Instability of plasma waves in structures with resonant tunneling through a double barrier

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We consider surface plasma waves in semiconducting structures with resonant tunneling through a double barrier. We discuss the spectrum and the stability of the surface plasma waves. Under well-defined conditions the waves are found to be absolutely unstable. The growth rate increases with increasing wavelength. We discuss the effect of this instability on the work done by radiation sources using resonant tunnel structures.

1. INTRODUCTION

Recently a number of papers have investigated plasma instabilities in synthetic semiconducting structures.[1–5] In most of these papers drift instabilities (known from the theory of a gaseous plasma) in two-dimensional channels and superlattices have been studied. Unfortunately, an experimental realization of a drift instability is made difficult by the low value ($\sim 10^7$ cm/s) of the velocity of the current carriers actually attainable in semiconductors.

In the present paper we consider a new type of plasma instability which can arise in special semiconducting structures with resonant tunneling (RTS) through a double barrier.[6,7]

In RTS there is a layer of an unalloyed semiconductor with a built-in quantum well between two semiconductor layers with a high conductivity (Fig. 1). The dependence of the tunneling current through the undoped layer on the potential on it has a decreasing part which corresponds to a negative differential conductivity (NDC). At the present time RTS are used for the production of high-frequency generators (resonant tunnel diodes).[7]

It is well known that a NDC facilitates the development of an instability in the system. We consider the problem of the stability of surface plasma waves in RTS. At first sight the frequencies of the plasma waves in the conducting regions of the RTS are much higher than those for which a NDC has been observed. Indeed, the dopant concentration of conducting regions in a RTS on a GaAs–AlAs base is usually $n_0 \approx 2\times 10^{18}$ cm^{-3}, which corresponds to frequencies of the bulk plasmons $\omega_p = (e^2 n_0/\varepsilon m)^{1/2} \approx 2\pi\cdot 10^{13}$ s^{-1} (Ref. 8). However, on the boundaries of the conducting regions there are surface plasma waves. If the plasma waves on different boundaries do not interact one another, their frequency is $\omega_{ps} = 2^{-1/2}\omega_p$. The interaction between waves on different surfaces leads to the presence of two branches in the spectrum. One branch corresponds to in-phase oscillations, the other to antiphase oscillations of the surface charge density on different boundaries. Two features of the antiphase oscillations are important for us. Firstly, their frequency must decrease as a function of wavelength, since the longer the wavelength the stronger the relative attenuation of the fields produced by charges on different surfaces. It may thus turn out that the frequency of such oscillations decreases in the region where the NDC remains. Secondly, thanks to the presence of the normal component of the electric field the plasma oscillations may be amplified or excited due to the presence of the NDC.

2. CALCULATION

The evaluation of the spectrum of surface plasma waves is carried out in the hydrodynamic approximation for the idealized system shown in Fig. 2. A layer of an undoped semiconductor ($-\Delta < z < \Delta$, Fig. 2) with a built-in quantum well (not shown in Fig. 2) lies between two identical highly doped layers ($-d < z < -\Delta$ and $\Delta < z < d$, Fig. 2). On the boundaries of the structure ($z = \pm d$, Fig. 2) there are metallic electrodes with a conductivity which is assumed to be infinite. A constant bias $\mathscr{E}$ is applied to the electrodes so that the potential on the undoped layer (U_0, Fig. 3) corresponds to the NDC:

$$G = \frac{dj}{dU}\bigg|_{U=U_0} < 0, \tag{1}$$

where j is the tunneling current density. We neglect the charge of the electron beam in the undoped region and assume that Eq. (1) is valid in some range of frequencies (below we use as an estimate the frequency range up to 10^{12} Hz). In what follows we shall be interested in the stability of the initial state of the system (the point O in Fig. 3). To describe the plasma in the conducting regions we use the hydrodynamic approximation and neglect retardation effects. The equations of motion and the continuity and Poisson equations take the form

$$\frac{\partial \mathbf{v}}{\partial t} - \frac{\partial \mathbf{v}}{\partial \mathbf{r}}\mathbf{v}_0 = -\frac{e}{m}\nabla\varphi - \nu\mathbf{v}, \tag{2}$$

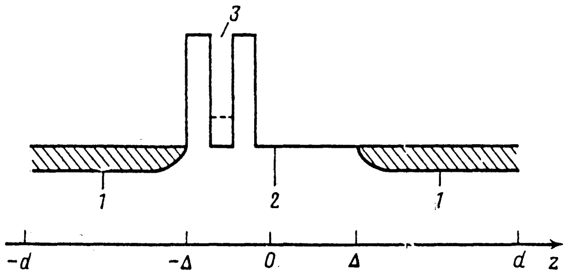

FIG. 1. Band diagram of a resonant tunneling structure: *1*—highly doped regions; *2*—undoped region; *3*—quantum well.

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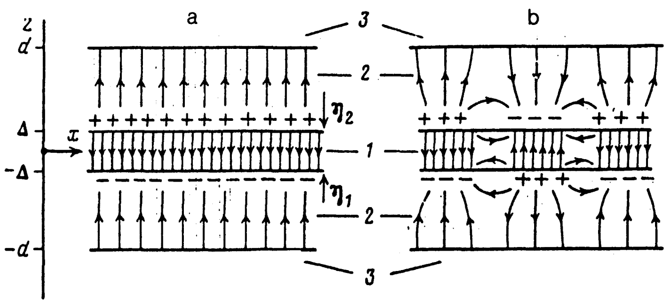

FIG. 2. Sketch of the structure and electric field distribution in uniform (a) and nonuniform (b) plasma waves: *1*, *2*—the same as in Fig. 1; *3*—metallic regions (contacts).

$$\frac{\partial n}{\partial t}+\nabla(n_0\mathbf{v}+n\mathbf{v}_0)=0, \tag{3}$$

$$\Delta\varphi=-4\pi ne/\varepsilon. \tag{4}$$

Here m and e are the electron mass and charge, ν is the collision frequency, n and v are the deviations of the electron density and velocity from their initial values n_0 and $\mathbf{v}_0$ ($\mathbf{v}_0$ is the drift velocity of the electrons due to the source of the potential $\mathscr{E}$), φ is the deviation of the electrostatic potential from its initial value, and ε is the dielectric permittivity (assumed to be independent of the spatial coordinates).

To describe the boundary conditions we assume that the carrier density at the boundaries ($z=\pm\Delta$, Fig. 2) of the conducting regions changes abruptly (i.e., that there is no transition layer) and that the Debye radius is much shorter than the characteristic scales for changes in the quantities which occur in Eqs. (2) to (4) (long-wavelength approximation). We can then introduce the concept of a surface charge density and write the boundary conditions in the form

$$\frac{\partial n_i}{\partial t}=\eta_i\left(n_0\mathbf{v}+n\mathbf{v}_0-\frac{1}{e}\mathbf{j}\right),\quad z=\pm\Delta, \tag{5}$$

$$\left.\frac{\partial\varphi}{\partial z}\right|_+-\left.\frac{\partial\varphi}{\partial z}\right|_-=4\pi n_i e,\quad z=\pm\Delta, \tag{6}$$

$$\varphi=0,\quad z=\pm d. \tag{7}$$

Here the n_i are the deviations of the surface charge density from their initial values, the index i numbers the boundaries ($i=1$ for $z=-\Delta$ and $i=2$ for $z=\Delta$), η_i is the unit vector perpendicular to the corresponding boundary (Fig. 2), and $\mathbf{j}$ is the deviation of the tunneling current from its initial value. Equation (7) describes the jump in the normal component of the electric field at the boundaries. We look for solutions of Eqs. (2) to (4) in the form $\propto\exp(i\mathbf{qr}+i\omega t)$. Equations (2) to (4) take the form

$$(i\omega+\nu-i\mathbf{q}\mathbf{v}_0)\mathbf{v}=(-e/m)\nabla\varphi, \tag{8}$$

$$n(\omega-\mathbf{q}\mathbf{v}_0)=n_0\mathbf{q}\mathbf{v}, \tag{9}$$

$$q^2\varphi=4\pi ne. \tag{10}$$

From Eqs. (8) to (10) we obtain

$$n[(i\omega+i\mathbf{q}\mathbf{v}_0+\nu)(\omega-\mathbf{q}\mathbf{v}_0)-i\omega_p^2]=0. \tag{11}$$

By putting the expression in the square brackets in (11) equal to zero we can obtain the dispersion relation for the bulk plasmons. However, in the discussion below (following the considerations given in the Introduction) we shall only be interested in surface plasmons. The solution describing surface plasmons corresponds to the relation $n=0$ in Eq. (11). In that case we get from (10)

$$q^2=q_x^2+q_z^2=0, \tag{12}$$

where for the sake of simplicity we put $q_y=0$. In what follows we use the notation $q_x\equiv k$.

It follows from (12) that we can write the spatial dependence of the potential in a form corresponding to surface waves:

$$\varphi\propto\exp(\pm kz-ikx). \tag{13}$$

We can write the boundary conditions (5) in the form

$$i\omega n_1(x)=n_0v_z(x,z=-\Delta)-\frac{G}{e}[\varphi(x,z=-\Delta)-\varphi(x,z=\Delta)],$$

$$i\omega n_2(x)=-n_0v_z(x,z=\Delta)+\frac{G}{e}[\varphi(x,z=-\Delta)-\varphi(x,z=\Delta)]. \tag{14}$$

Equations (14) together with Eqs. (8) and (10) and the boundary conditions (6) and (7) make it possible to find the dispersion relation for the surface plasma waves. In this case (as we shall show below) we can neglect the term $i\mathbf{q}\mathbf{v}_0=kv_0$ in comparison with ν in the equation of motion (8). Since this system is symmetric with respect to the $z=0$ plane there exist symmetric ($n_1=n_2$) and antisymmetric ($n_1=-n_2$) oscillations.

Neglecting the tunneling current and the damping ($G=0$, $\nu=0$) we find the following dispersion relations from the equations we have obtained for the symmetric oscillations:

$$\omega^{(+)}(k)=\omega_{ps}\left[1+e^{-2k\Delta}+\frac{e^{-2kd}(e^{2k\Delta}-e^{-2k\Delta})}{1+e^{-2k\Delta}}\right]^{1/2}, \tag{15}$$

where

$$\omega_{ps}=(2\pi e^2n_0/\varepsilon m)^{1/2},$$

and for the antisymmetric oscillations

$$\omega^-(k)=\omega_{ps}A^{1/2}(k), \tag{16}$$

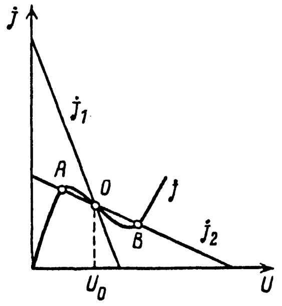

FIG. 3. Tunneling current density j through the structure as function of the potential U in the undoped region: $j_{1(2)}(U)=(\mathscr{E}-U)/R$, where R is the total resistivity (per 1 cm^2) of the conducting layers of the structure and $\mathscr{E}$ is the potential maintained by an external source on the metallic contacts.

where

$$A(k)=(1-e^{-2k\Delta})+\frac{1-e^{-4k\Delta}}{1-e^{-2kd}}e^{-2kd}. \qquad (17)$$

These solutions are shown schematically in Fig. 4. In what follows we shall only be interested in the low-frequency branch $\omega^{(-)}$. Including the damping ($\nu\neq 0$) and the tunneling current the dispersion relation it takes the form

$$\omega_{1,2}^{(-)}(k)=\frac{1}{2}\{i[\nu-\nu^*B(k)]$$
$$\pm[4\omega_{ps}^2A(k)-(\nu+\nu^*B(k))^2]^{1/2}\}, \qquad (18)$$

where

$$B(k)=(1-e^{-2k\Delta})(2\Delta k)^{-1}\left(1-e^{-2k(d-\Delta)}\frac{1-e^{-2k\Delta}}{1-e^{-2kd}}\right), \qquad (19)$$

$$\nu^*=-4\pi G\cdot 2\Delta/\varepsilon>0. \qquad (20)$$

We note that for $kd\ll 1$ and $\Delta\ll d$ we have

$$A(k)\approx 2\Delta/d+4k\Delta,$$

$$B(k)\approx 1-\Delta/d-k\Delta^2/d-\frac{1}{3}k^2d\Delta. \qquad (21)$$

For $k\Delta\ll 1$ and $kd\gg 1$ we have

$$A(k)\approx 2k\Delta, \quad B(k)\approx 1. \qquad (22)$$

3. DISCUSSION

Equation (18) determines the spectrum and the stability of the plasma waves. The waves become unstable for Im $\omega^{(-)}(k)<0$. We consider first of all the case $\nu>\nu^*B(k)$. It follows from Eq. (18) that even in that case there may be growing aperiodic (Re $\omega^{(-)}=0$) perturbations. Indeed, all perturbations for which

$$\nu\nu^*B(k)>\omega_{ps}^2A(k),$$

will grow with time, and it follows from (21) that the largest growth rate occurs for the uniform ($k=0$) solution. The evolution of a uniform perturbation causes the system to undergo a transition into one of the stable states (A or B in Fig. 3).

The system will be stable if

$$\nu\nu^*B(k=0)<\omega_{ps}^2A(k=0)$$

or

$$\nu\nu^*(1-\Delta/d)<\omega_{ps}^2 2\Delta/d. \qquad (23a)$$

Using (20) we can show that the stability condition (23a) is equivalent to the condition

$$|G^{-1}|>R, \qquad (23b)$$

where R is the resistivity (per 1 cm^2) of the conducting layers of the structure. Condition (23) means that the "load line" $[j_1(U)]$ decreases more steeply than the current-voltage characteristic of the RTS. Note that condition (23) must always be satisfied in a RTS used for the generation of electromagnetic radiation.[7] In that case Eq. (18) leads to the well known result[7] that a RTS not connected to an external resonant circuit is stable.

It therefore follows from Eq. (18) that for $\nu>\nu^*B(k)$ there can only be damped plasma waves in the system. Note that if $\nu-\nu^*B(k)$ is sufficiently small the frequencies of the weakly damped surface plasma waves can fall in the operating range of a generator based on a RTS. The excitation of such plasmons can lead to a significant reduction in the power of the generator.

We now consider the case $\nu^*B(k)>\nu$. Estimates given below show that such a situation can be reached in practice. It now follows from (18) that there always exists a growing solution, even when condition (23) is satisfied. It follows from Fig. 3 that when condition (23) is satisfied there are no other equilibrium states of the system (apart from the point O in Fig. 3). Thus auto-oscillations appear in the system even without an external oscillatory circuit. The nature of the auto-oscillations can be determined from Eq. (18). We consider the uniform ($k=0$) case, which is the most important from a practical point of view. Using (21) we then find that for

$$\nu\nu^*\left(1-\frac{\Delta}{d}\right)<\omega_{ps}^2\frac{2\Delta}{d}<\frac{[\nu+\nu^*(1-\Delta/d)]^2}{4}, \qquad (24)$$

we have Re $\omega^{(-)}=0$, Im $\omega^{(-)}<0$ and relaxation-type auto-oscillations arise in the system.

For

$$4\omega_{ps}^2\frac{2\Delta}{d}>\left[\nu+\nu^*\left(1-\frac{\Delta}{d}\right)\right]^2 \qquad (25)$$

the system is characterized by the eigenfrequency of the plasma waves:

$$\text{Re}\,\omega^{(-)}(k=0)=\left\{\omega_{ps}^2\frac{2\Delta}{d}+\left[\frac{\nu+\nu^*(1-\Delta/d)}{2}\right]^2\right\}^{1/2},$$

and the growth rate

$$\alpha=-\text{Im}\,\omega^{(-)}(k=0)=-\nu+\nu^*(1-\Delta/d).$$

The growth rate is for nonuniform oscillations equal to

$$-\text{Im}\,\omega(k)=-\nu+\nu^*B(k)$$

and decreases, according to (21), slowly with increasing k.

Taking into account plasma effects (i.e., the electron inertia in the conducting regions of the RTS) at $\upsilon^*B(k)>\upsilon$ thus leads to an essentially new result: auto-oscillations arise in the system even without an external circuit. The absence of an external electromagnetic circuit may make it possible to decrease the size of a generator based on RTS.

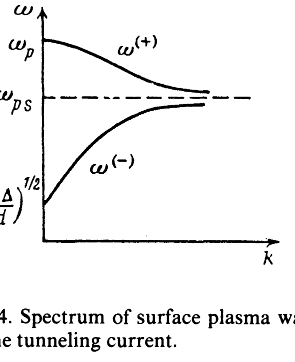

FIG. 4. Spectrum of surface plasma waves when one neglects damping and the tunneling current.

We give an estimate of the quantities occurring in the formulae given above. We use the data of Ref. 7, in which a generator based on RTS with a very high frequency is described. For a GaAs–AlAs RTS (Ref. 7) the negative differential conductance is equal to $6\times10^{-2}\ \Omega^{-1}$ for an area of $\sim10^{7}$ cm^2 and an undoped layer of thickness $2\Delta\approx700$ Å. This gives $\nu^*\approx2\pi\cdot5\times10^{11}\ \mathrm{s}^{-1}$. Starting from a mobility value $\mu=3000$ cm^2/V s we get $\nu=2\pi\cdot1.5\times10^{12}\ \mathrm{s}^{-1}$. Therefore we have $\nu\approx3\ \nu^*$. The value of ν^* may be larger for structures with a larger "peak-to-valley" ratio on the current-voltage characteristic. For the GaAs–AlAs structure in Ref. 7 this ratio was 1.4, a relatively small value. In the same paper a structure based on InGaAs–AlAs was discussed with a "peak-to-valley" ratio of more than 10. Hence, the condition $\nu^*>\nu$ may be reached in the very near future.

As we mentioned earlier, in Eq. (8) we neglected the term qv_0 in comparison with ν. Let us show that one can indeed do this. For a constant tunneling current $\sim10^5$ A/cm^2 and[7] $n_0=2\times10^{18}\ \mathrm{cm}^{-3}$ we find $v_0=3\times10^{5}$ cm/s. We estimate the typical value of q to be $q\sim2\pi/L$, where L is the lateral size of the structure for which one usually has $L\sim1$–$10\,\mu$m (Ref. 7). This leads to $qv_0\sim2\pi\cdot3\times10^{9}\ \mathrm{s}^{-1}$, considerably smaller than $\nu\approx2\pi\cdot10^{12}\ \mathrm{s}^{-1}$.

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[1]M. V. Krashennikov and A. V. Chaplik, Zh. Eksp. Teor. Fiz. **79**, 555 (1980) [Sov. Phys. JETP **52**, 279 (1980)].
[2]B. G. Martin, J. J. Quinn, and R. F. Wallis, Surface Sci. **105**, 145 (1981).
[3]B. G. Martin and R. F. Wallis, Phys. Rev. **B32**, 3824 (1985).
[4]J. Cen, K. Kempa, and P. Bakshi, Phys. Rev. **B38**, 10051 (1988).
[5]R. Gupta and B. K. Ridley, Phys. Rev. **B39**, 6208 (1989).
[6]R. Tsu and L. Esaki, Appl. Phys. Lett. **22**, 562 (1973).
[7]E. R. Brown and T. C. L. G. Sollner, 7th Intern. Workshop on Future Electron Devices, Superlattices and Quantum Functional Devices, Oct. 2–4 (1989), Toba, Japan.
[8]A. Mooradian and G. B. Wright, Phys. Rev. Lett. **16**, 999 (1966).

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