Electron magnetism in antiferromagnets

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(Submitted 19 September 1990; received 29 April 1991)
Zh. Eksp. Teor. Fiz. **100**, 915–923 (September 1991)

The magnetism of electrons in low-dimensional antiferromagnets is analyzed in the case without fluctuations. In a magnetic field transverse with respect to the antiferromagnetic order parameter, the spin-splitting factor $g_\perp$ vanishes at the boundary of the magnetic Brillouin zone. This circumstance broadens the ESR peak and also gives rise to a combined resonance. Under orbital quantization conditions, the transverse magnetic field does not lift the spin degeneracy of the Landau levels. Transitions not accompanied by spin flip are possible under ESR conditions.

1. INTRODUCTION

This paper concerns electron magnetism in reduced-dimensionality antiferromagnets.

A symmetry analysis of the electronic states[1] shows that the spin-splitting factor g breaks up into two components, $g_\parallel$ and $g_\perp$, in an antiferromagnet. These components correspond to longitudinal and transverse orientations of the magnetic field **H** with respect to the antiferromagnetic-order vector **n**. In general, $g_\parallel$ has no singularities, while $g_\perp$ acquires a momentum dependence and vanishes at the boundary of the magnetic Brillouin zone (Fig. 1). This circumstance leads to extremely unusual manifestations of many electron-magnetism effects.

First, it becomes possible to excite transitions between states with oppositely directed spins by means of an alternating electric field. This "combined resonance"[2] occurs both in the continuum of band states and in the discrete spectrum of Landau levels. In each case the shape of the curve and the resonant frequency depend strongly on the angle φ between **H** and **n** and also on the carrier density.

A dependence of this sort is also characteristic of electron spin resonance (ESR). In a system containing $1+\delta$ electrons per unit cell ($\delta \ll 1$), a variation of φ from 0 to $\pi/2$ is accompanied by a decrease in the resonant frequency from $\gamma_\parallel H$ to $\gamma_\perp H$. The magnitude of $\gamma_\parallel$ does not depend on δ, while $\gamma_\perp$ is proportional to δ^α, where α can take on a value of 1/2 or 1, depending on whether the minimum of the dielectric spectrum is at the X point or the M point at the boundary of the magnetic Brillouin zone (Fig. 1). Of particular interest is a two-dimensional antiferromagnet in which $g_\perp$ varies along the Fermi surface. In this case the ESR signal in a transverse field is nonzero at all frequencies below $\gamma_\perp H$, even in the absence of scattering and at absolute zero.

An unusual aspect of static spin magnetism is a suppression of the transverse Pauli susceptibility by the factor $g_\perp^2$. With a decrease in the doping level, the transverse susceptibility of the system thus vanishes in proportion to $\delta^{2\alpha}$ (see the discussion above). The longitudinal susceptibility has no singularities.

The momentum dependence of $g_\perp$ is also seen in the interaction of a magnetic field with the orbital motion of an electron: in a transverse field, there is no Zeeman splitting of the Landau levels. Degenerate states with oppositely directed spins differ either in the position of the center of the orbit (if the minimum of the conduction band lies at the X point) or in magnetic radius (if the minimum is at the M point). Under ESR conditions, transitions involving a change in the index of the Landau level, but without spin flip, are possible. This is another manifestation of the combined resonance.

Below we analyze these effects on the basis of a simple model which ignores fluctuations (both quantum and classical) of the antiferromagnetic vector and also ignores electron scattering. We assume that the Fermi surface in the metallic phase is close to the boundary of the magnetic Brillouin zone.

The Hamiltonian describing low-lying states in the conduction band is

$$\mathcal{H}=\eta(\mathbf{p})-(\mathbf{H}_\parallel\boldsymbol{\sigma})-\frac{1}{\Delta}\xi(\mathbf{p})(\mathbf{H}_\perp\boldsymbol{\sigma}). \tag{1}$$

This Hamiltonian can be constructed both by the effective-mass method and in the weak-coupling model, for a doubly commensurate spin density wave.[4,5] Here and below,

$$\mathbf{H}_\parallel=(\boldsymbol{\Delta}\mathbf{H})\frac{\boldsymbol{\Delta}}{\Delta^2},\qquad \mathbf{H}_\perp=\mathbf{H}-\mathbf{H}_\parallel.$$

In a two-dimensional antiferromagnet, we consider two possible positions of the minimum of the conduction band: at the M point and at the X point (Fig. 1). Within terms quadratic in the momentum, we have

$$\eta(\mathbf{p})=\frac{1}{2M}(p_x^2+p_y^2),\qquad \xi(\mathbf{p})=\frac{1}{2m}(p_x^2-p_y^2),$$

in the first case and

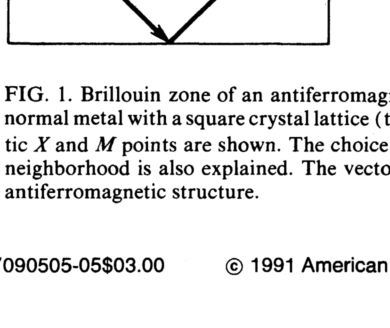

FIG. 1. Brillouin zone of an antiferromagnet (thick line) and that of a normal metal with a square crystal lattice (the thin line). The characteristic X and M points are shown. The choice of coordinate system in their neighborhood is also explained. The vector **G** is the wave vector of the antiferromagnetic structure.

 0038-5646/91/090505-05$03.00

$$\eta(\mathbf{p})=\frac{1}{2m_x}p_x^2+\frac{1}{2m_y}p_y^2,\quad \xi(\mathbf{p})=vp_y$$

in the second. In a one-dimensional antiferromagnet we have

$$\eta(p)=\frac{1}{2m}p^2,\quad \xi(p)=vp.$$

The following sections of this paper contain a calculation of the absorption in the effects described above.

2. ESR AND COMBINED RESONANCE IN THE CONTINUOUS SPECTRUM

As we mentioned in the Introduction, the momentum dependence of $g_\perp$ makes possible a combined resonance: an electric dipole excitation of transitions between states with oppositely directed spins.

Corresponding calculations can be carried out with the help of Fermi's golden rule[6] and from the Kubo formula.[7]

Evaluating the matrix elements of the magnetic-moment operator for the Hamiltonian (1) for the ESR regime as $T\to 0$, we find

$$\chi_{xx}^{\prime\prime(ESR)}(\omega)=\frac{\pi}{V}\omega\sum_{\mathbf{p}}\gamma^2(\mathbf{p})\delta(E_c(\mathbf{p})-\mu)\delta\times\{2(H_\parallel^2+\gamma^2(\mathbf{p})H_\perp^2)^{1/2}-\omega\},\tag{2}$$

$$\chi_{yy}^{\prime\prime(ESR)}(\omega)=\frac{\pi}{V}\omega\sum_{\mathbf{p}}\frac{\gamma^2(\mathbf{p})}{\cos^2\varphi+\gamma^2(\mathbf{p})\sin^2\varphi}\delta(E_c(\mathbf{p})-\mu)\times\delta\{2(H_\parallel^2+\gamma^2(\mathbf{p})H_\perp^2)^{1/2}-\omega\}.$$

Here V is the d-dimensional volume of the sample, $\cos\varphi\equiv(\mathbf{H}\cdot\boldsymbol{\Delta})/H\Delta$, μ is the chemical potential of the dopant electrons, reckoned from the bottom of the conduction band, and the unit vectors in spin space are defined by

$$\hat{\mathbf{z}}=\frac{\mathbf{H}}{H},\quad \hat{\mathbf{x}}=\frac{[\mathbf{H},\boldsymbol{\Delta}]}{|[\mathbf{H},\boldsymbol{\Delta}]|},\quad \hat{\mathbf{y}}=[\hat{\mathbf{z}},\hat{\mathbf{x}}].$$

Here and below,

$$\gamma^2(\xi)=\frac{\xi^2(\mathbf{p})}{\Delta^2}.$$

Under combined-resonance conditions, as $T\to 0$, we have

$$\chi_{aa}^{\prime\prime(EDSR)}(\omega)=\frac{\pi}{V}\omega\sum_{\mathbf{p}}\left\{\frac{2m_0c}{\omega}\partial_{p_a}\gamma(\mathbf{p})\right\}^2\times\frac{H_\parallel^2H_\perp^2}{H_\parallel^2+\gamma^2(\mathbf{p})H_\perp^2}\delta\{2(H_\parallel^2+\gamma^2(\mathbf{p})H_\perp^2)^{1/2}-\omega\}\delta(E_c(\mathbf{p})-\mu),\tag{3}$$

where m_0 is the mass of a free electron, c is the velocity of light, and the superscript EDSR ("electric dipole spin resonance") stands for the combined resonance. The susceptibility corresponding to the combined resonance has been determined in the unit vectors of momentum space, i.e., in a manner different from that in the ESR case.

Let us examine some results which follow from (2) and (3) in some specific cases.

One-dimensional antiferromagnet. After some calculations we find

$$\chi_{xx}^{\prime\prime(ESR)}(\omega)=\left(\frac{2m}{\Delta}\right)^{1/2}\frac{mv^2}{\Delta}\left(\frac{\mu}{\Delta}\right)^{1/2}H(\cos^2\varphi+\gamma_F^2\sin^2\varphi)^{1/2}\times\delta\{2(H_\parallel^2+\gamma_F^2H_\perp^2)^{1/2}-\omega\},$$

$$\chi_{yy}^{\prime\prime(ESR)}(\omega)=\left(\frac{2m}{\Delta}\right)^{1/2}\frac{mv^2}{\Delta}\left(\frac{\mu}{\Delta}\right)^{1/2}\frac{H}{(\cos^2\varphi+\gamma_F^2\sin^2\varphi)^{1/2}}\times\delta\{2(H_\parallel^2+\gamma_F^2H_\perp^2)^{1/2}-\omega\},$$

$$\chi^{\prime\prime(EDSR)}(\omega)=\frac{\omega}{2}\frac{(2m_0c)^2v}{\Delta^2}\frac{mv^2}{\Delta}\frac{1}{\gamma_F}\frac{H_\parallel^2H_\perp^2}{(H_\parallel^2+\gamma_F^2H_\perp^2)^2}\times\delta\{2(H_\parallel^2+\gamma_F^2H_\perp^2)^{1/2}-\omega\}.$$

Here γ_F is the value of

$$\gamma(p)=\frac{1}{\Delta}vp$$

at the Fermi level, and

$$\mu=\frac{1}{2m}p_F^2.$$

The intensity ratio of the ESR and combined resonances is, in order of magnitude,

$$\left(\frac{\mu}{\Delta}\right)\left(\frac{\Delta}{mc^2}\right)(H_\parallel^2+\gamma_F^2H_\perp^2)^2H_\parallel^{-2}H_\perp^{-2},$$

where c is the velocity of light. Even for gaps on the order of 1 eV, the ratio Δ/mc^2 is no greater than 10^{-6}.

Two-dimensional antiferromagnet: the X point. In this case we find

$$\chi_{yy}^{\prime\prime(ESR)}(\omega)=\frac{(m_xm_y)^{1/2}}{2\pi}\frac{1}{\sin^2\varphi}\frac{\left[\left(\frac{\omega}{2H}\right)^2-\left(\frac{\omega_{min}}{2H}\right)^2\right]^{1/2}}{\left[\left(\frac{\omega_{max}}{2H}\right)^2-\left(\frac{\omega}{2H}\right)^2\right]^{1/2}},$$

$$\chi_{xx}^{\prime\prime(ESR)}(\omega)=\chi_{yy}^{\prime\prime(ESR)}(\omega)\left(\frac{\omega}{2H}\right)^2,$$

$$\chi_{xx}^{\prime\prime(EDSR)}(\omega)=0,$$

$$\chi_{yy}^{\prime\prime(EDSR)}(\omega)=\frac{(m_xm_y)^{1/2}}{2\pi}\left(\frac{2m_0cvH_\parallel H_\perp}{\Delta\omega H}\right)^2\times\left\{\left[\left(\frac{\omega}{2H}\right)^2-\left(\frac{\omega_{min}}{2H}\right)^2\right]\left[\left(\frac{\omega_{max}}{2H}\right)^2-\left(\frac{\omega}{2H}\right)^2\right]\right\}^{-1/2},$$

where

$$\left(\frac{\omega_{min}}{2H}\right)^2=\cos^2\varphi,\quad \left(\frac{\omega_{max}}{2H}\right)^2=\cos^2\varphi+\gamma_{max}^2\sin^2\varphi,$$

and γ_{max}^2 is the maximum value of $\gamma^2(\mathbf{p})$ on the Fermi surface of the doped insulating phase.

The intensity ratio of the ESR and the combined resonance is, in order of magnitude,

$$\left(\frac{\Delta}{m_0cv}\right)^2\left(\frac{H}{H_\perp}\right)^4(H_\parallel^2+\gamma_F^2H_\perp^2)H_\parallel^{-2}.$$

The ratio $(\Delta/m_0cv)^2$ does not exceed 10^{-8} for any reasonable choice of parameter values.

Two-dimensional antiferromagnet: the M point. In this case we find

$$\chi_{yy}^{\prime\prime(ESR)}(\omega)=\frac{M}{2\pi}\frac{1}{\sin^2\varphi}\frac{\left[\left(\frac{\omega}{2H}\right)^2-\left(\frac{\omega_{min}}{2H}\right)^2\right]^{1/2}}{\left[\left(\frac{\omega_{max}}{2H}\right)^2-\left(\frac{\omega}{2H}\right)^2\right]^{1/2}},$$

$$\chi_{xx}^{''(ESR)}(\omega)=\chi_{yy}^{''(ESR)}(\omega)\left(\frac{\omega}{2H}\right)^2,$$

$$\chi_{yy}^{''(EDSR)}(\omega)=\frac{\mu}{2\pi}\left(\frac{4Mm_0cH_\parallel H_\perp}{m\omega\Delta H}\sin\psi_0\right)^2$$
$$\times\left\{\left[\left(\frac{\omega}{2H}\right)^2-\left(\frac{\omega_{min}}{2H}\right)^2\right]\left[\left(\frac{\omega_{max}}{2H}\right)^2-\left(\frac{\omega}{2H}\right)^2\right]\right\}^{-1/2},$$

$$\chi_{xx}^{''(EDSR)}(\omega)=\frac{\mu}{2\pi}\left(\frac{4Mm_0cH_\parallel H_\perp}{m\omega\Delta H}\cos\psi_0\right)^2$$
$$\times\left\{\left[\left(\frac{\omega}{2H}\right)^2-\left(\frac{\omega_{min}}{2H}\right)^2\right]\left[\left(\frac{\omega_{max}}{2H}\right)^2-\left(\frac{\omega}{2H}\right)^2\right]\right\}^{-1/2},$$

where

$$\cos^2\varphi+\left(\frac{M\mu}{m\Delta}\cos 2\psi_0\right)^2\sin^2\varphi=\left(\frac{\omega}{2H}\right)^2.$$

The quantities $\omega_{\min}$ and $\omega_{\max}$ are determined in the same way as in the case corresponding to the X point.

The imaginary parts of the susceptibility in the ESR and combined resonance regimes have a ratio

$$\left(\frac{m}{M}\right)\left(\frac{\Delta}{\mu}\right)\left(\frac{H}{m_0c^2}\right)\left(\frac{H}{H_\perp}\right)^4(H_\parallel^2+\gamma_F^2H_\perp^2)H_\parallel^{-2}.$$

The ratio Δ/m_0c^2 is less than 10^{-6}, but if the doping level is sufficiently low, and if the angle between H and Δ is small, the ESR and combined resonance intensities may become comparable in magnitude. This would be an extremely unusual result.

3. ESR AND COMBINED RESONANCE UNDER ORBITAL-QUANTIZATION CONDITIONS

In a quantizing magnetic field, the combined resonance is manifested in two ways.

First, there can be an electric dipole excitation of spin transitions, as in the continuous spectrum. A spin flip may be accompanied by a change in the index of the Landau level.

Second, transitions between orbital-quantization levels, including transitions which are not accompanied by a spin flip, become possible under ESR conditions.

Let us consider some specific cases.

Two-dimensional antiferromagnet: the X point. We write the Hamiltonian (1) in the Landau gauge:

$$H=\frac{p_x^2}{2m_x}+\frac{1}{2m_y}\left(p_y+\frac{e}{c}(\mathbf{Hn})x\right)^2$$
$$-(\mathbf{H}_\parallel\boldsymbol{\sigma})-\frac{v[p_y+(e/c)(\mathbf{Hn})x]}{\Delta}(\mathbf{H}_\perp,\boldsymbol{\sigma}),\qquad(4)$$

where $\mathbf{n}$ is the normal to the plane. In the case $H_\parallel=0$ we would have

$$H=\Omega\left\{\frac{p^2}{2}+\frac{1}{2}\left(q-\beta\left(\frac{\mathbf{H}_\perp}{|\mathbf{H}_\perp|}\boldsymbol{\sigma}\right)\right)^2\right\}-\frac{H_\perp^2}{2\Delta}\frac{m_yv^2}{\Delta},\qquad(5)$$

where

$$\Omega=\frac{e}{c}\frac{(\mathbf{Hn})}{(m_xm_y)^{1/2}},$$
$$p=p_xa_H,$$
$$q=\frac{1}{a_H}\left(x+\frac{cp_y}{e(\mathbf{Hn})}\right),$$
$$a_H=(m_x\Omega)^{-1/2},$$
$$\beta=\left(\frac{m_yv^2}{\Delta}\frac{H_\perp}{\Delta}\frac{H_\perp}{\Omega}\right)^{1/2}.$$

According to (5), a transverse magnetic field moves the centers of the orbits of the states with oppositely directed spins away from each other, without causing Zeeman splitting of the Landau levels.

We will use perturbation theory to study the effect of a longitudinal field. Defining unit vectors in spin space by means of

$$\hat{\mathbf{z}}=\frac{\mathbf{H}_\perp}{|\mathbf{H}_\perp|},\quad \hat{\mathbf{x}}=\frac{\boldsymbol{\Delta}}{\Delta},\quad \hat{\mathbf{y}}=[\hat{\mathbf{z}},\hat{\mathbf{x}}]$$

and writing the unknown wave function $\Psi_{n\sigma}$ in the form

$$\Psi_{n\sigma}=\alpha_\sigma|n\uparrow\rangle+\beta_\sigma|n\downarrow\rangle,$$

we find

$$\Psi_{n\uparrow}=(|n\uparrow\rangle-|n\downarrow\rangle),\ E_{n\uparrow}=\Omega(n+1/2)-|\langle n\uparrow|(\mathbf{H}_\parallel\boldsymbol{\sigma})|n\downarrow\rangle|,$$
$$\Psi_{n\downarrow}=(|n\uparrow\rangle+|n\downarrow\rangle),\ E_{n\downarrow}=\Omega(n+1/2)+|\langle n\uparrow|(\mathbf{H}_\parallel\boldsymbol{\sigma})|n\downarrow\rangle|,\qquad(6)$$

where $|n\uparrow\rangle$ and $|n\downarrow\rangle$ are eigenstates of Hamiltonian (5).

The matrix element in (6) is given by[8]

$$\langle n\uparrow|(\mathbf{H}_\parallel\boldsymbol{\sigma})|n\downarrow\rangle=H_\parallel L_n(2\beta^2)\exp(-\beta^2),\qquad(7)$$

where $L_n(2\beta^2)$ is the Laguerre polynomial.

Substituting wave functions (6) into the Schrödinger equation for the Hamiltonian (4), and requiring that the discarded terms be small in comparison with the Zeeman splitting, we find a condition for the applicability of (6) and (7):

$$\beta^2\ll 1.$$

Consequently, we can set β^2 equal to zero in (6) and (7):

$$E_n=\Omega(n+1/2)\pm H_\parallel.$$

The intensity of the combinational scattering is determined by the square of the matrix element of the dipole-moment operator

$$P_y(\omega)=\frac{(\mathbf{H}_\perp\boldsymbol{\sigma})}{\Delta}\frac{ev}{\omega}=2\frac{m_0vc}{\Delta}\frac{(\mathbf{H}_\perp\sigma)}{\omega}.$$

This matrix element is

$$\langle\Psi_{n\uparrow}|P_y(\omega)|\Psi_{m\downarrow}\rangle=2\frac{m_0vc}{\Delta}\frac{H_\perp}{H_\parallel}\delta_{mn},$$

where m_0 is the mass of a free electron (we have set the Bohr magneton equal to unity).

The ESR intensity is determined by the matrix elements of the spin operator. Here we reproduce the results of the corresponding calculations. The quantity λ is an estimated upper limit on the ratio of the intensity of the ESR to that of the combined resonance. We used the values $\Delta\sim 1$ eV, $v\sim 10^8$ cm/s, and $\Omega\sim 1$ T):

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{x}}\boldsymbol{\sigma})|\Psi_{m\downarrow}\rangle=\beta^{m-n}C_m^n\left(2^{m-n}\frac{n!}{m!}\right)^{1/2}\frac{1-(-1)^{m-n}}{2},$$
$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{x}}\boldsymbol{\sigma})|\Psi_{m\uparrow}\rangle=\beta^{m-n}C_m^n\left(2^{m-n}\frac{n!}{m!}\right)^{1/2}\frac{1+(-1)^{m-n}}{2},$$

$$\lambda\sim\beta^{2(m-n)}\left(\frac{\Delta}{m_0vc}\right)^2\left(\frac{H_\parallel}{H_\perp}\right)^2,$$

$$\left(\frac{\Delta}{m_0vc}\right)^2\leqslant10^{-8}.$$

$$\langle\Psi_{n\uparrow}(\hat{\mathbf{z}}\boldsymbol{\sigma})\frac{v(p_y+(e/c)(\mathbf{Hn})x)}{\Delta}|\Psi_{m\downarrow}\rangle$$

$$=\delta_{m,n+1}\left(\frac{n+1}{2}\right)^{1/2}\left(\frac{m_yv^2}{\Delta}\frac{\Omega}{\Delta}\right)^{1/2},$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{z}}\boldsymbol{\sigma})\frac{v(p_y+(e/c)(\mathbf{Hn})x)}{\Delta}|\Psi_{m\uparrow}\rangle\propto\delta_{m,n},$$

$$\lambda\sim\frac{\Omega}{m_0c^2}\left(\frac{H_\parallel}{H_\perp}\right)^2,\qquad\left(\frac{\Omega}{m_0c^2}\right)\leqslant10^{-10}.$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{y}}\boldsymbol{\sigma})\frac{v(p_y+(e/c)(\mathbf{Hn})x)}{\Delta}|\Psi_{m\downarrow}\rangle$$

$$=\frac{i}{2}\left(\frac{m_yv^2}{\Delta}\frac{\Omega}{\Delta}\right)^{1/2}\beta^{m-n-1}(m-n)C_m^n\left(2^{m-n}\frac{n!}{m!}\right)^{1/2}\frac{1-(-1)^{m-n}}{2},$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{y}}\boldsymbol{\sigma})\frac{v(p_y+(e/c)(\mathbf{Hn})x)}{\Delta}|\Psi_{m\uparrow}\rangle$$

$$=\frac{i}{2}\left(\frac{m_yv^2}{\Delta}\frac{\Omega}{\Delta}\right)^{1/2}\beta^{m-n-1}(m-n)C_m^n\left(2^{m-n}\frac{n!}{m!}\right)^{1/2}\frac{1+(-1)^{m-n}}{2},$$

$$\lambda\sim\frac{\Omega}{m_0c^2}\left(\frac{H_\parallel}{H_\perp}\right)^2\beta^{m-n-1}.$$

Two-dimensional antiferromagnet: the M point. Within terms quadratic in the momentum, the Hamiltonian (1) becomes

$$\mathcal{H}=\frac{1}{2M}\left\{p_x^2+\left(p_y+\frac{e}{c}(\mathbf{Hn})x\right)^2\right\}-(\mathbf{H}_\parallel\boldsymbol{\sigma})$$
$$-(\mathbf{H}_\perp\boldsymbol{\sigma})\frac{1}{2m\Delta}\left\{p_x^2-\left(p_y+\frac{e}{c}(\mathbf{Hn})x\right)^2\right\}.$$

In a transverse field we have

$$\mathcal{H}=\left(\frac{1}{2M}-\frac{1}{2m}\frac{(\mathbf{H}_\perp\boldsymbol{\sigma})}{\Delta}\right)p_x^2$$
$$+\left(\frac{1}{2M}+\frac{1}{2m}\frac{(\mathbf{H}_\perp\boldsymbol{\sigma})}{\Delta}\right)\left(p_y+\frac{e}{c}(\mathbf{Hn})x\right)^2.\qquad(8)$$

According to (8), the Landau levels remain degenerate in a transverse field, but the magnetic radius is found to depend on the spin:

$$a_H^2=\frac{c}{e(\mathbf{Hn})}\left(1-\frac{M}{m}\frac{(\mathbf{H}_\perp\boldsymbol{\sigma})}{\Delta}\right)^{1/2}\left(1+\frac{M}{m}\frac{(\mathbf{H}_\perp\boldsymbol{\sigma})}{\Delta}\right)^{-1/2}.$$

Proceeding as in the preceding case, we find

$$E_n=\Omega(n+1/2),\qquad\Omega=\frac{e(\mathbf{Hn})}{Mc}\left[1-\left(\frac{M}{m}\frac{H_\perp}{\Delta}\right)^2\right]^{1/2}.$$

This approximation is valid under the condition

$$\alpha=\frac{M}{m}\frac{H_\perp}{\Delta}\ll1.$$

The absorption intensity under combined resonance conditions is determined by the matrix element of the dipole-moment operator:

$$\langle\Psi_{n\uparrow}|P_{x,y}(\omega)|\Psi_{m\downarrow}\rangle$$
$$=2\frac{m_0}{m}\frac{H_\perp}{\Delta}\frac{c}{a_H(\Omega\pm2H_\parallel)}\delta_{m,n+1}\left(\frac{n+1}{2}\right)^{1/2},$$

$$\langle\Psi_{n\uparrow}|P_{x,y}(\omega)|\Psi_{m\uparrow}\rangle$$
$$=2\alpha\frac{m_0}{m}\frac{H_\perp}{\Delta}\frac{c}{a_H(\Omega\pm2H_\parallel)}\delta_{m,n+1}\left(\frac{n+1}{2}\right)^{1/2}.$$

We turn now to the result of a calculation of the matrix elements of the spin operator to within the first nonvanishing terms in an expansion in powers of α. The quantity λ is a measure of the ratio of the ESR and combined-resonance intensities. We find

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{x}}\boldsymbol{\sigma})|\Psi_{n+2m\downarrow}\rangle=\frac{\alpha^m}{2^mm!}\left(\frac{(n+2m)!}{n!}\right)^{1/2}$$
$$\times\begin{cases}\alpha n\left[1+1/2\frac{n-1}{m+1}\right], & m=2p,\\ 1, & m=2p+1,\end{cases}$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{x}}\boldsymbol{\sigma})|\Psi_{n+2m\uparrow}\rangle=\frac{\alpha^m}{2^mm!}\left(\frac{(n+2m)!}{n!}\right)^{1/2}$$
$$\times\begin{cases}1, & m=2p,\\ \alpha n\left[1+1/2\frac{n-1}{m+1}\right], & m=2p+1,\end{cases}$$

$$\lambda\sim\left(\frac{m}{m_0}\right)^2\alpha^m\frac{\Delta}{Mc^2}\frac{\Delta}{\Omega}\left(\frac{\Omega\pm2H_\parallel}{H_\perp}\right)^2,$$

$$\frac{\Delta}{Mc^2}\frac{\Delta}{\Omega}\sim10^{-2},$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{y}}\boldsymbol{\sigma})\frac{p_x^2-(p_y+(e/c)(\mathbf{Hn})x)^2}{2m\Delta}|\Psi_{n+2m\downarrow}\rangle$$
$$=\frac{i}{2}\frac{\Omega}{\Delta}[(m+1)(m+2)]^{1/2}\left[\frac{(n+2m)!}{(n+2)!}\right]^{1/2}$$
$$\times\frac{\alpha^{m-1}}{2^{m-1}(m-1)!}\begin{cases}\alpha(n+2)\left[1+1/2\frac{n+1}{m}\right], & m=2p,\\ 1, & m=2p+1,\end{cases}$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{y}}\boldsymbol{\sigma})\frac{p_x^2-(p_y+(e/c)(\mathbf{Hn})x)^2}{2m\Delta}|\Psi_{n+2m\uparrow}\rangle$$
$$=\frac{i}{2}\frac{\Omega}{\Delta}[(m+1)(m+2)]^{1/2}\left[\frac{(n+2m)!}{(n+2)!}\right]^{1/2}$$
$$\times\frac{\alpha^{m-1}}{2^{m-1}(m-1)!}\begin{cases}\alpha(n+2)\left[1+1/2\frac{n+1}{m}\right], & m=2p+1,\\ 1, & m=2p,\end{cases}$$

$$\lambda\sim\left(\frac{m}{m_0}\right)^2\frac{\Omega}{Mc^2}\left(\frac{\Omega\pm2H_\parallel}{H_\perp}\right)^2,$$

$$\frac{\Omega}{Mc^2}\sim10^{-10},$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{z}}\boldsymbol{\sigma})\frac{p_x^2-(p_y+(e/c)(\mathbf{Hn})x)^2}{2m\Delta}|\Psi_{m\downarrow}\rangle$$
$$=\frac{M}{m}\frac{\Omega}{\Delta}\begin{cases}1/2(m(m-1))^{1/2}, & n=m-2,\\ 1/2(n(n-1))^{1/2}, & n=m+2,\\ \alpha(n+1/2), & n=m,\end{cases}$$

$$\langle\Psi_{n\uparrow}|(\hat{\mathbf{z}}\boldsymbol{\sigma})\frac{p_x^2-(p_y+(e/c)(\mathbf{Hn})x)^2}{2m\Delta}|\Psi_{m\uparrow}\rangle\propto\delta_{m,n}$$

$$\lambda\sim\left(\frac{M}{m_0}\right)^2\frac{\Omega}{Mc^2}\left(\frac{\Omega\pm2H_\parallel}{H_\perp}\right)^2.$$

4. CONCLUSION

The results derived here apply to quasi-one-dimensional compounds with a spin density wave and also to the lanthanum-containing high T_c superconducting compounds in the insulating antiferromagnetic phase. In the latter case, experiments on the combined resonance and ESR might make it possible to determine whether the minimum of the conduction band is at the X point or at the M point of the magnetic Brillouin zone. The most obvious differences between these two cases should be seen under conditions of combined resonance in a quantizing magnetic field. If the minimum is at the X point, a transition accompanied by spin flip would occur without a change in the index of the Landau level. If the minimum is instead at the M point, a spin flip would be accompanied by a unit change in the index of the Landau level.

We note in conclusion that it seems quite likely that in a system describable by the Hamiltonian (4) it would be possible to observe an oscillatory dependence of the Zeeman splitting of the orbital-quantization levels on the magnitude and orientation of the field.[9] In addition to the similarity of the equations, this effect is also implied by Eq. (7), although that equation becomes inapplicable before $L_n(2\beta^2)$ begins to oscillate.

I wish to thank S. A. Brazovskiĭ for formulating the problem and for guidance in this study. I am also indebted to Yu. A. Bychkov, É. I. Rashba, and D. E. Khmel'nitskiĭ for a discussion of several of the topics involved here.

[1] S. A. Brazovskiĭ and I. A. Luk'yanchuk, Zh. Eksp. Teor. Fiz. **96**, 2088 (1989) [Sov. Phys. JETP **69**, 1180 (1989)].
[2] Z. I. Rashba, Usp. Fiz. Nauk **84**, 557 (1964) [Sov. Phys. Usp. **7**, 823 (1965)].
[3] G. L. Bir and G. E. Pikus, *Symmetry and Deformation Effects in Semiconductors*, Nauka, Moscow, 1972.
[4] N. I. Kulikov and V. V. Tugushev, Usp. Fiz. Nauk **144**, 643 (1984) [Sov. Phys. Usp. **27**, 954 (1984)].
[5] S. A. Brazovskiĭ, I. A. Luk'yanchuk, and R. R. Ramazashvili, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 557 (1989) [JETP Lett. **49**, 644 (1989)].
[6] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*, Nauka, Moscow, 1973 (Pergamon, New York, 1977).
[7] R. M. White, *Quantum Theory of Magnetism*, McGraw-Hill, New York, 1970 (Russ. Transl. Mir, Moscow, 1972).
[8] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic, New York, 1966.
[9] Yu. A. Bychkov, V. I. Mel'nikov, and É. I. Rashba, Zh. Eksp. Teor. Fiz. **98**, 717 (1990) [Sov. Phys. JETP **71**, 401 (1990)].

Translated by D. Parsons