Numerical simulation of a streamer discharge in a uniform field

N. V. Evlakhov, V. Yu. Kachorovskiĭ, and V. M. Chistyakov

A. F. Ioffe Physicotechnical Institute, Russian Academy of Sciences
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We report a numerical analysis of a streamer discharge in a uniform field. The analysis confirms the results of a previously proposed[4,5] qualitative theory. The calculations yielded the numerical coefficients in several of the relations derived in Refs. 4 and 5. Equations for the calculation of the streamer velocity in the electric fields in the front and in the rear of the discharge are proposed for the first time.

1. INTRODUCTION

The dynamics of the evolution of thin highly conducting filaments (streamers) in a discharge gap has been the subject of many studies. The growth of the streamer filament is due to intense impact ionization of the medium in the strong field present in the streamer header. Maxwellian relaxation in the highly conducting streamer channel causes the charge to spread over the filament and thus to maintain a strong field at the header.

The streamer velocity is extremely high ($\approx 10^8$–10^9 cm/s) and exceeds as a rule the drift velocity of the carriers contained in the multiplication region at the streamer header. In view of this circumstance, it has been postulated already in the early studies by Leeb, Meek, and Roether (for a survey of these studies see Refs. 1–3) that the streamer is an ionization wave whose propagation velocity is proportional to the dimension of the region of intense multiplication (and not of the carrier drift velocity in the strong field at the head), and can be high enough if the streamer head is large. For this premise to be true it is necessary that the streamer be preceded by free or weakly bound carriers capable of being multiplied by impact ionization. If the streamer propagates in a medium without prior ionization, such carriers can be produced by the streamer radiation through photoionization. The presence of a free-carrier streamer ahead of the front is experimentally confirmed by the fact that the propagation velocities of the anode and cathode streamers in a gas not previously ionized are of the same order of magnitude (a positively charged streamer could not propagate at all without electrons ahead of its front).

There is at present no rigorous quanitative streamer-discharge theory leading to expressions for the streamer parameters in terms of the discharge-gap voltage and of the characteristics of the medium. The main reason is the complexity of the system of nonlinear non-one-dimensional partial differential equations that describe the streamer discharge. Analytic solution of these equations is impossible even in the simplest case in which it is assumed that free carriers with uniform specified density n_0 are present ahead of the front (n_0 depends in general on the intensity of the photoionization by the streamer radiation and must be determined self-consistently).

In this situation one of the possible ways of developing a theory is to obtain for the streamer parameters a number of qualitative relations that are valid to within numerical coefficients, and then determine these coefficients by computer simulation. Equations obtained in this manner cannot be used for a quantitative description of the streamer discharge.

A qualitative streamer theory was proposed by M. I. D'yakonov and one of us.[4,5] Straightforward arguments yielded in these papers, for most streamer parameters, order-of-magnitude expressions based on equations in which account was taken of only the most significant processes (impact ionization ahead of the front and Maxwell relaxation behind the front). References 4 and 5 served as a basis for suggesting a method of theoretically determining the streamer velocity and the streamer-surface shape,[6] a theoretical study of a streamer discharge in an electronegative gas,[7] and an investigation of the evolution dynamics of a streamer discharged by a metallic needle point whose potential increases linearly with time.[8]

As to computer simulation, notwithstanding the numerous numerical analyses of a streamer discharge, the set of equations used in most of these studies included many secondary processes that do not play a substantial role in the streamer propagation, and was even quite complicated for numerical computations. They employed simplifying assumptions (one-dimensional analysis, the disk method, and others) and were not justified even qualitatively. A more correct approach, from our point of view, was chosen by Dhali and Williams,[9] based on simple equations that take into account only the main processes that lead to streamer development, but are on the other hand obtained with sufficient rigor without unfounded simplifications. The results of Ref. 9 provide a clear enough picture of the streamer-discharge evolution and agree qualitatively with the theory.[4,5] The calculations in Ref. 9, however, were made for specific conditions of discharge excitation and their results do not yield the numerical coefficients of the relations obtained in Refs. 4 and 5.

Our aim in this study was a computer simulation of a streamer discharge by analogy with Ref. 9, but for a specific purpose—to check the validity of the qualitative relations obtained in Refs. 4 and 5 and to calculate the unknown numerical coefficients in these relations. We consider here the simplest model of a streamer discharge (only impact ionization and Maxwellian relaxation are taken into account) and eschew the complicated question of the role of photoionization, assuming that the streamer front is preceded by free carriers having a specified uniform density n_0. A similar approach was used in Ref. 9 where, as in our calculation, the streamer parameters depended quite weakly on n_0.

2. FORMULATION OF PROBLEM

We investigated the evolution of a streamer between the electrodes of a gas-filled parallel-plate capacitor (uniform

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field). A low density n_0 of the free electrons in the gap was assumed, and hence a low initial gas conductivity $\sigma_0 = e\mu n_0$, where μ is the electron mobility. The discharge was initiated by a high-conductivity primer located near one of the capacitor electrodes. We used the simplest set of equations to describe the streamer discharge (see Ref. 5):

$$\partial\sigma/\partial t=\beta(E)\sigma, \quad (1)$$

$$\partial\rho/\partial t+\mathrm{div}(\sigma\mathbf{E})=0, \quad (2)$$

$$\Delta\varphi=-4\pi\rho, \quad (3)$$

where $\sigma = e\mu n$ is the gas conductivity, n the electron density, $\mathbf{E} = -\nabla\varphi$ the electric field, φ the electrostatic potential, ρ the charge density, and β the impact-ionization frequency. We assume for simplicity that the impact-ionization intensity depends only on the local value of the electric field and assume for β the expression[5,9]

$$\beta(E)=\beta_0(E/E_0)\exp(-E_0/E), \quad (4)$$

where β_0 and E_0 depend only on the sort of the gas and on the pressure, but not on the discharge-excitation conditions.

We neglect in Eqs. (1)–(3) a number of secondary slow processes that have little effect on the streamer development. We neglect the ion mobility and diffusion (i.e., we assume in fact immobile ions), as well as the electron diffusion. (The possible influence of electron diffusion on the streamer evolution is discussed in Ref. 4). In addition, we have discarded in the right-hand side of (1) the term $\mathrm{div}(\mu\mathbf{E}\sigma)$ describing the conductivity change due to electron drift. It is correct to neglect this term[5] if the streamer velocity is much higher than the electron-drift velocity in the strong field at the discharge head. Note that in this model the propagations of the anode and cathode streamers are perfectly identical.[5]

A qualitative theory based on Eqs. (1)–(3) was proposed in Refs. 4 and 5.[1)] We present below some of the numerically verified results of these references.

1) The electric field E_m at the streamer head is of the order of E_0:

$$E_m\sim E_0. \quad (5)$$

The numerical coefficient in this equation is unknown and must be determined either from an exact solution of Eqs. (1)–(3) or by computer simulation. Our symbol for this coefficient is C_0 (the coefficients C_1 and C_2 introduced below have a similar meaning) and rewrite (5) in the form

$$E_m=C_0E_0. \quad (6)$$

2) The conductivity σ_m directly behind the streamer front is given by

$$\sigma_m=\frac{1}{4\pi}\int_0^{E_m}\frac{\beta(E)}{E}dE. \quad (7)$$

Note that Eq. (7) contains no unknown numerical coefficients and is exact.

3) The streamer velocity V is directly proportional to the radius r_0 of its head

$$V=C_1\frac{\beta_0 r_0}{\Lambda_1}. \quad (8)$$

where

$$\Lambda_1=\ln\left[\frac{\sigma_m}{\sigma_0}\right]. \quad (9)$$

The argument of the logarithm in (9) is the ratio of the conductivities behind and ahead of the front, and amount to several orders of magnitude. Therefore Λ_1 is much larger than unity.

4) The electric field E_z directly behind the front of the streamer is given by

$$E_z=C_2\frac{E_0}{\Lambda_1}. \quad (10)$$

Relations (6)–(10) are valid independently of the geometry of the discharge gap. The dynamics of streamer evolution from sharp tip differs from than in a uniform field. In particular, if a uniform field $\mathscr{E}$ is applied to the gap, the streamer propagation is stationary[5,10,11] at a certain critical value $\mathscr{E} = \mathscr{E}_c$ (the head radius and velocity do not vary with time). At $\mathscr{E} > \mathscr{E}_c$ the radius and velocity of the streamer increase slowly with time [remaining proportional to one another in accordance with Eq. (8)], but they decrease slowly at $\mathscr{E} < \mathscr{E}_c$. As shown in Ref. 5, $\mathscr{E}_c$ and E_z are equal.

The purpose of the here-reported numerical simulation is to check on the qualitative relations (6)–(10) and to calculate the numerical coefficients C_0, C_1, and C_2. One more remark is needed to prevent misunderstandings. By solving (1)–(3) we obtain the spatial distributions of σ, ρ, and E at various instants of time, but not the quantities V, r_0, σ_m, E_m, and E_z directly. Moreover, these quantities (with the exception of the velocity V) are on the whole not determined unambiguously, since we do not know, for an arbitrary distribution of the conductivity σ. As noted in Ref. 4, however, the width of the streamer front is small compared with the radius of its head, and the highly conductive region has a sufficiently abrupt boundary. One can therefore introduce the concept of a streamer surface[4,6] and take r_0 to be the curvature radius of this surface at a point located on the discharge axis. In the present paper we define the streamer surface as conductivity constant-level line corresponding to $\sigma_m/2$. For σ_m and E_m we choose the maximum values of σ and E on the discharge axis, and E_z is taken to be the electric field on the discharge axis directly behind the streamer front.

3. COMPUTER SIMULATION

Equations (1)–(3) were numerically analyzed in the plane parallel interval $0<z<L$, $0<r<R$, where z and r are cylindrical coordinates. The following conditions were imposed on the potential φ at the gap boundary:

$$\varphi|_{z=0}=0,\ \varphi|_{z=L}=-\mathscr{E}L,\qquad \varphi|_{r=R}=-\mathscr{E}z, \quad (11)$$

where $\mathscr{E}$ is the uniform external field applied to the discharge gap.

The initial conditions for the potential and for the charge density were chosen in the form

$$\varphi|_{t=0}=-\mathscr{E}z,\ \rho|_{t=0}=0. \quad (12)$$

It was also assumed that a primer having high-conductivity primer σ_1 located near one of the capacitor electrodes was placed against the background of the low conductivity σ_0 initially existing in the gap. The initial conductivity distribution was correspondingly chosen in the form

$$\sigma|_{t=0}=\sigma_0+(\sigma_1-\sigma_0)\exp\left(-\frac{z^2+r^2}{2R_0^2}\right), \tag{13}$$

i.e., the conductivity is equal to σ_1 to the point $z=0$, $r=0$ (the center of one of the capacitor plates) and decreases exponentially to σ_0 at a distance on the order of R_0. In this case R_0 acts as the radius of the high-conductivity primer.

We found it convenient in the calculations and in the analysis of the results to make Eqs. (1)–(3) dimensionless by measuring the quantities t, r, z, E, φ, σ, and ρ in units of $(\beta_0)^{-1}$, z_0, z_0, E_0, E_0z_0, β_0, and E_0/z_0, respectively. The choice of z_0 as the unit of length calls for the following comments. As noted in Ref. 5, Eqs. (1)–(3) contain no parameter with the dimension of length, so that the nondimensionalizing scale z_0 can be chose arbitrarily. The characteristic streamer dimensions are determined in fact by the radius R_0 of the high-conductivity streamer R_0 (Ref. 5), and to a considerably lesser degree by the geometric dimensions L and R of the gap. We assume in the present paper that the dimensions L and R are fixed and are connected with one another and with the unit length z_0 by the relations $L=4R$, $\Gamma=20z_0$, and $R=5z_0$, whereas R_0 ranged from $0.5z_0$ to $2z_0$. It is important that the nondimensional system of equations (1)–(3) contains no parameters that depend in any way on the type of gas or on the pressure. The calculation results given in dimensionless units are therefore universal and only the measurement units depend on the specific characteristics of the gas (the quantities that depend on the type of gas and on the pressure are β_0 and E_0). We note also that the results make it possible to describe gaps of various lengths, since a severalfold change of the length-measurement unit z_0 is equivalent to an increase of R_0, L, and R by the same number of times.

The calculation procedure was the following. Using the values of σ and $\mathbf{E}$ on the kth time steps, we calculated from Eqs. (1) and (2) the values of σ and ρ on the $(k+1)$st time step. The values of ρ obtained in this manner were used to solve the Poisson equation (3) with boundary conditions (11), and to find the distribution of the potential and of the electric field on the $(k+1)$st step. The process was then repeated, with the conductivity of the medium smoothed out at definite time intervals to stabilize the numerical calculation (the smoothing was over the nearest sites of the numerical lattice). The interaction scheme used in the calculations is described in the Appendix.

Figures 1 and 2 show the calculation results for the case $R_0=1$, $\sigma_0=10^{-8}$ and for different values of the external field, viz., $\mathscr{E}=0.15$ (Fig. 1) and $\mathscr{E}=0.25$ (Fig. 2). Figures 1a and 2a show for various instants of time the distributions of σ along the discharge axis in logarithmic scale. The distribution of the z-projection of the electric field $\mathbf{E}$ on the discharge axis is shown in Figs. 1b and 2b (we have considered a cathode streamer, so that the values of the z-projections are negative), while Figs. 1c, d, e and 2c, d, e show families of conductivity constant-level lines plotted at various instants of time.

It is evident from the figures that the streamer motion becomes quasistationary after a certain stabilization period, and its parameters stay practically constant. A similar result was obtained in Ref. 9, but in our case we observed a more stable streamer evolution during the quasistationary stage. In particular, we were able to determine by numerical calculations, with good accuracy, stationary-motion parameters such as σ_m (see Figs. 1a and 2a), E_m and E_z (see Figs. 1b, 2b), r_0 (see Fig. 1b, d, e and 2b, d, e), the streamer velocity V for an external field $\mathscr{E}$ that ranged from 0.15 to 0.30 and for high-conductivity primer radii R_0 from 0.5 to 1.0. The choice of these intervals of R_0 and $\mathscr{E}$ was based on the following considerations. At R_0 values larger than 1.0 the calculation results come under the influence of the ratio of R_0 to the gap radial dimension $R=5$. If $R_0<0.5$ numerical instabilities set in because R_0 becomes comparable with the distance of the spacing between the numerical-lattice points (a spacing chosen to be 0.2 in the chosen system of units). For $\mathscr{E}$ larger than 0.30 the background value of the conductivity σ_0 increases strongly during the time of streamer motion, owing to ionization in the external field of frequency $\beta(\mathscr{E})$, so that the streamer parameters begin to change significantly in the quasistationary phase. Finally, if $\mathscr{E}$ is to small ($\mathscr{E}<0.15$) the stabilization slows down and the streamer reaches the opposite electrode of the capacitor before reaching the stationary development stage. (As seen from Fig. 1, at $\mathscr{E}=0.15$ the streamer reaches the stationary stage of its development in the immediate vicinity of the capacitor electrode.)

The computer-simulation results confirm the validity of relations (5), (6), and (8)–(10).

It is evident from the figures that the absolute value of the electric field directly and ahead of the front is constant in the quasistationary regime and is close to the unity (in terms of the dimensions x and E_0). A similar result is observed also at external parameters ($\mathscr{E}$,R_0) other than those in the figures. It can therefore be stated that Eqs. (5) and (6) are valid. The approximate value of the coefficient C_0 in (6) is

$$C_0=0,9\pm0,1. \tag{14}$$

It is noteworthy that, in general, in our calculations C_0 increases slowly with increase of the external field $\mathscr{E}$. The relative change of C_0 over its entire range (from 0.15 to 0.3) does not exceed 10% and is probably governed by numerical-simulation errors.

Comparison of the calculations with Eq. (8) requires allowance for the time dependence of the conductivity a head of the discharge front. In fact, if the logarithm Λ_1 in (8) is calculated using for σ_0 the unrenormalized background conductivity ($\sigma_0=10^{-8}$) the value of $(V\Lambda_1)/(\beta_0 r_0)$ obtained by computer simulation is not a constant as suggested in Eq. (8). The actual conductivity $\sigma_0(t)$ ahead of the streamer front increases slowly with time as a result of impact ionization in the external field (see Figs. 1a and 2a for the conductivity in logarithmic scale). Replacing the background conductivity σ_0 in the argument of the logarithm in Eq. (9) by $\sigma_0(t)$, we obtain good agreement between the calculations and Eq. (8) if the numerical coefficient C_1 is

$$C_1=0,18\pm0,02. \tag{15}$$

The electric field far behind the discharge front tends to the value of the external field $\mathscr{E}$. The field E_z directly behind the front, however, as seen from the figures, is weaker than $\mathscr{E}$ and very weakly dependent on $\mathscr{E}$. Equation (10) agrees well with the numerical experiment if σ_0 in the expression for the logarithm Λ_1 in (8) is replaced by $\sigma_0(t)$. The coefficient

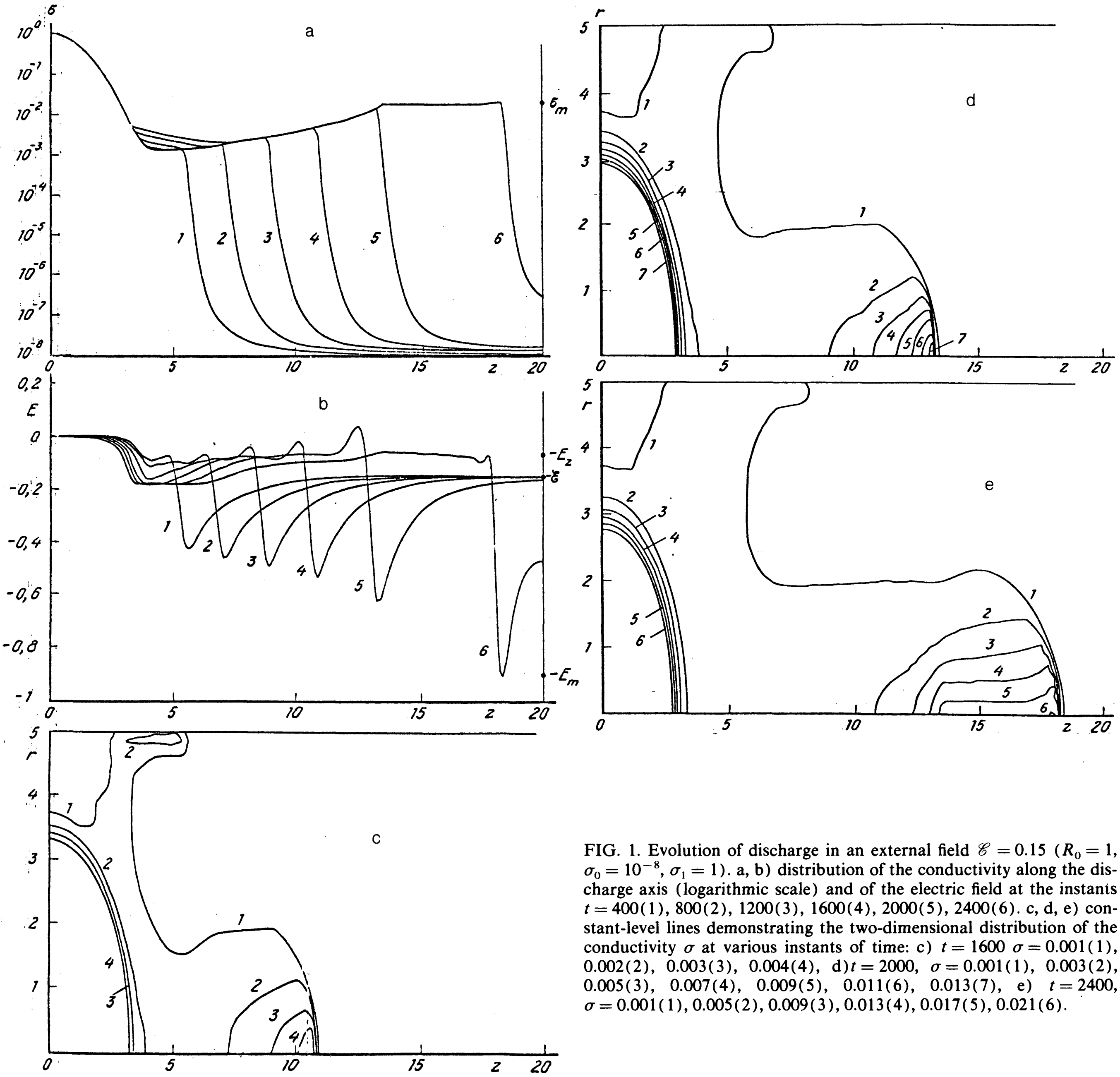

FIG. 1. Evolution of discharge in an external field $\mathscr{E} = 0.15$ ($R_0 = 1$, $\sigma_0 = 10^{-8}$, $\sigma_1 = 1$). a, b) distribution of the conductivity along the discharge axis (logarithmic scale) and of the electric field at the instants $t = 400(1)$, $800(2)$, $1200(3)$, $1600(4)$, $2000(5)$, $2400(6)$. c, d, e) constant-level lines demonstrating the two-dimensional distribution of the conductivity σ at various instants of time: c) $t = 1600$ $\sigma = 0.001(1)$, $0.002(2)$, $0.003(3)$, $0.004(4)$, d) $t = 2000$, $\sigma = 0.001(1)$, $0.003(2)$, $0.005(3)$, $0.007(4)$, $0.009(5)$, $0.011(6)$, $0.013(7)$, e) $t = 2400$, $\sigma = 0.001(1)$, $0.005(2)$, $0.009(3)$, $0.013(4)$, $0.017(5)$, $0.021(6)$.

C_2 that leads to the best agreement with the calculations is given by

$$C_2 = 0{,}95 \pm 0{,}1. \tag{16}$$

The coefficients C_0, C_1, and C_2 were determined here by averaging over numerical experiments with various $\mathscr{E}$ and R_0. The error in the determination of the coefficients was taken to be the rms deviation from the mean value. It is seen from (14)–(16) that the relative error in the calculation of C_0, C_1, and C_2 is small, of the order of $\sim 10\%$.

Relations (6), (8), and (19) are thus in good agreement with those of computer simulation. Jointly with (14)–(16), these relations make it possible to determine quantitatively the velocities of the streamer and of the electric field ahead and behind the discharge front. The situation with Eq. (7) is somewhat more complicated, since this relation is exact and contains no numerical coefficients. The conductivity σ_m behind the front, obtained by numerical calculation, is approximately double the value given by (7). Recognizing that the conductivities ahead and behind the front differ by several orders of magnitude, Eq. (7) can be taken to be in qualitative agreement with the numerical computation. The quantitative difference may be due to several factors. First, σ_m is the quantity most sensitive to commutation errors, since the conductivity ahead of the front increases exponentially and is altered by several orders of magnitude over a

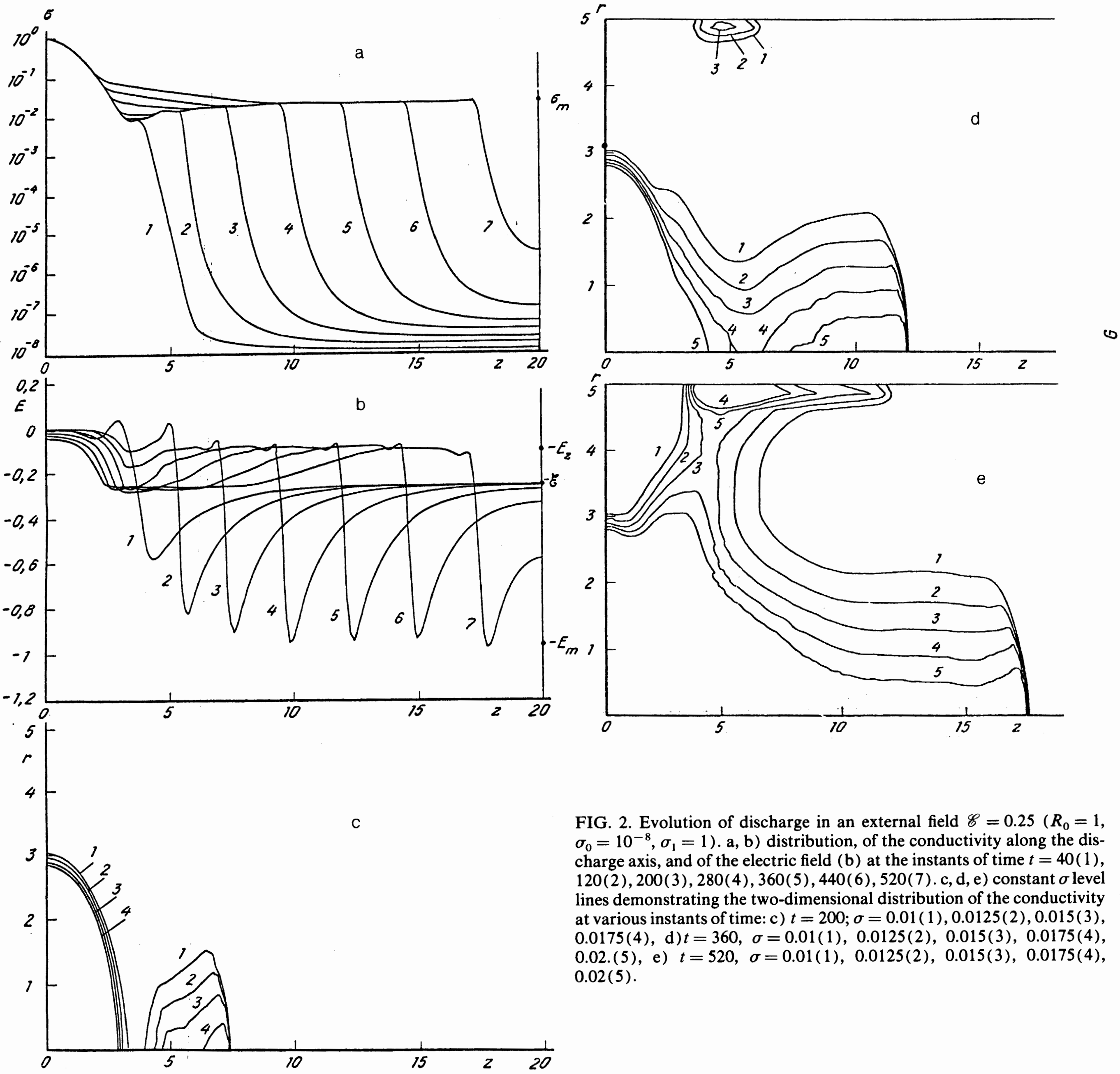

FIG. 2. Evolution of discharge in an external field $\mathscr{E} = 0.25$ ($R_0 = 1$, $\sigma_0 = 10^{-8}$, $\sigma_1 = 1$). a, b) distribution, of the conductivity along the discharge axis, and of the electric field (b) at the instants of time $t = 40(1)$, $120(2)$, $200(3)$, $280(4)$, $360(5)$, $440(6)$, $520(7)$. c, d, e) constant σ level lines demonstrating the two-dimensional distribution of the conductivity at various instants of time: c) $t = 200$; $\sigma = 0.01(1)$, $0.0125(2)$, $0.015(3)$, $0.0175(4)$, d) $t = 360$, $\sigma = 0.01(1)$, $0.0125(2)$, $0.015(3)$, $0.0175(4)$, $0.02.(5)$, e) $t = 520$, $\sigma = 0.01(1)$, $0.0125(2)$, $0.015(3)$, $0.0175(4)$, $0.02(5)$.

very short distance. Second, relation (7) is valid[5] if the width δ of the streamer front (the distance over which the conductivity changes by a factor of two) is small compared with the radius r_0 of the streamer head (the conductivity changes by several orders of magnitude over a distance r_0). The ratio δ/r_0 is of the order of $1/\Lambda_1$ (Ref. 4) and should be small when Λ_1 is large. In our calculations $\Lambda_1 \approx 10$, and the front width is approximately three or four times smaller than the head radius. It is possible that at higher values of the logarithm Λ_1 (and hence at lower values of δ/r_0) we shall obtain a more satisfactory agreement between (7) and the numerical calculation. Actually, however, when an attempt is made to increase substantially (this can be done by decreasing σ_0 by several orders of magnitude) the front width would become comparable with the distance between the numerical-lattice points and numerical instabilities would set in. We note finally that in the derivation of (7) it was assumed that the streamer motion is strictly self-similar, whereas in our case the streamer velocity varies slowly with time even in the quasistationary phase. The calculation results thus confirm relation (7) qualitatively, Although the possibility of a rigorous quantitative computation of σ_m on the basis of (7) remains uncertain.

The dynamics of streamer evolution at various values of the external field is of great interest. As shown in Ref. 5, stationary evolution is possible only at $\mathscr{E} = \mathscr{E}_c$, where $\mathscr{E}_c$ coincides with the field strength E_z behind the front. It can thus be stated that our present results allow the field $\mathscr{E}_c$ also to be calculated by using Eqs. (10) and (16). The value of $\mathscr{E}_c$ for a conductivity $\sigma_0 = 10^{-8}$ ahead of the front is approximately 0.1 ($0.1E_0$ in dimensional units). We were unfortunately unable to verify directly the statement that the streamer development is stationary at $\mathscr{E} = \mathscr{E}_c$. The point is that when the external field is weakened the time needed to reach that stationary phase of the development is increased. For fields weaker than 0.15, the transition period lengthened and the streamer passes through the discharge gap without entering into a stationary regime. The external fields investigated in the present study were therefore stronger than $\mathscr{E}_c$, the streamer motion was never strictly stationary, and slow increases of the streamer velocity and of the radius of its head were observed in the quasistationary phase. A slow increase of V and r_0 at $\mathscr{E} > \mathscr{E}_c$ was predicted in Ref. 5.

We note finally that, just as in Ref. 9, we observed that the streamer-head radius r_0 is proportional to the size R_0 of the high-conductivity nucleus.[2)]

4. CONCLUSIONS

On the basis of numerical calculations and a previously proposed qualitative theory we have derived here, for the first time ever, quantitative equations for the electric field in front and behind the front of a streamer discharge, as well as the streamer propagation velocity. In addition, we have confirmed qualitatively the relation (7) for the conductivity of the streamer channel. The conductivity and electric-field distributions obtained and presented here in dimensionless form are universal and are independent of the type of gas or of the pressure (only the measurement units depend on the specific gas characteristics). The calculation results make it also possible to describe intervals of different lengths by varying the nondimensionalizing scale z_0.

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APPENDIX

To determine the values of σ, ρ, φ, and E on in the $(k+1)$st time step we used the following numerical scheme

$$\sigma_{ij}^{k+1} = \sigma_{ij}^{k} \exp[\beta(E_{ij}^{k})\Delta t], \qquad \text{(A1)}$$

$$\rho_{ij}^{k+1} = \rho_{ij}^{k} \exp(-4\pi\sigma_{ij}^{k}\Delta t) + \frac{\exp(-4\pi\sigma_{ij}^{k}\Delta t) - 1}{4\pi\sigma_{ij}^{k}} \nabla \sigma_{ij}^{k} \mathbf{E}_{ij}^{k}, \qquad \text{(A2)}$$

$$\Delta\varphi_{ij}^{k+1} = -4\pi\rho_{ij}^{k+1}, \qquad \text{(A3)}$$

$$E_{ij}^{k+1} = -\nabla\varphi_{ij}^{k+1}, \qquad \text{(A4)}$$

where the subscripts i and j number the numerical-lattice points and Δt is the step in time. The differential operators (the Laplace operator and the Lagrange operator) in (A2–A4) were calculated by the standard difference scheme. Expression (A2) was obtained by integrating over the time interval from t_k to t_{k+1} (assuming σ and E to be constant) the equation

$$\frac{\partial\rho}{\partial t} + \nabla \sigma \mathbf{E} + 4\pi\sigma\rho = 0,$$

which is easily derived from (2) by expanding the divergence and using the relation $\operatorname{div}\mathbf{E} = 4\pi\rho$. Expression (A3) stands in fact for a system of inhomogeneous linear difference equations (with respect to the potential φ at different numerical lattice points), which was solved during each time to step. To stabilize the numerical calculations, the conductivity of the medium was smoothed over definite time intervals. The smoothing was over the nearest numerical-lattice points.

1) Reference 4 deals with a streamer discharge in a semiconductor. As noted in Refs. 4 and 5, however, the relations derived in Ref. 4 are valid, with insignificant modifications, also to a streamer in a gas.

2) This proportionality was observed only for low R_0 ($R_0 \ll R$). A dependence of r_0 on R set in when R_0 became comparable with R.

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