Classical mesoscopic effect due to scattering of conduction electrons by a rough boundary

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The effect on the conductance of correlations between events in which electrons undergo classical scattering from the rough boundary of a channel formed by a two-dimensional electron gas is discussed. The corresponding correlation contribution, which depends on the specific realization of the shape of the uneven boundary is calculated. This contribution causes the conductance to exhibit a stochastic oscillatory dependence on a weak magnetic field. This dependence exhibits fractal structure, whose fractal dimension is determined. At sufficiently high temperatures the classical correlation contribution can exceed the contribution of universal quantum fluctuations.

It has recently been shown that mesoscopic effects can have not only a quantum but also a purely classical nature.[1,2] Such "classical mesoscopics" is governed by the correlation between successive acts of scattering of an electron moving along a classical trajectory. The classical correlation contribution to the conductivity, as also the quantum interference contribution,[3–5] is manifested in stochastic oscillations of the conductivity in weak magnetic fields. Generally speaking, the correlation contribution is not small compared with quantum contributions and it can be distinguished from the quantum contribution because the correlation and quantum contributions have different temperature dependences.

The mesoscopic fluctuations predicted in Refs. 1 and 2 are associated with the scattering of electrons by volume defects whose fields have a short range. Scattering of electrons by random nonuniformities of the boundary of the sample should also lead to mesoscopic fluctuations of the conductivity. Since the relative importance of the surface relaxation channel increases as the dimensions of the sample decrease, the amplitude of mesoscopic fluctuations governed by surface scattering can exceed the amplitude of fluctuations governed by volume scattering.

We shall study surface mesoscopic effects for the case of a two-dimensional (2*D*) electronic channel, bounded by rough boundaries (Fig. 1).

Roukes *et al.*[6,7] recently showed that electron transport in such a channel can often be described classically, if the channel contains at least several levels of size quantization. The direction of motion of an electron, moving in the channel under the action of an electric field **E**, changes with each collision with the boundary and the trajectory of the electron is a jagged line whose segments have a random length. In the leading-order approximation the conductivity of the channel depends on the average characteristics of the rough boundary (for example, on the specularity parameter). The dependence on the specific realization of the random rough boundary appears in higher orders of perturbation theory in the roughness scale.

In an external magnetic field the points at which an electron collides with the boundary are shifted, i.e., the distribution of scatterers along the electron trajectory and the associated corrections to the conductivity change. When the magnetic field varies continuously, small-amplitude stochastic oscillations of the conductivity (called "grass") arise. The pattern of these oscillations is determined by the specific arrangement of the irregularities on the boundary of the channel and is thus an individual characteristic of the sample (fingerprint). The average amplitude and period of the oscillations depend on the statistical characteristics of the boundary—the height ζ and length L of the irregularities as well as the width a and length b of the channel.

The current density j_x in a 2*D* electron channel is determined by the standard formula

$$j_x(x,y)=2e\int\frac{d^2\mathbf{p}}{(2\pi\hbar)^2}v_x\frac{\partial f_0}{\partial\varepsilon}\chi. \tag{1}$$

Here e is the elementary charge, $\chi df_0/d\varepsilon$ is the nonequilibrium correction to the Fermi distribution function $f_0(\varepsilon)$, and the y axis is perpendicular to the mean boundary of the sample, which is also the plane $y=0$ (Fig. 1).

We give the classical kinetic equation, in which volume collisions are taken into account in the τ approximation, for the function χ:

$$\frac{\chi}{\tau}+v_x\frac{\partial\chi}{\partial x}+v_y\frac{\partial\chi}{\partial y}=e\mathbf{E}\mathbf{v}=eEv_x. \tag{2}$$

This equation is solved by the method of characteristics. We introduce the coordinates of the points where the electron collides with the boundary ($X_1, X_2,..., X_n, ...$) and the angles of incidence ($\theta, \theta_1, \theta_2, ..., \theta_n, ...$). On each section between the points X_n and X_{n+1} the velocity of the electron is constant, and in addition we have $v_x=v_F\cos\theta_n$ and $v_y=v_F\sin\theta_n$, where v_F is the Fermi velocity. Using the notation introduced above, we can write the solution of Eq. (2) satisfying the condition $\chi(x=\pm\infty)=0$ in the form

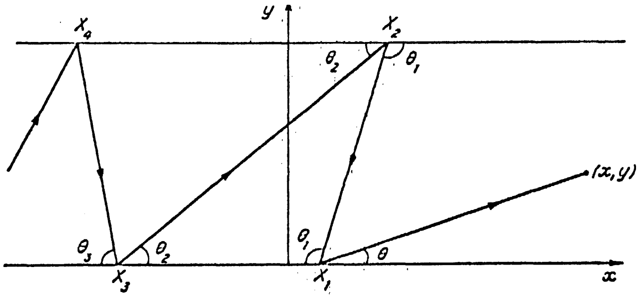

FIG. 1. Trajectory of an electron in a two-dimensional electron channel with diffuse boundaries.

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$$\chi=e\int_{X_1}^{x} dx'E\exp\left(\frac{x-x'}{l\cos\theta}\right)+e\sum_{n=1}^{\infty}\int_{X_{n+1}}^{X_n} dx'E$$
$$\times\exp\left\{-\frac{1}{l}\left[\frac{x-X_1}{\cos\theta}+\frac{X_1-X_2}{\cos\theta_1}+\ldots+\frac{X_n-x'}{\cos\theta_n}\right]\right\}, \quad (3)$$

where $l=v_F\tau$ is the mean free path length in the volume of the channel.

In what follows we study the strong size effect, when

$$l\gg a. \quad (4)$$

In this case

$$N=l/a\gg 1 \quad (5)$$

segments of the electron trajectory, on which the difference of the exponential factor from unity can be neglected, make the effective contribution to the sum over n in Eq. (3). This fact makes it possible to express the sum appearing in Eq. (3) in terms of the approach angles:

$$\int_{X_1}^{x} dx'+\int_{X_2}^{X_1} dx'+\ldots+\int_{X_{n+1}}^{X_n} dx'$$
$$=y\operatorname{ctg}\theta+a(\operatorname{ctg}\theta_1+\operatorname{ctg}\theta_2+\ldots+\operatorname{ctg}\theta_n). \quad (6)$$

Two successive values of θ (θ_{n-1} and θ_n) are related by a relation which is determined by the reflection law at the point $x=X_n$ of the random boundary. Without specifying this reflection law, we write this relation in the general form

$$\theta_{2n+1}=\psi_b(\theta_{2n}, X_{2n+1}),\ \theta_{2n}=\psi_u(\theta_{2n-1}, X_{2n}), \quad (7)$$

where the functions ψ_b and ψ_u prescribe the law of reflection of an electron from the bottom and top boundaries of the channel. The dependence of ψ_b and ψ_u on the angle is determinate (for example, the local angle of incidence is equal to the angle of reflection), while their dependence on the coordinate X_n is random. If the correlation length of the irregularities of the boundary is equal to L, then the characteristic size of the domain of ψ_b and ψ_u as functions of X_n is equal to the scale L.

Thus the formulas (4), (6), and (7) completely determine the nonequilibrium distribution function χ.

Substituting the expression for χ into the formula (1) we obtain the current density at a point with coordinates x and y:

$$j_x(x,y)=2e^2\int\frac{d^2\mathbf{p}}{(2\pi\hbar)^2}v_xE\frac{\partial f_0}{\partial\varepsilon}\{y\operatorname{ctg}\theta+a\operatorname{ctg}\psi_b(\theta,X_1)$$
$$+a\operatorname{ctg}[\psi_u(\psi_b(\theta,X_1),X_2]+\ldots\}+(\psi_b\rightleftharpoons\psi_u). \quad (8)$$

The first term in braces describes the average conductivity of a sample with a diffuse boundary (the contribution of the section of the electron trajectory after the last collision with the boundary is included). In accordance with the results of Fuks[8] it contains the factor $\ln(v_F\tau/a)$, arising when the logarithmic divergence in the integral over the angle θ is cut off. The second term is associated with the nonuniformity of the flux of electrons reaching the point of observation as a result of the last collision with the boundary at the point X_1. It is obvious that it does not contain information about the correlation between successive acts of scattering of an electron by the boundary. Such information is contained in the third term, which depends on the coordinates of the two collisions, X_1 and X_2. It will be shown below that it is this term that results in the appearance of small-scale fluctuations of the conductivity of the channel in a magnetic field. As for the correlation terms of higher orders, i.e., depending on the coordinates of three and more collisions, they also contain a contribution that oscillates in a magnetic field, but the amplitude of the oscillations is found to be small.

In calculating the integrals in Eq. (8) the coordinates of the collision points X_1 and X_2 must be viewed as functions of x and y:

$$X_1=x-y\operatorname{ctg}\theta,\ X_2=x-y\operatorname{ctg}\theta-a\operatorname{ctg}[\psi_b(\theta, x-y\operatorname{ctg}\theta)]. \quad (9)$$

It follows from Eqs. (8) and (9) that the third (correlation) term in Eq. (8) contains a random function of a complicated argument, which, in turn, contains a random function. It is obvious that the correlation scale of this complicated function with respect to the variable x is $a/L\gg 1$ times smaller than L. This property reflects the general principle that in the case of mechanical motion in a random potential the initial conditions are "forgotten." The correlation scale of the higher-order terms, which are omitted in Eq. (8) and correspond to taking into account a large number of collisions and thus to a higher level of the hierarchy of random functions, will be $(a/L)^n$ times smaller. Since Eq. (8) contains an integral, fast oscillations of the random functions will ensure that the corresponding terms in the conductivity are small.

In order to estimate the integrals in Eq. (8) we take into account the fact that the integral of a random function is also a random function, but with a larger correlation scale. Namely, if $\varphi(\xi)$ is a random function having a correlation scale of ξ_0, an amplitude of the order of unity, and zero mean, then

$$\int_{\alpha}^{\lambda+\alpha}\varphi(\xi)d\xi=(\lambda\xi_0)^{1/2}\gamma(\alpha.\lambda),\quad \lambda\gg\xi_0, \quad (10)$$

where $\gamma(\alpha,\lambda)$ is a random function with amplitude of order unity and characteristic scale of order λ as a function of α and λ.

Using Eq. (10) to carry out the integration over the angle θ in Eq. (8), we obtain

$$j_x(x,y)=\sigma_0E\frac{a}{v_F\tau}\left[\frac{y}{a}\ln\frac{v_F\tau}{a}+\left(\frac{L}{a}\right)^{1/2}\gamma_1(x,y)\right.$$
$$\left.+\frac{L}{a}\gamma_2(x,y)+\ldots\right]. \quad (11)$$

Here σ_0 is the two-dimensional conductivity of the electron gas, and γ_1 and γ_2 are random functions with characteristic x and y oscillation periods of order a.

The current density (11) contains a fluctuating correction, which depends on the two coordinates x and y. Generally speaking, this correction does not satisfy the electrical neutrality condition $\operatorname{div}\mathbf{j}=0$. The spatial fluctuations of the electric field ensure that this condition is satisfied. This field can be determined from the equation of electrical neutrality itself, substituting the total field $\mathbf{E}+\mathbf{E}'$ into the expression for the current (1) and (2), in place of the mean field $\mathbf{E}$, where $\mathbf{E}'$ is the fluctuating correction sought. It follows from Eq. (11) that the characteristic scale of the variation of the fluctuating correction $\mathbf{E}'$, averaged over the cross section of the channel, is of order a. Small-scale fluctuations $\mathbf{E}'$ arise only in a narrow layer near the boundary of the channel. This means that the fluctuating field must be taken into account only in the first term in Eq. (11); in higher-order terms the conductivity-field correlation effects are insignificant. Thus, in order to find the average current density it is sufficient to integrate the expression (11) over the volume of the sample, since with such averaging the contribution of the fluctuating fields vanishes. As a result we obtain the following estimate for the fluctuating corrections to the conductance of the channel:

$$\frac{\Delta G}{G}=\left(\frac{a}{b}\right)^{1/2}\left(C_1\left(\frac{L}{a}\right)^{1/2}+C_2\frac{L}{a}+\ldots\right)\Big/\ln\frac{l}{a}. \quad (12)$$

where b is the length of the channel and C_1, C_2, ... are coefficients of the order of unity, which depend on the details of the uneven boundary.

We note that although the first term in Eq. (12) is dominant, it can be observed only if the details of the sample change (i.e., the width a of the sample or the form of the irregularities of the random boundary), which for a two-dimensional electron gas could be brought about, for example, by a change in the voltage on the gate. Local changes in the properties of the boundary at distances on the order of the length L of the irregularities, however, change the conductance by an amount

$$\frac{\Delta G}{G}\sim\left(\frac{a}{b}\right)^{1/2}\frac{L}{a}. \quad (13)$$

We now discuss the effect of weak magnetic fields. In the leading order term in Eq. (8) the magnetic field starts to have an effect if

$$(Ra)^{1/2}<l, \quad (14)$$

where R is the Larmor radius. In terms describing the fluctuation corrections, the effect of the magnetic field is manifested in weaker fields, since corrections associated with the curvature of the trajectories

$$\Delta X_1=\frac{y^2}{R\sin^3\theta}\sim\frac{a^2}{R},$$
$$\Delta X_2\sim\frac{a}{L}\Delta X_1,\ldots,\Delta X_n\sim\left(\frac{a}{L}\right)^n\Delta X_n \quad (15)$$

appear for the coordinates $X_1, X_2, ..., X_n$. The correction ΔX_1 in the formula (8) appears everywhere combined with the coordinate x, and it can be interpreted as a displacement of the point of observation. When we average over the volume of the sample the effect associated with a change in the coordinate X_1 vanishes. This means that the first correction in the formula (8) is not affected by a weak magnetic field. On the other hand, the appearance of the correction ΔX_2 changes the correlation between the successive points of reflection, so that the condition $\Delta X_2 \gtrsim L$ actually means that the realization of the random function, appearing in the third term in Eq. (8), changes. It can be concluded that in magnetic fields determined by the condition

$$\Delta X_2 \gtrsim L, \quad (16)$$

the conductance fluctuates with amplitude determined by the formula (13). From the condition (16) we find that the characteristic period of the oscillations is equal to

$$H_{cl}=\Phi_0 L^2/\lambda a^3, \quad (17)$$

where $\Phi_0 = \pi hc/e$ and λ is the de Broglie wavelength of the electron.

The formulas (13) and (17) determine the main scale of the stochastic oscillations of the conductance in a weak magnetic field. A finer structure having a period a/L times smaller than (17) and amplitude $(a/L)^{1/2}$ times smaller than (13) is superposed on these oscillations. These fluctuations are described by the first dropped term, which depends on X_1, X_2, and X_3, in the formula (8). If an infinite number of terms is included in Eq. (8), i.e., an infinite number of collisions of an electron with the boundaries of the channel, then it can be shown that the curve $\Delta G(H)/G$ exhibits self-similar structure. Such curves are called stochastic fractals (see, for example, Ref. 9). It is easy to determine the dimension of this fractal curve. To do so the curve must be divided into segments which have length ε and completely cover the fractal. If $N(\varepsilon)$ is the number of such elementary segments, then the dimension d of the fractal is calculated according to the formula[9]

$$d=\lim_{\varepsilon\to 0}\frac{\ln N(\varepsilon)}{\ln 1/\varepsilon}. \quad (18)$$

The fractal dimension (18) for the curve $\Delta G(H)/G$ is equal to 1.5.

We note that since the number of collisions (5) is limited by the mean free path length in the volume of the channel, the shortest period of the stochastic oscillations of the conductance will be of the order of $H_{cl}(4/a)^n$, and their amplitude will be of the order of $(a/b)^{1/2}(4/a)^{N/2}$. On the smallest scales the curve $\Delta G(H)/G$ does not exhibit scaling and is a standard smooth curve and not a stochastic fractal.

Comparative analysis of the classical and universal quantum fluctuations of the conductance was performed in Ref. 1. Without repeating this analysis, we point out that, first, for comparatively small samples (which for our geometry corresponds to channel length b not too much greater than the channel width a) the classical contribution is at least comparable with the quantum contribution and can even exceed the latter. Second, the classical contribution is important in the region of quite high temperatures, when the quantum contribution becomes insignificant.

In conclusion we shall discuss the mesoscopic fluctuations in a channel with almost specular boundaries. Almost specular reflection means that the random functions ψ_u and ψ_b have a large determinate (specular) part and a small random part:

$$\psi(\theta, X)=\theta+\alpha(\theta, X), \quad |\alpha|\ll\theta. \quad (19)$$

The characteristic value of the random function α is called the width of the scattering phase function. The relation between the width of the scattering phase function and the geometric characteristics of the boundary (height and length of the irregularities) was determined in Refs. 10 and 11. In the case of almost specular reflection, the cotangent with random argument in the formula (8) can be expanded in powers of the quantity α. After this, the sum of all determinate terms must be replaced by the mean-free path length l. This sum forms the conductivity of the channel in the leading-order approximation, which for a channel with specular boundaries is equal to σ_0. The fluctuation corrections arising after the cotangent is expanded must be averaged, using the formula (10), in which the fact that the characteristic amplitude of the random function (in this case α) is not equal to unity must be taken into account. As a result, Eq. (12) is obtained (to within a logarithmic factor) for the relative correction to the conductance. The only difference is that the characteristic values of the constants C_1, C_2, ... now are not equal to unity, but rather to the width of the scattering phase function.

The case of classical motion of an electron along a trajectory, studied in the present work, corresponds to the Kirchhoff approximation in the theory of surface scattering.[10] In this approximation the electron wave packet is reflected almost specularly from a random surface, if the rms slope angle of the irregularities $\gamma = \xi / L$ is small, and the reflection is diffuse if $\gamma \sim 1$.[11] In the case of almost specular reflection the width of the scattering phase function is equal to the parameter γ (Ref. 11). Thus the magnitude of the relative fluctuations of the conductance in the case of almost specular reflection is $\gamma^{-1} \gg 1$ times smaller than in the case of diffuse reflection.

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