Multiscaling in randomly inhomogeneous media: effective conductivity, relative spectral density of 1/f noise, and higher-order moments

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The critical behavior of the concentration dependences of the higher order moments of the current distribution in randomly inhomogeneous media near the percolation threshold is studied. The critical exponents of the higher order moments are expressed in terms of the well-known indices of percolation theory. Scaling functions are proposed on the basis of the similarity hypothesis. The values of the higher-order moments of the current distribution at the percolation threshold are determined. The results are in good agreement with numerical calculations.

One of the main processes studied in randomly inhomogeneous media is current flow. The theoretical study of the current distribution in an inhomogeneous medium can be used both to explain the experimental results (for example, the spectral density of $1/f$ noise of inhomogeneous composite materials) and to construct an adequate model of inhomogeneous media that would make it possible to predict the experimental results and those obtained by numerical methods.

The main characteristic describing current flow in a randomly inhomogeneous medium is the effective conductivity σ_e, which by definition relates the volume-averaged electric current density **j** and electric field intensity **E**:

$$\langle \mathbf{j} \rangle = \sigma_e \langle \mathbf{E} \rangle. \tag{1}$$

If the medium varies macroscopically (the characteristic size a_0 of an inhomogeneity is much greater than the characteristic microscopic free paths), then it is assumed that Ohm's law holds locally: $\mathbf{j}(\mathbf{r}) = \sigma(\mathbf{r})\mathbf{E}(\mathbf{r})$. It can be shown immediately that the effective conductivity does not equal the volume-averaged conductivity $\langle\sigma\rangle$, and in the general case it is impossible to obtain an analytical expression for the effective conductivity as a function of the phase concentration with different values of the local conductivity. Maxwell[1] was one of the first to raise the problem of determining σ_e.

In the last few years two-phase ($\sigma_1 > \sigma_2$), strongly inhomogeneous ($h^{-1} = \sigma_1/\sigma_2 \gg 1$) media near the percolation threshold (see, for example, Refs. 2 and 3) have been under intense study. In such media σ_e exhibits critical behavior:

$$\sigma_e \approx \sigma_1 \tau^t (A_0 + A_1 h \tau^{-(t+q)} + \ldots), \quad \tau > 0, \quad \tau \gg \Delta, \tag{2}$$

$$\sigma_e \approx \sigma_2 |\tau|^{-q} (B_0 + B_1 h \tau^{-(t+q)} + \ldots), \quad \tau < 0, \quad |\tau| \gg \Delta, \tag{3}$$

$$\sigma_e \approx (\sigma_1^q \sigma_2^t)^{1/(t+q)} (C_0 + C_1 \tau / h^{1/(t+q)} + \ldots), \quad |\tau| \approx \Delta, \tag{4}$$

where $\tau = (p - p_c)/p_c$ is the proximity to the percolation threshold, p is the concentration of the phase having good conductivity, p_c is the percolation threshold, $\Delta = h^{1/(t+q)}$ is the width of the critical region, and t and q are the critical exponents (CEs). The values of the critical exponents, as a rule, are obtained by numerical modeling on a network of random resistances $r_1 = \sigma_1^{-1}$ and $r_2 = \sigma_2^{-1}$.

The classical percolation problem—determination of the critical behavior of the conductivity of a network of random resistances—is much more complicated than previously thought.[4–9] It turns out that the critical behavior of such a network is multifractal; i.e., it is described by an infinite collection of CEs. Each moment of the current or voltage distribution has its own CEs. From this viewpoint σ_e is only one, though a very important, characteristic of the critical behavior, associated with the first moment of the current distribution. As is well known (see, for example, Ref. 10),

$$\sigma_e = \frac{\langle (\mathbf{Ej}) \rangle}{\langle \mathbf{E} \rangle^2} \equiv \frac{\langle \sigma^{-1}(\mathbf{r}) \mathbf{j}^2(\mathbf{r}) \rangle}{\langle \mathbf{E} \rangle^2}. \tag{5}$$

We define the moment of nth order as follows:

$$C_e(n) = \frac{\langle C(n,\mathbf{r}) (\mathbf{Ej})^n \rangle}{(\langle \mathbf{E} \rangle \langle \mathbf{j} \rangle)^n}, \quad d = 2, 3, \ldots, \tag{6}$$

where d is the dimension of the problem and $C(n,\mathbf{r})$ is the local moment, whose physical meaning can be different for each n. For example, for $n = 1$ and any d we have $C(n,\mathbf{r}) = 1$ and $C_e(1) = 1$, whence follows Eq. (5).

The second moment is an equally important characteristic of inhomogenous media. As shown, for example, in Ref. 11, $C(2)$ is the relative spectral density of $1/f$ noise of the entire medium. In this case $C(2,\mathbf{r} = \{\delta\sigma\delta\sigma\}/\sigma^2$ is the Fourier component of the mean-square fluctuation of the local conductivity and determines the $1/f$ noise of a homogeneous material with conductivity σ. The more inhomogeneous the medium is the higher is the level of $1/f$ noise in it.[11–13] Model, numerical, and experimental investigations of $1/f$ noise in media close to the percolation threshold have shown that its amplitude exhibits critical behavior—it increases according to a power law as p approaches p_c from either side.

In order to determine the critical behavior of the moment $C_e(n,d)$ for any n it is necessary to know, as follows from Eq. (6), at least approximately, the distribution of the fields and currents. In randomly inhomogeneous media this is possible either by performing numerical modeling or by using a model of the medium for which an analytical calculation can be performed. The zeroth-order approximation in h is sufficient for calculating σ_e in the simplest case and the basic behavior of σ_e above and below the percolation threshold remains unchanged [the first terms in Eqs. (2)–(3)], but this approximation is no longer applicable for $n = 2$. In Ref. 14 it was pointed out that above the percolation threshold, when current flows primarily along the phase with good

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conductivity, taking into account the poorly conducting phase (σ_2) when calculating $1/f$ noise ($n = 2$) can result in a different critical exponent. In Ref. 15 the critical indices of $1/f$ noise were obtained above, below, and at the percolation threshold and the conditions under which the CEs change (crossover) were determined on the basis of the weak-link model (WLM), which takes into account current flow through both phases simultaneously. In Ref. 16 the h dependence of $C_e(2)$ at the percolation threshold was checked for $d = 2.3$ by numerical simulation. In Refs. 6 and 7, on the basis of numerical modeling, numerical values were found for the CEs of the nth moments ($n = 1,2,3$) and a scaling form was proposed for $C_e(n,d)$. In Ref. 9 scaling of the moments was examined and crossover above the percolation threshold was proved numerically for $d = 2$.

In the present work, analytical expressions are obtained for the CEs, making it possible to describe the critical behavior of $C_e(n)$ above, below, and at the percolation threshold for any n and d, on the basis of a hierarchical weak-link model. A comparison was made with the published results of numerical modeling. We note immediately that the numerical values of the analytical expressions obtained for the CEs on the basis of the WLM agree satisfactorily with all numerical results known to us and they thereby make it possible to express the infinite set of CEs, obtained by numerical modeling for $n = 1, 2, ...$ in terms of only the three main CEs—two CEs for the conductivity (t and q) and one CE for the correlation length (ν). At the end of this paper it is shown that instead of partially summing the terms in $C_e(n)$ over powers of h, an "exact" expression, an analog of the Dyson equation, can be written down for $C_e(n,d)$.

1. HIERARCHICAL WEAK-LINK MODEL

The first model of a percolation medium above the percolation threshold ($\tau > 0$) was the model constructed by Skal and Shklovskii (Ref. 17) and de Gennes (Ref. 18). This model, though it makes it possible to describe the most important aspect of such systems, their critical behavior, leads to a contradiction in the case $d = 2$. An analogous model, but for the case $\tau < 0$, was proposed in Ref. 19. The Skal–Shklovskii–de Gennes model was extended in a number of papers, where, for example, in the "colored" model,[20,21] additional CEs, which in turn must be found numerically, are introduced. In order to describe the kinetic properties, for example, σ_e, more accurately it was necessary to employ more complicated, fractal (self-similar) models, in particular, the drop model[22] for the case $\tau > 0$.

We note that in all of the models indicated above, first, the main problem is to determine the conductivity exponents t and q in terms of, for example, the critical exponents of the correlation length ν and, second, the case $h = \sigma_2/\sigma_1 = 0$ is studied. In the process it is assumed that in the case $\tau > 0$ the poorly conducting phase does not conduct current at all ($\sigma_2 = 0$) while in the case $\tau < 0$ the voltage drop across the phase with good conductivity can be neglected ($\sigma_1 = \infty$).

In Ref. 15 the so-called weak-link model was proposed. In this model, first, it is assumed that the critical exponents t and q are known and are used to determine the geometry of the model and, second, current flow through both phases simultaneously is taken into account. We examine this model in detail, since all of the numerical data can be described by calculating the critical exponents on the basis of this model.

Above the percolation threshold ($\tau > 0$) the first step in the hierarchy of the WLM is a long thin bridge between two bases—two sections of an infinite cluster, the voltage drop across which can be neglected. The difference between the WLM and the Skal–Shklovskii–de Gennes model and the NLB (node link-blob, where the bridge is called the link and the base is called the blob) model is fundamental and consists of the fact that the bridge length l is not sought on the basis of different probablistic considerations (as is the length of singly connected links), but rather it is postulated so as to have an expression for σ_e ($\tau > 0$) in the zeroth-order approximation in h. The area S of the thin interlayer consisting of the poorly conducting phase in the first step of the hierarchy of the WLM below the percolation threshold is also determined similarly. Thus, equating the resistance of the section of the medium of size $L \sim \xi$ (ξ is the correlation length) to the resistances of the bridge ($p > p_c$) and the interlayer ($p < p_c$) (see Fig. 1a), i.e., assuming that

$$\frac{1}{\sigma_1 \tau^t}\frac{\xi}{\xi^{d-1}} = \frac{1}{\sigma_1}\frac{l}{a_0^{d-1}}, \qquad \frac{1}{\sigma_2 |\tau|^{-q}}\frac{\xi}{\xi^{d-1}} = \frac{1}{\sigma_2}\frac{a_0}{S}, \tag{7}$$

and taking into account the fact that $\xi \sim a_0|\tau|^{-\nu}$, we obtain the following basic conditions of the WLM:

$$l \sim a_0 |\tau|^{-\zeta_R}, \quad S \sim a_0^{d-1}|\tau|^{-\zeta_G}, \tag{8}$$

where

$$\zeta_R = t-(d-2)\nu, \quad \zeta_G = q+(d-2)\nu, \tag{9}$$

and ζ_R and ζ_G are known exponents in percolation theory and are related to the average resistance and average conductance of the medium.

The second step in the WLM hierarchy takes into account the finite conductivity of the poorly conducting ($\sigma_2 \neq 0$) and well conducting ($\sigma_1 \neq \infty$) phases. This means that for $p > p_c$ not only the current in the bridge but also the current flowing through the interlayer oriented parallel to it (Fig. 1c) are taken into account. In the case $p < p_c$ the bridge is connected in series with the interlayer (Fig. 1d). This step of the hierarchy corresponds to the first two terms in Eqs. (2) and (3). The next, third, step is shown in Figs. 1e and f. The equivalent electrical circuits of these steps (in the absence of a magnetic field and thermo-emf) are presented in Fig. 2. Note that all bridges and interlayers have the same geometry [the same length and area (8)]; i.e., for any step in the WLM hierarchy the geometry of the weak links is determined by only two relations.

2. MOMENTS OF THE CURRENT DISTRIBUTION IN A PERCOLATION SYSTEM

We first study the current distribution for $p > p_c$, using the second step of the WLM hierarchy (Fig. 1c). According to Eq. (6),

$$C_c(n) \sim \frac{C_1(n) j_1^n E_1^n V_b/V + C_2(n) j_2^n E_2^n V_{\rm int}/V}{\sigma_e^n \langle E\rangle^{2n}}, \tag{10}$$

where j_1 and E_1 are the current and field in the bridge, j_2 and E_2 are the current and field in the interlayer; $V_b \approx a_0^{d-1} l$ and

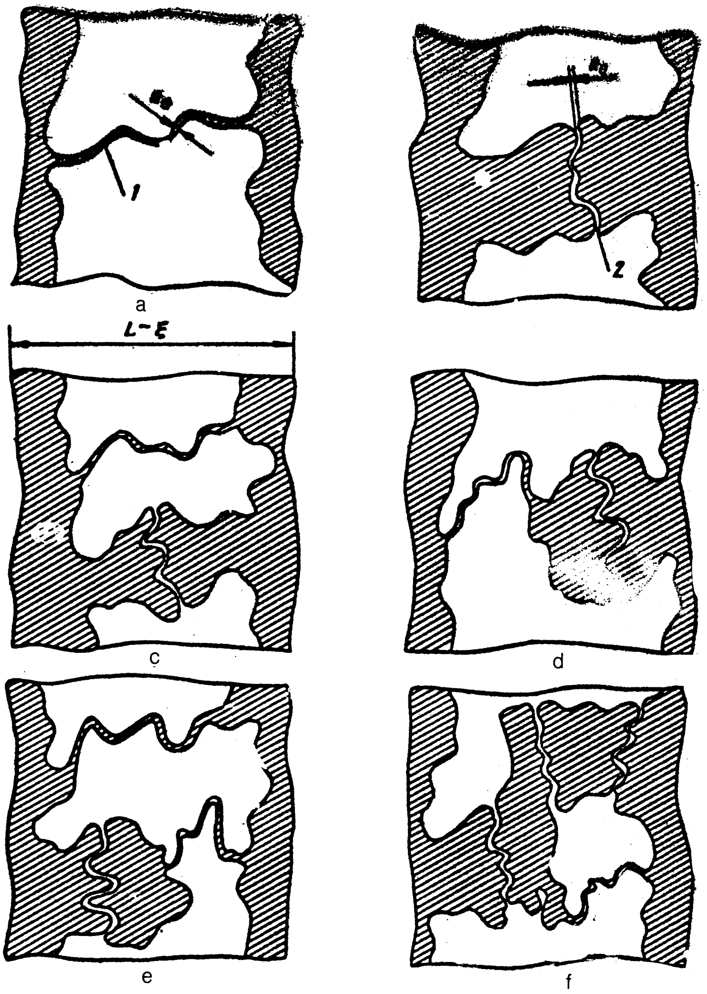


FIG. 1. Hierarchical weak-link model. The phase with good conductivity is hatched. 1—bridge, 2—interlayer, a_0—minimum size of the structure (in the lattice model—the link length); a, c, c, (b, d, f) are the first, second, and third steps in the hierarchy, which correspond to the first term, first two terms, and first three terms in Eqs. (2) and (3), respectively.

$V_{\rm int} \approx a_0 s_{\rm int}$ are the volumes of the bridge and the interlayer, respectively; $V \sim L^d$; and $E \approx \Delta\varphi / L$, where $\Delta\varphi$ is the voltage drop across the distance L. Turning to the equivalent circuit (Fig. 2), we find

$$j_1=\sigma_1 E_1 \approx \sigma_1 \frac{\Delta\varphi}{l}, \quad j_2=\sigma_2 E_2 \approx \sigma_2 \frac{\Delta\varphi}{a_0}. \tag{11}$$

Substituting Eqs. (11) into Eq. (10) and using Eqs. (8) and (9), after elementary transformations we obtain ($n > 1$)

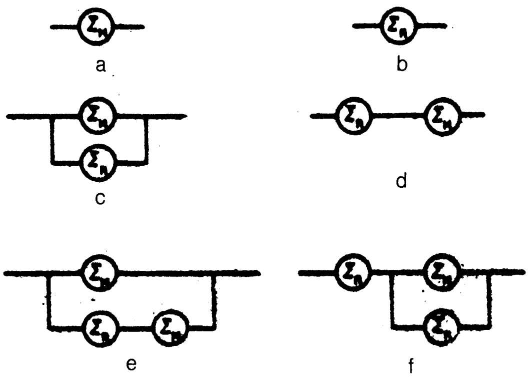


FIG. 2. Equivalent electrical circuits of the steps in the hierarchy.

$$C_e(n) \approx C_1(n)\tau^{-k_n} + C_2(n) h^n \tau^{-w_n}, \tag{12}$$

and similarly for the case $p < p_c$

$$C_e(n) \approx C_2(n)|\tau|^{-k_n'} + C_1(n) h^n |\tau|^{-w_n'}. \tag{13}$$

Here

$$k_n = [(2\nu(d-1) - (t)](n-1), \quad k_n' = (2\nu - q)(n-1),$$

$$w_n = k_n' + n(t+q), \quad w_n' = k_n + n(t+q). \tag{14}$$

We note that in order to determine the critical behavior it is likewise possible to use not $C_e(n)$, i.e., a specific characteristic, but rather $\mathscr{S}_e(n)$, characterizing the moment (for $n = 2$, $1/f$ noise) of the entire sample with volume V_0. In Ref. 13 analogs of Kirchhoff's laws were established for $\mathscr{S}_e(n \neq 2)$. These laws are easily extended to the case of arbitrary n:

$$\mathscr{S}_e = \sum_i \left(\frac{R_i}{R_e}\right)^n \mathscr{S}_i(n), \quad \mathscr{S}_e = \sum_i \left(\frac{R_e}{R_i}\right)^n \mathscr{S}_i(n), \tag{15}$$

where the first and second relations refer to resistances R_i connected in series and in parallel, respectively.

It is easy to find the relation between the notations adopted in Ref. 13 and C_e:

$$C_e(n)=V_0^{n-1}\mathcal{F}_e(n). \quad (16)$$

Taking into account the first nonzero in h terms makes it possible to establish the behavior of the moments $C_e(n)$ at the percolation threshold. These moments can be obtained by equating τ and Δ, i.e., the width of the critical region. In so doing, we obtain from both Eqs. (12) and (13)

$$C_e(n)\sim C_1(n)h^{-s_n}+C_n(n)h^{-z_n}, \qquad h=\sigma_2/\sigma_1, \quad (17)$$

where the critical exponents s_n and z_n are as follows:

$$s_n=k_n/(t+q), \qquad z_n=k_n'/(t+q). \quad (18)$$

Substituting into Eq. (18) the numerical values of the critical exponents t, q, and ν for any d, we can easily show that s_n and z_n are positive and therefore at the percolation threshold $1/f$ noise and other moments increase with increasing inhomogeneity of the medium, i.e., with increasing h^{-1}. We also note that for all d, except $d=2$, the relation $s_n-z_n\approx 0.3(n-1)$ is satisfied. As for 5the two-dimensional case, it is shown in Ref. 10 that at the percolation threshold the average values of the nth powers of the dissipated energies are the same in both phases and hence the difference in the terms in Eq. (17) is determined only by the difference between $C_1(n)$ and $C_2(n)$. Indeed, for $d=2$ we obtain from Eqs. (18)

$$s_n=z_n=(n-1)(2\nu-t)/(t+q)\approx(n-1)/2.$$

3. ANALYSIS OF RESULTS

We now compare the resulting critical exponents with the known results of numerical modeling. It follows from Eqs. (12), (13), and (17) that the second terms, which originate from the additional elements of the structure (parallel interlayer for $p>p_c$ and series bridge for $p<p_c$), can be comparable to and even greater than the first terms, depending on the value of the ratio $C_1(n)/C_2(n)$. For the case $n=2$ ($1/f$ noise) this fact was first noted in Refs. 14 and 15. In Ref. 15 the critical indices k_2, k_2', w_2 and w_2' were also found. Detailed numerical investigations of the behavior of the nth moments were performed in Refs. 7 and 8. Thus, for example, for the percolating regions ($\tau>0$) the nth moment is defined as

$$C_P(n,L)=G_P(n,L)C_1(n)+I_P(n,L)C_2(n), \quad (19)$$

where

$$G_P=\sum_\alpha (i_\alpha^2)^n C_\alpha(n), \qquad I_P=\sum_\alpha (i_\alpha^2)^n C_\alpha(n)$$

and in addition the summation in G_p extends over the well-conducting phase and the summation in I_p extends over the poorly conducting phase; i_α is the current flowing through the αth link; and a unit current is assumed to flow through the sample as a whole. The moment for the nonpercolating samples ($\tau<0$) G_{NP} is written similarly. In the same work a scaling transformation is proposed for G_P, I_P, G_{NP}, and I_{NP}, for example,

$$G_P(n,L,\Delta p,h)=\lambda^{-x_n}G_P(n,L/\lambda,\Delta p/\lambda^{-1/\nu},h\lambda^{\varphi}),$$
$$I_P(n,L,\Delta p,h)=\lambda^{y_n}(n\lambda^{\varphi})^{2n}I_P(n,L/\lambda,\Delta p/\lambda^{-1/\nu},h\lambda^{\varphi}), \quad (20)$$

where $\phi=-(x_1+y_1)\equiv(t+q)/\nu$ is the so-called crossover exponent.

With the help of the Monte Carlo method the functions

$$\lg[L^{x_n}G_P(n,L,p=p_c,h)],$$
$$\lg[L^{-y_n}(nL^{\varphi})^{-2n}I_P(n,L,p=p_c,h)]$$

and similar expressions for G_{NP} and I_{NP} as a function of $\log(hL^\phi)$ were obtained for $n=1, 2, 3$ and $d=2,3$. One of these functions is shown in Fig. 3. Numerical values of x_n and y_n were established on the basis of these calculations.[7,8]

In order to compare with the results of Refs. 7 and 8 it is necessary to find a relation between G_P, G_{NP}, ... and C_e. It is easy to show that this relation has the form

$$\sum_\alpha (i_\alpha^2)^n=\frac{1}{C}\left(\frac{L}{a_0}\right)^{4n-3}\frac{C_e(n)}{\sigma_e^n}. \quad (21)$$

It is understood that the summation or integration on both the left- and right-hand sides of Eq. (21) extends only over one of the phases. Comparing Eqs. (12) and (13) with Eq. (20), we find on the basis of Eq. (21) the following expressions for x_n and y_n:

$$-\nu x_n=t-(d-2)\nu\equiv\zeta_R,$$
$$-\nu y_n=(2n-1)[q+(d-2)\nu]\equiv(2n-1)\zeta_G. \quad (22)$$

The values of $-\nu x_n$ and $-\nu y_n$ obtained in Refs. 7 and 8 and with the help of the WLM (22) are presented in Table I. The agreement between these values is satisfactory in spite of the simplicity of the WLM. Using the $\varepsilon=6-d$ expansion ($d_c=6$ is the critical dimension[23]) and taking the critical exponents $t=3-0.24\varepsilon$ and $\nu=0.5+0.06\varepsilon$, we obtain $k_n\approx 2(n-1)-0.16(n-1)\varepsilon$ and $-\nu x_n\approx 1+0.02\varepsilon$. For the critical dimension $d_c=6$ we obtain for these critical exponents $k_n=2(n-1)$, $k_n'=n-1$, $-\nu x_n=1$, and $-\nu y_n=2(2n-1)$.

We note that using the analytical relations for the critical exponents y_n and x_n and the crossover exponent $\phi=-(x_1+u_1)$, we obtain from Eq. (22) for any dimension of the problem

$$y_n=-(2n-1)(x_n+\phi). \quad (23)$$

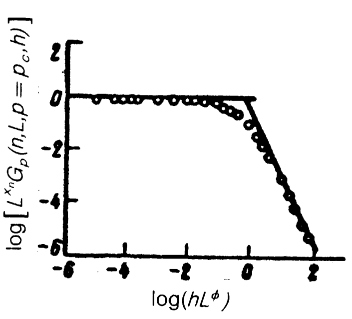


FIG. 3. Multifractal moment of a percolation sample for $n=3$, $d=2$: Monte Carlo results from Ref. 7. The straight lines correspond to the weak-link model.

TABLE I.

n	$-\nu x_n$		$-\nu y_n$	
	WLM (22)	Data from Refs. 6 and 7	WLM (22)	Data from Refs. 6 and 7
		$d=3$		
1	0,9	1,12	1,9	1,75
2	0,9	1,1	5,7	5,62
3	0,9	1,03	9,5	9,3
		$d=2$		
1	1,3	1,3	1,3	1,3
2	1,3	1,1	4,0	4,0
3	1,3	1,02	6,6	6,9

The equation (23) agrees well with the numerical results of Refs. 7 and 8.

4. SCALING OF HIGHER-ORDER MOMENTS

The series expansion of σ^e (2)–(4) is based on the similarity hypothesis,[2] according to which

$$\sigma_e(\tau,\ h)=\sigma_1 h^s F(\tau/h^m), \qquad (24)$$

where $s=t/(t+q)$, $m=s/t$, and $F(z)=1$ for $Z=0$ and is a power-law function as $Z\to\pm\infty$. The function $F(z)$ is customarily called a scaling function.

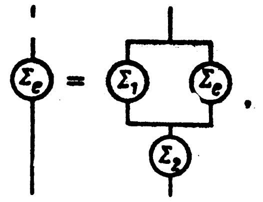
In the study of the critical behavior of the moments of the current distribution there naturally arises the question of how to write a relation $C_e(n)$ similar to Eq. (24) and the question of the asymptotic behavior of functions similar to $F(\tau/h^m)$. In order to answer these questions we turn to the WLM and we examine first the case $\tau>0$. The equivalent circuits presented in Figs. 2a, b, and e can be interpreted as partial sums of an infinite series. According to the WLM, the structure of the medium, i.e., the arrangement of the bridges and interlayers with respect to one another, is self-similar. For this reason, it is possible to write in the language of Fig. 2 a closed equation for the conductance Σ_e, the analog of the Dyson equation (see, for example, Ref. 24), to which there corresponds the quadratic equation

$$\Sigma_e^2-\Sigma_1\Sigma_e-\Sigma_1\Sigma_2=0. \qquad (25)$$

Since, according to Eqs. (7) and (8),

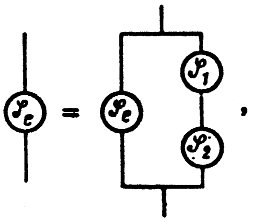
$$\Sigma_e=\sigma_e a_0^{d-2}\tau^{-\nu(d-2)},\quad \Sigma_1=\sigma_1 a_0^{d-2}\tau^{t_R},\quad \Sigma_2=\sigma_2 a_0^{d-2}\tau^{-t_G}, \qquad (26)$$

the solution of Eq. (25) can be expanded in a series of the small parameter $h\tau^{-(t+q)}$, whence Eq. (2) follows. A closed equation can also be written down for σ_e in the case $\tau<0$:



(27)

The analog of the Dyson equation can be obtained for $C_e(n)$ in a manner similar to the closed equation for σ_e. For example, for $\tau>0$



(28)

whence, according to Eq. (15),

$$\mathcal{S}_e^{(n)}=\left(\frac{\Sigma_1}{\Sigma_e}\right)^n\mathcal{S}_1+\left(\frac{\Sigma_2}{\Sigma_2+\Sigma_e}\right)^n\left[\left(\frac{\Sigma_e}{\Sigma_2+\Sigma_e}\right)^n\times\mathcal{S}_2^{(n)}+\left(\frac{\Sigma_2}{\Sigma_2+\Sigma_e}\right)^n\mathcal{S}_e^{(n)}\right]. \qquad (29)$$

We note that Eq. (29), in contrast to Eq. (25), is a linear equation. The equation (29) answers the question of scaling, since, rewriting Eq. (29) in the form

$$\mathcal{S}_e(n)=\mathcal{S}_1(n)\frac{(\Sigma_1/\Sigma_e)^n}{1+(\Sigma_2/(\Sigma_2+\Sigma_e))^{2n}}+\mathcal{S}_2(n)\frac{\Sigma_2^n\Sigma_e^n}{(\Sigma_2+\Sigma_e)^{2n}+\Sigma_2^{2n}}. \qquad (30)$$

we can express $\mathcal{S}_e$ in terms of $\mathcal{S}_1$ and $\mathcal{S}_2$ and the scaling functions Σ_e which are now known.

Switching to specific quantities and performing calculations similar to those performed above, we obtain for the case $\tau<0$

$$\mathcal{S}_e(n)=\mathcal{S}_2\left(\frac{\Sigma_e}{\Sigma_2}\right)^n\left(1+\frac{\Sigma_e}{\Sigma_1}\right)^{2n}+\mathcal{S}_1\left(\frac{\Sigma_e}{\Sigma_1}\right)^n. \qquad (31)$$

We designate the terms in parentheses in Eqs. (2) and (3) as $g_+(\tilde{h})$ and $g_-(\tilde{h})$, where $\tilde{h}=h\tau^{-(t+q)}$, i.e.,

$$\sigma_e(\tau>0)=\sigma_1\tau^t g_+(\tilde{h}),\quad \sigma_e(\tau<0)=\sigma_2|\tau|g_-(\tilde{h}).$$

The functions $g_\pm(\tilde{h})$ determine the scaling corrections and are related as follows to $F(\tau/h^m)$:

$$g_+(\tilde{h})=(h^m/\tau)^t F(\tau/h^m),\quad \tau>0,$$
$$g_-(\tilde{h})=(|\tau|/h^m)^q F(q/h^m),\quad \tau<0. \qquad (32)$$

In terms of $g_+(\tilde{h})$ and $g_-(h)$ the specific characteristics

$C_e(n)$, as follows from Eqs. (30) and (31), substituting Eq. (16), have the form

$$C_e(n)=C_1(n)\tau^{-k_n}f_1(g_+)+C_2(n)\tilde{h}^n\tau^{-k_n'}f_2(g_+),\quad \tau>0,$$
$$C_e(n)=C_2(n)|\tau|^{-k_n'}f_3(g_-)+C_1\tilde{h}^n|\tau|^{-k_n}f_4(g_-),\quad \tau<0, \tag{33}$$

where the small corrections (multipliers of order unity) f_i are expressed as follows in terms of $g_\pm$:

$$f_1(g_+)=\frac{1}{g_+^n}\cdot\frac{(g_++\tilde{h})^{2n}}{(g_++\tilde{h})^{2n}-\tilde{h}^{2n}},\qquad f_2=\frac{g_+^n}{(g_++\tilde{h})^n-\tilde{h}^{2n}},$$
$$f_3(g_-)=g_-(1+g_-)^{2n},\quad f_4(g_-)=g_-^n. \tag{34}$$

Thus the relations (33) answer the question of the scaling of the higher-order moments, since the corrections f_i are expressed in terms of the known scaling function $F(\tau/h^m)$.

In conclusion we note that it would be interesting to investigate the higher-order moments of the current distribution for media with no minimum size, for example, for the so-called "swiss cheese" media.[25] These investigations were initiated in Ref. 26. We hope that elaboration of the approach proposed here will help obtain results for this case also.

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[1] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon, Oxford, 1960.
[2] A. L. Afros and B. I. Shklovskii, Phys. Status Solidi B **76**, 475 (1976).
[3] D. Stauffer, *Introduction to Percolation Theory*, London (1985), p. 123.
[4] L. Arcangelis, S. Redner, and A. Coniglio, Phys. Rev. B **31**, 4725 (1986).
[5] L. Arcangelis, S. Redner, and A. Coniglio, Phys. Rev. B **34**, 4656 (1986).
[6] A.-M. S. Tremblay, Canad. Research, 11–14, 29–30 (1988).
[7] R. R. Tremblay, G. Albinet, and A.-M. S. Tremblay, Phys. Rev. B **43**, 11546 (1991).
[8] R. R. Tremblay, G. Albinet, and A.-M. S. Tremblay, Preprint, Centre de Recherche on Phys. du Solid, Univ. de Sherbrooke, Canada (1991).
[9] A. Kolek, Phys. Rev. B 1991 (to be published).
[10] A. M. Dykhne, Zh. Eksp. Teor. Fiz. **59**, 110 (1970) [Sov. Phys. JETP 32(1), **63** (1971)].
[11] R. Rammal, J. de Phys. Lett. **46**, L129 (1985).
[12] Sh. M. Kogan, Usp. Fiz. Nauk **145**, 285 (1985) [Sov. Phys. Usp. **28**(2), 170 (1985)].
[13] R. Rammal, C. Tannous, and A.-M. S. Tremblay, Phys. Rev. A **31**, 2662 (1985).
[14] A.-M. S. Tremblay, B. Fourcade, and P. Breton, Physica A **157**, 89 (1989).
[15] A. E. Morozovskii and A. A. Snarskii, Zh. Eksp. Teor. Fiz. **95**, 1844 (1989) [Sov. Phys. JETP **68**(5), 1066 (1989)].
[16] I. V. Bezsudnov, S. A. Kucherov, and A. A. Snarskii in *Abstracts of Reports at the 2nd All-Union Conference on Computational Physics and Mathematical Modeling, Volgograd,* 1989, Moscow (1990), p. 9.
[17] A. S. Skal and B. I. Shklovskii, Fiz. Tekh. Poluprovodn. 8, 1586 (1974) [Sov. Phys. Semicond. **8**, 1029 (1975)].
[18] P. G. de Gennes, J. de Phys. **37**, L1 (1976).
[19] D. C. Wright, D. G. Bergman, and Y. Kantor, Phys. Rev. B **33**, 396 (1986).
[20] H. E. Stanley, Lect. Not. Phys. **149**, 59 (1981).
[21] R. Pike and H. E. Stanley, J. Phys. A **14**, L169 (1981).
[22] A. K. Sarychev and A. P. Vinogradov, J. Phys. C **14**, L487 (1981).
[23] Y. Parck, B. Harris, and T. C. Lubensky, Phys. Rev. B **35**, 5048 (1987).
[24] R. Mattuck, *A Guide to Feynman Diagrams in the Many-Body Problem*, McGraw-Hill, New York, 1967.
[25] S. Feng, B. I. Halperin, and P. N. Sen, Phys. Rev. B **35**, 197 (1987).
[26] D. J. Bergman, Phys. Rev. B **29**, 4598 (1989).

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