Probabilities and cross sections for the ionization of hydrogen and of hydrogen-like ions by fast multiply charged ions

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(Submitted 22 November 1991)
Zh. Eksp. Teor. Fiz. **101**, 1198–1207 (April 1992)

The impact-parameter method and the Born version of the decay model are used to calculate the probability w and the cross section σ for the ionization of hydrogen-like particles by fast ions. The hydrogen-like particles have an atomic number Z_a and are initially in the $1s$ state. The projectile ions are structureless and have charges Z from 1 to 100. The collision velocities ar $v = 1$–100 a.u. Relativistic effects are ignored. Simple approximate expressions are derived for the Born ionization probability w_B and for the ratio of the ionization cross sections calculated in the Born version of the decay model ($\sigma_{\rm Bd}$) and the Born cross section (σ_B). These calculations are carried out over broad ranges of v and Z. Expressions are also derived for the ionization cross section $\sigma_{\rm Bd}$ at $v \geqslant 2.5Z_a$ and $Z > 5v$. At $1 \lesssim Z/v \leqslant v/Z_a$, the cross section $\sigma_{\rm Bd}$ is dominated by collisions with impact parameters b from $\sim Z/v$ to $\sim v/Z_a$ (in Coulomb units, with respect to Z_a). The cross sections $\sigma_{\rm Bd}$ calculated for the ionization of the hydrogen atom at $v \geqslant 2$ agree to within 5–10% with the experimental cross sections for $Z \leqslant 5$ and to within 15–20% for $Z = 7$–9.

1. INTRODUCTION

The single ionization of hydrogen atoms by fast charged particles of low charge Z can be described well by the first Born approximation. This case has been studied in detail (Refs. 1 and 2, for example). The ionization caused by multiply charged ions is a more complicated theoretical problem. The Born approximation cannot be used in this case, in contrast with cases of the collisions of light atomic systems (the range of applicability of the Born approximation is $Z/v \ll 1$, where Z and v are the charge and velocity of the ion). The few calculations of cross sections for the ionization of hydrogen atoms by fast multiply charged ions which have been carried out have used the approximation of a strong coupling of channels with three states,[3] the classical-trajectory Monte Carlo method,[4–6] or the approach of Refs. 7 and 8, which starts from a modification of the Keldysh method.[9]

A comparison of the calculated and experimental cross sections shows that the classical-trajectory Monte Carlo method underestimates the ionization cross sections at high ion velocities.[10] The reason lies in an inexact description of the electron density of the atom far from the nucleus.[8] In quantum-mechanical calculations of the cross sections for the ionization of hydrogen by multiply charged ions,[3,7,8] only the dipole part of the interaction of the fast ion with the electron which is stripped off is taken into account systematically. The contribution of multipole transitions to the ionization cross section was treated in Refs. 3 and 8 through the introduction of an empirical constant factor. However, in a study of the ionization of helium by multiply charged ions, it was shown[10] that the increase in the cross sections due to the incorporation of multipole transitions depends on the charge Z and the velocity v of the fast ions. At large values of the ratio Z/v, this increase should be much smaller than that used in Refs. 3 and 8. The calculations of the helium ionization cross section in Ref. 10 were carried out inthe Born version of the decay model (BVDM),[11] which takes the transitions of any multipolarity into account on an equivalent basis. The helium ionization cross sections calculated in the BVDM[10–13] for ion energies $E/Z \gtrsim 50$ keV/nucleon are no less successful in describing the experimental data than the cross sections found in more-complex calculations in the approximation of a strong coupling of channels[14] or the cross sections found through the use of a multipole expansion of the wave function of an individual electron determined at one center.[15]

We accordingly use the BVDM here to calculate the cross sections for the ionization of hydrogen atoms and of hydrogen-like ions by fast atomic particles and multiply charged ions. We assume that the internal structure of the particles causing the ionization can be ignored. We carry out the calculations for broad ranges of the charge Z and the velocity v of the fast particles. In the first stage of this study, we calculate the Born probabilities for ionization of hydrogen-like particles from the $1s$ state by nuclei at various impact parameters. Information on these probabilities is of interest in its own right. Before now, the probabilities for $1s$ ionization have been known only for $v \leqslant 6.7Z_a$ and $b \leqslant 6$–8, where Z_a is the atomic number of the particle which is ionized (Refs. 16 and 17; see also the references cited in Ref. 17). In the second stage, we calculate the ionization cross sections and compare the results with experimental data[18–21] and with theoretical results found in the approximation of a strong coupling of channels[3] and on the basis of a modification of the Keldysh method.[7,8]

2. BASIC EQUATIONS

In the impact-parameter method, the cross section σ^i for the ionization of unexcited hydrogen-like particles by fast ions is given by[22,23]

$$\sigma^i = 2\pi a_0^2 Z_a^{-2} \int w(b)\, b\, db, \qquad (2.1)$$

where b is the impact parameter in Coulomb units, $w(b)$ is the probability for the transition of an electron into the continuum, Z_a is the atomic number of the particle which is ionized, and a_0 is the first Bohr radius.

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In the BVDM the ionization probabilities w_{Bd} are related to the Born ionization probabilities w_B by[11]

$$w_{Bd}(b) = 1 - \exp[-w_B(n)]. \tag{2.2}$$

For ionization of hydrogen-like particles by structureless ions of atomic number Z, we have the following expression for $w_B(b)$ in the nonrelativistic case:[17]

$$w_B(b)=\left(\frac{Z}{v}\right)^2\sum_{\mu}\int d^3k\left|\int\frac{d^2q_\perp}{\pi q^2}\langle\mathbf{k}|e^{+i\mathbf{qr}}|1s\rangle J_\mu(bq_\perp)e^{-i\mu\varphi}\right|^2, \tag{2.3}$$

where $\mathbf{v}$ is the collision velocity in atomic units, $\mathbf{k}$ is the electron momentum in a state of the continuum, $\mathbf{q}$ is the momentum transfer, $q_\perp$ and φ are the orthogonal component of the momentum transfer and the azimuthal angle with respect to the velocity vector $\mathbf{v}$, $\langle\mathbf{k}|\exp(i\mathbf{qr})|1s\rangle$ is the Born matrix element of the $1s\to\mathbf{k}$ ionization transition, $\mathbf{r}$ is the radius vector of the electron with respect to the nucleus of the particle which is ionized, and $J_\mu(x)$ is the Bessel function of index μ. The quantities $\mathbf{k}$, $\mathbf{q}$, b, and $\mathbf{r}$ are expressed in Coulomb units with respect to the atomic number Z_a of the particle which is ionized.

The minimum value of the momentum transfer $\mathbf{q}$ is given by

$$q_{min}\equiv q_\parallel=\frac{1+k^2}{2s}, \tag{2.4}$$

where $s = \mathbf{v}/Z_a$ is the collision velocity in Coulomb units. For convenience and brevity, we will use both $\mathbf{v}$ and $\mathbf{s}$ to represent the velocity of the ions.

At large values of the impact parameter b and at high velocities s, the Born ionization probability $w_B(b)$ agrees[10] with the dipole ionization probability $w_D(b)$, which is given by (2.3) when we replace the exponential function $\exp(i\mathbf{qr})$ by $1 + i\mathbf{qr}$ in that expression. The dipole probability for ionization from the $1s$ state can be written as the single integral

$$w_D(b)=\frac{2^8}{3}\left(\frac{Z}{Z_a s^2}\right)^2\int_0^\infty\frac{k\,dk}{(1+k^2)^3}\frac{\exp[(-4/k)\operatorname{arctg}k]}{1-\exp(-2\pi/k)}\times[K_0^2(y)+K_1^2(y)], \tag{2.5}$$

where K_0 and K_1 are modified Bessel functions, and $y = b(1+k^2)/2s$.

3. IONIZATION PROBABILITIES

It follows from (2.3) and (2.4) that the reduced ionization probabilities $\tilde{w}_B = (v/Z)^2 w_B$ (i.e., the ionization probabilities which are independent of the atomic numbers Z_a and Z of the colliding particles) are determined by the values of b and s in the Born approximation. Figure 1 gives us an idea of the size of the reduced Born probabilities $\tilde{w}_B$ and the changes which occur in these probabilities when the impact parameter b or the collision velocity s changes. Shown in this figure are values of $\tilde{w}_B$ versus b over the interval $b = 0.02$–100 for six values of s, from 1 to 100.

At all values $s\geqslant 1$, the reduced Born probabilities $\tilde{w}_B$ fall off monotonically with increasing b. In the region $b<0.5$, the values of $\tilde{w}_B(b)$ depend only weakly on b [the values of $\tilde{w}_B(0.5)$ are lower than the values of $\tilde{w}_B(0)$ by less than 15%]. In the region $1<b<s$, these values are approximately

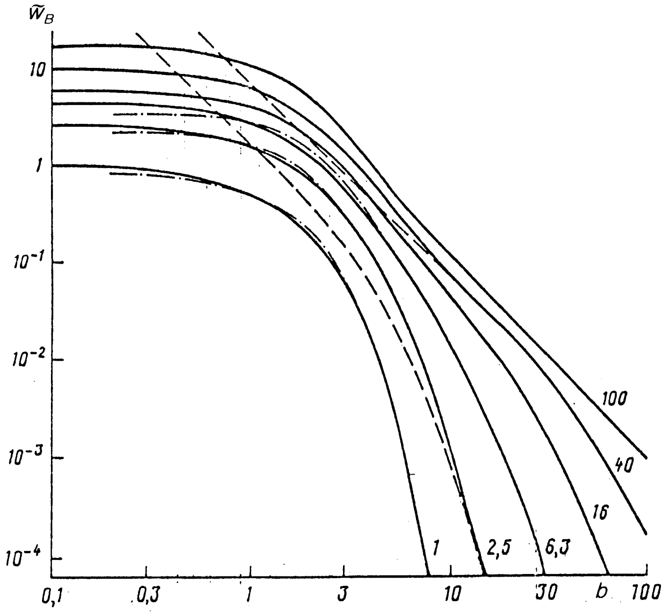

FIG. 1. The reduced ionization probability $\tilde{w}_B = w_B(v/z)^2$ versus the impact parameter b. Solid lines—Results of the present calculations in the Born approximation; dashed lines—results of the present calculations in the dipole approximation; dot-dashed lines—results of Ref. 16 found in the semiclassical approximation. The curve labels are the values of the velocity s. The values of $w(v/z)^2$ have been multiplied by $s^{1/2}$ in order to separate the curves.

proportional to b^{-2}. For $b\gtrsim 2s$, they fall off rapidly with increasing b [roughly as $\exp(-b/s)$].

At fixed values $b>3$, an increase in s is accompanied by a rapid increase in the probability $\tilde{w}_B(b,s)$ for $s>b$. This probability approaches the asymptotic value $\tilde{w}_B(b,s\to\infty)$. For $b\lesssim 2$, the values of $\tilde{w}_B(b,s)$ increase monotonically with increasing s in the intervals $1<s\lesssim 4$ and $15\leqslant s<100$, while they decrease by less than 20% as s increases from 6 to 15.

For $b\geqslant 15$ and $s\geqslant 7$, the Born values $\tilde{w}_B$ are essentially the same as the values $\tilde{w}_D$, found in the dipole approximation. In the interval $b = 1.2$–12 the Born values $\tilde{w}_B$ are larger than the dipole values (by a factor up to 1.5–2 at $s = 1.5$–4.5), while for $b<0.8$ they are less than the dipole values, since they depend weakly on b (while $\tilde{w}_D$ is proportional to b^{-2} for $b<s$).

For arbitrary values of b and s, the dipole probabilities can be approximated within 2% by the function

$$\tilde{w}_D=1{,}13b^{-2}f(b/s), \tag{3.1}$$

where

$$f(x)=\begin{cases}(1+0{,}39x^2)^{-1}, & x\leqslant 1\\ \dfrac{4{,}31}{e^x}\left[1+\dfrac{1{,}89}{x+0{,}57}\right]^{-1}, & x\geqslant 1\end{cases}. \tag{3.2}$$

For $s\geqslant 2.5$, the Born probabilities can be approximated within 3–10% by the function

$$\tilde{w}_B=\frac{1.13}{1+b^2}f\left(\frac{b}{s}\right)+\frac{0{,}57}{1+(b/2{,}2)^{4{,}25}}f(u), \tag{3.3}$$

where $u = (b/3.1)^{1.35}/s^{0.6}$. For $b\gg 1$, the first term in (3.3) corresponds to dipole transitions, and the second to multipole transitions (the contribution of s–s transitions is negligible even at $b > 1$; Refs. 17 and 24). The contribution of multipole transitions to w_B falls off rapidly at $b \lesssim 3$ if $s > b$. With $b = 7$ and $s \geqslant 7$, for example, this contribution is $\sim$22–26%, while for $b = 10$ and $s \geqslant 10$ it is only 10–14%.

For $b > 1$ and for all values of s, the probabilities $\tilde{w}_B$ reported here differ from those calculated in the semiclassical approximation[16] by less than 10–15%. At values $b < 1$, these values are higher 5–20% than those found in the semiclassical approximation (Fig. 1).The apparent reason for this discrepancy is that different methods were used to sum over the orbital angular momenta of the ejected electron in the calculations of the ionization probability in Ref. 16 and in the present study. The reason for the difference in summation method in turn lies in a different choice of the wave function for the final state. In Ref. 16 this wave function was chosen as an expansion in states with definite energy, with an orbital angular momentum l, and with an orbital-angular-momentum projection m. For this reason, the ionization probability was determined through a numerical summation of the partial probabilities for various values of l and m. The analysis of the present study uses an expression for the wave function of the final state with a definite energy, with a plane wave propagating along the **k** direction at infinity. In this case the summation over l is replaced by an integration over the electron emission angle θ.

Figure 2 gives an idea of the contribution of collisions with various impact parameters to the ionization cross section. This figure shows the reduced values $\tilde{w}_{\rm Bd} b^2$ versus log$b$ for $s = 8$ and 32 for ions with $Z = 1$, v, $2v$, $4v$, and $1.5vs$. The areas under the curves in Fig. 2 are proportional to the ionization cross sections. The values of $\tilde{w}_{\rm Bd} b^2$ for $Z = 1$ in the cases shown here ($s > 7$) are essentially the same as the Born values $\tilde{w}_B b^2$, which are independent of Z. It follows from Figs. 1 and 2 that for $s > 5$ and $Z < vs$ the ionization cross sections $\sigma^i_{\rm Bd}$ (and σ^i_B) are determined primarily by collisions with impact parameters b for which the ionization probability $\tilde{w}_{\rm Bd}$ (and $\tilde{w}_B$) is roughly proportional to b^{-2}. In the Born approximation, the limits on this region of impact parameters are $b_L \sim 1$ and $b_U \sim s$. In the BVDM, the upper limit is again $b_U \sim s$, while the lower limit is either $b_L \sim 1$, for $Z/v < 1$, or $b_L \sim Z/v$, for $Z/v > 1$. The reason why the ionization cross sections $\sigma^i_{\rm Bd}$ are smaller than the Born values σ^i_B at $1 \leqslant Z/v < s$ is the narrowing of this interval of impact parameters due to the shift of the lower boundary, b_L. The magnitude of this shift depends primarily on Z/v. In contrast, the *relative* decrease in the cross sections, $(\sigma^i_B - \sigma^i_{\rm Bd})/\sigma^i_B$ depends not only on Z/v but also on the dimensions of the region in which $\tilde{w}_B$ is proportional to b^{-2}. In other words it also depends on s. Accordingly, at fixed values $Z/v > 1$, the relative difference between the cross sections σ^i_B and $\sigma^i_{\rm Bd}$ decreases with increasing s.

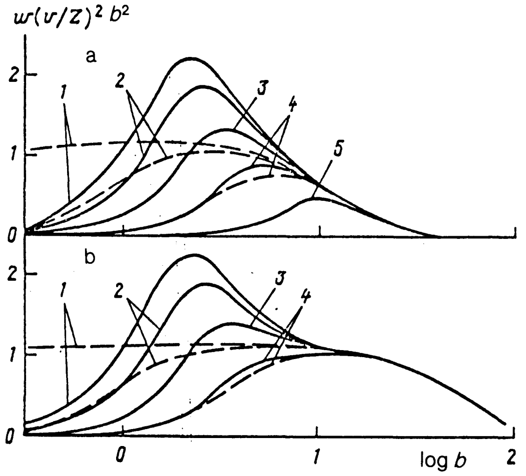

FIG. 2. The quantity $w(v/z)^2 b^2$ versus log b for velocities (a) $s = 8$ and (b) $s = 32$. Solid lines—Calculation in the Born version of the decay model; dashed lines—calculation in the dipole version of the decay model. Lines 1–5 correspond to ions with $Z = 1$, v, $2v$, $4v$, and $1.5vs$, respectively.

4. IONIZATION CROSS SECTIONS

We used the Born version of the decay model to calculate the cross sections for the ionization of hydrogen atoms by nuclei with charges Z from 1 to 100 and for collision velocities s from 1 to 100, working from Eqs. (2.1)–(2.4). These cross sections were also calculated in the Born approximation to bring out the effects due to the imposition of unitarity. The cross sections σ^i_B calculated in the Born approximation agree with those calculated previously in the plane-wave Born approximation.[25]

The results of these calculations are shown in Figs. 3 and 4 as the ratio $\mathscr{P}^i_{\rm Bd} = \sigma^i_{\rm Bd}/\sigma^i_B$, of the ionization cross section $\sigma^i_{\rm Bd}$ calculated in the BVDM to the Born cross section σ^i_B. In Fig. 3, the results are plotted against the ion velocity v; in Fig. 4 they are plotted against the parameters Z/v and Z/v^m, where $m = 1 + 0.18 \ln(1 + Z/v)$. To compare the calculated results with the experimental data, we show in the same figures the ratios $\mathscr{P}^i = \sigma_i/\sigma^i_B$ of the presently known experimental cross sections σ^i for the ionization of hydrogen atoms by nuclei and multiply charged ions[18–21] to the Born cross sections σ^i_B, as calculated in Ref. 25 and the present study.

We see from Fig. 3 for $v > 2Z$ the experimental ionization cross sections and those calculated in the BVDM differ from the Born cross sections by less than 10–15%, while for $v \lesssim 2Z$ the ratio of these cross sections to the Born cross sections decreases monotonically with decreasing velocity v. For $v \geqslant 2$, the experimental values of σ^i and $\mathscr{P}^i$ agree within the experimental errors with the results calculated in the BVDM, within 3–6% for $Z = 1$–3, within 5–10% for $Z = 5$, and within 15–20% for $Z = 7$–9. At lower velocities, these experimental values are significantly lower than the calculated values. In the latter cases ($v < 2$), because of the large cross sections (σ^c) for electron capture by the fast ions, the experimental cross sections for the loss of an electron, $\sigma^l = \sigma^i + \sigma^c$, are noticeably larger than the ionization cross sections σ^i. The ionization cross sections σ^i calculated in the BVDM, in contrast, lie between the experimental values of σ^i and the values of $\sigma^l = \sigma^i + \sigma^c$. The experimental ionization cross sections σ^i are smaller than the calculated values $\sigma^i_{\rm Bd}$ because of the competing effect of charge exchange. Cross sections for ionization and charge exchange in collisions of multiply charged ions with hydrogen atoms were carried out for ion velocities $v \lesssim 1.25$ in Ref. 8, where the competition between ionization and charge exchange was taken into account.

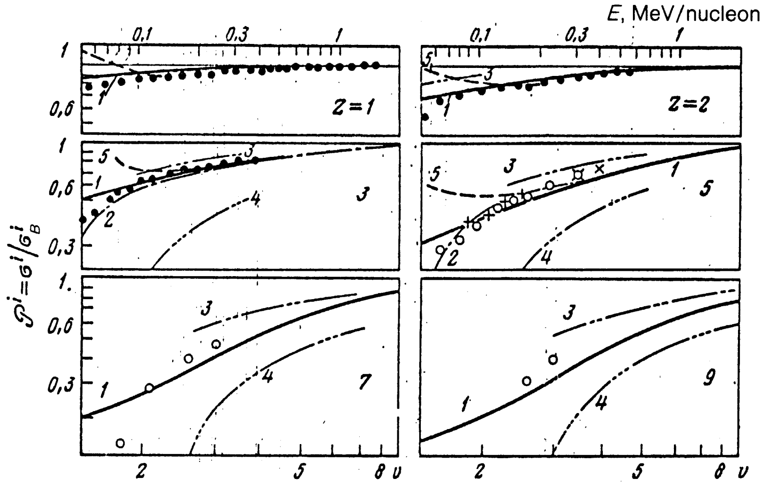

FIG. 3. The ratio $\mathscr{P}^i = \sigma^i/\sigma^i_B$, of the cross section for the ionization of hydrogen atoms by multiply charged ions to the Born cross sections, versus the velocity v for ions with atomic numbers $Z = 1$–9. All lines except the dashed lines are theoretical results. 1—Born version of the decay model (calculations of the present study); 2—calculations based on a modification of the Keldysh method[8] for $Z = 3$ and 5; 3—calculations from expression (55) of Ref. 8 for $Z^{1/2} < v < Z$; 4—calculations from expression (24) of Ref. 7. The points are experimental data. ●—Nuclei with $Z = 1$ and 2, from Ref. 18, and with $Z = 3$, from Ref. 20; $+$—N^{5+} ions, from Ref. 19, ×—C^{5+} ions, from Ref. 21; O—O^{5+} ions, from Refs. 19 and 21, and Ar^{7+} and Ar^{9+} ions, from Ref. 21. The dashed lines show the ratio of the experimental cross sections for electron loss, $\sigma^l = \sigma^i + \sigma^c$, to the Born ionization cross section σ^i_B. The values of σ^l were taken from Refs. 21, 26, and 27.

It can be seen from Fig. 4 that the ratios $\mathscr{P}^i_{\rm Bd} = \sigma_{\rm Bd}/\sigma^i_B$, of the ionization cross sections calculated in the BVDM to the Born cross sections, are approximately equal to unity according to (2.1), (2.2), (3.2), and (3.3) for all ions with $Z = 1$–100 and $v \geqslant 3$ under the condition $Z/v \lesssim 1/2$. For $Z/v < 0.35$ and $Z/v < 0.65$ we have $\mathscr{P}^i_{\rm Bd} > 0.97$ and $\mathscr{P}^i_{\rm Bd} > 0.90$, respectively. For $Z/v \gtrsim 2$, the values for $\mathscr{P}^i_{\rm Bd}$ fall off fairly rapidly with increasing Z/v, and they increase with increasing s. The decrease in $\mathscr{P}^i_{\rm Bd}$ with increasing Z/v is caused by an increase in the lower boundary b_L of the interval of impact parameters b which make a relatively large contribution to the ionization. The increase in $\mathscr{P}^i_{\rm Bd}$ with increasing s (at a fixed value of Z/v) results from a rise of the upper boundary of this interval, b_U (Fig. 2).

In this connection, the values of $\mathscr{P}^i_{\rm Bd}$ for ions with $Z = 1$–100 and $s = 2.5$–100 can be represented approximately as a function of Z/v^m, where the exponent m increases with increasing Z/v, from $m \approx 1.1$ at $Z/v = 0.5$–1 to $m \simeq 1.2$ at $Z/v = 2$ and then to $m \approx 1.35$ and 1.5 at $Z/v = 6$ and 15. These values (Fig. 4) are given within 5–10% by

$$\mathscr{P}^i_{\rm Bd} = [1 + a(Z/v^m)^{5/2}]^{-1}, \tag{4.1}$$

where $a = 0.53$ and $m = 1 + 0.18\ln(1 + Z/v)$. In contrast, the experimental values of $\mathscr{P}^i = \sigma^i/\sigma^i_B$ on the ionization of hydrogen atoms are described within 3–5% by (4.1) with $a = 0.44$ (Fig. 4b).

For ions with $s > 2$ and $Z/v > 1$, the ionization probabilities $w_{\rm Bd}$ are approximately one at small values of the impact parameter b, so the ionization cross sections can be written in the form

$$\sigma^i_{\rm Bd} = \pi a_0^2 Z_a^{-2}\,\eta\left[b_B^2 + \int_{b_B^2}^{\infty} w_B(b)\,db^2\right], \tag{4.2}$$

where the limiting values of the impact parameter b_B are determined by the relation $w_B(b_B) = (Z/v)^2\tilde{w}_B(b_B) = 1$. With $\eta = 1$, expression (4.2) corresponds to an ionization probability $w(b) = \min[1; w_B(b)]$, and from (2.2) we find

$$1 - 1/e < w_{\rm Bd}(b)/\min[1;\ w_B(b)] < 1.$$

For the value of η we thus find $0.63 < \eta < 1$ or $\eta \approx 0.8$ (within 25%).

For ions with $Z/v = 5 - s$, the values of b_B for $s > 6$ lie in the interval between 5 and s, in which the Born probabilities $w_B(b)$ are approximately the same as the dipole probabilities $w_D(b)$. Accordingly, the values of b_B in (4.2) for these ions are approximately the same as the values of b_D, found from the relation $w_D(b_D) = 1$.

In these cases we find from (3.1)–(3.3) and(4.2) the following result:

$$\sigma^i_{\rm Bd} = \sigma_0{}^i\eta\,\ln[4{,}7(1/\gamma + 0{,}47 - 0{,}09\gamma)], \tag{4.3}$$

where $\sigma^i_0 = 4\pi a_0^2 \cdot 0.2834(Z/vZ_a)^2$, and $\gamma = (Z/vs)^2$. In

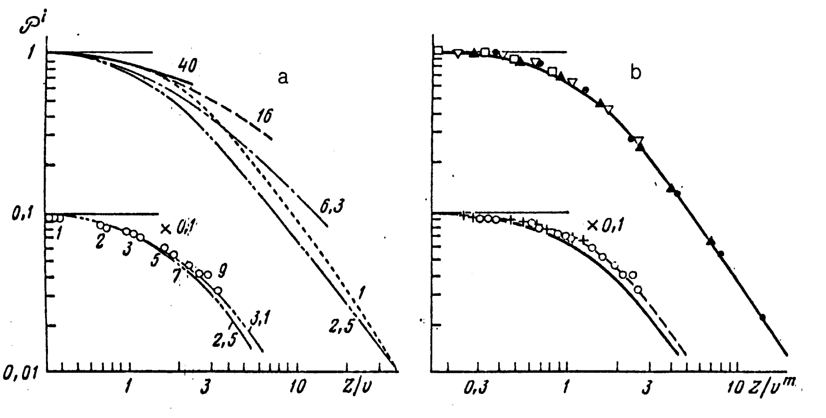

FIG. 4. Ratio of the cross sections for the ionization of hydrogen atoms by multiply charged ions to the Born cross sections as calculated in the Born version of the decay model and as found experimentally, versus the parameters Z/v and Z/v^m, for $m = 1 + 0.18\ln(1 + Z/v)$. a: Lines—calculated in the Born version of the decay model; circles—ratios found from the experimental data of Refs. 18–21 for $v = 2.5$–3.1. The curve labels are the ion velocities v; the labels on the circles are the ion charges Z. b: Points—calculated in the Born version of the decay model for $Z = 2^n$ ($n = 0$–7) with $v = 2.5$ (●), 6.3 (▲), 16 (▽), and 40 (□). O, $+$—Ratios found from the experimental data of Refs. 18–21 for $Z = 1$–9 and (O) $v = 2.5$–3.1 and ($+$) 3.4–4.1; lines—calculated from (4.1) for $a = 0.53$ (solid line) and 0.44 (dashed line).

the derivation of (4.3), after the integration in (4.2) we approximated the γ-dependent quantities

$$\varphi_1=b_B^2/1,134(Z/v)^2=1,131[(1+1,768\gamma)^{1/2}-1]/\gamma,$$

$$\varphi_2=\exp(\varphi_1-1)/\varphi_1$$

by linear expressions: $\varphi_1 = 0.98 - 0.24\gamma$ and $\varphi_2 = 1 + 0.04\gamma$. These expressions are accurate to within 2%.

A more accurate integration in (2.1) using (2.2), (3.2), and (3.3) leads to more complicated expressions for the cross sections $\sigma^i_{\rm Bd}$. However, these expressions can be approximated well by the simple formula

$$\sigma^i_{\rm Bd}=\sigma_0^i \ln(4v^2s^2/Z^2+1). \qquad (4.4)$$

For ions with higher charges Z, with $s \leqslant Z/v \leqslant 4s$, we have

$$\sigma^i_{\rm Bd}={}^5/_3\sigma_0^i(vs/Z). \qquad (4.5)$$

In other words, the cross sections $\sigma^i_{\rm Bd}$ are proportional to Z and independent of v.

The ionization cross sections $\sigma^i_{\rm Bd}$ calculated from (2.1)–(2.4) for ions with $s \geqslant 2.5$ and $Z > 5v$ can be described within 3–5% by (4.4) if $5 \leqslant Z/v \leqslant s$, and by (4.5) if $1 \leqslant Z/vs \leqslant 4$. For $v > 2.5$ and $Z > 5v$, the cross sections $\sigma^i_{\rm Bd}$ divided by Z/Z_a^3 depend only on the parameter ZZ_a/v^2 (as in the case in which only dipole transitions are taken into account[10]). For ions with $Z/v = 1$–5, they also depend on v (Fig. 5).

For ions with $s > 2$ and $Z/v \approx 1$–3, the cross section $\sigma^i_{\rm Bd}$ calculated from (2.1)–(2.4) are 15–40% larger than those given by (4.4) because (4.4) ignores multipole transitions. Because of these transitions, at $b \approx 1$–3 (the Born ionization probabilities w^i_B are larger than the dipole probabilities w^i_D by 15–40%, as can be seen from (3.2) and (3.3).

With $Z_a = 1$, expressions (4.2)–(4.4) are similar to the expressions given by Duman *et al.*[7] for the cross sections for the ionization of hydrogen atoms by nuclei with $Z/v^2 \ll 1$. On the other hand, these expressions are not identical. In particular, in expression (24) in Ref. 7 [which corresponds to expression (4.4) of the present study] the quantity $1.51v^2/Z$ is in the argument of a logarithm, while according to (4.4) the quantity $[1 + (2v^2/Z)^2]^{1/2}$ should be there. The reason for this difference is that the study in Ref. 7 dealt only with the limiting case $Z/v^2 \ll 1$, and the ionization probabilities $w(b)$ at small values of the impact parameter in cases with $Z/v > 1$ were taken to be 0.283. This choice corresponds to incorporating only direct dipole transitions, while these values are close to one according to the calculations by the Born version of the decay model in Ref. 10 and in the present study. As a result, expression (24) in Ref. 7 gives values of σ^i which are smaller than those given by expression (4.4) of the present study. The difference is 12–20% at $Z/v^2 = 0.2$–0.5 and a factor of 2 at $Z/v^2 = 1$ (Fig. 5).

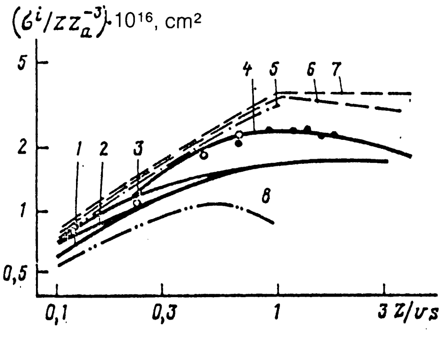

FIG. 5. The quantity σ^i/ZZ_a^{-3} versus the parameter Z/vs for the ionization of hydrogen-like ions by multiply charged ions. The lines are theoretical results. 1–4—The Born version of the decay model, for $v \geqslant 40$, $v = 16$, 6.3, and 2.1, respectively; 5—expression (55) in Ref. 8; 6,7—the Bohr formula[26] with $v = 2.5$ and $v \geqslant 8$, respectively; 8—expression (24) of Ref. 7. The points are results found from the experimental cross sections for the ionization of hydrogen atoms at $v = 2.1$. ○—For nuclei with $Z = 1$–3, from Refs. 18 and 20; ●—for Ar_2^{Z+} ions with $Z = 3$–8, from Ref. 21.

In the paper by Presnyakov and Uskov,[8] in expression (55) for the cross section for the ionization of the hydrogen atom by ions with $Z/v = 1 - v$, found in the dipole approximation, the constant coefficient in the argument of the logarithm was determined from a comparison with experimental cross sections for the ionization of hydrogen atoms by multiply charged ions. In this manner, various corrections to the dipole approximation, including the contribution of multipole transitions, are effectively taken into account. It follows from expression (55) in Ref. 8, with the empirical coefficient, that we have the following result for the cross section for the ionization of hydrogen atoms in cases with $1 < Z/v < v$:

$$\sigma^i=\sigma_0^i \ln(4,95v^2/Z)^2. \qquad (4.6)$$

This expression gives values for the cross sections which are larger than those given by (4.4) by a factor of 1.25 at $Z/v^2 \sim 0.1$ and by a factor of 1.9 at $Z/v^2 = 1$ (Fig. 5). A difference between the cross sections by a factor up to ~ 1.5 seems quite natural, since the Presnyakov-Uskov formula corresponds to cases in which the ion charges are relatively low ($Z/v \sim 2$–3), and multipole transitions contribute significantly to the ionization. Expression (4.4), in contrast, corresponds to cases $Z/v > 5$, in which multipole transitions can be ignored, as follows from the discussion above. Note, however, that even at $Z/v \sim 2$–3 the experimental cross sections for the ionization of hydrogen atoms by ions with $Z = 5$–7 at $v = 2.5$–3 are 20–25% lower than those found from expression (4.6) (Figs. 3 and 5). Expression (4.6) leads to cross sections which differ from those given by the Bohr formula[26] by only 5–15% (Fig. 5). Cross sections closer to the experimental cross sections are given by the formula of Voitkov and Pazdzersky,[28] in which [if this expression is written in the same form as (4.6)] the quantity $(3.1v^2/Z)^2$ is found in the argument of the logarithm. For ions with $Z/vs = 1$–4, the behavior of the cross sections calculated in the BVDM as a function of v and Z [Eq. (4.5)] is the same as in the Bohr formula, but the values of the cross section $\sigma^i_{\rm Bd}$ are considerably smaller than the Bohr values (Fig. 5).

5. CONCLUSION

The results of this study show that the Born version of the decay model (BVDM) leads to cross sections $\sigma^i_{\rm Bd}$ for the ionization of hydrogen atoms by multiply charged ions which, in the case $v \geqslant 2$, differ from the experimental cross sections by less than 3–6% for ion charges $Z = 1$–3, by 5–10% for $Z = 5$, and by 15–20% for $Z = 7$–9. A comparison of the experimental cross sections with the cross sections calculated in various models shows that, for ions with $Z > v$, the actual maximum values of the ionization probability $w(b)$, reached at small values of the impact parameter b, are not approximately equal to the value of 0.283, which corresponds to the incorporation of only direct dipole transitions into the continuum. They are instead close to a value of unity, which corresponds to the case in which an electron can also be removed from other bound states filled in the course of the collision.

Calculations of the probability for 1s ionization in the Born approximation and in the BVDM for broad ranges of the impact parameter b, the collision velocity v, and the ion charge Z show that at $Z/v < s$ the interval of impact parameters which dominates the ionization cross section $\sigma^i_{\rm Bd}$ has a lower limit $\sim \max[1;Z/v]$ and an upper limit $\sim s$. With increasing Z/v, this interval thus becomes narrower. At $Z/v > s$, this interval becomes a small interval of impact parameters near $b \sim s$. Multipole transitions increase both the Born ionization probabilities w_B by more than 15–20% only for impact parameters $b \approx 1.2$–5. Accordingly, for $Z > 5v$ and $s > 3$, they have no significant effect on the ionization cross section $\sigma^i_{\rm Bd}$.

For $v > 2.5$ and $Z > 5v$, the ionization cross sections $\sigma^i_{\rm Bd}$ divided by ZZ_a^{-3} depend only on the parameter ZZ_a/v^2. For ions with $Z/v = 1$–5, they also depend on v. For highly charged ions, with $Z/vs = 1$–4, the BVDM leads to qualitatively the same result as is predicted by the Bohr formula.[26] The ionization cross sections $\sigma^i_{\rm Bd}$ do not depend on the collision velocity v and are proportional to the ion charge Z.

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Translated by D. Parsons