Coherence effects in Thomson scattering

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The bremsstrahlung field radiated by a system of two identical charged particles as they scatter a monochromatic plane wave is examined theoretically. Explicit analytic expressions are derived for the intensity and angular distribution of this radiation as a function of the distance between the charges. The physical mechanisms responsible for coherence effects in these characteristics of the radiation field are discussed.

1. INTRODUCTION

The radiative interactions of moving charges with material media (including external fields) are the primary source of electromagnetic radiation in microwave electronics.[1–5] A coherence of the elementary radiators in corresponding generators and amplifiers of electromagnetic fields causes the radiative energy loss of intense charged-particle fluxes to be far higher (by many orders of magnitude) than the sum of the radiative energy losses of the particles making up these fluxes. This assertion, which follows directly from a comparison of the absolute values of these losses in standard microwave electronics devices, has found confirmation in a direct derivation of the spatial and temporal growth rates of several coherent radiative instabilities of electron fluxes from the spontaneous radiation fields of the individual electrons.[6–8] This assertion has also been confirmed by an analysis[1)] of the experimental data which have been accumulated on devices of the free-electron-laser type.[9]

The primary thrust in theoretical microwave electronics today is a search for ways to significantly reduce the wavelength of this intense coherent electromagnetic radiation (Refs. 11 and 12, for example). Here it is necessary to identify the conditions which must be arranged to achieve coherence among the elementary radiators in the given wavelength region. This question is most important for sources of coherent microwave radiation of the free-electron-laser type, which are expected to eventually generate coherent electromagnetic radiation at wavelengths on the order of hundreds or even tens of angstroms.[12] However, analysis of the experimental data available,[9] along with a semiphenomenological description of the collective radiative instability in the intensity of an electron beam in an undulator,[8] has shown that the degree of coherence of the radiation emitted by the electrons in an undulator falls off substantially with decreasing wavelength $\lambda_* = D/2\gamma^2$ of the electromagnetic field radiated by these electrons (D is the undulator period, and γ is the relativistic factor of the electron). Such a decrease in coherence is to be expected, since the number of elementary radiators in a given coherent bunch is being reduced.[8] Furthermore, it is clear on physical grounds that in the limit $\gamma \to \infty$ ($\lambda_* \to 0$) the total radiative power loss of an electron flux of finite density in an undulator must be equal to the sum of the intensities of the magnetobremsstrahlung (the undulator radiation) of the individual electrons making up this flux.

Under these conditions, determining the lower wavelength limit on the coherent emission of an idealized monoenergetic flux of electrons with a zero transverse divergence and a given density is extremely important for the physics and technology of intense sources of ultrashort-wave electromagnetic radiation. To estimate this limit, we consider an extremely simple model in which the degree of coherence of the electrons (the radiators) can be changed in a controllable way, and the corresponding response of various characteristics of the radiated field can be described most simply in an explicit analytic fashion.

As this basic model we consider the problem of the Thomson scattering of a monochromatic plane electromagnetic wave by two identical charged particles. To some extent, this model corresponds to the interaction of two electrons of a beam with the field of a plane undulator in the frame of reference in which the electrons are at rest, since the undulator field configuration in this frame is similar to the field configuration of a monochromatic plane wave. In addition, the electrons emit bremsstrahlung in both frames. Finally, by altering the distance between the scattering charges (thereby modeling a change in the beam density), we can change the degree of coherence and the total intensity of the Thomson scattering of the pair of charges. We can furthermore alter the directional pattern of their resultant bremsstrahlung. For these reasons, a quantitative analysis of the characteristics of the scattered field in this key problem should give us a comprehensive answer to the question of the conditions required for achieving coherence of the elementary radiators in microwave-electronics systems operating on the basis of the bremsstrahlung of moving electrons.

2. FORMULATION AND SOLUTION OF THE PROBLEM

We thus consider two identical charged particles, each having a charge q and a mass m. They are separated by a distance $d = 2a$. We assume that a monochromatic plane electromagnetic wave with a frequency ω_0 and an electric field amplitude E_0 is propagating along the z axis of the system. This axis passes through the equilibrium positions of the scattering charges on this axis: $\bar{x}_\pm = \bar{y}_\pm = 0$, $\bar{z}_\pm = \pm a$. We assume that the xz plane is parallel to the electric vector of the field of the scattered wave:

$$\mathscr{E}_x^{ext}(z,t) = E_0 \cos(\omega_0 t - k_0 z), \quad k_0 c = \omega_0.$$

Under these conditions we are to determine the functional relationships between the distance between charges and the following basic characteristics of the scattered radiation field:

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- the total intensity of the scattered radiation (or, equivalently, the scattering cross section) and
- the angular distribution of the energy flux of this radiation.

A. Radiation intensity. We find the total intensity of the bremsstrahlung of this system of scatterers as a function of the distance between scatterers by calculating the radiative energy loss of the radiating charges in the bremsstrahlung fields which are acting on these charges (in their near zones):[13]

$$I_{tot}(\theta)=-\left\langle\int d\mathbf{r}\,\mathbf{j}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)\right\rangle. \qquad (1)$$

Here $I_{\text{tot}}(\theta)$ is the intensity of the radiative loss of the system of charges ($[I]$ = watts); $\theta=k_0d$ is the distance between the scattering charges, divided by the wavelength of the incident wave, $\lambda_0=1/k_0$; $\mathbf{j}(\mathbf{r},t)$ is the total current density of the charges, which is determined by their oscillatory transverse motion in the electric field of the wave which they are scattering,

$$\mathbf{j}(\mathbf{r},t)=q\mathbf{n}_x\{v_-(t)\delta[x-\tilde{x}_-(t)]\delta(z+a)$$
$$+v_+(t)\delta[x-\tilde{x}_+(t)]\delta(z-a)\}\delta(y), \qquad (1\text{a})$$

$$\tilde{x}_\pm(t)=\alpha\cos(\omega_0t\mp k_0a),\quad v_\pm(t)=\dot{\tilde{x}}_\pm(t),$$
$$\alpha=-qE_0/m\omega_0^2, \qquad (1\text{b})$$

and $\mathbf{E}(\mathbf{r},t)$ is the electric component of the electromagnetic field radiated by the current in (1a), which is determined by the solution of Maxwell's equations stimulated by this current.

If one of the charges oscillates about its equilibrium point $\bar{x}=\bar{y}=0$, $\bar{z}=b$ in accordance with the expression $\tilde{x}(t)=\alpha\cos(\omega_0t-k_0b)$, the configuration of the field E_x which this charge produces at the frequency $\omega=\omega_0$ near the z axis is (Ref. 13, for example)

$$E_x(z,t;b)=-\frac{R_0}{\lambda_0}E_0\left\{\frac{\cos\psi(z,t)}{k_0|z-b|}[1-(k_0|z-b|)^{-2}]\right.$$
$$\left.+\frac{\sin\psi(z,t)}{(k_0|z-b|)^2}\right\},\quad \psi(z,t)=\omega_0t-k_0b-k_0|z-b|, \qquad (1\text{c})$$

where $R_o\equiv q^2/mc^2$ is the classical radius of the scattering charge.

Substituting (1a)–(1c) into the right side of (1), we find an explicit expression for the total bremsstrahlung energy flux as a function of the distance between the scattering charges:

$$I_{tot}^{(2)}(\theta)=\frac{2}{3}I_0\left\{1+\frac{3}{2}\left[\frac{\sin\theta}{\theta}\left(1-\frac{1}{\theta^2}\right)+\frac{\cos\theta}{\theta^2}\right]\cos\theta\right\},$$
$$I_0=q^2\alpha^2\omega_0^4/c^3. \qquad (2)$$

The superscript 2 here is the number of scatterers.

B. Directional pattern. The simplest way to calculate the angular distribution of the scattered radiation for this system is to work from the classical expressions for dipole radiation in the far zone:[13]

$$dI=\frac{1}{4\pi c^3}\langle[\ddot{\mathbf{P}}(t'),\mathbf{n}']^2\rangle d\Omega, \qquad (3\text{a})$$

where $\mathbf{P}(t')$ is the dipole moment of the system, which depends on the retarded time t'; $\mathbf{n}'$ is a unit vector toward the observation point; $d\Omega\equiv\sin\vartheta d\vartheta d\varphi$ is the element of the corresponding solid angle; and the angle brackets mean a time average over the field period $T_0=2\pi/\omega_0$.

It is simple to show that the effective dipole moment of this system of oscillating charges, $P_x(t)$, calculated from the field in the common far zone of the charges [i.e., at a distance $r\equiv(x^2+y^2+z^2)^{1/2}\gg a$ from the center of symmetry of the system, $x=y=z=0$], is

$$P_x(t')=2q\alpha(\cos\omega_0t')\left\{\cos\left[\frac{\theta}{2}(1-\cos\vartheta)\right]\right\},$$
$$\cos\vartheta=z/r,\quad t'=t-r/c. \qquad (3\text{b})$$

Substituting (3b) and (3a), we find the energy flux density of the scattered radiation as a function of the angle ϑ and of the dimensionless distance θ between charges:

$$dI^{(2)}(\vartheta,\theta)={}^1/_4I_0(1+\cos^2\vartheta)$$
$$\times\{1+\cos[\theta(1-\cos\vartheta)]\}\sin\vartheta\,d\vartheta. \qquad (3)$$

As expected, the integral of the right side of (3) over all ϑ ($0\leqslant\vartheta\leqslant\pi$) is exactly equal to the right side of (2).

3. DISCUSSION OF RESULTS

Expressions (2) and (3) give a complete solution of the problem formulated above (in Sec. 2). Let us examine the physical content of the results described by these expressions.

We should first point out that the first term inside the braces on the right side of each of these expressions does not depend on the parameter θ. There is an unambiguous physical explanation for this result: These are terms which quantitatively describe the incoherent scattering of the incident wave by the charges, due to the effect of their own radiative-friction forces (in their near zones) on their transverse motion. Only these fields, which are proportional to the third time derivatives of the corresponding transverse dipole moments of these charges,[13]

$$E_x^{(R)}(z=\pm a,\ t)=\frac{2q}{3c^3}\dddot{\tilde{x}}_\pm(t), \qquad (4\text{a})$$

make θ-independent contributions to the radiative energy loss of the system:

$$I_{ncoh}^{(2)}=-\frac{2q^2}{3c^3}\langle\dot{\tilde{x}}_-\dddot{\tilde{x}}_-+\dot{\tilde{x}}_+\dddot{\tilde{x}}_+\rangle=\frac{2}{3}I_0. \qquad (4\text{b})$$

Furthermore, the overall intensity of the scattered radiation is precisely twice the intensity of the Thomson scattering of the same way by a single charge, $I_T^{(1)}$ (Ref. 13):

$$I_{ncoh}^{(2)}=2I_T^{(1)},\qquad I_T^{(1)}=I_0/3.$$

Finally, the angular distribution of this radiation is precisely the same as that for a single charge:

$$dI_{ncoh}^{(2)}=\frac{I_0}{4}(1+\cos^2\vartheta)\frac{d\Omega}{2\pi}. \qquad (4\text{c})$$

The first term in braces on the right side of both (2) and (3) thus does indeed describe the characteristics of the incoherent scattering of the field of the incident wave by these charged particles. The second terms in these sums, which do depend on the parameter θ, should be identified as the result of the interaction of these particles with the fields produced in their near zones by their neighbors. The nature of this interaction and its resultant intensity are determined unambiguously by the phase relations between the bremsstrahlung field of each charge and the current of its neighbor in the near zone of the latter. To demonstrate this point, we consider the partial energy losses of each of the scatterers in the field of the oscillatory motion of its neighbor. Substituting currents (1a) and fields (1c) into the right side of Eq. (1), we find the total energy losses of each of the scatterers: (a) for the charge on the left,

$$I_-^{(1)}(\theta)=-q\langle v_-(t)\bar{E}_x(z=-a,t;\ b=+a)\rangle$$
$$=\frac{1}{2}I_0\left[\frac{\sin 2\theta}{\theta}\left(1-\frac{1}{\theta^2}\right)+\frac{\cos 2\theta}{\theta^2}\right], \qquad (5a)$$

(b) for the charge on the right,

$$I_+^{(1)}(\theta)=-q\langle v_+(t)\bar{E}_x(z=a,\ t;\ b=-a)\rangle=+I_0/2\theta^2. \qquad (5b)$$

It is easy to see that the sum $I_-^{(1)}(\theta)+I_+^{(1)}(\theta)$ is precisely equal to the difference between the total bremsstrahlung intensity of the scattering charges given by (2) and the incoherent component of this radiation given by (4b). For this reason, the θ-dependent part of this total bremsstrahlung intensity of the system of charges is indeed the result of a coherent "exchange" interaction of these charges through the electric fields which they produce:

$$I_{coh}^{(2)}(\theta)=I_{tot}^{(2)}(\theta)-I_{ncoh}^{(2)}=I_-^{(1)}(\theta)+I_+^{(1)}(\theta). \qquad (5c)$$

A point which deserves particular emphasis is that in the dipole approximation which we have used here, and which involves the strong inequality $k_0^2\alpha^2\ll\theta^2$, the radiator on the right (along the propagation path of the wave being scattered) always loses energy $[I_+^{(1)}(\theta)>0]$, although the loss is small at large distances from the scatterer on the left (at $\theta^2\gg1$). In the region $k_0^2\alpha^2\ll\theta^2\ll1$, the loss increases, but it is offset by the absorption of energy, at the same intensity, by the scatterer on the left: $I_-^{(1)}(\theta\ll1)\simeq-I_0/2\theta^2=-I_+^{(1)}(\theta)$.

It follows from (1c) that this aspect of the "exchange" interaction of these charges is determined unambiguously by the behavior of the quasistatic part of the fields produced by the charges in their near zones.

As was mentioned earlier (Sec. 1), the degree of coherence of the system of elementary scatterers is a matter of particular interest for the theory and applications of processes involving Thomson scattering by systems of charged particles. We accordingly introduce a quantitative measure of this effect: a coherence factor. We define it as the ratio of the total intensity of the bremsstrahlung of these scatterers to the intensity of their incoherent radiation:

$$K_{tot}^{(2)}(\theta)\equiv\frac{I_{tot}^{(2)}(\theta)}{I_{ncoh}^{(2)}}$$
$$=1+\frac{3}{2}\left[\frac{\sin\theta}{\theta}\left(1-\frac{1}{\theta^2}\right)+\frac{\cos\theta}{\theta^2}\right]\cos\theta. \qquad (6a)$$

It follows from this expression that in the limit $\theta\to0$ this factor tends to an absolute maximum of 2:

$$K_{tot}^{(2)}(\theta)=2-{}^7/_{10}\theta^2+O(\theta^4),\qquad \theta^2\ll1. \qquad (6b)$$

The latter result can be found (in the limit $\theta\to0$) by doubling the mass and charge of the scattering particle in the classical expression for the cross section for Thomson scattering,[13] $\sigma_T^{(1)}=2\pi R_0^2/3$. In the limit $\theta\to\infty$, on the other hand, the coherence of the radiation in the system is completely destroyed, as is demonstrated by the fact that the right side of (6a) asymptotically approaches unity

$$K_{tot}^{(2)}(\theta)=1+\frac{3}{2}\frac{\sin 2\theta}{2\theta}+O(\theta^{-2}),\qquad \theta\gg1. \qquad (6c)$$

The way in which the degree of coherence of the radiation by this system of charges decreases with increasing θ according to this expression can be explained unambiguously by the particular way in which the amplitude of the bremsstrahlung field of the charge on the right decreases near the point $\bar{z}=-a$, which is the position of the charge on the left [see (5a)].

For clarity, Fig. 1 shows the functional dependence of the coherence factor $K_{tot}^{(2)}$ versus the dimensionless distance between charges, θ. We see from this figure that the only substantial increase in the total bremsstrahlung intensity of this system of charges as a result of their coherence occurs at small values of θ, specifically, $\theta\lesssim\pi/3$. As θ increases to $\theta\gtrsim1$, the degree of coherence of the bremsstrahlung of the elementary scatterers become smaller. At the first maximum (after $\theta_0=0$), near the point $\theta_1=5\pi/4$, the increase in scattering due to the coherent effect is already down to about 25%, and at the second maximum ($\theta_2=9\pi/4$) the corresponding increase is no more than 10%.

Of particular interest from the physical standpoint is how the angular distribution of the bremsstrahlung of this system of scattering charges varies with the coherence parameter θ. For this distribution we can again introduce a coherence factor, $K^{(2)}(\nu,\theta)$, determining it from (3) as the ratio of the energy flux density of the radiation field at the angle ϑ (for a given distance between charges, θ) to the energy flux density of the field of the incoherent radiation in the same direction:

$$K^{(2)}(\vartheta,\theta)\equiv\frac{dI^{(2)}(\vartheta,\theta)}{dI_{ncoh}^{(2)}(\vartheta)}=1+\cos[\theta(1-\cos\vartheta)]. \qquad (7)$$

In the limit $\theta\to0$, this factor reaches an absolute maxi-

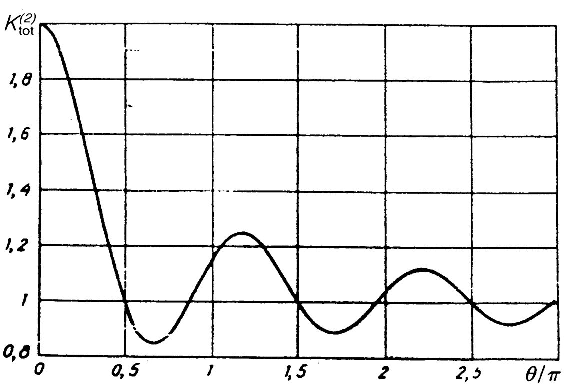

FIG. 1. The coherence factor in the total energy flux of the scattered radiation, $K_{tot}^{(2)}(\theta)$, versus the distance θ between charges.

mum of 2, which is independent of ϑ. An increase in the distance between charges leads to a modulation of the spectra and angular distribution of the energy flux density of the bremsstrahlung of the charges. The depth of this modulation is one. The physical meaning of this result is that there exist directions in which the coherence of the scatterers either doubles the flux density of their incoherent radiation $[K^{(2)}(\vartheta^{(l)}_{\max},\theta) = 2]$ or causes the resultant flux to vanish $[K^{(2)}(\vartheta^{(n)}_{\min},\theta) = 0]$. The corresponding angles are

$$\vartheta^{(l)}_{max} = \arccos(1-2\pi l/\theta),$$

$$l=0, 1, 2, \ldots, l_{max} = [\theta/\pi], \tag{7a}$$

$$\vartheta^{(n)}_{min} = \arccos\{1-(2n+1)\pi/\theta\},$$

$$n=0, 1, 2, \ldots, n_{max} = [\theta/\pi - 1/2], \tag{7b}$$

([...] means the largest integer). These angles depend on θ, i.e., on the distance between charges, and the number of extrema in the flux density increases in proportion to $[\theta/\pi]$ as this distance increases.

Figure 2 shows the right side of (7) as a function of the angle ϑ for three different values of the parameter θ. We see from these figures [and also directly from (7)] that the energy flux density is very anisotropic at $\theta \approx \pi$ (there is a significant change in absolute value when ϑ changes by an amount $\bar{\vartheta} = \pi - \vartheta$). Physically, this anisotropy can be attributed to a particular feature of the phase relations between the oscillations (the currents) of the radition charges and the fields which they radiate. In particular, at $\vartheta = 0$, at which the phase shift of the stimulating field $\mathscr{E}^{\text{ext}}(z,t)$ is exactly equal to the phase difference between the fields radiated by the charges, regardless of the distance between the charges, the bremsstrahlung of these charges is completely coherent $[K^{(2)}(0,\theta) = 2]$. For other directions, the difference between the phases of the bremsstrahlung fields of the two scatterers depends strongly on θ. It is this dependence which ultimately leads to the θ dependence of the right side of (7).

As expected, the anisotropy disappears as $\theta \to 0$, in which limit the two charges radiate as a single unit. In precisely the same way, the ansiotropy disappears at values of θ corresponding to integer numbers of half-wavelengths ($\theta = \theta_s = \pi s$, where $s = 1, 2, 3, \ldots$). In this case the right side is symmetric under the replacement of ϑ by $\bar{\vartheta}$. Finally, at $\theta \gg 1$, this anisotropy weakens substantially because there is no coherent exchange interaction between the charges. As follows from (6c), each charge radiates completely independently in this case, so there is no amplification of the bremsstrahlung of the system $[K^{(2)}_{\text{tot}}(\theta) = 1 + O(\theta^{-1})]$. The extrema in the spectrum and angular distribution of the energy flux of their radiation lead to only a redistribution of the fixed total (incoherent) radiation flux between minima ($\vartheta^{(n)}_{\min}$) and maxima ($\vartheta^{(l)}_{\max}$). Since the number of these extrema is large, the maximum at the point[2)] $\vartheta = 0$ (with a half-width on the order of $\theta^{-1/2} \ll 1$) turns out to be relatively small, on the order of θ^{-1} (curve *3* in Fig. 2).

Our analysis of the dependence of the total intensity and the spectral and angular distribution of the flux density of the bremsstrahlung in a system of two scatterers thus shows that an increase in the intensity (or cross section) of the scattering due to the coherence of these scatterers can occur only if the distance between the scatterers is smaller than the wavelength ($\theta \ll 1$).

4. CONCLUSION

The basic results of this analysis can be formulated as follows.

Analytic expressions have been derived for the total intensity of the bremsstrahlung of two charges and for the angular distribution of the energy flux of this radiation as a function of the distance between charges.

The physical mechanisms which are responsible for the coherent intensification of the radiation at a relatively small distance between the charges and which are responsible for the decrease in the radiation as this distance increases have been identified. The mechanism responsible for the nonmonotonic dependence of the energy flux density of the radiation on the angle ϑ has also been determined.

The most important conclusion to be drawn from these results concerns the questions, raised back in Sec. 1, regarding the conditions required for achieving coherence in the radiation by electrons in ultrarelativistic microwave-electronics devices of the free-electron-laser type. This conclusion is based on the established fact that there is a substantial

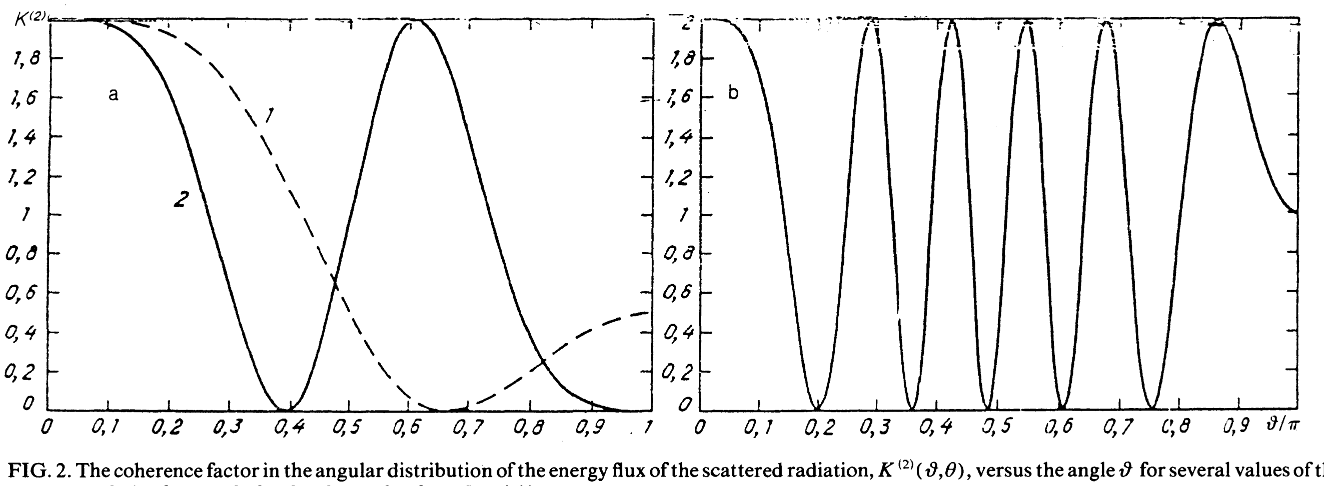

FIG. 2. The coherence factor in the angular distribution of the energy flux of the scattered radiation, $K^{(2)}(\vartheta,\theta)$, versus the angle ϑ for several values of the parameter θ. *1*—$\theta = 2\pi/3$; *2*—$\theta = 3\pi/2$; *3*—$\theta = (5 + 1/4)\pi$.

decrease in the intensity of the radiation by the charges as the distance between them increases, because of the decrease in the amplitude of the fields radiated by the scattering charges themselves in their wave zones (the decrease is in inverse proportion to this distance). Since it is this field which is responsible for the coherent interaction of the scattering charges, it follows from this fact that the density of electrons in their rest frame must satisfy the condition $n_0'\lambda_0'^3 \gg 1$, where λ_0' is the wavelength of the radiated wave in this frame, if there is to be any substantial increase in the degree of coherence and in the resultant intensity of the radiation by the elementary sources (electrons) in microwave-electronics systems which operate on the basis of the bremsstrahlung of electrons. In the particular case of free-electron lasers, this inequality leads to the following expression for the lower limit on the wavelength for coherent emission, $\lambda_{\rm min}^{\rm coh}$, as a function of the beam density n_0 and the undulator period D in the laboratory frame of reference:

$$\lambda_{min}^{coh} = (2/n_0 D)^{1/2}. \tag{8a}$$

The energy of the electron beam must be, in order of magnitude (and in units of m_0c^2),

$$\gamma_* = (D/2\lambda_{min}^{coh})^{1/2}. \tag{8b}$$

It follows from these estimates that with $D = 2$ cm, for example, achieving coherent undulator radiation with a wavelength $\lambda_0 = 10$ Å (Ref. 12) will require producing a beam of electrons with an energy $\mathscr{E}_e \approx 1.5$ GeV ($\gamma_* \approx 3\cdot 10^3$) and a peak current density up to 1 MA/cm^2. We should stress that this estimate refers to the idealized model of a monoenergetic beam with a zero transverse divergence, and the improvement in the radiation intensity due to the coherence factor is relatively small. If, on the other hand, these conditions are not satisfied, the total power of the magnetobremsstrahlung of the electrons of the beam in the undulator will be essentially no greater than the sum of the intensities of the incoherent undulator radiation of these electrons.

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[1)]It follows in particular from this analysis that the increase due to the coherence factor was more than eight orders of magnitude in the experiments of Ref. 10, where an electron efficiency $\eta_e \approx 36\%$ (at $\lambda_0 \approx 8$ mm) was achieved. This is the record high efficiency for free-electron lasers.

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