Broadband noise suppression in light propagating through a multiphoton absorber

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We consider propagation of light having arbitrary statistics in a medium capable of multiphoton absorption or enhancement. The limiting shot-noise suppression level is 33% for two-photon absorption and 50% for m-photon absorption as $m \to \infty$. The noise is lowered in a broad frequency band. The Langevin approach is used to analyze the transport equation for the electromagnetic-field density matrix.

INTRODUCTION

Many proposals to generate light in a nonclassical state (NCS), i.e., having sub-Poisson statistics or in a squeezed state, are based on the use of optical systems with cavities containing a nonlinear medium.[1–5] Just such systems were realized in a number of experiments[6–8] in which squeezed states of light with suppressed noise were obtained. Practical interest attaches not only to the level but also to the band of noise suppression. In cavity systems, the band is limited usually by the cavity width. In cavity-free cases, when the light propagates through a medium, the noise suppression band may turn out to be substantially different.

We consider here the variation of the statistical properties of light propagating through a nonlinear absorber. Multiphoton absorption is extensively discussed in the study of an NCS field. It is the subject of many studies, e.g., Refs. 9 and 10, an extensive list of which is given in Ref. 11. Thus, the photon-number dispersion and the distribution of the number of photons emerging from the medium have been investigated in detail. However, properties such as the spectrum of the radiation noise or the spectrum of the photocurrent, which are usually measured in experiments on NCS-field generation, have not been considered. The reason is that light propagation in a medium is far from a trivial problem in quantum electrodynamics. The standard approach is based on a spatiotemporal analogy in which the temporal coordinate is replaced by a spatial one, $t \to z/c$, and the initial conditions are replaced by boundary ones. As a result, the spatial evolution is obtained from the temporal one. In the spirit of this analogy, we have considered all the multiphoton-interaction problems known to us. This approach, however, does not permit spectral properties to be calculated, particularly the noise spectrum, for which one needs to know the unequal-time propagators or the correlation functions taken at one spatial point.

We calculate the radiation noise by the transport-equation method,[12] which permits the propagation of light in a medium to be described in the context of quantum electrodynamic. This method was used in Ref. 13 to consider the transformation of the statistical properties of light in a parametric medium. Since its development in the case of multiphoton absorption is not unitary, the procedure used in Ref. 13 to calculate the observed properties cannot be applied directly. To analyze a non-unitary transport equation we used the Langevin equations that follow from the Fokker–Planck equation for the Glauber and the generalized quasiprobabilities. The complications encountered in the description of NCS with the aid of a P-function are well known. They are due to the singular behavior of these quasiprobabilities, which leads to a negative diffusion matrix in the Fokker–Planck equation. It was shown in Ref. 14 that a transition from such an equation to Langevin equations can be rigorously justified in the framework of the Ito procedure. Note that the entire justification is only a question of interpretation.

Our results for the photon-number dispersion in m-photon absorption agree with the known data.[15] The obtained limiting noise-suppression level is $[m/(2m-1)]i_0$, where i_0 is the shot-noise level. The noise is lowered in a wide frequency band $\Delta\omega$, which can exceed substantially the width of the excess noise of the initial light. This result depends neither on the statistics of the input light nor on the type of the working transition in which the absorption takes place.

To formulate and verify the Langevin approach, we consider in Sec. 1 a test problem for which an exact solution of the Heisenberg equations of motion was obtained in Ref. 13. In Sec. 2 we calculate on the basis of this approach the statistical properties of the light and present numerical estimates for a two-photon interaction. Section 3, where a generalized P-representation is used, contains the results for m-photon interaction.

1. LANGEVIN APPROACH FOR THE TRANSPORT EQUATIONS

Using the test problem dealt with in Ref. 13, we consider methods of defining correlators of Langevin forces by starting from a Fokker–Planck equation written in transport form.

We recall first the principal premises of quantum transport theory, which we cite for the simple case of propagation of one-mode light along the z axis. By forming packets of plane waves with wave numbers k in the interval $[k_0 - \pi/l, k_0 + \pi/l]$ and centered about a selected mode $k_0 = 2\pi\lambda_0^{-1}$ ($\hbar = c = e = 1$), where l is the cell dimension in the auxilary normalization volume LS_0 ($\lambda_0 \ll l < L$), the operator of the electric field can be expressed in the Heisenberg representation as

$$E(z,t) = -i\sum_{k\sim k_0}\left(\frac{\omega_k}{2LS_0\varepsilon_0}\right)^{1/2} a_k(t)\exp(ikz) + \text{H.a.}$$

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$$\approx -i\left(\frac{\omega_0}{2lS_0\varepsilon_0}\right)^{1/2}\tilde{C}(z,t)\exp(ik_0z)+\text{H.a.},$$

$$\tilde{C}(z,t)=\sum_{k\sim k_0}\left(\frac{l}{L}\right)^{1/2}a_k(t)\exp[i(k-k_0)z].$$

Here $[a_k(t),a_{k'}(t+\tau)]=\exp(i\omega_k\tau)\delta_{kk'}$, and for the operators $\tilde{C}^+$ and $\tilde{C}$, which can be interpreted as photon creation and annihilation operators at the point z (space cell with dimension l), we obtain the commutation relations

$$[\tilde{C}(z,t),\tilde{C}^+(z,t+\tau)]=\exp(i\omega_0\tau)l\delta_l(\tau), \tag{1}$$

$$l\delta_l(\tau)=\sum_{k\sim k_0}\frac{l}{L}\exp(i\omega_k\tau),$$

$$\lim_{L\to\infty}l\delta_l(\tau)=\frac{\sin(\tau/2l)}{\tau/2l}.$$

The operators $\tilde{C}^+$ and $\tilde{C}$ can be used to express all the necessary mean values, particularly the radiation photocurrent-spectrum $i^{(2)}(\Omega)$ or the noise spectrum. This is precisely the quantity measured in experiments on light statistics. The expression obtained in Ref. 16 for $i^{(2)}(\Omega)$ takes in the plane-wave approximation the form

$$i^{(2)}(\Omega)=A\left[\langle n(z)\rangle l^{-1}+2ql^{-2}\,\text{Re}\int_0^\infty d\tau\, e^{i\Omega\tau}g(z,\tau)\right],$$

$$\langle n(z)\rangle=\langle\tilde{C}^+(z,t)\tilde{C}(z,t)\rangle,$$

$$g(z,\tau)=\langle\tilde{C}^+(z,t)\tilde{C}^+(z,t+\tau)\tilde{C}(z,t+\tau)\tilde{C}(z,t)\rangle,$$

where q is the quantum efficiency of the photogeometry whose geometry is accounted for by the factor $A\propto q$.

A resonance variant of parametric frequency division or of subharmonic generation in an extended medium was considered in Ref. 13 for the case of a classical pump wave whose frequency ω is at resonance with the atomic transition ω_{ab}: $\omega=\omega/2+\omega/2\approx\omega_{ab}$. The transport equation obtained for the density matrix ρ of a signal-wave subharmonic of frequency $\omega/2$ corresponds to the unitary development:

$$(\partial_t+\partial_z)\rho(z,t)=-i[V,\rho],$$

$$V=\lambda(C^+)^2+\text{H.a.}, \tag{2}$$

$$\tilde{C}=C\exp(-i\omega_0t),$$

where the interaction constant λ is proportional to the complex amplitude of the pump wave, the development of which can be considered independently. A solution of (2) for specified boundary conditions is easily obtained by changing to the Heisenberg equations of motion for the operators C:

$$(\partial_t+\partial_z)C(z,t)=-2i\lambda C^+.$$

As a result of which all the observable quantities of interest at the exit from the medium are expressed in terms of mean value at the entry.

The method of Langevin equations, which can be determined from the transport equation, is a varient of the Heisenberg picture for non-unitary development (and incidentally, also for the unitary). To formulate it we rewrite (2) in the P-representation:

$$(\partial_t+\partial_z)P(\alpha,z,t)=(\partial_\alpha\Lambda_1+\partial_\alpha\partial_\alpha\Lambda_2)P,$$

$$\Lambda_1=2i\lambda\alpha^*,\ \Lambda_2=-i\lambda. \tag{3}$$

Note that Eq. (3) is exact.

We introduce the polar coordinates $\alpha=r\exp(i\varphi)$ and make the change of variables $\varphi\to\psi=2\varphi-\varphi_H$, where $\varphi_n=\arg\lambda$ is, apart from an additive constant, the phase of the pump wave. We assume that the signal-wave field fluctuates weakly about the semiclassical values:

$$\psi=\psi_0(z,t)+\mu,\ \mu\ll\psi_0,$$

$$r^2=n(z,t)+\varepsilon,\ \varepsilon\ll n. \tag{4}$$

The conditions (4) make it possible to linearize the coefficients in (3) and to obtain for ψ_0 and n the semiclassical equations

$$(\partial_t+\partial_z)\psi_0(z,t)=-4|\lambda|\sin\psi_0,$$

$$(\partial_t+\partial_z)n(z,t)=4|\lambda|n\cos\psi_0 \tag{5}$$

with specified boundary conditions. The system (5) has an integral

$$n(z)\sin\psi_0(z)=\text{const}. \tag{6}$$

Equation (3) can be linearized in the form

$$(\partial_t+\partial_z)P=[\partial_\varepsilon(-\Gamma\varepsilon-\Gamma_1\mu)+\partial_\mu(\Gamma\mu)$$

$$+\partial_\varepsilon\partial_\mu Q_{\varepsilon\mu}+\partial_\varepsilon\partial_\varepsilon Q_{\varepsilon\varepsilon}+\partial_\mu\partial_\mu Q_{\mu\mu}]P.$$

$$\Gamma=4|\lambda|\cos\psi_0,\ \Gamma_1=-4|\lambda|n\sin\psi_0, \tag{7}$$

$$Q_{\varepsilon\varepsilon}=2|\lambda|n\cos\psi_0,\ Q_{\varepsilon\mu}=-2|\lambda|\sin\psi_0,$$

$$Q_{\mu\mu}=-2|\lambda|n^{-1}\cos\psi_0.$$

It is quite evident that the diffusion coefficients can be negative here, thus attesting to the NCS of the field, since we are using here the Glauber quasiprobability.

Let us write down the Langevin equations corresponding to (7). As shown in Ref. 14, the transition procedure can be rigorously substantiated. These equations take the form

$$(\partial_t+\partial_z)\varepsilon(z,t)=\Gamma\varepsilon+\Gamma_1\mu+f_\varepsilon(z,t),$$

$$(\partial_t+\partial_z)\mu(z,t)=-\Gamma\mu+f_\mu(z,t). \tag{8}$$

Given the boundary conditions, we choose the random-force correlators in the form

$$\langle f_k(z_1,t_1)f_k(z_2,t_2)\rangle=2Q_{kk}\delta(z_2-z_1)l\delta_l(t_2-t_1),\ k=\varepsilon,\mu.$$

$$\langle f_\varepsilon(z_1,t_1)f_\mu(z_2,t_2)\rangle=Q_{\varepsilon\mu}\delta(z_2-z_1)l\delta_l(t_2-t_1). \tag{9}$$

All other correlators are equal to zero.

Equations (8) and (9) make it possible to calculate all the necessary mean values. Thus,

$$g(z,\tau)=n^2(z)+\langle\varepsilon(z,t)\varepsilon(z,t+\tau)\rangle,$$

where $n(z)$ are determined by the solution of the semiclassical problem.

It follows from (8) that

$$\varepsilon(z,t)=\varepsilon(0,t-z)\exp\left\{\int_0^z dz_1\Gamma(z_1,t-z)\right\}$$

$$+\int_0^z dz_1\left[\exp\left\{\int_{z_1}^{z} dz_2\Gamma(z_2,t-z)\right\}\right]$$

$$\times\{\Gamma_1(z_1,t-z)\mu(z_1,t-z)+f_e(z_1,t-z)\},$$

where $\varepsilon(0,t-z)$ are the values on the boundary of the medium. To calculate $g(z,\tau)$ we use a simple situation in which the phase difference on the front boundary is $\psi_0(0)$: $\sin\psi_0(0)=0$. It follows from (6) that it is conserved as the light propagates through the medium: $\sin\psi_0(z)=0$. As a result we have

$$\langle\varepsilon(z,\ t)\varepsilon(z,\ t+\tau)\rangle=K^2\langle\varepsilon(0,\ t-z)\varepsilon(0,\ t+\tau-z)\rangle$$
$$+K(K-1)n(0)l\delta_l(\tau),$$

where $K=n(z)/n(0)$ and $n(0)$ is the average number of photons on the boundary. The correlator at the input is assumed given. It can be expressed by specifying the field statistics on the boundary. To confine ourselves only to physical states of the field, we consider cavity sources that generate light both in the classical state and in the NCS. A model of such a source was considered, for example, in Ref. 17. We have for it

$$\langle\varepsilon(0,\ t_1)\varepsilon(0,\ t_2)\rangle=(Cl)^2n\xi\exp(-\Gamma_a|t_2-t_1|). \tag{10}$$

Here Γ_a is the width of the amplitude-fluctuation spectrum, n and ξ are respectively the stationary number of photons and the statistics parameter $(\langle n^2\rangle-\langle n^2\rangle=\langle n\rangle(1+\xi))$ inside the source, and C is the cavity width. We have used in (10) the connection between the mean value inside the cavity and on its boundary:

$$n(0)=Cln,\ \xi(0)=Cl\xi.$$

As a result, omitting the terms independent of τ, we have

$$l^{-2}g(z,\ \tau)=C^2g_2(\tau)+Cg_1(\tau)\delta_l(\tau),$$
$$g_2(\tau)=K^2n\xi\exp(-\Gamma_a\tau), \tag{11}$$
$$g_1(\tau)=nK(K-1).$$

Calculation of $g(z,\tau)$ with the aid of exact solutions of the Heisenberg equations leads in (11) to an additional term $g_0(l\delta_l(\tau))^2$ (Ref. 13). It describes spontaneous parametric scattering, i.e., the onset of subharmonics at $n(0)=0$. This difference is due to the use of the small-fluctuations approximation, in the framework of which purely spontaneous processes with $n(0)=n(z)=0$ cannot be directly analyzed. This is the basic limitation of the considered method. The approximations employed, however, can be readily monitored and do not lead to unphysical singularities. Thus, the noise spectrum corresponding to (11) is of the form

$$i^{(2)}(\Omega)=KP(0)(1-q)+2qK^2P(0)\left(\frac{1}{2}+\xi\frac{C}{\Gamma_a}\frac{\Gamma_a^2}{\Gamma_a^2+\Omega^2}\right),$$

where we have introduced the power of the light incident on the medium, $P(0)=\omega_0n(0)l^{-1}$. For the chosen source models we have $\xi\geqslant-1/2$ (the case $\xi<0$ correspond to sub-Poisson statistics of the photons). For $\xi=-1/2$ and $C=\Gamma_a$ we have $i^{(2)}(\Omega)>0$.

Note that the above method is based on the Fokker–Planck equation for Glauber quasicoherence. In this case the P-function is certainly singular because of the NCS of the field. The Langevin approach obviates the need for explicit determination of the P-function to calculate the observables and obtain directly normally ordered mean values.

2. PROPAGATION OF CLASSICAL AND SUBPOISSON LIGHT IN A MEDIUM WITH TWO-PHOTON ABSORPTION

Let the light incident on the medium have a frequency ω_0, a power $P(0)$, and arbitrary statistics. After passing through a layer of the nonlinear medium the light is incident, after going through a filter tuned to the frequency ω_0 and having a pass band $\Delta\omega$, lands on a photoreceiver, and the noise spectrum $i^{(2)}(\Omega)$ is subsequently measured (Fig. 1a).

The medium is simulated by a set N of identical immobile atoms with a working transition $a\leftrightarrow b$ at a frequency ω_{ab}. The energy scheme of the atom is shown in Fig. 1b. Incoherent pumps Λ_a and Λ_b ensure stationary populations φ_a and φ_b of the upper and lower levels. This model, used for the model discussed in Ref. 4 for four-wave mixing processes, makes it possible to consider two important physical cases. These are the Lamb–Scully two-level system with an intermediate-level working transition, and a two-level model of the Haken type, where the lower level is the ground state.

To describe the two-photon interaction we use the traditional effective Hamiltonian

$$v(t)=\sum_{\alpha=1}^{N}v_\alpha(t), \tag{12}$$

$$v_\alpha(t)=\sum_{k_1,k_2}f_{k_1k_2}a_{k_1}a_{k_2}s_\alpha^+\exp[i(\omega_{ab}-\omega_{k_1}-\omega_{k_2})t]+\text{H.a.},$$

where s_α^+ and s_α are single-atom operators corresponding to atom transition between working levels, $f_{k_1k_2}$ is the coupling constant,[18]

$$f_{k_1k_2}=\frac{1}{2}\sum_n[(g_{an})_{k_1}(g_{nb})_{k_2}(\omega_{nb}-\omega_{k_1})^{-1}$$
$$+(g_{an})_{k_2}(g_{nb})_{k_1}(\omega_{an}-\omega_{k_1})^{-1}],$$

$$(g_{mn})_{k_\mu}=i\left(\frac{\omega_{k_\mu}}{2LS_0\varepsilon_0}\right)^{1/2}d_{mn}\exp(i\mathbf{k}_\mu\mathbf{r}_\alpha),\quad \mu=1,2.$$

Using the standard procedure of adiabatic exclusion of fast atomic variables, of the Lamb–Scully[19,20] or of the Haken[21,22] type, we write down the kinetic equation for the field in the P-representation. To change over to the transport equation, we form a packet of plane waves with central frequency $\omega_0\approx\omega_{ab}/2$ and ω_1, ω_2: and with frequencies $\omega_1+\omega_2\approx\omega_{ab}$. We consider the two-photon interaction in the lowest order in the coupling constant f. In this case the modes with frequencies ω_1 and ω_2 as well as the mode ω_0 separated by a filter evolve independently. We can therefore confine ourselves to the simple one-mode approximation. As a result we obtain for the P-function describing the propagation of the mode ω_0

$$(\partial_t+\partial_z)P(\alpha,\ z,\ t)=(\partial_\alpha\Lambda_1+\partial_\alpha\partial_\alpha\Lambda_2+\partial_\alpha\partial_{\alpha^*}\Lambda_3)P+\text{c.c.},$$

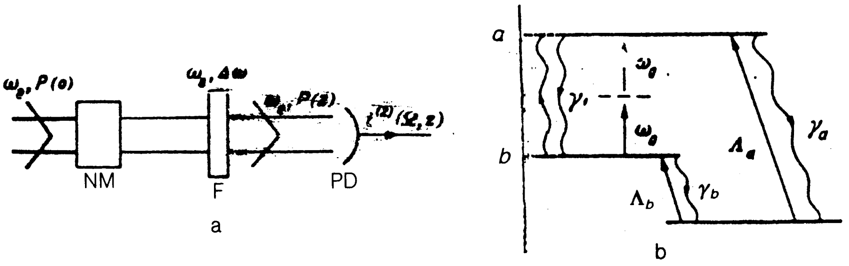

FIG. 1. System with multiphoton absorption: a—Optical scheme (NM—nonlinear medium, F—filter, PD—photodetector); b—energetic structure of the working levels of the absorber.

$$\Lambda_1=-2\alpha|\alpha|^2 b(1-i\delta),$$
$$\Lambda_2=\alpha^2 b(1-i\delta),\ \Lambda_3=4b|\alpha|^2\varphi_a(\varphi_a-\varphi_b)^{-1}, \tag{13}$$
$$\delta=(\omega_{ab}-2\omega_0)/\gamma_{ab},\ b=N(\varphi_a-\varphi_b)|f|^2/\gamma_{ab}(1+\delta^2).$$

Here γ_{ab} is the transverse-relaxation constant of the working transition, and the quantity b, for which we write the dimensional expression ($[b]=c^{-1}$),

$$b=\frac{\varphi_a-\varphi_b}{1+\delta^2}\frac{\tilde{b}}{lS_0},$$
$$\tilde{b}=\left(\frac{\hbar\omega_0}{2\varepsilon_0}\right)^2\frac{3}{2\hbar}(-\mathrm{Im}\,\varkappa^{(3)}),$$

is connected with the cubic susceptibility $\varkappa^{(3)}$ (Ref. 23) describing two-photon resonance processes. The stationary populations $\varphi_{a,b}$ are expressed in terms of a set of relaxation constants and of the pumping rate:

$$\varphi_b=(\gamma_\downarrow+\gamma_a')\{(\gamma_\uparrow+\gamma_b')(1+\gamma_a/\Lambda_a)+(\gamma_\downarrow+\gamma_a')(1+\gamma_b/\Lambda)\}^{-1},$$
$$\gamma_{a,b}'=(1-\Lambda_{a,b}/\Lambda)\gamma_{a,b},$$
$$\Lambda=\Lambda_a+\Lambda_b.$$

The quantity φ_a is obtained from Φ_g by making the substitutions $\gamma_\downarrow\to\gamma_\uparrow$, $\gamma_\uparrow\to\gamma_\downarrow$, $a\to b$, $b\to a$.

Equation (13) is written in the lowest approximation in the two-photon interaction, so that the effects connected with the change of the level populations by the field are not taken into account. This imposes on the field strengths constraints that are of no importance in what follows. As a consequence of this approximation, the structure of (13) is independent of the type of transition. We have confined ourselves in (13) to derivatives with respect to α of order not higher than second; i.e., we have used the diffusion approximation.

Let us linearize (13), assuming the fluctuations to be small:

$$\alpha=[n(z,t)+\varepsilon]^{1/2}\exp[i\varphi_0(z,t)+i\mu],$$
$$\varepsilon\ll n,\ \mu\ll\varphi_0.$$

For n and φ_0 Eq. (13) leads to the quasiclassical boundary-value equations

$$(\partial_t+\partial_z)n=4bn^2,$$
$$(\partial_t+\partial_z)\varphi_0=-2bn\delta. \tag{14}$$

It is convenient to express the solution for n in terms of dimensional powers $P(0)$ and $P(z)$ of entry into and exit from the medium:

$$K=P(z)/P(0)=[1-I(0)zB^{-1}]^{-1}, \tag{15}$$

where we have introduced the dimensional light intensity at the entrance boundary:

$$I(0)=P(0)/S_0,$$
$$B=\frac{12\tilde{Q}(1+\delta^2)}{(\varphi_a-\varphi_b)},\quad \tilde{Q}=\hbar\omega_0c^2/48\tilde{b}. \tag{16}$$

After linearization, Eq. (13) takes the same form as (7);

$$(\partial_t+\partial_z)P=[\partial_\varepsilon(-\Gamma_1\varepsilon)+\partial_\mu(\Gamma_2\mu)+\partial_\varepsilon\partial_\varepsilon Q_{\varepsilon\varepsilon}+\partial_\varepsilon\partial_\mu Q_{\varepsilon\mu}+\partial_\mu\partial_\mu Q_{\mu\mu}]P,$$
$$\Gamma_1=8bn,\ \Gamma_2=2b\delta,$$
$$Q_{\varepsilon\varepsilon}=4bn^2\left(\frac{1}{2}+\frac{2\varphi_a}{\varphi_a-\varphi_b}\right),\quad Q_{\varepsilon\mu}=-bn\delta,$$
$$Q_{\mu\mu}=b\left(-\frac{1}{2}+\frac{2\varphi_a}{\varphi_a-\varphi_b}\right).$$

The corresponding Langevin equations are

$$(\partial_t+\partial_z)\varepsilon=\Gamma_1\varepsilon+f_\varepsilon(z,t),$$
$$(\partial_t+\partial_z)\mu=-\Gamma_2\varepsilon+f_\mu(z,t), \tag{17}$$

where the nonzero random-force correlators are defined in accordance with (9). It is seen from (17) that the intensity fluctuations do not depend on the phase fluctuations. In the presence of detuning ($\delta\neq0$) the phase fluctuations are influenced by the intensity fluctuations. With account taken of the relation

$$\exp\left(\int_{z_1}^{z_2}\Gamma_1\,dz\right)=\left[\frac{n(z_2)}{n(z_1)}\right]^2,$$

which follows directly from (14), the solutions of (17) with the specified boundary conditions will be

$$\varepsilon(z,t)=K^2\varepsilon(0,t-z)+\int_0^z dz_1\left(\frac{n(z)}{n(z_1)}\right)^2 f_\varepsilon(z_1,t-z),$$
$$\mu(z,t)=\mu(0,t-z)+\int_0^z dz_1[-\Gamma_2\varepsilon+f_\mu(z_1,t-z)]. \tag{18}$$

K is defined in (15). It has the meaning of gain or loss. We have $K=1$ at the boundary of the medium, meaning at $z=0$, $K>1$ for amplification, when $\varphi_a>\varphi_b$, and $K<1$ for absorption. Using (18), we get

$$\langle\varepsilon(z,t)\varepsilon(z,t+\tau)\rangle=K^4\langle\varepsilon(0,t-z)\varepsilon(0,t+\tau-z)\rangle$$
$$-{}^1/_3 n(z)(1-K^3)[1+4\varphi_a/(\varphi_a-\varphi_b)]l\delta_l(\tau).$$

From this we obtain for $\tau=0$ an expression for the normal dispersion of the number of photons leaving the medium $(\langle n^2(z)\rangle - \langle n(z)\rangle^2)\langle n(z)\rangle^{-1} - 1 = \langle \varepsilon(z,t)\varepsilon(z,t)\rangle/n(z)$, which agrees with the value obtained in Refs. 10 and 15.

For field states on the forward boundary, which are defined in accordance with (10), we obtain

$$i^{(2)}(\Omega,z)=i_0\left[1+2qK^3\xi\frac{C}{\Gamma_a}\frac{\Gamma_a^2}{\Gamma_a^2+\Omega^2} - \frac{q}{3}(1-K^3)\left(1+4\frac{\varphi_a}{\varphi_a-\varphi_b}\right)\right], \quad i_0\propto P(z). \tag{19}$$

This expression shows how the light noise is transformed either classically ($\xi \geqslant 0$) or with sub-Poisson statistics ($\xi<0$) in the case of two-photon interaction with a layer of the medium. Thus, as the light propagates the excess noise of the initial radiation [the second term in (19)] is altered by addition of a broadband component [last term in (19)]. Amplification only increases the noise. In the absorption regime ($K<1$) the excess noise of the source decreases in proportion to K^3 and the broadband component tends to its limiting value $qi_0/3$. Thus, in the case of a long absorbing medium, when $K \ll 1$, the noise-suppression level can amount to 33% of the shot-noise level. The noise-suppression band $\Delta\omega$ can be estimated from two conditions. First, the rate of field evolution, $4bn$ in this case [see Eq. (14)], should be much slower than the atom-development rate $\gamma_{\rm AT}$ which reduces to the atomic relaxation constants $\gamma_{\rm AT}=\{\gamma'_a,\gamma'_b,\gamma_{ab}\}$. Second, we have assumed in the transport equations that the field is practically constant over the length l of the auxiliary cell, viz., $I(0)lB^{-1} \ll 1$. As a result it follows for $\Delta\omega \propto l^{-1}$ that

$$I(0)c/B \ll \Delta\omega \ll \gamma_{\rm AT}.$$

Figure 2 shows the noise spectrum for $\Omega=0$ and $q=1$ as a function of K, i.e., of the length of the medium, when coherent light ($\xi=0$) or sub-Poisson light ($\xi=-1/2$) with fully suppressed noise ($C=\Gamma_a$) is incident on the boundary.

We present now numerical estimates. Let the photoreceiver be ideal, i.e., $q=1$, $-\operatorname{Im}\varkappa^{(3)} \sim 10^{-32}$ A·s·V^{-3} (Ref. 23). $\lambda_0=0.5\ \mu$m. For the case of absorption under conditions of exact resonance (φ_a, $\delta=0$) we have in Eq. (16) $B=6\cdot 10^8$ W/cm. At an absorber length $l=1$ m and at entry intensities $I(0)\sim 6$–54 MW/cm^2 the value of K^3 is 10^{-1}–10^{-3}. This means the excess noise of the source is suppressed by a factor 10–10^3. For a long medium, e.g., $l=1$ km, the same values of K are assured at much lower intensities, $I(0)\sim 12$–54 kW/cm^2. Note that for sources of power 10 mW–10 W and for a beam diameter on the order of 10 μm (fiber optics) the intensity is 11 kW/cm^2–11 MW/cm^2.

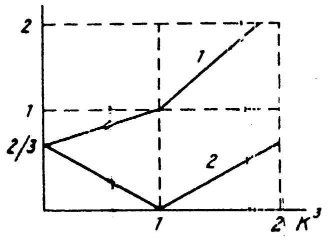

FIG. 2. Dependence of noise spectrum on the length of the medium: 1—coherent light incident on the boundary of the medium; 2—light with fully suppressed noise is incident on the boundary of the medium.

3. *m*-PHOTON INTERACTION

We use for this case the effective Hamiltonian (12), where

$$f_{k_1k_2}a_{k_1}a_{k_2}\exp[i(\omega_{ab}-\omega_{k_1}-\omega_{k_2})t] \to \exp(i\omega_{ab}t)f^{(m)}\prod_{i=1}^{m}a_{k_i}\exp(-i\omega_{k_i}t).$$

We calculate the noise spectrum using the generalized P-representation:[24]

$$\rho(z,t)=\int \Lambda(\alpha,\beta)P(\alpha,\beta;z,t)\,d\mu(\alpha,\beta),$$
$$\Lambda(\alpha,\beta)=|\alpha\rangle\langle\beta^*|(\langle\beta^*|\alpha\rangle)^{-1}, \tag{20}$$

The integration measure $d\mu(\alpha,\beta)=\delta(\alpha-\beta^*)$ in (20) corresponds to the Glauber quasiprobability, while $d\mu(\alpha,\alpha^+)=d^2\alpha\,d^2\alpha^+$, where $\alpha^+=\beta^*$ and α are two independent variables, corresponds to a positive P-representation with quasiprobability $P(\alpha,\alpha^+)$. The latter is in an analytic function of the variables α and α^+. The use of the generalized P-representation makes it possible to formulate the problem in the framework of traditional diffusion processes.

In the considered case we obtain for $P(\alpha,\alpha^+)$ an equation containing high-order derivatives with respect to α and α^+. The diffusion approximation, in which derivative of order not higher than the second are used, leads to a Fokker–Planck equation with a non-negative diffusion matrix:

$$(\partial_t+\partial_z)P(\alpha,\alpha^+;z,t)=[\partial_\alpha\Lambda_1+\partial_\alpha\partial_\alpha\Lambda_2 + \partial_\alpha\partial_{\alpha^+}\Lambda_3+(\alpha\to\alpha^+,\ \alpha^+\to\alpha,\ \text{c.c.})]P(\alpha,\alpha^+;z,t).$$

Here

$$\Lambda_1=-\alpha b_m m\alpha^{m-1}(\alpha^+)^{m-1}(1-i\delta_m),$$
$$\Lambda_2={}^1\!/_2\alpha^2 b_m m(m-1)\alpha^{m-2}(\alpha^+)^{m-2}(1-i\delta_m),$$
$$\Lambda_3=b_m m^2\alpha^{m-1}(\alpha^+)^{m-1}\varphi_a(\varphi_a-\varphi_b)^{-1},$$
$$\delta_m=(\omega_{ab}-m\omega_0)\gamma_{ab}^{-1},$$
$$b_m=N|f^{(m)}|^2(\varphi_a-\varphi_b)[\gamma_{ab}(1+\delta_m^2)]^{-1}.$$

The corresponding Langevin equation is

$$(\partial_t+\partial_z)\begin{pmatrix}\alpha\\ \alpha^+\end{pmatrix}=-\begin{pmatrix}\Lambda_1\\ \Lambda_1^+\end{pmatrix}+D_\alpha^{1/2}\eta_\alpha(z,t), \tag{21}$$

where the diffusion-matrix elements are

$$D_{\alpha\alpha}=2\Lambda_2,\ D_{\alpha^+\alpha^+}=2\Lambda_2^+,\ D_{\alpha^+\alpha}=D_{\alpha\alpha^+}=2\Lambda_3.$$

For a random source

$$\eta_\alpha(z,t)=\begin{pmatrix}\eta(z,t)\\ \eta^+(z,t)\end{pmatrix},$$

which describes in this case a Wiener process, the nonzero correlators are defined by the relations

$$\langle \eta(z,t)\eta(z',t')\rangle = \langle \eta^+(z,t)\eta^+(z',t')\rangle = l\delta_1(t-t')\delta(z-z').$$

Methods of noise-spectrum calculation with the aid of an equation such as (21) are well known, see, e.g., Ref. 25. They duplicate formally the derivations of the preceding sections. Thus, introducing the polar variables $I = \alpha\alpha^+$, $\Phi = (1/2i)\ln(\alpha^+/\alpha)$ we use for the calculation of the mean value $g(z,\tau) = \langle I(z,t)I(z,t+\tau)\rangle$, the small-fluctuation approximation: $I = n + \varepsilon$, $\varepsilon \ll n$. For n we obtain from (21) the quasiclassical equation

$$(\partial_t + \partial_z)n = 2mb_m n^m.$$

As a result, the expression for the noise spectrum takes the form

$$i^{(2)}(\Omega,z) = i_0\left[1 + 2qK^{2m-1}\xi\frac{C}{\Gamma_a}\frac{\Gamma_a^2}{\Gamma_a^2+\Omega^2} + q(K^{2m-1}-1)\left(\frac{m-1}{2m-1} + \frac{2m}{2m-1}\frac{\varphi_a}{\varphi_a-\varphi_b}\right)\right],$$

where $K = P(z)/P(0)$.

It follows hence that NCS fields with suppressed noise appear only in the case of absorption at $m > 1$. Independently of the statistics of the field at the entrance in the cae of a long medium ($K \ll 1$, $\varphi_a \approx 0$).

$$i^{(2)}(\Omega,z) = i_0\left(1 - q\frac{m-1}{2m-1}\right).$$

The shot noise can thus be suppressed at $q \approx 1$ by a factor $m/(2m-1)$. As $m \to \infty$ the suppression level of the shot component of the noise is 50%.

Note that Eq. (21) is precisely the differential equation obtained by using the diagonal representation with $\alpha^+ \to \alpha^*$. Calculations using the Glauber quasiprobability for the noise spectrum lead to the same result as those based on $P(\alpha,\alpha^+)$. This circumstance allows us to assume that a diagonal representation can be used to calculate the observables if the field is in an NCS.

CONCLUSIONS

Light propagation in a medium produces broadband structures in the intensity-fluctuation spectrum. For a multiphoton absorber this lowers the noise output to below the shot value. The obtained shot-noise suppression level is 33% for two-photon absorption and 50% for m-photon absorption as $m \to \infty$. These values are not the theoretical limit, which can reach 100%. But the noise suppression takes place in a broad frequency band $\Delta\omega$ whose value can exceed substantially the width of the excess-noise contour of the initial source.

Effective suppression of excess noise of light calls for a long absorber, $I(0)zB^{-1} \gg 1$. The latter is ensured either by high input light intensity $I(0)$ or by using an extended medium with large z. Numerical estimates show that a reduction of the excess noise by, say, a factor of one hundred takes place at $I(0)zB^{-1} \approx 3.6$; i.e., for $z = 1$ m we have $I(0) = 22$ MW/cm^2, for $z = 100$ m we obtain $I(0) \approx 220$ kW/cm^2.

The necessary condition for suppressing shot noise of light is the onset of an NCS of the field. In multiphoton absorption there is produced a sub-Poisson light which under the above conditions is squeezed in amplitude. Our analysis of the phase fluctuations shows that states squeezed in phase are produced neither in absorption nor in amplification. This is due to the absence of a physical mechanism that stabilizes the phase fluctuations.

A Langevin approach was proposed for the calculation of the statistical characteristics of light propagating in a medium. It yields observable properties without an intermediate calculation of the quasiprobabilities. The results of the diagonal representation and of the generalized P-representation agree. This attests to the possibility of using the diagonal representation along with others in problems with an electromagnetic field in the NCS.

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