Production of supersymmetric particles in ($\nu\tilde{\gamma}\tilde{\nu}$)-interactions in an external field with allowance for the anomalous magnetic and electric dipole moments of the neutrino and photino

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The three-particle processes $\tilde{\nu}\to\nu\tilde{\gamma}$, $\nu\to\tilde{\nu}\tilde{\gamma}$, and $\tilde{\gamma}\to\tilde{\nu}\nu$ are studied in the field of a linearly polarized plane wave and in a constant crossed field, with allowance for the anomalous magnetic and electric dipole moments of the neutrino ν and photino $\tilde{\gamma}$, and general expressions for their probabilities, threshold values, and ultrarelativistic asymptotic behavior are derived. In the case of the asymptotic behavior the balance equations for the number of particles and superparticles are solved and the possible effect of the channels on the formation of relative concentrations in the early stages of the evolution of the universe is studied.

1. INTRODUCTION

The idea of supersymmetry is one of the most promising concepts on the basis of which it is apparently possible to build a closed and self-consistent field theory with unification of all types of interaction. The superpartners of the neutrino ν and the photon γ in the standard Glashow–Weinberg–Salam model are the chargeless scalar neutrino $\tilde{\nu}$ and the photino $\tilde{\gamma}$, which has spin $\frac{1}{2}$. At present there are no experimental indications of the existence of these or other superparticles. It is quite possible, however, that their masses are fairly small compared to those of the charged superpartners W and e^-, which at least in principle manifest themselves in electromagnetic interactions with matter, but which have a production threshold that is practically unattainable in the existing experimental and astrophysical situations.[1] Hence, a theoretical study of the interconversion processes for these chargeless particles and superparticles might prove highly promising for discovering traces of this type of interaction and substantiating the supersymmetry hypothesis. For instance, in Refs. 2–4 the basic variant of the model was applied to the $\tilde{\nu}\to\nu\tilde{\gamma}$ process and experimental consequences were discussed. But the process in the free case, as in the case with crossing processes, can be carried out only for a certain particle mass ratio, and the corresponding conclusions are preliminary.

When external fields come into the picture, the interaction of chargeless fermions is stimulated and new channels open owing to contributions from higher-order diagrams that incorporate the effective values of the anomalous magnetic moment (AMM) μ and, possibly, the electric dipole moment (EDM) ε of the fermions. In the standard model, as is well known,[5] the massive neutrino carries an exceptionally low AMM:

$$\mu\sim 10^{-19}\frac{m_\nu}{\text{eV}}\mu_B,$$

with μ_B the Bohr magneton; there are also certain arguments in favor of larger values.[6] As for EDM, its existence is related to *CP* invariance breaking in extended versions of the standard model. There are no strong arguments against the possibility of the photino having AMM (and EDM), although, to our knowledge, theoretical calculations of single-loop contributions to the photino's AMM have never been done. Of course, the fairly small AMM and EDM values require ultrahigh electromagnetic fields for the effect to be observable. The usual arguments refer to the magnetic fields in the vicinity of collapsed astrophysical objects (up to 10^{13} G and higher), laser fields at a beam focus ($\sim 10^8$ G), and the fields of oriented crystals. Yet calculations of the probabilities of the $\nu\to\nu\gamma$ and $\nu_1\to\nu_1\nu_2\bar{\nu}_2$ reactions involving "ordinary" particles in an external field[8–10] do not encourage the optimistic conclusion that such reactions can be observed.

Nonetheless, the $\nu\to\tilde{\nu}\tilde{\gamma}$, $\tilde{\nu}\to\nu\tilde{\gamma}$, and $\tilde{\gamma}\to\tilde{\nu}\nu$ channels in the field of a linearly polarized plane wave and in constant crossed fields are of interest in relation to their possible effect on the particle–superparticle balance in the cosmological aspect and of possible discovery of traces of supersymmetry in connection with further progress in laser engineering.

The invariant method of calculations developed in Refs. 8 and 10 allowing for AMM and EDM in fields of the plane-wave type is applied here for the first time to three-particle processes involving superparticles. Section 2 gives the effective Lagrangian and the wave functions of fermions carrying AMM and EDM. Sections 3 and 4 are devoted to calculations of the probabilities of processes in various channels in the field of a linearly polarized plane wave and in a constant crossed field, with exact account taken of all masses and calculation of the threshold values and ultrarelativistic asymptotics. The results are analyzed in Sec. 5.

2. THE LAGRANGIAN AND THE WAVE FUNCTIONS

For the interaction involving the left neutrino and its scalar superpartner the effective Lagrangian is

$$\mathscr{L}=ig\,[\overline{\Psi}_{\tilde{\gamma}}(1+\gamma^5)\,\Psi_\nu]\,\Psi_{\tilde{\nu}}+\text{H.c.},\tag{1}$$

where g is the dimensionless effective coupling constant. In the case of electron neutrinos and the single-loop approximation of the supersymmetry model with the standard set of particles, the coupling constant has been determined in Refs. 2 and 4 ($e^2/4\pi=1/137$),

$$g=e^3/(32\pi^2\cdot 2^{1/2}\sin^2\theta_W)\mathscr{F},\tag{2}$$

where for the accepted value of the Weinberg angle we have

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$$g^2 \approx 8\cdot 10^{-8}\mathscr{F}^2, \quad (2a)$$

where $\mathscr{F}$ is a dimensionless function of the particle masses in the set whose form for m_ν, $\tilde{m}_\gamma \ll \tilde{m}_e$, $\tilde{m}_\nu$, and $\tilde{m}_W$ is given in the same papers; the tilde signifies that the respective quantity refers to a superparticle. In the calculations below we use the solution found in Ref. 8 for the generalized Dirac equation for a massless particle carrying AMM and EDM and moving in the field of a plane wave $A_\alpha = a_\alpha f(kx)$:

$$\Psi = [\cos z + \sin z(\mu' M_+ - i\varepsilon'\gamma^5 M_-)]\frac{u(p)}{(2p_0)^{1/2}}\exp[-i(px)], \quad (3)$$

$$M_\pm = \frac{1}{2(kp)(-a^2)^{1/2}}(\hat{k}\hat{a}\hat{p} \pm \hat{p}\hat{k}\hat{a}), \quad (3a)$$

$$z = [-(\mu^2+\varepsilon^2)a^2 f^2]^{1/2}, \quad (3b)$$

$$\mu' = \frac{\mu}{(\mu^2+\varepsilon^2)^{1/2}}, \quad \varepsilon' = \frac{\varepsilon}{(\mu^2+\varepsilon^2)^{1/2}}, \quad (3c)$$

where $u(p)$ is the Dirac spinor. There also exists an exact invariant solution in the field of a circularly polarized plane,[8] but studying this type of polarization adds nothing new.

3. PROBABILITIES OF PROCESSES IN THE FIELD OF A LINEARLY POLARIZED PLANE WAVE

In the case of a linearly polarized plane wave $f = \sin(kx)$, standard methods[11] lead, after averaging and summation over spins are carried out, to the following expression for the square of the matrix element of the $\nu \to \tilde{\nu}\tilde{\gamma}$ reaction:

$$\frac{1}{2}\sum |\langle f|S|i\rangle|^2 = \frac{g^2(2\pi)^4}{4p_0\tilde{p}_0\tilde{\varkappa}_0}\sum_s \delta(q-\tilde{p}-\tilde{\varkappa}) \quad (4)$$

$$\times [A_s^{(+)}(\tilde{\varkappa}p) - \tilde{m}_\gamma m_\nu(\mu_\nu'\tilde{\mu}_\gamma' + \varepsilon_\nu'\tilde{\varepsilon}_\gamma')A_s^{(-)}], \quad (4a)$$

$$q = p + sk,$$

$$A_s^{(\pm)} = J_s^2(x_+) \pm J_s^2(x_-), \quad (4b)$$

$$x_\pm = [-a^2(\mu_\nu^2+\varepsilon_\nu^2)]^{1/2} \pm [-a^2(\tilde{\mu}_\gamma^2+\tilde{\varepsilon}_\gamma^2)]^{1/2}, \quad (4c)$$

where J_s is a Bessel function; s is the number of photons captured from the wave; p, $\tilde{p}$, and $\tilde{\varkappa}$ are the momenta of the neutrino, scalar neutrino, and photino; and the tilde, as before, signifies that the respective quantity refer to a superparticle.

Note that (4) contains no pseudoscalar term, the expression is symmetric with respect to AMM and EDM, and even in the differential probability the contributions of the two cannot be separated. This represents a marked contrast to four-neutrino processes, where a similar situation manifests itself only in the expression for the total probability:[9,10]

$$W(\nu\to\tilde{\nu}\tilde{\gamma}) = \frac{g^2}{8\pi p_0}\sum_{s=s_m}^{\infty}\frac{1}{q^2}[(q^2-\tilde{m}_\gamma^2-\tilde{m}_\nu^2)^2 - 4\tilde{m}_\gamma^2\tilde{m}_\nu^2]^{1/2}$$

$$\times\left[A_s^{(+)}\frac{pq}{2q^2}(q^2+\tilde{m}_\gamma^2-\tilde{m}_\nu^2) - \tilde{m}_\gamma m_\nu(\mu_\nu'\tilde{\mu}_\gamma'+\varepsilon_\nu'\tilde{\varepsilon}_\gamma')A_s^{(-)}\right], \quad (5)$$

$$s_m = \frac{1}{2(kp)}[(\tilde{m}_\nu+\tilde{m}_\gamma)^2 - m_\nu^2]. \quad (5a)$$

For $m_\nu > \tilde{m}_\nu + \tilde{m}_\gamma$ the value of s_m is negative and the process can proceed both by yielding photons to the wave and by capturing photons from the wave. Otherwise, only capture is possible, and W is nonzero only in the presence of an external field.

The integral probabilities of crossing processes, obtained in a similar manner, are

$$W(\tilde{\nu}\to\nu\tilde{\gamma}) = \frac{g^2}{4\pi\tilde{p}_0}\sum_{s=s_m}^{\infty}\frac{1}{q^2}[(q^2-m_\nu^2-\tilde{m}_\gamma^2)^2 - 4m_\nu^2\tilde{m}_\gamma^2]^{1/2}$$

$$\times\left[\frac{1}{2}A_s^{(+)}(q^2-m_\nu^2-\tilde{m}_\gamma^2) + \tilde{m}_\gamma m_\nu(\mu_\nu'\tilde{\mu}_\gamma'+\varepsilon_\nu'\tilde{\varepsilon}_\gamma')A_s^{(-)}\right], \quad (6)$$

$$q = \tilde{p} + sk, \quad s_m = \frac{1}{2(k\tilde{p})}[(m_\nu+\tilde{m}_\gamma)^2 - \tilde{m}_\nu^2], \quad (6a)$$

and the expression for $W(\tilde{\gamma}\to\nu\tilde{\nu})$ can be obtained from (5) by interchanging $\tilde{m}_\gamma \leftrightarrow m_\nu$ and $p \leftrightarrow \tilde{\varkappa}$ with appropriate changes in the sign of s_m. In the absence of the field and for $m_\nu = \tilde{m}_\gamma = 0$, the result of Ref. 4 follows from (6).

4. PROBABILITIES OF PROCESSES IN A CONSTANT CROSSED FIELD

In the case of a constant crossed field $f = (kx)$, which in the ultrarelativistic case approximates constant fields of other configurations,[12] the following expression can be obtained for the square of the matrix element of the $\nu \to \tilde{\nu}\tilde{\gamma}$ process:

$$\frac{1}{2}\sum |\langle f|S|i\rangle|^2 = \frac{g^2(2\pi)^4}{2p_0\, 2\tilde{p}_0\, 2\tilde{\varkappa}_0}\sum_{\xi,\rho=\pm 1}\delta(p+\xi k(\rho)-\tilde{p}-\tilde{\varkappa})$$

$$\times\Big\{(\tilde{\varkappa}p) - \rho\tilde{m}_\gamma m_\nu(\mu_\nu'\tilde{\mu}_\gamma'+\varepsilon_\nu'\tilde{\varepsilon}_\gamma') + \frac{\xi m_\nu}{(kp)(-a^2)^{1/2}}$$

$$\times[(\varepsilon_\nu'((\tilde{\varkappa}k)(ap)-(\tilde{\varkappa}a)(kp)) + \mu_\nu'(\tilde{\varkappa}kap)]$$

$$+\frac{\xi\rho\tilde{m}_\gamma}{(k\tilde{\varkappa})(-a^2)^{1/2}}[\tilde{\varepsilon}_\gamma'(kp)(a\tilde{\varkappa}) - (ap)(k\tilde{\varkappa})) + \tilde{\mu}_\gamma'(pka\tilde{\varkappa})]\Big\}, \quad (7)$$

where

$$k(\rho) = \{[-a^2(\mu_\nu^2+\varepsilon_\nu^2)]^{1/2} + \rho[-a^2(\tilde{\mu}_\gamma^2+\tilde{\varepsilon}_\gamma^2)]^{1/2}\}k, \quad (7a)$$

$$\times(abcd) \equiv e^{\mu\nu\alpha\beta}a_\mu b_\nu c_\alpha d_\beta, \quad (7b)$$

and the other notation was specified earlier.[1)] In contrast to the result obtained in Sec. 3, Eq. (7) and the differential probability contain a pseudoscalar term that gives rise to a difference in the contributions of AMM and EDM. It is easy to see that this term vanishes after phase-volume integration. After such integration it is convenient to write the final result for the probability in terms of the following invariant positive parameter:

$$\chi \equiv \chi(\rho) = \frac{1}{m_\nu^2}\{[(\mu_\nu^2+\varepsilon_\nu^2)^{1/2} + \rho(\tilde{\mu}_\gamma^2+\tilde{\varepsilon}_\gamma^2)^{1/2}]^2(pF^2p)\}^{1/2}, \quad (8)$$

where F is the field-strength tensor. After several transformations we get [where $\theta(x)$ is the well-known step function]

$$W(\nu\to\tilde{\nu}\tilde{\gamma}) = \sum_{\xi,\rho=\pm 1} W(\xi,\rho)\theta(\xi\chi-\chi_0), \quad (9)$$

$$\chi_0 \equiv \chi_0(\xi) = \frac{1}{2}[(\Delta_\gamma+\Delta_\nu)^2-1]\theta[\xi(\Delta_\gamma+\Delta_\nu-1)], \quad (9a)$$

$$\Delta_\gamma = \tilde{m}_\gamma/m_\nu, \quad \Delta_\nu = \tilde{m}_\nu/m_\nu, \quad (9b)$$

$$W(\xi,\rho)=\frac{g^2m_\nu^2}{16\pi p_0}(1+2\xi\chi)^{-1}\left\{\chi-\frac{\xi}{2}[(\tilde{\Delta}_\gamma+\xi\tilde{\Delta}_\nu)^2-1]\right\}^{1/2}$$
$$\times\left\{\chi-\frac{\xi}{2}[(\tilde{\Delta}_\gamma-\xi\tilde{\Delta}_\nu)^2-1]\right\}^{1/2}\left\{\frac{1+\xi\chi}{2(1+2\xi\chi)}(1+2\xi\chi+\tilde{\Delta}_\gamma^2-\tilde{\Delta}_\nu^2)\right.$$
$$\left.-\rho\tilde{\Delta}_\gamma(\mu_\nu'\tilde{\mu}_\gamma'+\varepsilon_\nu'\tilde{\varepsilon}_\gamma')\right\}. \quad (10)$$

The value $\xi=1$ corresponds to a situation in which energy is "extracted" from the field. This occurs for any mass ratio, provided that the value of χ is above the threshold, in contrast to the case without an external field, where for $\tilde{\Delta}_\gamma+\tilde{\Delta}_\nu>1$ the process is forbidden energetically. At $\xi=-1$ energy is "pumped" into the field, which is possible only if $\tilde{\Delta}_\gamma+\tilde{\Delta}_\nu<1$, with the process taking place in the absence of a field as well.

As Eqs. (8)–(9b) imply, in the most interesting case (from the standpoint of the effect of a field) $\tilde{\Delta}_\gamma+\tilde{\Delta}_\nu>1$, the threshold value of χ is determined by the condition

$$\chi(\rho=1)\equiv\chi_{\rm thr}=\frac{1}{2}[(\tilde{\Delta}_\gamma+\tilde{\Delta}_\nu)^2-1], \quad (11)$$

or

$$(pF^2p)_{min}=\frac{[(\tilde{m}_\gamma+\tilde{m}_\nu)^2-m_\nu^2]^2}{4[(\mu_\nu^2+\varepsilon_\nu^2)^{1/2}+(\tilde{\mu}_\gamma^2+\tilde{\varepsilon}_\gamma^2)^{1/2}]^2}. \quad (11a)$$

An expression similar to (9) and (10) for the probability of the crossing process $\tilde{\nu}\to\nu\tilde{\gamma}$ has the form

$$W(\tilde{\nu}\leftrightarrow\nu\tilde{\gamma})=\sum_{\xi,\rho=\pm1}W(\xi,\rho)\theta(\xi\chi-\chi_0), \quad (12)$$

$$W(\xi,\rho)=\frac{g^2\tilde{m}_\nu^2}{8\pi\tilde{p}_0}(1+2\xi\chi)^{-1}\left\{\chi-\frac{\xi}{2}[(\Delta_\nu+\xi\Delta_\gamma)^2-1]\right\}^{1/2}$$
$$\times\left\{\chi-\frac{\xi}{2}[(\Delta_\nu-\xi\Delta_\gamma)^2-1]\right\}^{1/2}\left\{\frac{1}{2}(1+2\xi\chi-\Delta_\nu^2-\Delta_\gamma^2)\right.$$
$$\left.+\rho\Delta_\nu\Delta_\gamma(\mu_\nu'\tilde{\mu}_\gamma'+\varepsilon_\nu'\tilde{\varepsilon}_\gamma')\right\}, \quad (13)$$

where we have introduced the notation $\Delta_\nu=m_\nu/\tilde{m}_\nu$ and $\Delta_\gamma=\tilde{m}_\gamma/\tilde{m}_\nu$ and redefined χ and χ_0 appropriately. As in the case of a linearly polarized plane wave, in crossed fields the probability of the $\tilde{\gamma}\to\nu\tilde{\nu}$ process can be obtained from (8)–(10) by interchanging $\tilde{m}_\gamma\leftrightarrow m_\nu$, $p\leftrightarrow\tilde{\varkappa}$.

In accordance with (11), the behavior of the probability (9) and (10) near the threshold is specified by the expression ($\tilde{m}_\nu$, $\tilde{m}_\gamma\neq0$):

$$W(\nu\to\tilde{\nu}\tilde{\gamma})\approx W(1,1)\approx\frac{g^2(2\tilde{m}_\gamma^3\tilde{m}_\nu)^{1/2}}{16\pi p_0}\left(\frac{m_\nu}{\tilde{m}_\nu+\tilde{m}_\gamma}\right)^2$$
$$\times\left[\frac{(\tilde{m}_\gamma+\tilde{m}_\nu)^2+m_\nu^2}{2m_\nu(\tilde{m}_\gamma+\tilde{m}_\nu)}-(\mu_\nu'\tilde{\mu}_\gamma'+\varepsilon_\nu'\tilde{\varepsilon}_\gamma')\right](\chi-\chi_{\rm thr})^{1/2}, \quad (14)$$

and that of the crossing process by the expression (m_ν,$\tilde{m}_\gamma\neq0$)

$$W(\tilde{\nu}\to\nu\tilde{\gamma})\approx W(1,1)$$
$$\approx\frac{g^2\tilde{m}_\nu}{16\pi\tilde{p}_0}\frac{(2m_\nu\tilde{m}_\gamma)^{3/2}}{(m_\nu+\tilde{m}_\gamma)^2}(1+\mu_\nu'\tilde{\mu}_\gamma'+\varepsilon_\nu'\tilde{\varepsilon}_\gamma')(\chi-\chi_{\rm thr})^{1/2}, \quad (15)$$

where χ and $\chi_{\rm thr}$ are redefined appropriately.

In the ultrarelativistic limit ($\chi\gg\tilde{\Delta}_\gamma^2$, $\tilde{\Delta}_\nu^2$) we have

$$W(\nu\to\tilde{\nu}\tilde{\gamma})=\sum_{\rho=\pm1}W(1,\rho)\approx\frac{g^2}{32\pi p_0}\mu_{eff}(pF^2p)^{1/2}, \quad (16)$$

$$\mu_{\rm eff}=\max\left[(\mu_\nu^2+\varepsilon_\nu^2)^{1/2},(\tilde{\mu}_\gamma^2+\tilde{\varepsilon}_\gamma^2)^{1/2}\right], \quad (16a)$$

and in the analogous approximation for the $\tilde{\nu}\to\nu\tilde{\gamma}$ process we have

$$W(\tilde{\nu}\to\nu\tilde{\gamma})=\sum_{\rho=\pm1}W(1,\rho)\approx\frac{g^2}{8\pi\tilde{p}_0}\mu_{eff}(\tilde{p}F^2\tilde{p})^{1/2}. \quad (17)$$

Equations (16) and (17) do not contain functions of particle masses and are valid for arbitrary constant fields.

5. DISCUSSION AND ANALYSIS OF RESULTS

Estimates of the possibilities of stimulating and identifying four-neutrino processes in existing laser fields done in Ref. 9 remain valid for the channels considered above involving superparticles and are highly pessimistic. As for constant fields, according to Eq. (11a) the threshold value of the field that opens the $\nu\to\tilde{\nu}\tilde{\gamma}$ channel ($\tilde{m}_\nu\gg m_\nu,\tilde{m}_\gamma$) is

$$F_{\rm thr}\sim\left(\frac{\tilde{m}_\nu}{m_e}\right)^2\left(\frac{m_e}{p_0}\right)\frac{F_0}{\delta}, \quad (18)$$

where

$$F_0=m_e^2/e_0=4{,}41\cdot10^{13}\ \mathrm{G}$$

is the Schwinger field, and we have put

$$\mu_{eff}=\delta\mu_B.$$

Since the value of $\tilde{m}_\nu$ is, apparently, on the order of several dozen GeV, for an upper bound $\delta\sim10^{-10}$ (Ref. 6) and reasonable values of p_0 the threshold value is several orders of magnitudes higher than the most optimistic estimates of magnetic fields in the vicinity of neutron stars.

More interesting are the ideas concerning the role of the processes under discussion in forming the balance between ν, $\tilde{\nu}$, and $\tilde{\gamma}$ in the early stages of the evolution of the universe. More precisely, it can be assumed that at the time of the Big Bang the magnetic field was high enough for the asymptotic behavior specified by (16) and (17) to manifest itself. Then, at ultrarelativistic energies and with an isotropic distribution, Eqs. (16) and (17) are momentum-independent and linked through the following relations:

$$W(\nu\to\tilde{\nu}\tilde{\gamma})\approx W(\tilde{\gamma}\to\tilde{\nu}\nu)\approx{}^1/_4W(\tilde{\nu}\to\nu\tilde{\gamma}), \quad (19)$$

where

$$W(\tilde{\nu}\to\nu\tilde{\gamma})\equiv\frac{1}{t_0}\approx\frac{g^2F}{8\pi}\mu_{eff}. \quad (19a)$$

Let us find the time dependence of the concentration of such particles and superpartners forming as a result of these reactions. Clearly, the differential balance equations have the form

$$\begin{cases}\dfrac{d\tilde{n}_\nu}{d\tau}=\dfrac{1}{4}(n_\nu+\tilde{n}_\gamma)-\tilde{n}_\nu,\\[2mm] \dfrac{d\tilde{n}_\gamma}{d\tau}=\dfrac{1}{4}(n_\nu-\tilde{n}_\gamma)+\tilde{n}_\nu,\\[2mm] \dfrac{dn_\nu}{d\tau}=\dfrac{1}{4}(\tilde{n}_\gamma-n_\nu)+\tilde{n}_\nu.\end{cases} \quad (20)$$

Here $\tau = t/t_0$, and the other notation is obvious. Solving this system of equations by standard methods, we obtain:

$$\tilde{n}_\nu = \left[\frac{1}{4\cdot 3^{1/2}}(n_\nu^{(0)}+\tilde{n}_\gamma^{(0)})+\frac{1}{3+3^{1/2}}\tilde{n}_\nu^{(0)}\right]\exp(c_2\tau) + \left[-\frac{1}{4\cdot 3^{1/2}}(n_\nu^{(0)}+\tilde{n}_\gamma^{(0)})+\frac{1}{3-3^{1/2}}\tilde{n}_\nu^{(0)}\right]\exp(c_3\tau), \quad (21)$$

$$\left\{\begin{matrix} n_\nu \\ \tilde{n}_\gamma \end{matrix}\right\} = \pm\frac{1}{2}(n_\nu^{(0)}-\tilde{n}_\gamma^{(0)})\exp(c_1\tau) + \left[\frac{1}{2(3-3^{1/2})}(n_\nu^{(0)}+\tilde{n}_\gamma^{(0)})+\frac{1}{3^{1/2}}\tilde{n}_\nu^{(0)}\right]\exp(c_2\tau) + \left[\frac{1}{2(3+3^{1/2})}(n_\nu^{(0)}+\tilde{n}_\gamma^{(0)})-\frac{1}{3^{1/2}}\tilde{n}_\nu^{(0)}\right]\exp(c_3\tau),$$

$$c_1 = -\frac{1}{2}, \quad \left\{\begin{matrix} c_2 \\ c_3 \end{matrix}\right\} = \frac{1}{2}(-1\pm 3^{1/2}). \quad (22)$$

The superscript "(0)" indicates concentrations at an initial time $t = 0$.

If the above conditions for the reactions are retained over a certain period $t > t_0$, the term with c_2 dominates in (21), and the density ratio is

$$\frac{n_\nu}{\tilde{n}_\nu} = \frac{\tilde{n}_\gamma}{\tilde{n}_\nu} = \frac{\frac{1}{2(3^{1/2}-1)}(n_\nu^{(0)}+\tilde{n}_\gamma^{(0)})+\tilde{n}_\nu^{(0)}}{\frac{1}{4}(n_\nu^{(0)}+\tilde{n}_\gamma^{(0)})+\frac{1}{1+3^{1/2}}\tilde{n}_\nu^{(0)}} > 1. \quad (23)$$

But if the given conditions are retained over a period $t \gg t_0$, the initial time can be chosen so that (23) remains valid for the initial densities as well, whence

$$\frac{n_\nu}{\tilde{n}_\nu} = \frac{\tilde{n}_\gamma}{\tilde{n}_\nu} = \frac{n_\nu^{(0)}}{\tilde{n}_\nu^{(0)}} = \frac{\tilde{n}_\gamma^{(0)}}{\tilde{n}_\nu^{(0)}} \equiv x,$$

and from (23) we find that

$$x = 1+3^{1/2}. \quad (24)$$

Other numerical estimates appear to be quite indeterminate because the situation is the same for the masses and AMM and EDM of particles and superparticles and in the values of fields at earlier stages in the evolution of the universe. For instance, it follows from (19a) that

$$t_0 \approx \frac{(F_0/F)}{\delta\mathcal{F}^2}\cdot 4\cdot 10^{-13}\ \text{s},$$

Assuming, for the sake of an estimate, that $\mathcal{F} \sim 1$, we find that t_0 varies from $(F_0/F)\times 10^5$s ($m_\nu \sim 10$ eV and $\delta \sim 10^{-18}$) to a minimum value of the order of $\sim (F_0/F)\times 10^{-3}$ s ($\delta \sim 10^{-10}$; see Ref. 6).

In any case, the above reactions can dominate in forming the balance of ν, $\tilde{\nu}$, and $\tilde{\gamma}$ since these particles are evidently the lightest of all particles and superparticles (three-particle reactions involving a photon do not change the type of particle and the respective density). Experimental verification of the "relic" relation (24) would be extremely interesting from the cosmological standpoint.

[1)] The parameters ρ and ξ allow for the contribution to the matrix elements of various terms that appear when we expand the trigonometric functions in Eq. (3) by the Euler formulas and then integrate, with respect to coordinates, the products of these functions of the type $\exp[\pm i(z_\nu \pm \tilde{z}_\gamma)]$, with the value of ρ corresponding to the relative sign of z_ν and $\tilde{z}_\gamma$ and the value of ξ to the general sign of their combination.

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