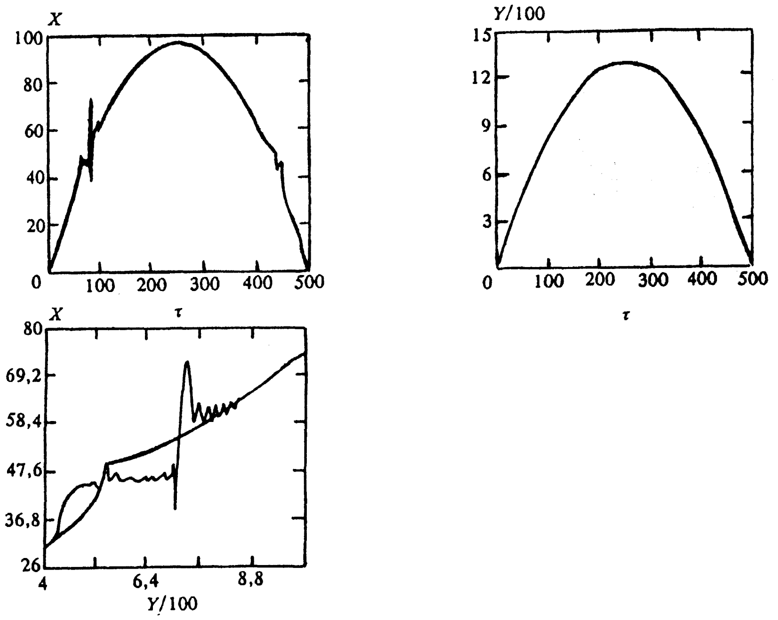

FIG. 12. Dynamic instability from external parabolic pulse, $\tau = 500$, $F = \pi/2 + 2\pi n$.

the radiation incident on the cavity into pulsations.

Experimental investigations of OB frequently reveal not static but dynamic OB resulting from comparison of the time-dependent external pumping with the corresponding response of the system.

OB of this type were first considered by Bischofsberger and Shen.[24,25] They investigated theoretically and experimentally the behavior of a nonlinear Fabry–Perot interferometer filled with a Kerr medium and acted upon by pulses of various shapes. They obtain excellent agreement of theory with experiment.

As for dynamic OB in a system of excitons, photons, and biexcitons, it remains an unresolved problem to this day. We have performed a computer experiment in which the nonlinear differential-equations system (16)–(21) describing the dynamics of coherent excitons, photons, and biexcitons is solved numerically with allowance for the boundary conditions for a ring cavity, and with the external pump $Y(\tau)$ a function of the time of barabolic form. The result of this experiment are shown in Figs. 11 and 12. At low values of the duration of the incident pulse, deformation in time is insignificant and the response is far from stationary. As τ increases, the initial pulse is deformed and the system tends to a stationary behavior (Fig. 11). Figure 12 shows the dynamic bistability as a function of the external pulse at $F = (\pi/2) + 2n\pi$. At these parameter values both the lower and the upper branches of the second hysteresis loop have sections with self-pulsations, owing to the unstabilities of the system. Note that the system of coherent excitons, photons, and biexcitons is much richer than the model two-level atoms.

We discuss in conclusion the feasibility of observing the predicted effects in experiment. The model studied by us is most valid for crystals such as CdS and CdSe, in which the exciton-binding energy is low ($I_m < 3$ meV), and if the spectral width of the radiation incident on the crystal is $\hbar\Delta\omega \sim I_m$ quantum transitions are possible both in the region of radiative recombination of the biexcitons, and transitions from the ground state of the crystal into a biexciton state. Photons from one and the same pulse create excitons from the ground state of the crystal and convert them into biexcitons. By way of example we give the numerical estimates for crystals of the CdS type, where $L = 750$ Å, $\gamma_{ex} = 10^{11}$ s^{-1}, $\gamma_{biex} = 10^{12}$ s^{-1} $G = 1.25\cdot 10^8$ cm$^{-3/2}$, $g = 5\cdot 10^{13}$ s^{-1}, $\omega_{ex} = 4\cdot 10^{15}$ s^{-2}, $c = 10$. We find then that the critical power at which the investigated nonlinear phenomena can be observed are of the order of 150 kW/cm^2, the upward and downward switchover times are $\tau_\uparrow = 20$ ps and $\tau_\downarrow = 25$ ps, respectively, and the switchover energies are of the order of ~ 50 fmJ. Our numerical estimates allow us to conclude that it is really possible to observe optical hysteresis, switchovers, and self-pulsations in a system of coherent excitons and biexcitons in condensed media.

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Translated by J. G. Adashko