Phenomenological description of magnetic relaxation in magnets containing rare-earth ions

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The effective equations of motion for the magnetization of the rare-earth (R) sublattices in ferrite-type magnets are analyzed. The relaxation part of the equations is found from general considerations of symmetry and the hierarchy of interactions in a magnet and consists of two terms corresponding to two different relaxation processes of the R sublattice magnetization, transverse and longitudinal. The general expressions for the dissipative functions of these processes for arbitrary small-amplitude spin waves and low-frequency nonlinear magnetization oscillations are found. To apply them to the description of various magnetic excitations, it is necessary to represent the exchange field at R-ions in terms of the macroscopic averages of the iron subsystem in a particular magnetic crystal. The anisotropy of the domain walls (DW) mobility in ferrite garnets, previously unexamined theoretically, which is caused by the longitudinal relaxation, is described. The R-ion contribution to spin wave damping and DW braking in orthoferrites is calculated and analyzed. It is shown that in one type of possible spin wave and DW modes dissipation is caused only by transverse relaxation and in another only by longitudinal relaxation. In the angular phase of an orthoferrite a crossover of the spin wave relaxation mechanisms is found. The phenomenological approach considered may be generalized to describe the damping of other excitations, for instance, elastic ones in crystals not only with R-ions but with other dopants having internal degrees of freedom of any nature.

1. INTRODUCTION

Relaxation of magnetization perturbations (spin waves, domain walls, and others) in magnets such as ferrites with rare-earth ions attracts considerable scientific and practical interest. At the present time, the basic laws of this relaxation can be regarded as understood (see the monograph by Gurevich[1]). The dissipation of magnetic perturbations in the presence of rare-earth (R-) ions is attributed to two different relaxation mechanisms—longitudinal (or slow) and transverse (or fast). This division into two mechanisms was revealed initially in the microscopic theory.[2]

The mechanism of transverse relaxation can be described on the basis of the phenomenological equations of the dynamics of the R-sublattice magnetization $\mathbf{M}^{(R)}$ with a standard relaxation term in the Landau–Lifshitz or Hilbert form[4] (see also Refs. 1 and 3). The results of the two approaches are in agreement and lead, in particular, to the conclusion that the damping of the lowest spin-wave modes has a weak frequency dependence (a small temporal dispersion).

The situation is different with the description of longitudinal relaxation, which is the most pronounced in ferrites. At the present time this mechanism is described only microscopically, as formulated by Van Vleck[2,5] for spin-wave damping (see Refs. 6 and 7 concerning relaxation of nonlinear perturbations of the moving domain-wall (DW) type). The microscopic approach is particularly complicated and unwieldy at sufficiently high temperature $T > \Delta$, where Δ is a quantity on the order of the ion-level splitting energy in a crystal field (usually, $\Delta \sim 100$ K), for in this case several levels are effectively excited and the simple doublet model cannot be used. A characteristic feature of longitudinal relaxation is strong temporal dispersion, causing many authors to state that it cannot be described phenomenologically (see Ref. 3).

We shall show that longitudinal relaxation, like transverse, enters naturally into the phenomenological description if a relaxation term, constructed by the approach proposed by Bar'yakhtar,[8] is used in the magnetization-dynamics equations for the R-sublattice.

We present answers here and analyze them for spin-wave damping and DW deceleration in the magnets most frequently investigated in experiment, iron garnets and orthoferrites. This approach is most effective for high temperatures $T > \Delta$, when general equations can be obtained for any R-ion. The results of our approach agree qualitatively with those of the microscopic theory for iron garnets with R-ions, but are much simpler to obtain and permit analysis of many important details, particularly the question of the anisotropy of DW mobility, as well as investigation of more complicated magnets such as orthoferrites.

2. PHENOMENOLOGICAL EQUATIONS OF MOTION AND DISSIPATIVE FUNCTIONS OF A MAGNETIC R-SYSTEM

Consider a magnet containing R-ions in nonequivalent crystalline positions numbered α ($\alpha = 1,2,...,n$). The state of an R-ion with a total angular momentum j is determined, generally speaking, by $4j(j+1)$ variables, of which three determine the magnetization dynamics and the rest are known as the multipole variables, see Ref. 9 for details. We consider the case in which the interaction between the R- and Fe ions consists of exchange. It can be shown then that in the high-temperature approximation ($T > \Delta$) of interest to us the Fe-ion spins excite primarily the R-ion dipole variables connected with the change of the magnetization, and the excitation of multipole variables of higher order can be neglected. (Interestingly, the condition that the quadrupole variable have low excitation is considerably less stringent and takes the form $T > \varepsilon_e$, where $\varepsilon_e \ll \Delta$ is the exchange splitting of the R-ion level; see Ref. 10.)

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For the phenomenological description, we group all the R-ions in an αth crystal position into a single R-sublattice and characterize it by a magnetization $\mathbf{M}^{(\alpha)} = \mathbf{M}^{(\alpha)}(\mathbf{r},t)$. The equations of motion for $\mathbf{M}^{(\alpha)}$ can be obtained from Ref. 8, with allowance for the specific sublattice symmetry that includes elements of the αth position and is as a rule lower than the symmetry of the entire crystal. Unification of all the R-sublattices into one can lead therefore to the loss of certain effects (more below).

We write the equation of motion for $\mathbf{M}^{(\alpha)} = \mathbf{M}^{(\alpha)}(\mathbf{r},t)$ in the form

$$\dot{\mathbf{M}}^{(\alpha)} = -g_{\mathrm{R}}^{(\alpha)}[\mathbf{M}^{(\alpha)}, \mathbf{F}^{(\alpha)}] + \mathbf{R}^{(\alpha)}, \tag{1}$$

where $\mathbf{F}^{(\alpha)}$ is the effective field, defined as the variation of the free energy W of the magnet with respect to $\mathbf{M}^{(\alpha)}$, $\mathbf{R}^{(\alpha)}$ is the relaxation term, and $g_{\mathrm{R}}^{(\alpha)}$ is the magnetomechanical ratio of the αth sublattice. Note that $g_{\mathrm{R}}^{(\alpha)}$ should, generally speaking, be regarded as a tensor, but its tensor character is of no importance for the relaxation of magnetic excitation. This tensor character can be taken into account automatically by using in place of (1) the effective equation $\mathbf{J}^{(\alpha)} = g_{\mathrm{R}}^{-1}\mathbf{M}^{(\alpha)}$ for the total moment $\mathbf{J}^{(\alpha)}$. In this equation the effective field $\mathbf{F}^{(\alpha)}$ is redefined: $\mathbf{F}^{(\alpha)} = -\delta W/\delta\mathbf{J}^{(\alpha)}$, and, in particular, the exchange field $\mathbf{H}_e^{(\alpha)}$ is replaced by $g_{\mathrm{R}}\mathbf{H}_e^{(\alpha)}$. The equation for $\mathbf{J}^{(\alpha)}$ then takes the form (1) with $g_{\mathrm{R}}^{(\alpha)} = 1$ and with appropriate substitutions.

For the specific form of $\mathbf{F}^{(\alpha)}$ we note that direct interaction between R-ions in ferrites is as a rule negligibly small, and these ions can be regarded as paramagnetic. Taking this into account and neglecting the excitation of the quadrupole variables, we express the free energy of the R-ions in the form

$$W = \sum_{\alpha}^{n} W^{(\alpha)}, \quad W^{(\alpha)}(\mathbf{M}^{(\alpha)}) = \int d\mathbf{r}\{-\mathbf{H}_e^{\alpha}\cdot\mathbf{M}^{(\alpha)} + f(\mathbf{M}^{(\alpha)})\}. \tag{2}$$

In this equation $\mathbf{H}_e^{(\alpha)}$ is the exchange field applied by the Fe ions to the R-ions. Since the magnetizations of the Fe-ions are strongly correlated, $\mathbf{H}_e^{(\alpha)}$ is expressed in terms of the simplest mean values of the Fe-subsystem spin variables. In particular, in all the uncompensated ferrimagnets such as iron garnets, $\mathbf{H}_e^{(\alpha)}$ can be expressed in terms of the summary magnetization $\mathbf{M}$ of the Fe subsystem. For collinear antiferromagnets and weak ferromagnets (such as orthoferrites) we can accordingly define $\mathbf{H}_e^{(\alpha)}$ in terms of the summary magnetization vector $\mathbf{M}$ and the Fe-subsystem antiferromagnetism vector $\mathbf{L}$ (see Sec. 4 below).

For the term $f(\mathbf{M}^{(\alpha)})$ in (2), which is determined by the entropy contribution at temperatures $T > \Delta$, we can write

$$f(\mathbf{M}^{(\alpha)}) = (M^{(\alpha)})^2/2\chi_\alpha, \tag{2a}$$

where $\chi_\alpha \sim 1/T$ is the paramagnetic susceptibility of the αth R-sublattice $M^\alpha = |M^{(\alpha)}|$. If the nonequivalent crystalline positions are distributed with equal probability, the corresponding values of χ_α are the same for all the sublattices, $\chi_\alpha = \chi$. This is precisely the case we shall consider below.

Thus the tensors g and χ are isotropic in our model, but the anisotropy of the R-ions is taken into account in the R–Fe interaction (more below). The anisotropies of χ and $\mathbf{H}_e^{(\alpha)}$ are not on a par because the latter differ in their physical nature, being relativistic and exchange-relativistic, respectively.

The relaxation term $\mathbf{R}^{(\alpha)}$ is determined, in accordance with the approach of Ref. 8, in terms of the components of $\mathbf{F}^{(\alpha)}$ and its spatial derivatives. By virtue of the above arguments, we can neglect the spatial-dispersion arguments and write $\mathbf{R}^{(\alpha)} = \hat{\lambda}^{(\alpha)}\mathbf{F}^{(\alpha)}$, where $\hat{\lambda}^{(\alpha)}$ is the tensor of the relaxation constants. The actual form of $\hat{\lambda}^{(\alpha)}$ is chosen from symmetry considerations; see Ref. 8. Since the only preferred direction in the model (2), (2a) is that of the exchange field $\mathbf{H}_e^{(\alpha)}$, the tensor $\hat{\lambda}^{(\alpha)}$ has only two independent components, $\lambda_\parallel$ and $\lambda_\perp$. Accordingly, the dissipation function Q which determines the rate $\dot{W}_{\mathrm{R}}$ of dissipation of the R-subsystem energy ($\dot{W}_{\mathrm{R}} = -2Q$), in this approximation takes the form of the sum

$$Q = \sum_{\alpha=1}^{n} Q^{(\alpha)},$$

where

$$Q^{(\alpha)} = {}^1/_2 \int d\mathbf{r}\{\lambda_\parallel(\mathbf{F}^{(\alpha)}\mathbf{e}_3^{(\alpha)})^2 + \lambda_\perp[(\mathbf{F}^{(\alpha)})^2 - (\mathbf{F}^{(\alpha)}\mathbf{e}_3^{(\alpha)})^2]\} \tag{3}$$

and $\mathbf{e}_3^{(\alpha)}$ is a unit vector along $\mathbf{H}_e^{(\alpha)}$.

We proceed now to analyze the contribution of the R-ions to the relaxation of magnetic excitations such as spin waves, moving DW, and others, which are determined primarily by the oscillations of the Fe-ion spins. To calculate this contribution we must express the dissipative function of the R-subsystem in the form of a functional of the Fe-system magnetization (or of the vectors $\mathbf{M}$ and $\mathbf{L}$), i.e., express the quantities $\mathbf{F}^{(\alpha)}$ in terms of $\mathbf{H}_e^{(\alpha)}$, and then use $\mathbf{H}_e^{(\alpha)}$ expressed in terms of $\mathbf{M}$ or of $\mathbf{M}$ and $\mathbf{L}$ for the specific magnetic crystal. The difficulty is that $\mathbf{F}^{(\alpha)}$ depends on the R-sublattice magnetization $\mathbf{M}^{(\alpha)}$, which is determined in turn by the form of $\mathbf{H}_e^{(\alpha)}$ on the basis of Eq. (1). To find Q it is convenient to write the dynamic equations directly for the field components $\mathbf{F}^{(\alpha)}$. Using the relation $\mathbf{F}^{(\alpha)} = \mathbf{H}_e^{(\alpha)} - \mathbf{M}^{(\alpha)}/\chi$ and Eq. (1) we obtain the equation

$$\dot{\mathbf{F}}^{(\alpha)} + g_{\mathrm{R}}[\mathbf{F}^{(\alpha)}, \mathbf{H}_e^{(\alpha)}] + (\hat{\lambda}^{(\alpha)}/\chi)\mathbf{F}^{(\alpha)} = \dot{\mathbf{H}}_e^{(\alpha)}, \tag{4}$$

which relates $\mathbf{F}^{(\alpha)}$ with the field $\mathbf{H}_e^{(\alpha)}$ (we assume that all $g_{\mathrm{R}}^{(\alpha)}$ are equal: $g_{\mathrm{R}}^{(\alpha)} = g_{\mathrm{R}}$). No simple equations for $\mathbf{F}^{(\alpha)}$ in terms of $\mathbf{H}_e^{(\alpha)}$ can be obtained in general, but this problem can be solved in most interesting cases.

It is convenient to transform to a moving coordinate frame with unit vectors $\mathbf{e}_1^{(\alpha)}$, $\mathbf{e}_2^{(\alpha)}$, and $\mathbf{e}_3^{(\alpha)}$, where

$$\mathbf{e}_3^{(\alpha)} = \mathbf{H}_e^{(\alpha)}/H_e^{(\alpha)}, \quad \mathbf{e}_2^{(\alpha)} = -\dot{\mathbf{e}}_3^{(\alpha)}/|\dot{\mathbf{e}}_3^{(\alpha)}|,$$
$$\mathbf{e}_1^{(\alpha)} = [\mathbf{e}_2^{(\alpha)}\mathbf{e}_3^{(\alpha)}], \quad H_e^{(\alpha)} = |\mathbf{H}_e^{(\alpha)}|.$$

We obtain then for the components $(\mathbf{F}^{(\alpha)}\mathbf{e}_i^{(\alpha)}) = F_i^{(\alpha)}$, with allowance for (4),

$$\dot{F}_1^{(\alpha)} + \Omega^{(\alpha)}F_2^{(\alpha)} + \gamma_\perp F_1^{(\alpha)} = 0, \tag{4a}$$

$$\dot{F}_2^{(\alpha)} - \Omega^{(\alpha)}F_1^{(\alpha)} + \gamma_\perp F_2^{(\alpha)} = -|\dot{\mathbf{e}}_3^{(\alpha)}|(H_e^{(\alpha)} - F_3^{(\alpha)}), \tag{4b}$$

$$\dot{F}_3^{(\alpha)} + \gamma_\parallel F_3^{(\alpha)} + |\dot{\mathbf{e}}_3^{(\alpha)}|F_2^{(\alpha)} = \dot{H}_e^{(\alpha)}, \tag{4c}$$

where

$$\Omega^{(\alpha)} = \omega_e^{(\alpha)} + (\mathbf{e}_1^{(\alpha)}\dot{\mathbf{e}}_2^{(\alpha)}), \quad \omega_e^{(\alpha)} = g_{\mathrm{R}}H_e^{(\alpha)}.$$

The quantities $\gamma_\perp = \lambda_\perp/\chi$ and $\gamma_\parallel = \lambda_\parallel/\chi$ have the meanings of the transverse and longitudinal relaxation frequencies of the R-sublattice magnetization vector, which are indicative, respectively, of the vector relaxation in direction (with $F_1^{(\alpha)} = F_2^{(\alpha)} = 0$) and in length ($M^{(\alpha)} \to M_{eq}^{(\alpha)} = \chi H_e^{(\alpha)}$). With (3) taken into account, these mechanisms are characterized, respectively, by the dissipative-function densities $q_\perp^{(\alpha)}$ and $q_\parallel^{(\alpha)}$, where

$$q_\perp^{(\alpha)} = (\lambda_\perp/2)\,[(F_1^{(\alpha)})^2 + (F_2^{(\alpha)})^2], \quad q_\parallel^{(\alpha)} = (\lambda_\parallel/2)\,(F_3^{(\alpha)})^2.$$

The system (4) can be solved exactly for a small-amplitude spin wave of arbitrary frequency ω, and also for magnetization perturbations that are not small and have an arbitrary amplitude but a characteristic frequency ω lower than the exchange frequency ω_e, $\omega \ll \omega_e$.

Let us solve the system (4) for a low-amplitude spin wave. In the general case we express the exchange-field in the form

$$\mathbf{H}_e^{(\alpha)}(\mathbf{r},t) = [H_{e0}^{(\alpha)} + {}^1/_2(\delta H_e^{(\alpha)} \exp[i(\omega t - \mathbf{kr})] + \text{c.c.})]\,\mathbf{e}_\xi^{(\alpha)}$$
$$+ {}^1/_2[(h_{e1}^{(\alpha)} \mathbf{e}_\rho^{(\alpha)} + i h_{e2}^{(\alpha)} \mathbf{e}_\mu^{(\alpha)}) \exp[i(\omega t - \mathbf{kr})] + \text{c.c.}],$$

where $\mathbf{k}$ is the wave vector of the spin wave, the amplitudes $\delta H_e^{(\alpha)}$, $h_{e1}^{(\alpha)}$, and $h_{e2}^{(\alpha)}$ are small compared with $H_{e0}^{(\alpha)}$, $\mathbf{e}_\xi^{(\alpha)}$, $\mathbf{e}_\rho^{(\alpha)}$, and $\mathbf{e}_\mu^{(\alpha)}$ is the orthonormalized basis, and at $h_{e2} = 0$ and $h_{e1} = h_{e2}$ the field $\mathbf{H}_e^{(\alpha)}$ is determined by a linearly or circularly polarized wave, respectively. For $F^{(\alpha)} = F_1^{(\alpha)} + iF_2^{(\alpha)}$ we obtain from (4a) and (4b)

$$F^{(\alpha)} = (-i\omega)\,[(\gamma_\perp - i\omega_e^{(\alpha)})^2 + \omega^2]^{-1} \exp\left\{-i \operatorname{arctg}\left(\operatorname{tg}\varphi \frac{h_{e1}^{\alpha}}{h_{e2}^{\alpha}}\right)\right\}$$
$$\times \{\sin\varphi[(\gamma_\perp - i\omega_e^{(\alpha)}) h_{e1}^{(\alpha)} + i\omega h_{e2}]$$
$$-\cos\varphi[\omega h_{e1}^{(\alpha)} - i h_{e2}^{(\alpha)} (\gamma_\perp - i\omega_e^{(\alpha)})]\}, \tag{5}$$

where $\omega_e^{(\alpha)} = g_{\rm R} H_e^{(\alpha)}$ is the exchange amplitude and $\varphi = \omega t - \mathbf{k}\cdot\mathbf{r}$. This yields readily

$$q_\perp^{(\alpha)} = (\lambda_\perp \omega^2/4)\,\{[(h_{e1}^{(\alpha)})^2 + (h_{e2}^{(\alpha)})^2]\,(\gamma_\perp^2 + (\omega_e^{(\alpha)})^2 + \omega^2)$$
$$- 4\omega_e^{(\alpha)} \omega h_{e1}^{(\alpha)} h_{e2}^{(\alpha)}\}\,\{[\gamma_\perp^2 + \omega^2 - (\omega_e^{(\alpha)})^2] + (2\omega_e^{(\alpha)}\gamma_\perp)^2\}^{-1}. \tag{6}$$

For the component $F_3^{(\alpha)}$ we obtain from (4c) and from the contribution to $q_\parallel$

$$F_3^{(\alpha)} = {}^1/_2\left\{\frac{i\omega\,\delta H_e^{(\alpha)}}{i\omega + \gamma_\parallel} \exp[i\varphi] + \text{c.c.}\right\}, \quad q_\parallel^{(\alpha)} = \frac{\lambda_\parallel |\delta H_e^{(\alpha)}|^2 \omega^2}{4(\omega^2 + \gamma_\parallel^2)}. \tag{7}$$

The contribution of the R-ions to the spin-wave damping decrement Γ_k is determined from the equation (W_k is the spin-wave energy density)

$$\Gamma_k = \sum_{\alpha=1}^{n} (q_\perp^{(\alpha)} + q_\parallel^{(\alpha)})/W_k. \tag{8}$$

To calculate Γ_k we must specify the form of $\delta H_e^{(\alpha)}$ and find W_k. The calculation of W_k is greatly simplified when the R-ion density is so low that the entire spin-wave energy is concentrated in the Fe sublattice. This is typical of the magnets of practical importance—dilute rare-earth garnets. If, however, the R-ion densities are not low, the problem is very complicated, since W_k must be calculated from a coupled system of equations for the R- and Fe-sublattice magnetizations. Such calculations were made in Refs. 11 and 12 directly by analyzing the equations, without the use of a dissipative function. We confine ourselves henceforth in the analysis of the spin wave only to the case of low densities (other small parameters come into play for DW), when the use of a dissipative function actually simplifies the calculation of Γ_k.

We proceed now to the nonlinear perturbations. Assume that the characteristic time $t_0 \sim 1/\omega$ of magnetization change is long compared with $1/\omega_e^{(\alpha)}$. We can then neglect in (4a) and (4b) the derivatives with respect to time and write

$$F_1^{(\alpha)} = (H_e^{(\alpha)} - F_3^{(\alpha)})\,\{\omega_e^{(\alpha)} |\dot{\mathbf{e}}_3^{(\alpha)}|\,[(\omega_e^{(\alpha)})^2 + \gamma_\perp^2]^{-1}\}, \tag{9a}$$
$$F_2^{(\alpha)} = -(\gamma_\perp/\omega_e^{(\alpha)})\,F_1^{(\alpha)}. \tag{9b}$$

Using these equations we obtain for $F_3^{(\alpha)}$ to first order in $(\omega/\omega_e)^2$ the closed equation

$$\dot{F}_3^{(\alpha)} + \gamma_\parallel F_3^{(\alpha)} = \dot{H}_e^{(\alpha)} + H_e^{(\alpha)} (\dot{\mathbf{e}}_3^{(\alpha)})^2 \gamma_\perp/[(\omega_e^{(\alpha)})^2 + \gamma_\perp^2], \tag{10}$$

which demonstrates clearly the main property of dissipation by R-ions, viz., the appearance of a strong temporal dispersion for sufficiently small ω, namely, for $\omega \sim \gamma_\parallel$. It has an explicit solution for arbitrary changes of $\mathbf{H}_e^{(\alpha)}$ in both magnitude and direction, as well as for arbitrary ratio of ω and $\gamma_\parallel$. In the important case $\omega \ll \gamma_\parallel$ the solution simplifies to

$$F_3^{(\alpha)} = (1/\gamma_\parallel)\,\{\dot{H}_e^{(\alpha)} + H_e^{(\alpha)} \gamma_\perp (\dot{\mathbf{e}}_3^{(\alpha)})^2/[(\omega_e^{(\alpha)})^2 + \gamma_\perp^2]\}. \tag{11}$$

In (11) we have $F_3^{(\alpha)} \ll H_e^{(\alpha)}$ and the equations for $F_{1,2}^{(\alpha)}$ and q also simplify:

$$q = \sum_{\alpha=1}^{n} (q_\parallel^{(\alpha)} + q_\perp^{(\alpha)}),$$
$$q_\perp^{(\alpha)} = (\lambda_\perp/2)\,(\dot{\mathbf{e}}_3^{(\alpha)} H_e^{(\alpha)})^2/[(\omega_e^{(\alpha)})^2 + \gamma_\perp^2], \tag{12}$$

$$q_\parallel^{(\alpha)} = (\chi^2/2\lambda_\parallel)\,\{\dot{H}_e^{(\alpha)} + \gamma_\perp H_e^{(\alpha)} (\dot{\mathbf{e}}_3^{(\alpha)})^2/[(\omega_e^{(\alpha)})^2 + \gamma_\perp^2]\}^2. \tag{13}$$

Knowledge of the density of the dissipative function q permits calculation of the relaxation characteristics of various nonlinear magnetic perturbations (see Refs. 14 and 15). For example, the friction force $F_{\rm fr}$ acting on a moving domain wall [and on any magnetic soliton such as a simple wave, in which $\mathbf{M} = \mathbf{M}(x - vt)$] is given by

$$F_{\rm fr} = (2/v) \int d\mathbf{r}\, q.$$

Note that for the case $v \ll v_c = \gamma x_0$, where x_0 is the DW thickness, one can use for $\mathbf{M} = \mathbf{M}(x - vt)$ the static solution obtained for the DW with account taken of the renormalization of the Fe-sublattice magnetic energy due to interaction with the R-sublattices (see Ref. 13 for renormalization of anisotropy constants). If, however, $\gamma x_0 \ll v \ll \omega_e x_0$ holds, the situation is more complicated and the connection between the magnetizations of the R and Fe sublattices is no longer the same as in the static case. It is then necessary either to solve a coupled system of equations for $\mathbf{M}$ and $\mathbf{M}^{(\alpha)}$, a task outside the scope of our paper, or assume a small R-ion density and neglect the reaction of the R sublattices to the Fe sublattice.

Hereafter, unless specially stipulated, we shall assume a small R-ion density in analyses of spin-wave damping and DW deceleration at $v \gg \gamma x_0$.

3. MAGNETIC RELAXATION IN IRON GARNETS WITH R-IONS

Magnetic relaxation has been most thoroughly investigated (experimentally as well as theoretically) for iron garnets (IG). This permits comparison of the results of our approach with the microscopic results known for these magnets. We present the results for spin-wave relaxation in IG with R-ions.

In a cubic IG with R-ions one can distinguish six nonequivalent R-sublattices ($n=6$), each with rhombic symmetry and unit vectors $\mathbf{x}^{(\alpha)}, \mathbf{y}^{(\alpha)}, \mathbf{z}^{(\alpha)}, \alpha = 1$–6, that coincide with the rhombic axes and are oriented along the IG axes [$\mathbf{z}^{(\alpha)}$ along [100] axes, $\mathbf{x}^{(\alpha)}$ and $\mathbf{y}^{(\alpha)}$ along [110] axes; see Chap. 13 of Ref. 16]. We represent the exchange field $\mathbf{H}_e^{(\alpha)}$ in the form

$$\mathbf{H}_e^{(\alpha)}=\hat{J}^{(\alpha)}\mathbf{M}. \tag{14}$$

With allowance for the sublattice symmetry with respect to the axes $\mathbf{x}^{(\alpha)}$, $\mathbf{y}^{(\alpha)}$, and $\mathbf{z}^{(\alpha)}$ the tensor $\hat{J}^{(\alpha)}$ indicative of the R–Fe interaction is diagonal [$\hat{J}^{(\alpha)} = J\hat{\varepsilon}^{(\alpha)} = \mathrm{diag}(1+\varepsilon_1, 1, 1+\varepsilon_3)$, J is the exchange integral], but is isotropic.

We begin with the analysis of the damping of spin waves. If the magnetization $\mathbf{M}$ of the iron sublattice is directed along the vector $\mathbf{n}$, the oscillations of the magnetization in a spin wave with circular polarization correspond to

$$(\mathbf{M}\boldsymbol{\tau})+i(\mathbf{M}\boldsymbol{\nu})=M\xi\exp[i(\omega_k t-\mathbf{kr})],$$

$\boldsymbol{\tau}$ and $\boldsymbol{\nu}$ are unit vectors perpendicular to $\mathbf{n}$, $\xi \ll 1$ is the dimensionless wave amplitude, and $M=|\mathbf{M}|$. Calculating $\mathbf{H}_e^{(\alpha)}$ with allowance for (14), using (7), (8), and the equation $W_k=\omega_k M\xi^2/2g$, for the spin-wave energy density (g is the effective gyromagnetic ratio), we obtain for the damping rate $\Gamma_{\parallel,k}$ due to the R-ion longitudinal relaxation

$$\Gamma_{\parallel,k}=[2g\chi\varepsilon_*^2(\mathbf{n})J^2M][\gamma_\parallel\omega_k/(\gamma_\parallel^2+\omega_k^2)], \tag{15}$$

where $\varepsilon_*^2(\mathbf{n})$ is the effective constant that determines the anisotropy of the R–Fe interaction. For an arbitrary form of $\hat{J}^{(\alpha)}$ in (14) (even without assuming that $\hat{J}^{(\alpha)}$ is diagonal) we have for $\varepsilon_*^2(\mathbf{n})$

$$\varepsilon_*^2(\mathbf{n})=(2JM)^{-2}\sum_{\alpha=1}^{6}\{[\boldsymbol{\nu}(\partial H_e^{(\alpha)}(\mathbf{n})/\partial\mathbf{n})]^2 +[\boldsymbol{\tau}(\partial H_e^{(\alpha)}(\mathbf{n})/\partial\mathbf{n})]^2\}. \tag{16}$$

If the tensor $\hat{J}^{(\alpha)}$ is taken to be diagonal and the anisotropy is assumed to be low ($\varepsilon_1, \varepsilon_3 \ll 1$), the expression for $\varepsilon_*(\mathbf{n})$ simplifies to

$$\varepsilon_*^2(\mathbf{n})=(\varepsilon_1^2/2)-\bar{\varepsilon}\Big(1-\sum_{i=x,y,z}(\mathbf{n}\mathbf{e}_i)^4\Big),\qquad \bar{\varepsilon}=(\varepsilon_3-\varepsilon_1)^2-{}^3/_2\varepsilon_1^2.$$

According to (6), the equation for the contribution of the transverse relaxation $\Gamma_{\perp,k}$ is the same as in Ref. 4, and will therefore not be discussed here. We consider the contribution of the longitudinal relaxation.

Expression (15) for $\Gamma_{\parallel,k}$ has the same characteristic features as the equations of the microscopic theory of longitudinal relaxation—both in the doublet model[5] and in the more complicated cases of the multilevel model.[2,17] The phenomenological approach describes the temporal dispersion of $\Gamma_{\parallel,k}$ at $\omega_k=\gamma_\parallel$, and also the fact that the longitudinal relaxation exists only in the presence of anisotropy of the R–Fe interaction, which exists in turn in a cubic magnet of the IG type only when the symmetry of the R-ion position is lower than the crystal symmetry, and each position has a corresponding sublattice.

Thus, in the linear approximation, the phenomenological and the microscopic approaches yield qualitatively the same results. What is more important is that it can be easily used to analyze the relaxation of nonlinear perturbations such as moving DW. In particular, for low-frequency perturbations of arbitrary type ($\omega \ll \gamma_\parallel$, for a DW $v \ll v_c = \gamma_\parallel x_0$, and x_0 is the DW thickness[1)]) one can obtain a simple expression for the dissipative function in the form of a functional of $\mathbf{m}$ and $\dot{\mathbf{m}}$, where $\mathbf{m}=\mathbf{M}/M$. Using (12)–(14) we obtain

$$Q=\int q\,d\mathbf{r},\qquad q=(\lambda_1/2g^2)\dot{\mathbf{m}}^2+(\lambda_2/2g^2)\sum_{i=x,y,z}m_i^2\dot{m}_i^2, \tag{17}$$

where g is the magnetomechanical ratio of the Fe sublattice, and the effective relaxational constants λ_1 and λ_2 are equal for $\varepsilon_1, \varepsilon_3 \ll 1$ to

$$\lambda_1=6\lambda_\perp(g/g_R)^2+2\varepsilon_1^2g^2(\chi JM)^2/\lambda_\parallel,\quad \lambda_2=4(\chi JM)^2\bar{\varepsilon}/\lambda_\parallel. \tag{18}$$

(Note that we have $q>0$, although the constant λ_2 can be negative.)

Recognizing that at sufficiently high temperatures the magnetizations of the Fe and R sublattices are practically collinear, we can take $\mathbf{M}$ to mean the total magnetization of the IG. The first term in (17) coincides in form with the Landau–Lifshitz dissipative function, and the effective constant contains a contribution from both the longitudinal and the transverse relaxations. The second (anisotropic) term is determined only by the longitudinal relaxation and has cubic symmetry. Note that there is no longitudinal-relaxation contribution for an isotropic ion, in accord with Refs. 18 and 19. Note also that an anisotropic term in q can be obtained in principle from Bar'yakhtar's symmetry approach when equations are derived for the total magnetization[8] of an IG with allowance for its cubic symmetry, but then the ratio λ_2/λ_1 is undetermined. Arguments favoring $|\lambda_2| \ll \lambda_1$ are advanced in Ref. 8. In our approach, the value of $|\lambda_2|/\lambda_1$ is determined by the R-sublattice parameters and is in general not small. For example, for an R-ion with axial symmetry ($\varepsilon_3 \gg \varepsilon_1$) we have $\lambda_2 \gg \lambda_1 > 0$.

Using the dissipative function (17) we can calculate the viscous-friction coefficient η of a DW, defined as $\eta = -F_{\mathrm{fr}}/Sv$, where S is the DW area. It turns out that the longitudinal-relaxation contribution to η depends on the orientation of the magnetization rotation in the DW relative to the crystallographic axes. For 180-deg DW in epitaxial IG films with R ions having an easy-magnetization axis perpendicular to the film, we obtain for $\eta_\parallel$ the equation

$$\eta_\parallel=\frac{(\chi\varepsilon_{\mathrm{eff}}JM)^2}{2\lambda_\parallel x_0},\qquad \varepsilon_{\mathrm{eff}}={}^1/_2\varepsilon_1^2+\bar{\varepsilon}\sum_i[{}^2/_5(\nu_i^2+\tau_i^2)^2-{}^1/_3(\nu_i\tau_i)^2], \tag{19}$$

where $\nu_i=(\boldsymbol{\nu}\cdot\mathbf{e}_i)$, $\tau_i=(\boldsymbol{\tau}\cdot\mathbf{e}_i)$, $i=x,y,z$, $\boldsymbol{\nu}$ is a unit vector along the easy axis, $\boldsymbol{\tau}$ is a unit vector in the planes of the DW and of the film, and $\boldsymbol{\tau}\times\boldsymbol{\nu}$ is the normal to the DW. In magnetic films with substrates of type (111), (110), and (100), respectively, we have

$$\varepsilon_{\text{eff}}^2=\begin{cases}\varepsilon_1^2/2+2\bar{\varepsilon}/3, & (111),\\ \varepsilon_1^2/2+65\bar{\varepsilon}/120+(\bar{\varepsilon}/120)(9\cos 4\varphi-2\cos 2\varphi), & (110),\\ \varepsilon_1^2/2+7\bar{\varepsilon}/10+(\bar{\varepsilon}/10)\cos 4\varphi & (100),\end{cases} \quad (20)$$

where φ is the angle between the DW plane and the IG fourfold axis lying in the plane of the film. It can be seen that the anisotropy of the viscosity coefficient

$$\Delta\eta=(\eta_{max}-\eta_{min})/[(\eta_{max}+\eta_{min})/2]$$

need not be small and can reach (at $\varepsilon_1 \ll \varepsilon \approx \varepsilon_3$) 34% and 28% for films of type (110) and (100), respectively, and $\Delta\eta=0$ only for a (111) film. A value of $\Delta\eta$ at a 30% level was observed in IG films, see Refs. 3 and 20, but cannot be described by the existing theories[21] (a microscopic calculation of $\Delta\eta$ is very complicated and was not given in Refs. 6 and 7).

Note that Eq. (19) agrees, to within a constant factor, with the expressions obtained for DW mobility in the microscopic calculation.[6,7] Thus, the results of the proposed phenomenological approach agree well for IG with those obtained earlier in the microscopic approach, but are much easier to obtain.

4. MAGNETIC RELAXATION IN ORTHOFERRITES WITH R-IONS

To our knowledge, there is no microscopic theory of DW relaxation in antiferromagnets and weak ferromagnets of the orthoferrite (OF) type with R-ions, and a theory of spin-wave relaxation in these magnets was developed in Refs. 12 and 22 only in the framework of the doublet model. On the other hand, experimental data are available on the damping of magnons[23] and on domain-wall slowing[24,25] in OF, with the DW slowing investigated in the range from 150 to 400 K, where the doublet model is not applicable at all. Let us use the phenomenological approach to describe magnetic relaxation in these magnets.

In the simplest model of an OF rare-earth one can distinguish two iron sublattices and two R-sublattices.[13] The magnetizations of the Fe sublattices $\mathbf{M}_1$ and $\mathbf{M}_2$, $|\mathbf{M}_1|=|\mathbf{M}_2|=M_0$, almost cancel one another, and the total OF magnetization $\mathbf{M}$ is small when the Dzyaloshinskiĭ interaction (DI) is small. It is convenient to describe the dynamics of the magnetizations of Fe sublattices using effective equations for the normalized antiferromagnetism vector $\mathbf{l}=(\mathbf{M}_1-\mathbf{M}_2)/|\mathbf{M}_1-\mathbf{M}_2|$ (Ref. 25), with the magnetization given by

$$\mathbf{M}=(2/\delta)\{(2/g)[\mathbf{l},\dot{\mathbf{l}}]+[\mathbf{H}_D,\mathbf{l}]\}. \quad (21)$$

Here δ is the exchange constant ($\delta M_0/2$ is the OF exchange field) $\mathbf{H}_D=H_D\mathbf{e}_y$ is the DI field, and the axes x, y, and z coincide with axes a, b, and c of the OF. We confine ourselves in (21) to the simplest version of the DI of type ($\mathbf{H}_D[\mathbf{M},\mathbf{l}]$), disregarding the relativistic terms compared with the exchange-relativistic ones, and also neglect the anisotropy of the Fe-sublattice susceptibility $\chi=4/\delta$, which can be done at sufficiently high temperatures.[13] The dynamics of $\mathbf{l}$ is determined by the Lagrangian[25]

$$L=T-U=(M_0^2\tilde{\alpha}/2c^2)\int d\mathbf{r}\dot{\mathbf{l}}^2-M_0^2\int d\mathbf{r}[(\tilde{\alpha}/2)(\nabla\mathbf{l})^2+W_a(\mathbf{l})], \quad (22)$$

where c is the phase velocity of the spin waves, $\tilde{\alpha}$ is the inhomogeneous exchange constant, and $W_a(\mathbf{l})$ is the anisotropy energy. Note that (21) and (22) contain effective constants renormalized by the R–Fe interaction. The energy of the field $\mathbf{l}$ is defined in the usual manner, $E=T+U$.

The R-ions in an orthoferrite are located in two nonequivalent crystalline positions with C_s point symmetry. We combine them to form two sublattices $\mathbf{M}^{(\alpha)}$, $\alpha=+,-$. The Fe-sublattice exchange field at an R-ion in position α depends both on $\mathbf{M}$ and on $\mathbf{l}$:

$$\mathbf{H}_e^{(\alpha)}=\hat{J}_1^{(\alpha)}\mathbf{M}+\hat{J}_2^{(\alpha)}\mathbf{L},\quad \mathbf{L}=M_0\mathbf{l}, \quad (23)$$

where $\hat{J}_{1,2}^{(\alpha)}$ are exchange tensors whose form is determined from symmetry considerations,[26,13] and demonstrates the substantial anisotropy of the R–Fe interaction in OF:

$$\hat{J}_1^{(\pm)}=\begin{pmatrix} J_{1xx} & \pm J_{1xy} & 0\\ \pm J_{1yx} & J_{1yy} & 0\\ 0 & 0 & J_{1zz}\end{pmatrix},$$

$$\hat{J}_2^{(\pm)}=\begin{pmatrix} 0 & 0 & J_{2xz}\\ 0 & 0 & \pm J_{2yz}\\ J_{2zx} & \pm J_{2zy} & 0\end{pmatrix}.$$

Taking (21) into account, we can represent the exchange fields $\mathbf{H}_e^{(\alpha)}$ in terms of only $\mathbf{l}$ and $\dot{\mathbf{l}}$ and the effective exchange constants

$$J_x=J_{2xz}+(2H_D/\delta M_0)J_{1xx},\quad J_y=J_{2yz}+(2H_D/\delta M_0)J_{1yx},$$

$$J_z=J_{2zx}-(2H_D/\delta M_0)J_{1zz},\quad p=J_{2zy}/J_z,\quad \alpha=\pm 1,$$

$$\mathbf{H}_e^{(\alpha)}=M_0\{(J_x\mathbf{e}_x+\alpha J_y\mathbf{e}_y)l_z+J_z(l_x+\alpha p l_y)\mathbf{e}_z\}+(4/\delta g)\hat{J}_1^{(\alpha)}[\mathbf{l},\dot{\mathbf{l}}]. \quad (24)$$

A specific feature of an OF is that the effective exchange constants $J_{x,y,z}$ in (24) are smaller than those of an IG (the exchange field at the R ion does not exceed several tens of kOe; see Ref. 16), and the spin-wave frequencies $\omega_{1,2}=gM_0(\delta\beta_{1,2})^{\frac{1}{2}}/2$, (where β_1 and β_2 are the anisotropy constants) contain the exchange constant δ and are quite large [the field $H_{EA}\approx(\delta\beta_{1,2})^{\frac{1}{2}}M_0/2$ is of the order of several tens of kOe]. The damping of spin waves should therefore be analyzed for an arbitrary relation between $\omega_{1,2}$ and $\omega_e\sim J_i/\hbar$, $\gamma_\parallel$ and $\gamma_\perp$ (recall that our analysis is valid only for low R-ion densities in OF with nonmagnetic R-ions, say in $YFeO_3$.

Let us examine the damping of magnons in the most abundant Γ_4 phase of OF ($\mathbf{l}\parallel\mathbf{e}_x$, and $\mathbf{m}\parallel\mathbf{e}_z$ in the ground state).[13] It corresponds to an anisotropy energy in the form

$$W_a={}^1/_2(\beta_1 l_z^2+\beta_2 l_y^2),\quad \beta_2>\beta_1>0.$$

In the two spin-wave modes the oscillations of the vector $\mathbf{l}$ are linearly polarized in the planes ac and ab, respectively

$$\mathbf{l}(\mathbf{r},t)=\mathbf{e}_x+[(\boldsymbol{\xi}_0/2)\exp[i(\omega_k t-\mathbf{kr})]+\text{c.c.}],$$
$$\omega_k=[\omega_0^2+(c\mathbf{k})^2]^{1/2}, \quad (25)$$

where we have for the ac mode

$$\boldsymbol{\xi}_0=\xi\mathbf{e}_z,\quad \omega_0=\omega_{ac}=gM_0(\beta_1\delta)^{1/2}/2,$$

and for the ab mode

$$\boldsymbol{\xi}_0=\xi\mathbf{e}_y,\quad \omega_0=\omega_{ab}=gM_0(\beta_2\delta)^{1/2}/2.$$

Using (25) and (24) we can find $\mathbf{H}_e^{(\alpha)}$ and calculate from Eqs. (6) and (7) the spin-wave energy dissipation rates $q^{(ab)}$ and $q^{(ac)}$. It turns out that the $\mathbf{H}_e^{(\alpha)}$ for these two polarization have substantially different forms. For the *ab* mode the field $\mathbf{H}_e^{\alpha}$ varies only over its length, ($\dot{\mathbf{e}}_3^{(\alpha)}=0$), has therefore no contribution from the transverse relaxation. The situation is reversed for the *ac* mode: accurate to ξ^2, there is no contribution of the longitudinal relaxation. We present an expression for the damping rates Γ_k of these modes, calculated as $\Gamma_k = q/W_k$, where $W_k = 2\xi_0\omega_k^2/g^2\delta^2$ is the wave energy density [see (22)]. We ultimately have

$$\Gamma_k^{(ab)}=\Gamma_{\parallel,k}=\lambda_\parallel(gMJ_z)^2\delta[p^2+r^2(2\omega_k/g\delta M_0)^2][4(\omega_k^2+\gamma_\parallel^2)]^{-1}, \tag{26}$$

$$\Gamma_k^{(ac)}=\Gamma_{\perp,k}={}^1/_8\lambda_\perp(gM_0J_z)^2\delta[(1+\varepsilon)[D(\omega_k)+D(-\omega_k)] - 8f\omega_k(2\omega_e/g\delta M_0)D(\omega_k)D(-\omega_k)], \tag{27}$$

where

$$r=J_{1zz}/J_z,\quad f=J_{1yy}J_x/J_z^2,\quad \varepsilon=(J_x^2+J_y^2-J_z^2)/J_z^2,$$
$$D(x)=[\gamma_\perp^2+(\omega_e+x)^2]^{-1},$$

and $\omega_e = g_{\mathrm{R}}J_zM_0$ is the frequency describing the exchange field at the R-ion. Note that ω_e and ω_k are small compared with the characteristic frequency $g\delta M_0/2$ describing the exchange field of the Fe subsystem. Therefore, in particular, the term with ω_k^2 in (26) can be substantial only for $r\gg p$, i.e., $J_{1zz}\gg J_{2zy}$. We have no data for these constants, but they correspond to substantially different R–Fe interactions, and one cannot exclude the satisfaction of such an inequality.

The frequency dependence of the contribution of the longitudinal relaxation to the magnon damping rate in OF ($\Gamma_{\parallel,k}\sim(\omega_k^2+\gamma_\parallel^2)^{-1}$, and not $\Gamma_{\parallel,k}\sim\omega_k(\omega_k^2+\gamma_\parallel^2)^{-1}$ as in a ferromagnet at $\omega\ll\omega_e$) is the same as in the microscopic calculations.[22] A more detailed comparison is impossible within the framework of the doublet model, since the impurity symmetry and the character of the interaction of the impurity level with the Fe subsystems were not specified in Ref. 22. In the doublet model of an Ho^{3+} ion in an OF, proposed by the authors of Ref. 12, there is no longitudinal relaxation at all in the Γ_4 phase. The damping of a mode of type (*ab*) was attributed by the authors to dissipation in the Fe sublattice. However, even allowance for the upper level would make a significant contribution to the relaxation, something taken into account automatically in our approach. In this approach, even without assuming a low R-ion density, when a coupled system of equations of motion must be solved for the R and Fe sublattices, the damping $\Gamma^{(ab)}$ of the *ab* mode will be determined only by the longitudinal-relaxation mechanism, and $\Gamma^{(ac)}$ by the transverse relaxation (neglecting, of course the proper damping of the Fe sublattice). This follows from the very form of $\mathbf{H}_e^{(\alpha)}$ as a functional of $\mathbf{l}$ (24).

In the OF canted phase, which appears in the spin-flip region, longitudinal and transverse relaxation are both essential for damping of the two modes, and in the transition to the OF collinear phases Γ_4 and Γ_2 ($\mathbf{l}\parallel\mathbf{c}$) one of these contributions vanishes. In particular, no contribution is made to the *ac* mode by the longitudinal relaxation to either collinear phase. The situation is more interesting for the *ab* mode: as noted above, nonzero contributions are made only by the longitudinal relaxation in the Γ_4 phase, only by the transverse in the Γ_2 phase, and by both in the canted phase Γ_{24}. The theory thus predicts changes in the spin-wave relaxation mechanism in the passage through the spin-flip region, and this should cause an abrupt change in the character of the relaxation—in the values of the damping rates, in their temperature and frequency dependences, etc. (see Ref. 10 for details).

In the case of low frequencies, $\omega\ll\omega_e, \gamma_\parallel, \gamma_\perp$, it is necessary to carry out, just as in the case of the IG, a more general analysis and write down for an arbitrary nonlinear wave the dissipative function, with the aid of Eqs. (12), (13), and (24), in the form of a functional of the vectors $\mathbf{l}$ and $\dot{\mathbf{l}}$. The density of the dissipative function is a sum of two terms corresponding to longitudinal ($q_\parallel$) and transverse ($q_\perp$) relaxation, but in view of the strong anisotropy of the R–Fe interaction the expressions obtained for $q_\parallel$ and $q_\perp$ are more unwieldy:

$$q_\perp=(\lambda_\perp/g_{\mathrm{R}}^2)[(\mathbf{l}\hat{s}\mathbf{l})^2-4S_{xy}^2l_x^2l_y^2]^{-2}\{[(\mathbf{l}\hat{s}\mathbf{l})(\dot{\mathbf{l}}\hat{s}\dot{\mathbf{l}}) +4S_{xy}^2l_xl_xl_yl_y-(\mathbf{l}\hat{s}\dot{\mathbf{l}})^2] \times[(\mathbf{l}\hat{s}\mathbf{l})^2+4S_{xy}^2l_x^2l_y^2]-8(\mathbf{l}\hat{s}\dot{\mathbf{l}})S_{xy}^2[l_xl_y(\dot{\mathbf{l}}\hat{s}\dot{\mathbf{l}})+l_xl_y(\mathbf{l}\hat{s}\mathbf{l}) -(\mathbf{l}\hat{s}\dot{\mathbf{l}})(l_xl_y)^{\cdot}]\}, \tag{28}$$

$$q_\parallel=(\chi^2M_0/\lambda_\parallel)[(\mathbf{l}\hat{s}\mathbf{l})^2+4S_{xy}^2l_x^2l_y^2]^{-1}\{[(\mathbf{l}\hat{s}\dot{\mathbf{l}})^2 +S_{xy}^2(((l_xl_y)^{\cdot})^2)(\mathbf{l}\hat{s}\mathbf{l})-4(\mathbf{l}\hat{s}\dot{\mathbf{l}})S_{xy}^2(l_xl_y)^{\cdot}l_xl_y\}, \tag{29}$$

where

$$\hat{s}=\mathrm{diag}(S_{xx}, S_{yy}, (S_{xx}+S_{yy})),\ S_{xx}=J_xJ_z,\ S_{yy}=J_yJ_zp,$$
$$S_{xy}={}^1/_2J_z(J_y+pJ_x).$$

This expression simplifies in many cases of practical interest, particularly moving domain walls. The motion can be regarded as low-frequency and the dissipative function (28), and (29) can be used for DW moving with velocity v under the condition $v\ll v_c$, where $v_c\approx x_0\gamma_\parallel$ and x_0 is the DW thickness. For $v\ll v_c$ the friction force is $F_{\mathrm{fr}}=-\eta v$.

It is known (see Ref. 25) that in the collinear phase Γ_4 of an OF there can be two types of DW. In one case the vectors $\mathbf{M}$ and $\mathbf{l}$ are rotated in the *ac* plane:

$$\mathbf{l}=\mathbf{e}_x\cos\vartheta+\mathbf{e}_z\sin\vartheta$$

(DW of type *ac*). In the second, the vector $\mathbf{l}$ rotates in the *ab* plane

$$\mathbf{l}=\mathbf{e}_x\cos\vartheta+\mathbf{e}_y\sin\vartheta,$$

while $\mathbf{M}$ only varies in length. For both DW we have

$$\cos\vartheta=\mathrm{th}[(y-vt)/x_0(1-(v/c)^2)^{1/2}],$$

where $x_0=(\tilde{\alpha}/\beta_1)^{1/2}$ for the DW of type *ac* and $x_0=(\tilde{\alpha}/\beta_2)^{1/2}$ for the DW of type *ab*. In most OF there is a DW of type *ac* at room temperature, while for dysprosium OF the *ab*-type DW is stable at $T<150$ K. The DW thickness in the OF is $x_0\approx10^{-6}$ cm, while at $\gamma_\parallel\approx10^{11}\ \mathrm{s}^{-1}$ we have $v_c\approx1$ km/sec, which is smaller by an order of magnitude than the DW velocity limit. The value of $\gamma_\parallel$, meaning also v_c, decreases when the temperature is lowered.

Calculation shows that in the *ab* DW, just is in a spin wave of *ab* type, there is no transverse-relaxation contribution and the deceleration of the DW is determined only by the longitudinal relaxation:

$$\eta^{(ab)}=\eta_{\parallel}^{(ab)}=A_{\parallel}^{(ab)}(\chi J_z M_0)^2/\lambda_{\parallel} x_0, \qquad (30)$$

where $A_{\parallel}^{(ac)}=(8/3)(l+p^2/2)$. In fact, owing to the Lorentz contraction of the DW, Eq. (30) contains in place of x_0 the quantity $x_0(v)=x_0(1-(v/c)^2)^{1/2}$, but this can be neglected for $v \ll c$.

For an *ac*-type DW, the contribution to the viscosity from either the longitudinal or the transverse relaxation differs from zero, $\eta^{(ac)}=\eta_{\parallel}^{(ac)}+\eta_{\perp}^{(ac)}$,

$$\eta_{\parallel}^{(ac)}=A_{\parallel}^{(ac)}(\chi J_z M_0)^2/\lambda_{\parallel} x_0, \qquad \eta_{\perp}^{(ac)}=A_{\perp}^{(ac)}\lambda_{\perp}/g_R^2 x_0, \qquad (31)$$

where

$$A_{\parallel}^{(ac)}=4\{1+\varepsilon/3-[(1+\varepsilon)^{1/2}/|\varepsilon|^{1/2}]\varkappa_1(\varepsilon)\},$$

$$A_{\perp}^{(ac)}=2\{1+[(1+\varepsilon)|\varepsilon|]^{-1/2}\varkappa_1(\varepsilon)\},$$

and we have $\varkappa_1(\varepsilon)=\sinh^{-1}\varepsilon^{1/2}$ for $\varepsilon>0$ and $\varkappa_1(\varepsilon)=\sin^{-1}|\varepsilon|^{1/2}$ for $-1<\varepsilon<0$. The quantity ε is indicative of the anisotropy of the R–Fe interaction, and as $\varepsilon\to 0$ we have $A_{\parallel}^{(ac)}\simeq 8\varepsilon^2/15$, and $A_{\perp}^{(ac)}\simeq 4$. It is assumed here and below, for simplicity, that $\gamma_{\parallel}, \gamma_{\perp} \ll \omega_e$.

Calculation of the deceleration of DW in OF with allowance for the anisotropy of the magnetic susceptibility χ leads to qualitatively the same results as obtained with $\chi_{ik}=\chi\delta_{ik}$. For an *ab*-type DW, in particular, $\eta_{\perp}=0$ and an exact equation for $\eta_{\parallel}$ is obtained from (30) by the substitution $\chi\to\chi_{zz}$. For an *ac*-type DW the equation for η becomes more unwieldly, but agrees qualitatively with (31). This allows us to state that allowance for the anisotropy of χ is of no fundamental interest.

We consider now the limit of high velocities, $v>v_c$ (recall that such an analysis is meaningful for low R-ion densities, when they influence little the Fe sublattice). For the calculation we must use a general system of Eqs. (9) and (10) with allowance for the explicit form of $\mathbf{H}_e^{(\alpha)}$. It follows from (10), in particular, that for $v \gg v_c$ we have

$$F_3^{(\alpha)}(t)=H_e^{(\alpha)}(t)-H_e^{(\alpha)}(-\infty)$$

(at $t=-\infty$, $F_3^{(\alpha)}=0$). In this case the friction force per unit area of an (ab)-type domain wall is

$$F_{\rm fr}^{(ab)}=(1/v)B_{\parallel}^{(ab)}\lambda_{\parallel}(J_z M_0)^2 x_0(v), \qquad v\gg v_c, \qquad (32)$$

where

$$B_{\parallel}^{(ab)}=4[\ln(4(1+p^2))+p^2-1-p(\pi-\operatorname{arctg} p)],$$

and decreases with increase of velocity.

The change in the character of the $F_{\rm fr}(v)$ dependence is a direct consequence of the temporal dispersion characteristic of longitudinal-relaxation processes. This behavior ("turning-off" a dissipation mechanism when the DW mobility is increased 5–7 times) was observed in Ref. 27 for DW in yttrium orthoferrite at $v\approx 2$ km/s, which agrees with the estimate of v_c. It would be of interest to relate this effect to turning off the relaxation due to small uncontrollable R-ion impurities that can be present in $YFeO_3$. For (ac)-type DW, both mechanisms contribute to the relaxation, so we have $F_{\rm fr}^{(ac)}=F_{\perp}^{(ac)}+F_{\parallel}^{(ac)}$. $F_{\parallel}^{ac}$ is "turned-off" for $v\gg v_c$ in accordance with the same law as for *ab* DW ($F_{\rm fr}^{(ac)}\sim 1/v$); see Eq. (32) with $B_{\parallel}^{(ab)}$ replaced by

$$B_{\parallel}^{(ac)}=4[\ln(1+\varepsilon)+\varepsilon+2\varkappa_2(\varepsilon)|\varepsilon|^{1/2}],$$

where $\varkappa_2(\varepsilon)=\tanh^{-1}\sqrt{|\varepsilon|}$ for $-1<\varepsilon<0$ and $\varkappa_2(\varepsilon)=-\tan^{-1}\sqrt{\varepsilon}$ for $\varepsilon>0$. As to the behavior of the transverse-relaxation mechanism, the situation is substantially different. The turning-off mechanism is absent, i.e., no dependence of type $F_{\rm fr}=F_{\perp}\sim 1/v$ sets in up to velocities $v\sim\omega_e x_0\sim 10$ km/sec, but when v goes through the value $v_c=\gamma_{\parallel}x_0$ only the coefficient of proportionality between $F_{\perp}^{ac}$ and v changes

$$F_{\rm fr}^{ab}=B_{\perp}^{(ac)}\lambda_{\perp}v/g_R^2 x_0(v), \qquad (33)$$
$$B_{\perp}^{(ac)}=1+{}^3/_2(1+\varepsilon)+3\varkappa_2(\varepsilon)/2(|\varepsilon|(1+\varepsilon)^3)^{1/2},$$

Let us compare the contributions of the two mechanisms to the DW mobility. It follows from (31) that $(\eta_{\parallel}^{(ac)}/\eta_{\perp}^{(ac)})\approx\omega_e^2/\gamma_{\perp}\gamma_{\parallel}\gg 1$ and the basic mechanism is longitudinal relaxation (a similar inequality appears also in the microscopic IG theory[7]). Let us estimate numerically the DW mobility, defined in OF as $\mu=2M/\eta$. Recognizing that $\chi=\mu_{\rm eff}^2 c/3T$, where $\mu_{\rm eff}$ is the effective magnetic moment of the R-ion, $\gamma_{\parallel}=\lambda_{\parallel}/\chi$, and c is the R-ion density, we obtain for the mobility

$$\mu=2M\gamma_{\parallel}x_0 T/AH_e^2 c\mu_{\rm eff}^2,$$

where the constant $A\sim 1$ is defined in (30) and (31). With allowance for the numerical values $x_0\simeq 10^{-6}$ cm, $\gamma_{\parallel}\simeq 10^{11}$ s^{-1}, $H_e=0.5\times 10^4$ Oe, $M=10$ G (Ref. 13) we find that at $T=300$ K we get $\mu\simeq 10^2/y$ (cm/s·Oe), where y is the number of R ions per formula unit. This value agrees with experimental data[24,25] according to which the value of μ for different rare-earth orthoferrites at room temperature ranges from 150 to 800 cm/s·Oe. A more detailed comparison of the value of μ at $T=300$ K and also of the temperature dependences of μ is impossible in view of the uncertainty in the values of the parameter $\gamma_{\parallel}$.

Investigation of the deceleration of DW in the canted phase of OF (which occurs in the spin-flip region) is described in detail in Ref. 10 and will not be discussed here. We note only that at the flip point itself the contribution of the transverse relaxation for the energetically preferred small-angle DW is much larger than that of the longitudinal one.

CONCLUSION

Our approximations have yielded for the spin-wave damping constants and domain-wall deceleration general equations that are valid for any ion. Specific, "individual" properties of each R ion are determined by the minimum number of phenomenological parameters $\gamma_{\parallel}$, $\gamma_{\perp}$, χ, ω_e and the anisotropy parameter ε.

These approximations relate primarily to the allowing only for the magnetization in the description of the R-ion state and neglecting higher odd multipoles (quadrupole variables, as already mentioned, are unimportant for $T\gg\varepsilon_e\sim 20$ K). Neglect of the anisotropy of the R-ion susceptibility is also significant. These two approximations are valid at sufficiently high temperatures $T>\Delta$, where $\Delta\geqslant 10^2$ K is the level splitting in the crystal field.

In our opinion these constraints on the temperature are of no fundamental significance. For each specific problem, allowance for the anisotropy of χ, while cumbersome, is possible. It is impossible only to write general equations analogous to these in the isotropic approximation. Allowance for the anisotropy of χ, carried out by us in certain cases (e.g., for DW deceleration in OF) does not change the results qualitatively (and not even quantitatively for *ab*-type DW).

It is more important that at low temperatures, when only two levels are excited, one can use the generally acceptable doublet model[5] that permits a microscopic description in terms of the spin density $\boldsymbol{\sigma}$ associated with the effective spin 1/2 (Ref. 12). A theory based on the doublet model has a high-temperature constraint (we have noted above that allowance for the next higher level of the Ho^{3+} ion can substantially alter the result of the calculation in this model. Our theory has a low-temperature limit. It seems to us that the use of alternative theories extends the possibilities of describing rare-earth magnets.

This method can also be generalized to include other problems. In our approach it is easy to describe the damping of elastic perturbations (both sound waves and nonlinear perturbations such as moving dislocations, crowdions, etc.). It suffices for this purpose to express the effective field $\mathbf{H}_{\text{eff}}^{(\alpha)}$ (the analog of the exchange field $\mathbf{H}_e^{(\alpha)}$) acting on the αth R-sublattice in terms of the deformation and distortion tensors.

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[1)] At the characteristic paramagnetic-relaxation frequencies $\gamma_{\parallel} \approx 10^{10}\ \text{s}^{-1}$ (Ref. 1) and at DW thickness $x_0 \approx 10^{-6}$ cm we have $v_c \approx 10^4$ cm/s. The DW velocities realized in experiments with rare-earth garnets are usually considerably lower.

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