Spatiotemporal distributions of solitons and dislocations in charge-density waves

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The equations of the dissipative dynamics of charge-density waves (CDW) in the presence of a continuous distribution of dislocations or solitons are derived. The response functions of the fields and the related correlation function of the currents for the process of spontaneous conversion of electrons into solitons are found. The one-dimensional development of a current pulse from a narrow injecting contact in a thin sample is investigated in detail. The problem is solved in the purely dissipative regime of the CDW dynamics and in the diffusion approximation for the gas of solitons. It is found that, at first, over very short times, the nominal values of the CDW current j_∞, CDW phase velocity $\beta_\infty = -\pi j_\infty$, and electric field $E_\infty \propto j_\infty$ are established over the whole sample. However, as the diffusion front of the gas of solitons passes through with constant velocity c and the soliton concentration ρ_s increases ($c = bE_\infty$, where b is the mobility of the solitons), the CDW velocity $\beta(x,t)$ and electric field $E \propto \beta$ decrease. In a characteristic regime, $j(x,t) \propto \rho_s^{-1} \propto t^{-1/3}$. Stationary distributions are also found for injectionless generation of solitons in the pinning layer upon passage of a CDW current.

1. THE BASIC EQUATIONS

1. From the point of view of elasticity theory a charge-density wave (CDW) is a crystal possessing only uniaxial strains (see the literature in Refs. 1 and 2):

$$\omega = \nabla\varphi, \tag{1}$$

where ω is the locally defined continuous gradient of the CDW phase φ, which can be restored from (1) as a many-valued continuous function. The vector ω is conjugate to the stress vector $\boldsymbol{\sigma}$:

$$\sigma \propto \hat\Lambda\omega \propto \frac{\delta W}{\delta\omega}, \qquad \omega \propto \hat\Lambda^{-1}\sigma = -\frac{\delta U}{\delta\sigma}, \tag{2}$$
$$U = W - \int \omega\sigma d^3r,$$

where $\hat\Lambda$ is the elasticity tensor, and W and U are the thermodynamic potentials as functions of ω and σ, respectively.

The equilibrium state of the system is determined by minimization of the potential U, which ensures the correct sign (repulsion) of the interaction of the dislocations. Minimization of W would lead, in analogy with the case for the currents in magnetostatics, to attraction of dislocations of the same sign. This fact somehow escapes attention in the literature, and the sign is corrected at the level of the calculation of the forces.

In the harmonic approximation, which is adequate to our problem, the functional W has the form

$$W\{\omega,\Phi\} = \frac{v}{4\pi s}\int d^3r\left[\omega\hat\Lambda_0\omega + \frac{2}{v}\omega_x\Phi - \frac{2}{\varkappa^2 v^2}(\nabla\Phi)^2\right], \tag{3}$$

where Φ is the potential of the electric field, s is the cross-sectional area of one chain,

$$\varkappa^2 = 8e^2/\hbar v s = r_D^{-2},$$

e is the electron charge, and v and r_D are the Fermi velocity and Debye radius of the initial metal. Henceforth, $\hbar = 1$.

If we are not interested in the boundary conditions related directly to the electric field, we can eliminate Φ from (3) and obtain

$$W\{\omega\} = \frac{v}{4\pi s}\int d^3r\omega\hat\Lambda\omega, \tag{4}$$

$$\Lambda_{ik} = (\Lambda_0)_{ik} - \frac{\varkappa^2}{\Delta}n_i n_k, \qquad \Lambda_0 = \mathrm{diag}(1, \alpha_y, \alpha_z), \qquad \Delta = \nabla^2. \tag{5}$$

Here, $\mathbf{n} = (1,0,0)$ is the unit vector in the direction of the axis x of the chain; α_y and α_z are the anisotropies of the elastic moduli. It follows from (2) and (5) that

$$\sigma = \hat\Lambda\omega = \hat\Lambda_0\omega + 2\mathbf{n}\Phi. \tag{6}$$

The quantity

$$\mathbf{F} = \frac{v}{2\pi}[\sigma d\mathbf{l}] \tag{7}$$

determines the force acting on an element of length dl of the dislocation.[3,4] The component

$$V = \mathbf{n}\sigma = \omega_x + 2\Phi/v \tag{8}$$

plays the role of the potential energy for 2π-solitons, while $V/2$ plays this role for above-gap electrons and for amplitude π-solitons. Finally, static equilibrium is determined by the condition

$$\delta = 0, \qquad \delta = \nabla\sigma = \hat\nabla\omega - \frac{2}{v}\mathbf{E}\mathbf{n}, \qquad \mathbf{E} = -\nabla\Phi, \tag{9}$$

where $\mathbf{E}$ is the electric-field intensity and

$$\hat\nabla = \Lambda_0\nabla, \quad \nabla\hat\nabla = \hat\Delta, \quad \hat\nabla\omega = \hat\Delta\varphi, \tag{10}$$

where $\hat\nabla$ and $\hat\Delta$ are the anisotropic gradient and anisotropic Laplacian. (Here and below, all notation corresponds to that in Refs. 1 and 2).

There is a fundamental difference between the dynamics of CDW and the dynamics of normal crystals. Owing to friction against the underlying lattice, dissipation appears that is directly proportional to the velocity $\dot\varphi = \partial\varphi/\partial t$ and not to its gradients. The complete system of equations of the dynamics of the medium for a given instantaneous distribution of defects is formed in terms of the density ρ and flux $\mathbf{I}$ of the dislocations:[3,4]

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$$[\nabla\omega]=-\rho, \tag{11}$$

$$\nabla\beta=\dot{\omega}-\mathbf{I},\quad \beta=\varphi, \tag{12}$$

$$\nabla\rho=0,\quad \dot{\rho}+[\nabla\mathbf{I}]=0, \tag{13}$$

$$\nabla\sigma=\gamma\beta+\ddot{\beta}/u^2,\quad \nabla\sigma=\delta, \tag{14}$$

where $\dot{f}\equiv\partial f/\partial t$ and γ is the damping coefficient. Thus, for a dislocation line element $\mathbf{dl}=\boldsymbol{\tau}dl$ being displaced with velocity $\mathbf{V}_0$ we have

$$\rho=-2\pi\boldsymbol{\tau}\delta(\boldsymbol{\xi}),\quad \mathbf{I}=[\mathbf{V}_0\rho],\quad \boldsymbol{\xi}=\mathbf{r}-(\boldsymbol{\tau}\mathbf{r})\boldsymbol{\tau}. \tag{15}$$

Henceforth, for brevity, we shall omit the inertial term $\ddot{\beta}/u^2$ in (14), where u is the phase velocity of the CDW, bearing in mind that, where necessary, it can be restored by means of the replacement

$$\gamma\beta\rightarrow\gamma\beta+\ddot{\beta}/u^2. \tag{16}$$

The kinematic equations are simplified if the constraints (13) are solved in terms of the dislocation moment $\mathbf{P}$ (Refs. 3, 4), so that

$$\rho=[\nabla\mathbf{P}],\quad \mathbf{I}=-\dot{\mathbf{P}}. \tag{17}$$

Equations (11)–(13) reduce to the single equation

$$\omega+\mathbf{P}=\nabla\varphi, \tag{18}$$

which relates the elastic, plastic, and total deformations. We recall that only the quantity ω is a state variable, in view of its relation to σ. The functions $\mathbf{P}$ and $\nabla\varphi$ have jumps on discontinuities along certain surfaces that abut on dislocation lines. These surfaces can be defined arbitrarily at a certain initial moment, and only their evolution by virtue of (13) is determined uniquely over the surface of the cylinders that are formed by the moving dislocation lines.

Our formulas (11)–(14) describe both the dynamics of individual dislocation lines [with the functions ρ, $\mathbf{I}$, and $\mathbf{P}$ determined by Eqs. (15) and (17)] and the dynamics of the continuous distribution of dislocation loops. The function $\mathbf{P}$ in the latter case is an average over a macroscopic volume containing a large number of closed dislocation lines. As is well known,[3] the averaged quantity $\mathbf{P}$ no longer has nonphysical discontinuities, and becomes a state variable. For a system of closed dislocation loops with total area $\mathbf{S}$ in volume $d\mathscr{V}$, it is determined from the condition

$$\mathbf{P}d\mathscr{V}=-2\pi d\mathbf{S}/s. \tag{19}$$

For a system of $\pm 2\pi$-solitons with linear densities $\rho_\pm$,

$$\mathbf{P}=(P_x,0,0),\quad P_x=2\pi(\rho_--\rho_+). \tag{20}$$

We shall now introduce closed equations for the quantities δ and V. Eliminating β from Eqs. (12) and (14), we obtain

$$\nabla(\nabla\sigma)=\gamma(\dot{\omega}-\dot{\mathbf{I}}). \tag{21}$$

We multiply (21) by Λ and make use of (6):

$$(\hat{\Lambda}\nabla)\nabla\sigma=\gamma(\dot{\sigma}-\hat{\Lambda}\mathbf{I}). \tag{22}$$

By multiplying (22) by ∇, taking (5), (6), and (10) into account we find a closed equation for δ:

$$\mathbb{K}\delta=\gamma\varkappa^2(\mathbf{n}\nabla)(\mathbf{nI})-\gamma\Delta(\hat{\nabla}\mathbf{I}), \tag{23}$$

where

$$\mathbb{K}=-\gamma\Delta\frac{\partial}{\partial t}+K,\quad K=\Delta\nabla\hat{\Lambda}\nabla=\Delta\hat{\Delta}-\varkappa^2\frac{\partial^2}{\partial x^2}. \tag{24}$$

Although the second term in the right-hand side of (23) is small in the parameter $\Delta/\varkappa^2$ in comparison with the first, it can become important, since the first term vanishes for motion of the dislocation along $\mathbf{n}$, when $\mathbf{n}\cdot\mathbf{I}=0$.

Multiplying (22) by $\mathbf{n}$, we obtain an equation relating $\mathbf{V}$ to δ:

$$\gamma\Delta\dot{V}=(\Delta-\varkappa^2)(\mathbf{n}\nabla)\delta+\gamma(\Delta-\varkappa^2)(\mathbf{n}\dot{\mathbf{I}}). \tag{25}$$

In the right-hand side of Eq. (25), for small gradients, we can always neglect Δ in comparison with $\varkappa^2$. An independent equation for V can be obtained by acting on (25) with the operator $\mathbb{K}$ and making use of (24). After transformations we obtain an equation that admits lowering of the order, as a result of which we obtain

$$\mathbb{K}V=(\varkappa^2-\Delta)([\hat{\nabla}\mathbf{n}]\rho)+\gamma(\varkappa^2-\Delta)(\mathbf{nI}). \tag{26}$$

The first term in the right-hand side corresponds to the static problem, and the second is realized in the case of transverse motion of the dislocation or when solitons are created, i.e., in cases of increase of the charge.

In terms of $\mathbf{P}$, Eqs. (23) and (25) acquire the form

$$\mathbb{K}\delta=\gamma\frac{\partial}{\partial t}[\Delta\hat{\nabla}\mathbf{P}-\varkappa^2(\mathbf{n}\nabla)(\mathbf{nP})], \tag{27}$$

$$\mathbb{K}V=(\varkappa^2-\Delta)\left[\left(-\gamma\Delta\frac{\partial}{\partial t}+\hat{\Delta}\right)(\mathbf{nP})-(\mathbf{n}\nabla)(\hat{\nabla}\mathbf{P})\right]. \tag{28}$$

For $\mathbf{P}=P\mathbf{n}$, the last two terms in (28) have the form $\hat{\Delta}_\perp P$, where

$$\hat{\Delta}_\perp\equiv\hat{\Delta}-\partial_x{}^2=\alpha_y\partial^2/\partial y^2+\alpha_z\partial^2/\partial z^2,$$

in accordance with the static problem.

Besides Eqs. (27) and (28), we can consider the equations for the phase φ, which are easily obtained from (21) by means of (18) and (6):

$$\mathbb{K}\varphi=-\varkappa^2(\mathbf{n}\nabla)(\mathbf{nP})+\Delta(\hat{\nabla}\mathbf{P}). \tag{29}$$

We note that the time-dependent equations for φ and Φ and Eq. (29) are easily obtained by generalizing the static equations found in Refs. 1 and 2 for the case of one dislocation. Introducing the dislocation moment $\mathbf{P}$ in accordance with (17) and (18) and varying not the effective Hamiltonian of the CDW but the Lagrangian, we obtain, with allowance for damping, the following equations:

$$(\mathbf{n}\nabla)\left(-\frac{\ddot{\varphi}}{u^2}+\gamma\dot{\varphi}+\hat{\nabla}\omega+\frac{2}{v}(\mathbf{n}\nabla)\Phi\right)=0, \tag{30}$$

$$\frac{2}{\varkappa^2 v}\Delta\Phi+\mathbf{n}\omega=0. \tag{31}$$

Eliminating the potential Φ in (30) by means of (31), we arrive at Eq. (29).

2. SOLITON SOLUTIONS

As an application of the theory we shall consider particular solutions of the equations with a point source. We shall find the fundamental solution $\mathscr{E}$ of the operator $\mathbb{K}$:

$$\mathbb{K}\mathscr{E}=\delta(\mathbf{r})\delta(t). \tag{32}$$

Fourier-tranforming with respect to the spatial variables, we obtain, obviously,

$$\mathscr{E}(k,t)=\frac{\theta(t)}{\gamma k^2}\exp\left(-\frac{K(k)}{\gamma k^2}t\right), \tag{33}$$

where $K(\mathbf{k}) = k^4 + \varkappa^2 k_x^2$ is the Fourier transform of the operator $\mathbb{K}$. Here and below, we consider the case $\alpha_x = \alpha_y = 1$. The results for $\alpha_y = \alpha_z = \alpha \ll 1$ are obtained by making the replacements

$$t\to\alpha t,\ x\to\alpha^{1/2}x,\ y\to y,\ z\to z,\ \Phi\to\alpha^{1/2}\Phi,\ \varphi\to\varphi.$$

Going over in (33) to the coordinate representation, we obtain

$$\mathscr{E}=\frac{\theta(t)}{(2\pi)^3}\left(\frac{\pi}{\gamma t}\right)^{1/2}\int\limits_{|\mathbf{m}|=1} d\mathbf{m}\exp\left\{-\frac{(\mathbf{rm})^2}{4t}-\gamma(\mathbf{nm})^2\frac{t\varkappa^2}{\gamma}\right\}. \tag{34}$$

In the region

$$\frac{r_\perp^2\varkappa^2}{2}\gg\frac{r^2\gamma}{4t}+\frac{t\varkappa^2}{\gamma}\gg1,\qquad r^2=r_\perp^2+x^2,\qquad r_\perp^2=y^2+z^2$$

the expression (34) can be integrated by the method of steepest descent:

$$\mathscr{E}\approx\frac{\theta(t)}{8\pi^{1/2}(\gamma t)^{1/2}\varkappa r_\perp},\qquad \frac{\partial\mathscr{E}}{\partial x}\approx-\frac{\theta(t)\gamma^{1/2}x}{2^7\pi^{1/2}t^{5/2}}. \tag{35}$$

We note that in the zeroth approximation in $(\varkappa r_\perp)^{-1}$ the function $\mathscr{E}$ does not depend on x.

In the opposite limiting case ($r^2\gamma/4t \ll 1$), we obtain for $t\varkappa^2/\gamma \gg 1$

$$\mathscr{E}\approx\frac{\theta(t)}{8\pi t\varkappa}\left(1-\frac{r_\perp^2\gamma}{8t}\right), \tag{36}$$

while for $t\varkappa^2/\gamma \ll 1$ we obtain

$$\mathscr{E}=\frac{\theta(t)}{(2\pi)^2}\left(\frac{\pi}{\gamma t}\right)^{1/2}\left(1-\frac{r^2\gamma}{12t}-\frac{t\varkappa^2}{3\gamma}\right). \tag{37}$$

For $r_\perp = 0$ the integral (34) can be calculated exactly:

$$\mathscr{E}=\frac{\theta(t)}{4\pi^2\gamma^{1/2}}\left(\frac{t\varkappa^2}{\gamma}+\frac{x^2\gamma}{4t}\right)^{-1/2}\operatorname{erf}\left[\left(\frac{t\varkappa^2}{\gamma}+\frac{x^2\gamma}{4t}\right)^{1/2}\right]. \tag{38}$$

The general solution of Eq. (29) is equal to the convolution of the right-hand side of the equation with the fundamental solution $\mathscr{E}$. If the dipole moment is oriented along the x axis, i.e., $\mathbf{P} = (P,0,0)$, then

$$\varphi=-(\varkappa^2-\Delta)\frac{\partial}{\partial x}\mathscr{E}*P(\mathbf{r},t), \tag{39}$$

where

$$f*g=\int f(\mathbf{r}-\mathbf{r}',t-t')g(\mathbf{r},t)d\mathbf{r}'\,dt'.$$

We shall define the coherent CDW current j in terms of its phase velocity $\dot\varphi$, as in the absence of defects. The total CDW current J and soliton current j_s are determined by the conservation laws in terms of the charge density ρ and the defect density ρ_s (determined from the effective soliton density), respectively. We have

$$j=-\frac{1}{\pi}\dot\varphi,\quad \rho=\frac{\partial U}{\partial\Phi}=\frac{1}{\pi}\omega_x,\quad \rho_s=-\frac{1}{2\pi}P_x,$$
$$\frac{\partial J}{\partial x}+\frac{\partial\rho}{\partial t}=0,\quad \frac{\partial j_s}{\partial x}+\frac{\partial\rho_s}{\partial t}=0. \tag{40}$$

From (18) and (40) it follows that

$$J=j+2j_s. \tag{41}$$

Henceforth, if this is not specially stipulated, by the CDW current we shall mean the quantity j (a characteristic of the CDW velocity). From (39) and (40) we obtain

$$j=\frac{1}{\pi}(\varkappa^2-\Delta)\frac{\partial}{\partial x}\mathscr{E}*\dot P(\mathbf{r},t). \tag{42}$$

A typical process is the rapid transformation of electrons into π-solitons (see the literature in Refs. 5 and 6). The characteristic time $\omega_{\rm ph}^{-1}$ of the process ($\omega_{\rm ph}$ is the phonon frequency) can be assumed to be such that the process is instantaneous, i.e., occurs in times shorter than the diffusion times:

$$P=-\pi\sum\delta(\mathbf{r}-\mathbf{r}_i)\theta(t-t_i), \tag{43}$$

where r_i and t_i are the coordinate and time of the creation of the ith soliton. For the current we obtain

$$j\approx-\sum\varkappa^2\frac{\partial}{\partial x}\mathscr{E}(\mathbf{r}-\mathbf{r}_i,t-t_i). \tag{44}$$

For a random process of creation of solitons the correlator of $\dot P$ has a δ-function form:

$$\langle[\dot P(\mathbf{r},t)-\langle\dot P\rangle][\dot P(\mathbf{r}',t')-\langle\dot P\rangle]\rangle=\upsilon\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')f(\mathbf{r}), \tag{45}$$

where

$$\upsilon=\langle(\dot P-\langle\dot P\rangle)^2\rangle,$$

and $f(\mathbf{r}) = 1$ for $\mathbf{r}\in\mathscr{V}$ and $f(\mathbf{r}) = 0$ for $\mathbf{r}\notin\mathscr{V}$ ($\mathscr{V}$ is the volume in which the solitons are created). The average value of the current is related to the quantity $\langle\dot P\rangle$ by $\langle j\rangle = -\langle\dot P\rangle\mathscr{V}/\pi$.

For the correlator of the currents we obtain from (43)–(45)

$$\mathscr{D}(\zeta,\zeta')\equiv\langle[j(\mathbf{r},t)-\langle j\rangle][j(\mathbf{r}',t')-\langle j\rangle]\rangle$$
$$=\frac{\upsilon\varkappa^4}{\pi^2}\int d^3\mathbf{y}\,d\tau\tilde{\mathscr{E}}(\mathbf{r}-\mathbf{y},t-\tau)\tilde{\mathscr{E}}(\mathbf{r}'-\mathbf{y},t'-\tau)f(\mathbf{y}), \tag{46}$$

where

$$\zeta=(\mathbf{r},t),\quad \tilde{\mathscr{E}}\equiv\partial\mathscr{E}/\partial x,\quad \mathbf{r}=(x,y,z).$$

We shall consider two limiting cases.

1. Solitons are created over all space, i.e., $\mathscr{V}\to\infty$ and $f = 1$. In the momentum representation, for the correlator (46) we obtain

$$\mathscr{D}(\mathbf{k},\omega)=\frac{\upsilon\varkappa^4}{\pi^2}\tilde{\mathscr{E}}(\mathbf{k},\omega)\tilde{\mathscr{E}}(-\mathbf{k},-\omega)$$
$$=\frac{\upsilon\varkappa^4}{\pi^2}\frac{k_x^2}{(k^4+\varkappa^2k_x^2)^2+\gamma^2k^4\omega^2}.$$

2. Solitons are created in a finite volume V. If we are interested in the correlations of currents at spatial points far from the volume in which injection of charge occurs with subsequent transformation into solitons, then in Eq. (46) we can set $f(\mathbf{y}) = \mathscr{V}\delta(\mathbf{y})$. Substituting this into (46), in the momentum representation we obtain

$$\mathscr{D}(\mathbf{k},\omega;\mathbf{k}',-\omega) = \frac{V v \varkappa^4}{\pi^2}\tilde{\mathscr{E}}(\mathbf{k},\omega)\tilde{\mathscr{E}}(\mathbf{k}',-\omega)$$
$$= \frac{\mathscr{V} v \varkappa^4}{\pi^2}\frac{k_x k_x'}{(k^4+\varkappa^2k_x^2+i\gamma\omega)(k'^4+\varkappa^2k_x'^2-i\gamma\omega)}.$$

Going over to the coordinate representation and integrating over the transverse coordinates $r_\perp$ and $r_\perp'$, we obtain for the correlator of $j_\Sigma(x) = \int d^2r_\perp j(x,\mathbf{r}_\perp)$ (the current integrated over the cross section) the following expression, valid in the region $x, x' \gg 1/\varkappa$:

$$\langle[j_\Sigma(x)-\langle j_\Sigma\rangle][j_\Sigma(x')-\langle j_\Sigma\rangle]\rangle$$
$$\approx \frac{v\mathscr{V}\varkappa^2}{8\gamma}\exp\left(-\frac{\varkappa^2}{\gamma}|t-t'|\right)\operatorname{sign}(xx').$$

In the dissipative approximation under consideration ($u\to\infty$) the correlator obtained does not depend on x or x', indicating the strong correlation of the currents at $t = t'$.

3. ONE-DIMENSIONAL DIFFUSION OF SOLITONS

We shall investigate the one-dimensional CDW-current propagation induced in a crystal by creation of solitons that is uniform over the cross section, e.g., as a result of injection of electrons by a narrow contact in a thin sample. Averaging Eqs. (20), (30), and (31) over the transverse coordinates, we obtain the following system of equations:

$$\left(\frac{1}{u^2}\frac{\partial^2}{\partial t^2}+\gamma\frac{\partial}{\partial t}\right)(\tilde{\varphi}+F)-\tilde{\varphi}''+\varkappa^2\tilde{\varphi}=0, \tag{47}$$

$$\frac{2}{\varkappa^2 v}\Phi'+\tilde{\varphi}=0, \tag{48}$$

where

$$F\equiv\int_{-\infty}^{x}P\,dx,\qquad \mathbf{P}=(P,0,0),$$

$$\tilde{\varphi}\equiv\int_{-\infty}^{x}\omega\,dx,\qquad \boldsymbol{\omega}=(\omega,0,0),\qquad f'=\partial f/\partial x.$$

The functions F and $\tilde{\varphi}$ have the meanings of the plastic and elastic components of the deformation of the phase.

The CDW current, according to (40) and (41), is equal to

$$j=-\frac{1}{\pi}\dot{\varphi}=-\frac{1}{\pi}(\dot{F}+\dot{\tilde{\varphi}}). \tag{49}$$

The electric field is determined using (48):

$$E=\frac{\varkappa^2 v}{2}\tilde{\varphi}. \tag{50}$$

In the dissipative regime ($u\to\infty$) Eq. (47) can also be written as the definition of the CDW current in terms of the elastic deformations:

$$\pi\gamma j=+\varkappa^2\tilde{\varphi}-\tilde{\varphi}''.$$

In the general case, Eqs. (47) and (48) describe the excitation of CDW plasma oscillations under the action of injection and the displacement of the topological defects that are characterized by the difference $\delta j_s = -\dot{F}/2\pi$ of the corresponding currents [see Eq. (53) below]. The oscillations are characterized by the following parameters: the frequency $\Omega = u\varkappa$, the relaxation time $\tau_0 = \gamma/\varkappa^2 = 4\pi\sigma$, where σ is the CDW conductivity (see below), and the time τ_1 of transition from the dissipative to the inertial regime: $\tau_1 = 1/\gamma u^2 = 1/\tau_0\Omega^2$. Of these quantities, only one (Ω) depends weakly on the temperature. For typical CDW the following values are characteristic (see the reviews in Refs. 7 and 8):

$$\varkappa\sim10^7\ \text{cm}^{-1},\ u\sim10^6\ \text{cm/sec},\ \Omega\sim10^{-1}\omega_{ph}\sim10^{13}\ \text{sec}^{-1},\ \Omega\sim10\omega_0.$$

The quantity τ_1^{-1} obviously lies between the phonon frequencies $\sim10^{12}$ Hz, where the damping is small, and the experimental range 10^9–10^{10} Hz. It may be postulated (see the discussion in Ref. 8) that $\tau_1^{-1}\sim10^{10}$–10^{11} sec^{-1}; consequently, $\tau_0^{-1}=\Omega^2\tau_1\sim10^{15}$ sec^{-1}, i.e., is a few orders of magnitude greater than the highest achievable CDW frequencies $\Delta\sim10^{13}$ Hz. The estimates given for τ_0 correspond to a CDW conductivity

$$\sigma=2/\pi\gamma v\sim10^{14}\ \text{sec}^{-1}\sim10^3/\Omega\ \text{cm}.$$

For the parameter γ we obtain the following estimates:

$$\gamma\sim\tau_0\varkappa^2\sim10^{-1}\ \text{sec/cm}^2,\quad \gamma\sim1/\tau_1^2u\sim10^{-1}-10^{-3}\ \text{sec/cm}^2,$$
$$\tau_0/\tau_1\sim10^{-2}-10^{-6}.$$

Thus, we may assume that the scales τ_0 and $\varkappa^{-1}$ are extremely small and neglect the changes of quantities over these intervals. Consequently, Eq. (47) reduces to a local equation:

$$\pi\gamma j=\varkappa^2\tilde{\varphi}=(2/v)E, \tag{51}$$

signifying that both the CDW current and its elastic polarization are proportional to the electric field. Thus, the response of a CDW to an electric field, from the point of view of its total polarization, is similar to that of a metal, while from the point of view of its elastic polarization it is similar to that of a dielectric (with permittivity ~1). By writing Eq. (47) for $u\to\infty$ in the form

$$\tilde{\varphi}+\tau_0(\dot{\tilde{\varphi}}+\dot{F})=0,$$

we note that the contribution of the elastic deformation ($\propto\dot{\tilde{\varphi}}$) to the current can be neglected in comparison with the plastic current $\propto\dot{F}$.

Finally, in the approximation under consideration we can assume that the potential energy V of the solitons is determined primarily by the Coulomb part:

$$V=v\tilde{\varphi}'+2\Phi=2(1-\varkappa^{-2}\partial^2/\partial x^2)\Phi\approx2\Phi,$$

as in the case of static problems.[1,2]

The dynamics of the function P involves the injection of matter (the transformation of electrons into solitons) and the redistribution of P (the aggregation of solitons into dislocations and diffusion of the dislocations). For definiteness, we shall consider the case when, in a certain cross section, continuous creation of 2π-solitons occurs, e.g., as a result of injection of electrons by a narrow contact in a thin sample.

Assuming that the entire electron current $I(t)$ goes over into a soliton current over a sufficiently short distance, we obtain

$$\frac{dP}{dt}=-2\pi\delta(x)I(t), \quad I(t)=I_0\theta(t),$$

where it is assumed that the current is switched on at time $t=0$.

The redistribution of P is induced by the diffusion of the solitons, even if a considerable part of P is present in the form of dislocation loops. In fact, the change of P is due to a local change of the area of a dislocation loop, requiring the attachment of a $\pm 2\pi$-soliton in analogy with vacancies or adatoms in ordinary crystals. The mechanisms of the mobility of solitons can be various, and are complicated by a two-stage conversion involving intermediate π-solitons (see the literature in Refs. 5 and 6). It may be supposed that the longitudinal redistribution of interest to us is determined by the mobility of the solitons along the chain. An upper bound on the magnitude of the mobility b can be obtained by considering the dissipation of energy as a consequence of the same mechanism as for the friction coefficient γ. One can obtain for the soliton-diffusion coefficient the estimate

$$D=bT=(\gamma')^{-1}\approx\frac{T}{T_c}\gamma^{-1}, \quad \gamma'>\gamma, \tag{52}$$

where $T_c\sim E_s$ is the temperature of the three-dimensional CDW transition and E_s is the soliton-activation energy.

We shall take the motion of the solitons into account in the diffusion approximation. The soliton density $\rho_s=-P/2\pi$ satisfies the continuity equation:

$$\frac{\partial\rho_s}{\partial t}+\frac{\partial j_s}{\partial x}=-\frac{1}{2\pi}\frac{dP_s}{dt}, \quad j_s=-D\rho_s'+b\rho_s\mathscr{F}, \tag{53}$$

where $\mathscr{F}$ is the longitudinal force acting on the soliton:

$$\mathscr{F}=-\partial(v\omega+2\Phi)/\partial x. \tag{54}$$

We confine ourselves here to the case of unipolar diffusion, when it is sufficient to take solitons of the same sign into account. This situation is realized at low temperatures, when the injection level exceeds the concentration P_∞ in the bulk or if, in the bulk, carriers of the same sign, coinciding with the sign of the injection, dominate.[1)]

From Eqs. (53) and (49) we obtain

$$\dot{F}-2\pi j_s=\dot{F}-bTF''-\frac{\gamma bv}{2}(\dot{F}+\dot{\tilde{\varphi}})F'=-I_0\theta(t)\,[\theta(x)-f(t)], \tag{55}$$

where the continuous function $f(t)$ is the constant of integration of Eq. (53) over x. Without any special conditions, we shall consider the case $f(t)=0$, when to the left of the contact, at $x\to-\infty$, there is no electric field or current:

$$E(-\infty,t)=-\Phi'=0, \quad j(-\infty,t)=0.$$

Otherwise, for an injecting contact with symmetrically placed sinks at $x\to\pm\infty$ it is necessary to make the replacement $\theta(x)\to\frac{1}{2}\mathrm{sign}\,x$ in the right-hand side of (55), i.e., $f(t)=\frac{1}{2}$.

Neglecting the term $\dot{\tilde{\varphi}}$ in (55), we obtain a closed equation for the phase F corresponding to plastic deformation of the CDW:

$$\dot{F}-bTF''-\frac{\gamma bv}{2}\dot{F}F'=-I_0\theta(t)\theta(x). \tag{56}$$

The important point is that Eq. (56) no longer contains the microscopic parameters $\tau_0=\gamma/\varkappa^2$ and $\varkappa^{-1}$ with the microscopic scale $\varkappa$. The remaining quantities are expressed in terms of F by the formulas

$$P=-2\pi\rho_s=F', \quad \tilde{\varphi}\approx-\tau_0\dot{F},$$

$$\frac{\partial\tilde{\varphi}}{\partial x}\approx 2\pi\rho\approx-\tau_0\dot{P}, \quad E\approx-\frac{\gamma v}{2}\dot{F},$$

$$j\approx-\frac{1}{\pi}\dot{F}, \quad j_s\approx+\frac{1}{2\pi}\dot{F}+\frac{1}{2}J, \quad J\approx J_0=\frac{1}{2\pi}I_0\theta(t)\theta(x).$$

Thus, outside the injection layer the variations of the total current J and density ρ are negligibly small, i.e., the zeroth order in τ_0 corresponds to the approximation of local electrical neutrality. The changes of the CDW current j and soliton current j_s cancel each other.

We consider first the homogeneous equation corresponding to (56), i.e., the case $I_0=0$. Physically, it describes the CDW after the injection pulse has been switched off or outside the interval between the sink-source contacts. This equation is characterized by the following length scale x_0, time scale t_0, and velocity scale c:

$$x_0=\gamma bv=l_TD\gamma, \; l_T=v/T; \; t_0=x_0^2/D; \; c=x_0/t_0=T/v\gamma=1/\gamma l_T.$$

When the condition (51) is fulfilled we have $x_0\sim l=v/T_c$ (the length of the soliton), while t_0 is the time of diffusion of the soliton over its length, and is measured directly in NMR.

In the variables x/x_0 and t/t_0 Eq. (56) acquires the form

$$\dot{F}-F''-F'\dot{F}=\nu\theta(t)\theta(x), \quad \nu=I_0t_0, \tag{57}$$
$$E\propto v\gamma/t_0\sim T/l, \quad \Phi\propto v\gamma c\sim T.$$

When (52) is fulfilled it is obvious that $\nu\ll 1$. Equation (57) contains only one dimensional parameter—the injection rate ν. It can be eliminated if we bring (56) to dimensionless form in other scales for the time, length, and deformation, respectively:

$$t_1=1/I_0^2t_0=t_0/\nu^2,$$
$$x_1=(Dt_1)^{1/2}=1/I_0\gamma l_T=x_0/\nu, \tag{58}$$
$$F=G/\nu.$$

Here,

$$P=x_0^{-1}G', \quad \pi j=(\nu/t_0)\dot{G}. \tag{59}$$

We emphasize that, although the scales t_1 and x_1 are reduced with increase of the injection rate ν, the diffusion relation $D=x_1^2/t_1$ remains invariant, as too (and this is important) does the defect density $P=G'/x_0$. In the variables $X=x/x_1$ and $T=t/t_1$ we have, in the general case,

$$\dot{G}-G''-G'\dot{G}=-\theta(t)\,[\theta(x)-f(t)] \tag{60}$$

with the boundary conditions

$$G'(\pm\infty,T)=P_\infty/x_1=p,$$
$$\dot{G}(\pm\infty,T)=\dot{G}_\pm=\pi(t_0/\nu)j(\pm\infty,T). \tag{61}$$

It follows from (60) and (61) that

$$f=\dot G_+(1-p)+1=\dot G_-(1-p).$$

Thus, although we have not undertaken a full investigation of Eq. (56), we can determine the characteristic scales of the distributions as functions of the current. We have not succeeded in finding the general solution of the system, although we can obtain approximate solutions for practically the whole plane of the variables X and T, with the exception of a number of crossover regions in which small expansion parameters do not exist.

We shall investigate Eq. (60) for $f=0$. In the linear region, in which the nonlinear term $\dot F F'$ is small in comparison with the other terms of the equation:

$$F'\dot F \ll \dot F,\ F'', \tag{62}$$

the solution of the equation has the standard form:

$$\frac{\partial F}{\partial x}=P=-\theta(t)I_0\int_0^t dt'\mathscr{E}_2(t',x),$$
$$\frac{\partial P}{\partial t}=-I_0\theta(t)\frac{1}{2(\pi Dt)^{1/2}}\exp\left(-\frac{x^2}{4Dt}\right). \tag{63}$$

Using the conditions (59) and (62), we obtain from (63) the following results:

1. For $x^2 \ll Dt$ and $t \ll t_1$,

$$P\approx-\theta(t)I_0\frac{t^{1/2}}{(\pi D)^{1/2}},$$
$$j=-\frac{\dot F}{\pi}\approx-\theta(t)\frac{I_0}{\pi}\left[\frac{1}{2}+\frac{x}{2(\pi Dt)^{1/2}}\right], \tag{64}$$
$$\varphi\approx-t_0\dot F.$$

2. For $x^2 \gg Dt$, $x/t \gg v_1=x_1/t_1$ or for $x^2 \ll Dt$, $x<0$,

$$P\approx-\theta(t)I_0\frac{D^{1/2}}{\pi^{1/2}}\frac{t^{3/2}}{x^2}\exp\left(-\frac{x^2}{4Dt}\right),$$
$$j\approx\frac{I_0}{\pi}\theta(x)-I_0\frac{D^{1/2}}{\pi^{1/2}}\frac{t^{1/2}}{x^2}\exp\left(-\frac{x^2}{4Dt}\right) \tag{65}$$

In the nonlinear regime we consider the region of the (X,T) plane in which the diffusion term DF'' is unimportant:

$$DF''\ll\dot F,\quad (\gamma bv/2)F'\dot F. \tag{66}$$

The dimensionless equation (60) takes the form

$$\dot G(1-G')=-\theta(X)\theta(T). \tag{67}$$

It admits the solution

$$G=Tg(X/T).$$

The function $g(\xi)$ satisfies the Clairaut equation for $X>0$:

$$g=\xi g'+(g'-1)^{-1}, \tag{68}$$

the solution of which has the form

$$g=\begin{cases}-2\xi^{1/2}+\xi, & 0<\xi<1,\\ -1, & \xi>1,\end{cases} \tag{69}$$

Hence, taking (66) into account, we obtain for P and j the following result:

3. For $(x/x_1)^3 \gg t/t_1$,

$$P=\begin{cases}-\left(\dfrac{I_0t}{\Gamma x}\right)^{1/2}+\dfrac{1}{\Gamma}, & 0<x/t<v_1,\\ 0, & x/t>v_1,\end{cases} \tag{70}$$
$$j=\begin{cases}\dfrac{1}{\pi}\left(\dfrac{I_0x}{\Gamma t}\right)^{1/2} & 0<x/t<v_1,\\ P_0, & x/t>v_1,\end{cases}$$

where $\Gamma\equiv\gamma bv/2$.

Finally, in the region in which the term $\dot F$ is small, we obtain the equation

$$G''+G'\dot G=\theta(X)\theta(T). \tag{71}$$

It is easy to see that Eq. (71) admits a solution in the form

$$G=T^{2/3}g(X/T^{1/3}),$$

where the function $g(\eta)$ satisfies the equation

$$g''+g'(2g-g')/3=\theta(\eta),\quad \eta=X/T^{1/3}. \tag{72}$$

For small $\eta \ll 1$ we seek the solution of (72) in the form of a series in powers of η. We find that

$$g\approx A+B\eta+\ldots, \tag{73}$$

where the constants $A<0$ and B are determined by matching to the solutions in the adjacent regions. Finally, using the solutions 1–3, we obtain from (73) the following result:

4. For $t \gg t_1$, $(x/x_1)^3 \ll t/t_1$, $x>0$,

$$P\approx-\frac{I_0^{2/3}}{\Gamma^{1/3}D^{1/3}}t^{1/3},\quad j\approx\frac{D^{1/3}I_0^{1/3}}{\pi\Gamma^{2/3}}\frac{1}{t^{1/3}}, \tag{74}$$

while for $x<0$, $x^2 \ll Dt$, and $(x/x_1)^3 \gg t/t_1$,

$$P=-\frac{2D}{\Gamma}\frac{t}{x^2},\quad j=-\frac{2D}{\Gamma\pi}\frac{1}{x}.$$

We recall that in all the regions considered the current and electric field are related by (60).

The regions of the solutions 1–4 cover the entire (X,T) plane with the exception of the crossover regions near the lines $X^3=T$ for $T>1$, $X^2=T$ for $T<1$, and $T=1$, $X<1$ (see Fig. 1). The regions 2 and 3 overlap on the set $X>T$, where the solutions 2 and 3 coincide with exponential accuracy.

From the solutions found it follows that for any finite T for $X \gg T$ we have $j \approx I_0/\pi$ and $P \approx 0$, i.e., everywhere ahead of the front moving with constant velocity $v_1 \sim bE$ the current has the nominal value for a moving CDW in the absence of solitons.

We shall consider the development in time of the fields at a given point $X \gg \varkappa^{-1}$. First, in a time of order τ_0, the maximum current $j=I_0/\pi$ is established, the soliton density being $P \approx 0$. Subsequently, the time dependences differ for the cases $X>1$ and $X<1$.

For $X>1$ for times $T \ll X$ the current remains constant, equal to $j=I_0/\pi$, and the function P is an exponentially small quantity. In the time interval $X \ll T \ll X^3$ the current decreases, and $|P|$ increases in a power-law manner: $j \propto T^{-1/2}$ and $P \propto T^{1/2}$, while for $T \gg X^3$ the decrease of j and

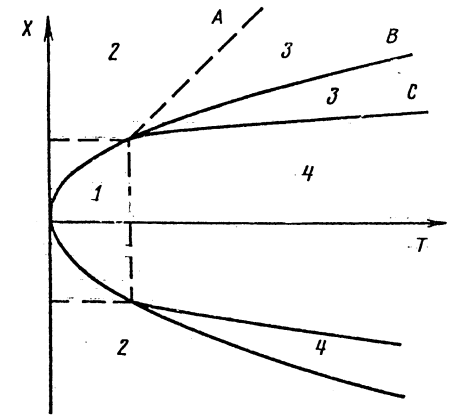

FIG. 1. Characteristic regions in the (X,T) plane. Line A: $X = T$; line B: $X^2 = T$; line C: $X^3 = T$.

increase of $|P|$ are slower: $j \propto T^{-1/3}$ and $P \propto T^{1/3}$.

For $X < 1$ for times $T \ll X^2$ the current remains equal to $j = I_0/\pi$, and the function P is exponentially small. Furthermore, in the interval $X^2 \ll T \ll 1$ the current is equal to half the maximum ($j \approx I_0/2\pi$), and $|P|$ increases in a power-law manner: $P \propto T^{1/2}$. For $T \gg 1$ we find the same dependences as for $X > 1$ ($P \propto T^{1/3}, j \propto T^{-1/3}$).

For negative x in the region $x^2 \gg Dt$ the current and soliton density are exponentially small, while in the region $x^2 \ll Dt$, on account of the diffusion of solitons, the soliton density increases by a power law and the current decreases.

The results of numerical calculations for the functions $P(X,T)$ and $j(X,T)$ are given in Figs. 2 and 3.

The general conclusion is as follows. The region of the defect distribution propagates from the contact with the constant drift velocity $v_1 = bE$ of the solitons. With increase of the concentration of solitons there is a decrease of the current and electric field, so that, over large times, the current is inversely proportional to the soliton concentration: $j \propto 1/P$.

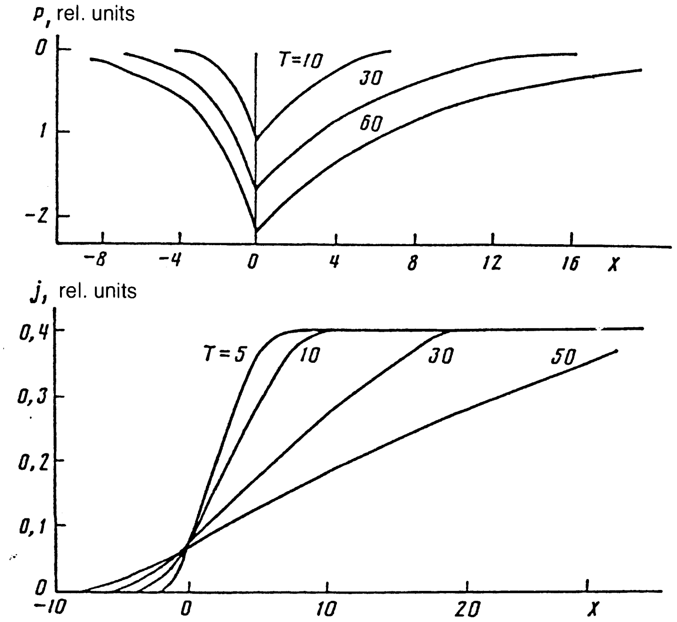

FIG. 2. Results of numerical calculations of $P(X)$ and $j(X)$ at different times T.

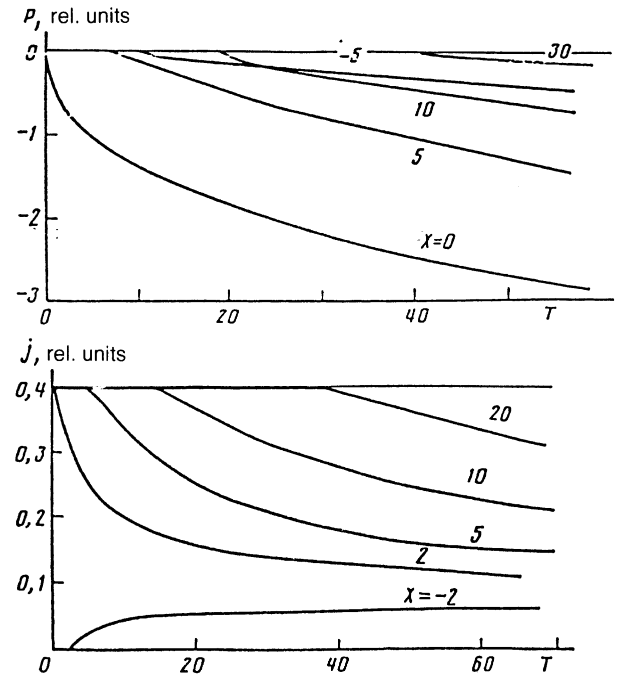

FIG. 3. Results of numerical calculations of $P(T)$ and $j(T)$ at different spatial points X.

We now consider the case when there is an nonzero soliton concentration $P_\infty < 0$ in the medium. Introducing the function

$$F_1 = \int_{-\infty}^{x} (P - P_\infty)\,dx, \qquad j - j_{-\infty} = -\dot{F}_1, \tag{75}$$

we obtain the generalization of Eq. (56):

$$\dot{F}_1 - DF_1'' - \Gamma \dot{F}_1 (F_1' + P_\infty) + \Gamma j_{-\infty} F_1' = -I_0 \theta(t)\,\theta(x). \tag{76}$$

In the problems under consideration with $j_{-\infty} = 0$ the solutions of Eq. (76) and all functions determined in terms of F are obtained from those found for the case $P_\infty = 0$ by the replacement

$$\begin{aligned} &P \to P - P_\infty, \quad D \to D/(1 - \Gamma P_\infty), \\ &I_0 \to I_0/(1 - \Gamma P_\infty), \quad \Gamma \to \Gamma/(1 - \Gamma P_\infty). \end{aligned} \tag{77}$$

We note that the magnitude of the total current J in leading order ($J \approx J_0$) is not related to the function F, i.e., does not depend on the transformation (77), and is determined by the unrenormalized value I_0. Because of this, the soliton current J_s does not vanish for $x \gg t$ but tends to

$$2j_s(x \gg t) \approx -P_\infty \pi j = -\Gamma P_\infty I_0/(1 - \Gamma P_\infty).$$

By recalling that E is uniquely related to j, we find that the coherent part of the conductivity j/E does not depend on P_∞, whereas the total conductivity J/E is proportional to $1 - \Gamma P_\infty$.

4. THE STATIONARY SOLUTIONS

We have convinced ourselves that the evolution of a sharp injection pulse leads to solutions that remain nonstationary even after the diffusion front has passed. However, it cannot be excluded that, under certain conditions, the system may emerge into a stationary regime of currents and (or) soliton concentration. We shall consider Eq. (76), or even the complete system (47), (55). It is easy to show that, amongst the truly stationary solutions ($\partial j/\partial t = 0, \partial \rho_s/\partial t = 0$) there is only the trivial solution $j(x,t) = \text{const}$, corresponding to an unchanged CDW velocity. We have

$$j_0 = -\frac{\dot{\varphi}}{\pi}\frac{2}{\pi\gamma v}E = \frac{\varkappa^2}{\pi\gamma}\dot{\varphi} = j = j_0 = \text{const}, \quad \rho \propto \dot{\varphi}' = 0. \tag{78}$$

For the concentration of solitons and their current we obtain from (76) for $\dot{F}_1 = 0$

$$P = -\frac{I_0}{bE}\exp\left[-\frac{|x|\theta(-x)E}{T}\right] + P_\infty, \quad P_{+\infty} - P_{-\infty} = \frac{I_0}{bE},$$
$$j_s = \frac{I_0}{2\pi}\theta(x). \tag{79}$$

We have shown that for $x < 0$ the soliton density tends to the thermal equilibrium concentration $\rho_{-\infty} = -P_{-\infty}/2\pi$ in accordance with the Boltzmann law in the field $2Ex$, so that the diffusion current and drift current cancel. For $x > 0$ we have a constant, but nonequilibrium, soliton concentration

$$\rho_\infty = -P_{+\infty}/2\pi = \rho_{-\infty} + I_0/2\pi E.$$

We note that the CDW conductivity σ_{CDW} does not depend on the injection conditions, whereas the additional conductivity σ_{inj} in the injection channel is proportional to a ratio of currents:

$$\sigma_{CDW} = \frac{j}{E} = \frac{2}{\pi\gamma v}, \quad \sigma_{inj} = \frac{J-j}{E} = \frac{2j_s}{E} = \frac{I_0}{2\pi E}. \tag{80}$$

Finally, the CDW phase is

$$\varphi = -\pi j_0 t - \frac{I_0}{bE^2}\exp\left[-\frac{|x|\theta(-x)E}{T}\right] - \frac{I_0}{bE}x\theta(x) + P_\infty x. \tag{81}$$

If we confine ourselves to the condition that the currents are steady and assume that the soliton concentration increases linearly with time, we obtain only the nontrivial solution:

$$j = -j_0 \,\text{th}\left(\frac{\Gamma j_0}{2D}x\right), \quad E(x) = \Gamma j(x),$$
$$P(x,t) = P_\infty + \frac{j_0^2\Gamma}{2D}\frac{t}{\text{ch}^2(\Gamma j_0 x/2D)}, \tag{82}$$
$$I_0 = -2\pi|j_0|(1-\Gamma P_\infty), \quad J = \frac{1}{2\pi}I_0 \,\text{sign}\, x.$$

The solution found corresponds to an extremely specific regime of draining off ($I_0 < 0$) of charge and solitons because their depletion (relative to the bulk concentration $\rho_\infty = -P_\infty/2\pi$) in a local region of size $\sim D/\Gamma j_0$ increases linearly with time. The regime under consideration can be realized with a low rate of injection of minority carriers, recombining with bulk-equilibrium particles. However, in the framework of our unipolar model we cannot trace the development of the solutions over times

$$t > t^* = 4\pi\rho_\infty/\Gamma j_0^2, \quad P = (0, t^*) = 0,$$

when the density P passes through zero.

5. CONCLUSION

Thus, we have derived equations describing the dissipative dynamics of CDW in the presence of a continuous distribution of dislocations or solitons (Sec. 1). We have found the response functions of the fields and the related correlation function of the currents for the process of spontaneous conversion of electrons into solitons (Sec. 2). The principal experimentally significant results are related to the investigation of the one-dimensional problem of the development of the current pulse (Sec. 3). The simplest formulation corresponds to a narrow injecting contact in a thin sample. We have solved the problem in the purely dissipative regime of the CDW dynamics, and in the diffusion approximation for a gas of solitons. With these restrictions we ignore both the possible diffusion of electrons ahead of their conversion into solitons, and the two-stage character of the conversion involving an intermediate distribution of amplitude π-solitons (see the discussion in Refs. 1 and 2). These processes effectively smear out the injection layer. We have also confined ourselves for the moment to the case of unipolar injection, which removes the possibility of considering regimes of depletion relative to both the bulk thermal and the near-contact (see Ref. 10) bare concentrations of solitons.

We have found that at first, for very short times, the nominal values of the CDW current j_∞ and the electric field $E_\infty \propto j_\infty$ are established in the whole sample. However, as the diffusion front of the gas of solitons passes through with constant velocity c and the soliton concentration ρ_s increases ($c = bE_\infty$, where b is the mobility of the solitons), the local value of the current $j(x,t)$ decreases. In a characteristic regime, $j(x,t) \propto \rho_s^{-1} \propto t^{-1/3}$. We have also found the stationary distributions for injectionless generation of solitons in a pinning layer during passage of a CDW current.

We draw attention to the fact that for $x < 0$, i.e., outside the sink-source interval, the region of the distribution of the soliton field, soliton current, and soliton concentration also increases. This result correlates with the experimental observations of nonlocal effects in Ref. 11.

As we should expect, our equations cannot describe the periodic oscillations that accompany the conversion of the current into CDW. The point is that we have not taken into account the possibility of aggregation of solitons into dislocations and we allow an unbounded growth of the soliton concentration. We can present two scenarios of the development.

1. Suppose that the Coulomb constant and (or) the structural-anisotropy coefficient $\alpha_{\min}$ in one direction are (is) small in comparison with the coefficient $\alpha_{\max}$ in the transverse direction with the greatest CDW stiffness, so that (see Refs. 1, 2, and 10) the following inequality is fulfilled:

$$\mu_0 \approx \alpha_{min}^{1/2}\,\omega_p < E_s \approx (\alpha_{max}/s)^{1/2}, \tag{83}$$

i.e.,

$$e^2/\hbar v\varepsilon_\infty < \alpha_{max}/\alpha_{min}.$$

Here, μ_0 is the energy of an isolated dislocation, ω_p is the plasma frequency, and ε_∞ is the permittivity without allowance for Coulomb effects. Then there is a critical line, like the dew-point line for a gas-liquid phase diagram and defined in terms of the local chemical potential of the gas of solitons:

$$\mu(\rho_s, T)=\mu_0-E_s, \quad \mu\propto T\ln\rho_s. \qquad (84)$$

As this line is crossed, with increase of ρ_s, the gas condenses into macroscopic dislocation loops or into dislocations that split off from the side surface (see the literature in Refs. 1, 2, and 10).

2. The second scenario is realized when the inequality (83) is violated, or, with small probability, in the gas region abutting the line (84). It involves the nucleation of large dislocations, with transverse dimensions $R>\lambda^{-1}$. To be more precise, the following stronger inequality, determined by the dislocation chemical potential $\mu_d(R)$, should be fulfilled for the size:

$$\mu_d(R)\propto\mu_0/\lambda R<\mu(\rho_s).$$

In principle, this regime can always be realized, but, in view of the large sizes, and since direct coalescence of solitons is blocked by their repulsion at distances shorter than λ^{-1}, it should have an essential dependence on the possibility of nucleation.

In both scenarios, the periodicity of the process is determined by the alternation of regimes of depletion and restoration of the concentration in the cloud of solitons.

[1)] Asymmetry of the thermal distribution of $\pm 2\pi$-solitons is induced by violation of charge symmetry. Its most common source is phonon dispersion,[9] leading to splitting of the activation energy: $E_{\pm}=E_s+\xi$, with $\xi\pm c_s$ (c_s is the sound velocity).

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