Interference of spatially multimodal squeezed states of radiation and noise-free optical wavefront control

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It is shown that when light waves in a spatially multimodal squeezed state interfere in an optical mixing device, whose transmission coefficients vary (are controlled externally) in time and space, optical images without photon (shot) noise can be obtained. The conditions for matching the phases of the waves in the sources of the "squeezed" light and in the interference device are determined. The characteristic scales of suppression of quantum noise, including taking diffraction into account, are discussed. It is proposed that Faraday rotation and birefringence be employed for controlling optical wavefronts without introducing quantum noise.

1. INTRODUCTION

Until recently it was assumed that light fields in a nonclassical state (squeezed), with sub-Poisson photon statistics) could be employed primarily for increasing the sensitivity and reliability in optical investigation of phenomena and processes which unfold in time.[1-3] For example, temporal fluctuations of the photoelectron flux can be suppressed when a sub-Poisson photon flux, temporal fluctuations of the light pressure on a test body in laser detection of gravitational waves,[4] etc., are received efficiently.

The fundamental possibilities of performing measurements with nonclassical light fields, however, encompass a wider range of phenomena. Suppression of fluctuations in the intensity spectrum of a pulse reflected from a test body was discussed in Ref. 5. Increasing attention is being devoted to the behavior of light in a nonclassical state in space.[6-8] In Refs. 6 and 7 it was shown that when spatially multimodal squeezed states of light are observed by the method of optical heterodyning the photon (shot) noise of the radiation can be suppressed not only in time, but simultaneously in space also, i.e., in the transverse cross section of the light beam.

A number of questions arise about the behavior of such "quantum-noise-free" light waves in many optical phenomena for which spatiotemporal (i.e., nonuniform) propagation is important. Examples are the production and reception of optical images, optical computation, holography, etc.

In this paper the problem of obtaining optical images without photon (shot) noise is solved. A transversely oriented interference mixing device is employed to achieve wavefront control of the light in a spatially multimodal squeezed state without the introduction of quantum noise. Thus a three-dimensional (spatiotemporal) extension of the method of controlling nonclassical light fields with the help of interference, which has been studied theoretically and experimentally[4,9-11] from the viewpoint of optical modulation in time, is proposed. The interference of spatially multimodal squeezed states under homogeneous conditions (no image) was studied in Ref. 12.

The conditions, sufficient for noise-free control, for matching the phases of the "squeezed" and reference radiations as well as the scattering coefficients of the interferometer are found. For physical realization it is proposed that devices that transform the state of polarization of the incident light waves be employed. We have in mind Faraday rotation or birefringence controlled in space and time. The spatiotemporal and spectral estimates of the characteristic scales of noise suppression in an optical image are given. The role of the diffraction of "squeezed" radiation and the possibility of achieving the maximum resolution of "noise-free" control in space are discussed.

2. INTERFERENCE OF LIGHT WAVES IN A SPATIALLY MULTIMODAL SQUEEZED STATE

We shall study the scheme shown in Fig. 1 for performing interference mixing of two light beams in a spatially multimodal squeezed state. The nonlinear crystals NC_m ($m = 1,2$) are independent sources of wideband "squeezed" radiation with orthogonal polarizations (for example, linear).

As a result of three-wave parametric interaction there occurs parametric scattering of the pump wave $\omega_p,\mathbf{k}_p$, i.e., division of the frequency $\omega_p \to \omega_1 + \omega_2$, where $\omega_1 = \omega_p/2 + \Omega$, $\omega_2 = \omega_p/2 - \Omega$. The optical system is assumed to be oriented transversely, so that the parametrically scattered photons are emitted in conjugate directions: $(\mathbf{k}_1)_\perp = (-\mathbf{k}_2)_\perp = \mathbf{q}$. The reference heterodyne wave $\omega_h,\mathbf{k}_h$ ($\omega_h = \omega_p/2, \mathbf{k}_h \| \mathbf{k}_p$), whose polarization corresponds to that of the parametric scattering, is incident on the input surface of each crystal.

We denote by $a_m(x,\boldsymbol{\rho},t)$, $b_m(x,\boldsymbol{\rho},t)$, and $e_m(x,\boldsymbol{\rho},t)$, where $\boldsymbol{\rho} = (y,z)$, the slowly varying, relative to $\exp \times\{i(k_h x - \omega_h t)\}$, amplitudes of the electric field in the polarizations $m = 1,2$ with respect to NC_m, after NC_m, and after the interference mixer I. We denote the quantum averages of the amplitudes a_m, b_m, and e_m by α_m, β_m, and ε_m; for example, $\alpha_m = \langle a_m \rangle$, etc. In this section we neglect free diffraction, i.e., we assume that the distance between the components in the scheme shown in Fig. 1 is small (see Sec. 4).

The starting quantum state $|\text{in}\rangle$ is the vacuum state, apart from the heterodyne waves, whose state at the input is coherent with amplitudes α_m:

$$a_m(\boldsymbol{\rho},\ t)\,|\text{in}\rangle = \alpha_m|\text{in}\rangle. \tag{1}$$

The commutation relations for the operators of the ampli-

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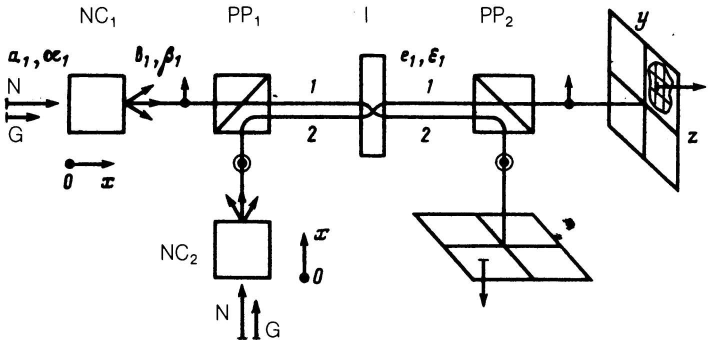

FIG. 1. The optical scheme used for interference mixing of "squeezed" orthogonally polarized light beams: NC are nonlinear crystals, I is an interference device, and PP_1 and PP_2 are polarization prisms, which combine and separate in space light waves with distinguished polarizations. A dense array of efficient photon counters is shown, in part, in one of the detection planes.

tudes of the free input fields are as follows:

$$[a_{m_1}(\mathbf{q}_1, \Omega_1), a_{m_2}^+(\mathbf{q}_2, \Omega_2)] = (2\pi)^3 c^{-1}\delta_{m_1, m_2}\delta(\mathbf{q}_1-\mathbf{q}_2)\delta(\Omega_1-\Omega_2). \quad (2)$$

The evolution of the slow Fourier amplitude ($\boldsymbol{\rho}\to\mathbf{q}, t\to\Omega$) from the input to the output of NC$_m$, i.e., the spatially multimode squeezing transformation has the form (see Refs. 6, 7, and 13)

$$b_m(\mathbf{q}, \Omega) = U_m(\mathbf{q}, \Omega)a_m(\mathbf{q}, \Omega) + V_m(\mathbf{q}, \Omega)a_m^+(-\mathbf{q}, -\Omega). \quad (3)$$

The average amplitude of the heterodyne at the input of NC$_m$ is

$$\beta_m = U_m(0, 0)\alpha_m + V_m(0, 0)\alpha_m^*. \quad (4)$$

The coefficients U and V in Eq. (3) depend on the strength of the parametric interaction, the amplitude of the pump wave (constant), the length of the crystal, and the matching of the phases of the interacting waves (spatial synchronism). The transformation (3) of spatially multimode squeezing can be clearly explained in terms of modulation of the quantum fluctuations of the field simultaneously in space and time. A fluctuation of the amplitude of the field with polarization $m = 1,2$ moves in the complex plane at the frequencies $\mathbf{q}$, Ω on the average inside an inhomogeneity ellipse whose semimajor and semi-minor axes are in the ratio $\exp\{r_m(\mathbf{q},\Omega)\}$ and $\exp\{-r_m(\mathbf{q},\Omega)\}$. The direction of the semi-major axis is determined by the angle $\Psi_m(\mathbf{q},\Omega)$, where

$$\exp\{\pm r_m(\mathbf{q}, \Omega)\} = |U_m(\mathbf{q}, \Omega)| \pm |V_m(\mathbf{q}, \Omega)|,$$
$$\psi_m(\mathbf{q}, \Omega) = {}^1/_2 \arg\{U_m(\mathbf{q}, \Omega)V_m(-\mathbf{q}, -\Omega)\}. \quad (5)$$

Knowing the behavior (dispersion) of the squeezing ellipse as a function of the frequencies q and Ω, it is possible to describe clearly the spectral properties of the fluctuations of the "squeezed" light, including also in the case of photodetection.

The polarization prism PP_1 combines in space the wavefronts of the heterodyne waves as well as the parametrically scattered waves, which arrive from the inputs $m = 1, 2$. It is assumed that the interference mixer I is linear and passive, does not introduce any losses, and is described by a unitary scattering matrix

$$\hat{R}(\boldsymbol{\rho}, t) = \begin{pmatrix} \exp(i\varphi_1) & 0 \\ 0 & \exp(i\varphi_2) \end{pmatrix} \times \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} \exp(-i\varkappa_1) & 0 \\ 0 & \exp(-i\varkappa_2) \end{pmatrix}. \quad (6)$$

The real parameters φ_i, $\varkappa_i$, c, and s, where $c^2 + s^2 = 1$, which prescribe the transformation of the phases and amplitudes, depend on the time and the coordinates (they are controlled from the side), as a result of which an optical image is formed in the scattered waves. The analyzer (a polarization prism PP_2) directs the scattered waves on the photodetection plane, where the image is detected. For the operator amplitude of the field on the detector $n = 1,2$ we have

$$e_n(\boldsymbol{\rho}, t) = \sum_{m=1}^{2} R_{nm}(\boldsymbol{\rho}, t) b_m(\boldsymbol{\rho}, t). \quad (7)$$

The numerical (average) amplitudes ε_n and β_m are related by an analogous equation.

The natural noise of photodetection in space in the detection plane and in time can be measured by the frequency and spatial-frequency spectrum of the fluctuations $\delta i_n(\boldsymbol{\rho},t)$ of the photocurrent density $i_n(\boldsymbol{\rho},t)$ (in cm$^{-2}\cdot$s^{-1}):

$$\delta i_n(\boldsymbol{\rho}, t) = i_n(\boldsymbol{\rho}, t) - \langle i_n(\boldsymbol{\rho}, t)\rangle. \quad (8)$$

The spectrum is determined as

$$(\delta i_n^2)_{\mathbf{q},\Omega} = \int d\boldsymbol{\rho}\, dt \left\langle \frac{1}{2}\overline{\{\delta i_n(0,0), \delta i_n(\boldsymbol{\rho}, t)\}_+} \right\rangle \times \exp[i(\Omega t - \mathbf{q}\boldsymbol{\rho})], \quad (9)$$

where $\{\ldots, \ldots\}_+$ is the anticommutator. A particular image, generated by the transformation (7), is localized in space and in time, so that the parameters of the quantum noise depend on the point and moment of observation. In order to draw more general conclusions, we shall study below a statistical ensemble of images that is homogeneous in space and time. We shall denote by an overbar averages over this ensemble.

It is well known from the quantum theory of photodetection (see Ref. 2) that the correlation functions of the current density and the slowly varying amplitude of the field are related as follows:

$$\langle i_n(\boldsymbol{\rho}, t)\rangle = \eta c\langle e_n^+(\boldsymbol{\rho}, t)e_n(\boldsymbol{\rho}, t)\rangle,$$
$$\langle {}^1/_2\{i_{n_1}(\boldsymbol{\rho}_1, t_1), i_{n_2}(\boldsymbol{\rho}_2, t_2)\}_+\rangle = \langle i_{n_1}(\boldsymbol{\rho}_1, t_1)\rangle\delta_{n_1, n_2}\delta(\boldsymbol{\rho}_{21}) \times\delta(t_{21}) + \eta^2c^2\{\langle e_{n_1}^+(\boldsymbol{\rho}_1, t_1)e_{n_2}^+(\boldsymbol{\rho}_2, t_2)e_{n_2}(\boldsymbol{\rho}_2, t_2) \times e_{n_1}(\boldsymbol{\rho}_1, t_1)\rangle\theta(t_{21}) + (1\leftrightarrow 2)\}. \quad (10)$$

Here $\eta \leqslant 1$ is the quantum efficiency of detection and $\theta(t)$ is the Heaviside function. Illumination is assumed to be close to normal. The average $\langle e^+ e\rangle$ corresponds to the energy of the field per unit volume at $\hbar\omega_h$.

In order to calculate the spectrum (9) the amplitude of the field, appearing in Eq. (10), at the output of the system must be expressed in terms of the input amplitudes, taking into account the transformation of the field in the nonlinear crystal and in the mixer; see Eqs. (3) and (7). The quantum averaging is performed using the relations (1) and (2). Keeping the contributions proportional to the power of the reference wave, we obtain

$$(\delta i_n^2)_{\mathbf{q},\Omega}=\overline{\langle i_n(\rho,t)\rangle}(1-\eta)+c\eta^2\sum_{m=1}^{2}(2\pi)^{-3}\int d\mathbf{q}'\,d\Omega'$$
$$\times\{G_{nm}^{(1)}(\mathbf{q}-\mathbf{q}',\Omega-\Omega')[\cos^2\theta_{nm}(\mathbf{q},\mathbf{q}',\Omega,\Omega')\exp\{2r_m(\mathbf{q}',\Omega')\}$$
$$+\sin^2\theta_{nm}(\mathbf{q},\mathbf{q}',\Omega,\Omega')$$
$$\times\exp\{-2r_m(\mathbf{q}',\Omega')\}]+(G^{(1)}\to G^{(2)},\cos\leftrightarrow\sin)\}. \quad (11)$$

The contribution $\sim(1-\eta)$ describes the partial reconstruction of the shot noise under conditions of nonideal photodetection. In order to determine the contribution of wave beats to the spectrum (11), we shall define the fluctuations $\delta e_n(\rho,t)$ of the field at the output as

$$\delta e_n(\rho,t)=e_n(\rho,t)-\langle e_n(\rho,t)\rangle. \quad (12)$$

In the classical description the energy of the beats is proportional to the quantity

$$\mathrm{Re}\{\delta e_n^*(\rho,t)\varepsilon_n(\rho,t)\}$$
$$=\mathrm{Re}\Big\{\sum_{m=1}^{2}\sum_{p=1}^{2}\delta b_m^*(\rho,t)R_{nm}^*(\rho,t)R_{np}(\rho,t)\beta_p\Big\}. \quad (13)$$

We shall call the effective field of the image appearing in Eq. (13) the following combination of amplitudes of the reference waves and transmission coefficients:

$$B_{nm}(\rho,t)=\sum_{p=1}^{2}R_{nm}^*(\rho,t)R_{np}(\rho,t)\beta_p. \quad (14)$$

Since the ensemble of images is homogeneous in space and time, we have

$$\overline{B_{nm}(\rho_1,t_1)\dots B_{lp}^*(\rho_k,t_k)}=\mathrm{inv}\ (\rho\to\rho+\Delta\rho,\ t\to t+\Delta t). \quad (15)$$

The correlation functions $G_{nm}^{(p)}, p=1,2$, arising in the calculation of the spectrum (11) can be written in terms of the effective field of the image as

$$G_{nm}^{(p)}(\mathbf{q},\Omega)={}^1\!/_2[G_{nm}(\mathbf{q},\Omega)-(-1)^p|(B_{nm}^2)_{\mathbf{q},\Omega}|], \quad (16)$$

where

$$G_{nm}(\mathbf{q},\Omega)={}^1\!/_2[(B_{nm}^*B_{nm})_{\mathbf{q},\Omega}+(B_{nm}B_{nm}^*)_{\mathbf{q},\Omega}]. \quad (17)$$

The cross-spectral density of the classical complex amplitudes in Eqs. (16) and (17) is defined as

$$(AB)_{\mathbf{q},\Omega}=\int d\rho\,dt\,\overline{A(0,0)B(\rho,t)}\exp\{i(\Omega t-\mathbf{q}\rho)\}. \quad (18)$$

It is shown in the Appendix that the quantities $G_{nm}^{(p)}(\mathbf{q},\Omega)$, $p=1,2$, arise when the amplitudes $B_{nm}(\rho,t)$ are expanded in quadrature components. The direction of the coordinate axes in the complex plane of the amplitude, with respect to which the expansion is made, is determined (for the coordinates 1) by the angle

$$\Phi_{nm}(\mathbf{q},\Omega)={}^1\!/_2\arg(B_{nm}^2)_{\mathbf{q},\Omega}. \quad (19)$$

The correlation functions $G_{nm}^{(p)}(\mathbf{q},\Omega)$ are proportional to the average energy of the quadratures $p=1,2$ of the effective field of the image which correspond to the frequencies $\mathbf{q}$ and Ω.

Thus the contribution of wave beats to the noise spectrum (11) is proportional to the energy of the quadrature components of the field of the image and the energy of the quadrature components of the field of the quantum fluctuations in the wide-band squeezed state and it depends on the shift of the optical phase between the two "characteristic" sets of quadrature components of the radiation

$$\theta_{nm}(\mathbf{q},\mathbf{q}',\Omega,\Omega')={}^1\!/_2\arg\{U_m(\mathbf{q}',\Omega')V_m$$
$$\times(-\mathbf{q}',-\Omega')\}-\Phi_{nm}(\mathbf{q}-\mathbf{q}',\Omega-\Omega'). \quad (20)$$

As one can see from Eqs. (11) and (20) the harmonics $\mathbf{q}$ and Ω of the photocurrent fluctuations are generated by the beats of the harmonics q', Ω' of the "squeezed" field of fluctuations with the harmonics $\mathbf{q}-\mathbf{q}'$, $\Omega-\Omega'$ of the effective field of the image. In order to suppress quantum noise in photodetection it is necessary to select sufficiently wide-band sources of light in the squeezed state in the required frequency range and also to take into account the spectral composition of the effective field of the image.

3. THE PHYSICAL POSSIBILITIES OF QUANTUM-NOISE-FREE CONTROL OF OPTICAL WAVEFRONTS

In this section we shall study methods for producing dynamical optical images without photon (shot) noise in the case of both weak and strong spatiotemporal modulation and we shall discuss the application of the phenomena of Faraday rotation of the polarization plane and birefringence for this purpose.

If the scattering coefficients of the interference mixer are weakly modulated in space and time (they deviate only slightly from their average values), then the quantum fluctuations of the photocurrent can be assumed to be independent of the weak modulation associated with the image. Neglecting the modulation, we find from the preceding relations

$$B_{nm}(\rho,t)=R_{nm}^*\varepsilon_n=\mathrm{const}\,(\rho,t),$$
$$\Phi_{nm}=\arg\varepsilon_n-\arg R_{nm}, \quad (21)$$
$$G_{nm}^{(1)}(\mathbf{q},\Omega)=(2\pi)^3\delta(\mathbf{q})\delta(\Omega)|\varepsilon_nR_{nm}^*|^2,\quad G_{nm}^{(2)}(\mathbf{q},\Omega)=0.$$

The spectrum of photocurrent fluctuations (11) assumes the form

$$(\delta i_n^2)_{\mathbf{q},\Omega}=c\eta|\varepsilon_n|^2\Big\{1-\eta+\eta\sum_{m=1}^{2}|R_{nm}|^2$$
$$\times[\cos^2\theta_{nm}(\mathbf{q},\Omega)\exp\{2r_m(\mathbf{q},\Omega)\}$$
$$+\sin^2\theta_{nm}(\mathbf{q},\Omega)\exp\{-2r_m(\mathbf{q},\Omega)\}]\Big\}. \quad (22)$$

Here

$$\theta_{nm}(\mathbf{q},\Omega)={}^1\!/_2\arg\{U_m(\mathbf{q},\Omega)V_m(-\mathbf{q},-\Omega)\}-\Phi_{nm}. \quad (23)$$

The squeezed light beams on the two inputs are statistically independent of one another. The intensity of the beats of the fluctuations of the field with the reference wave ε_n on the surface of the detector is proportional to the transmission coefficients $|R_{nm}|^2$. In order to suppress noise at some frequencies $\mathbf{q}$, Ω the phase of each of the "squeezed" light beams must be matched independently with the phase of the reference wave, so that the condition $\theta_{nm}(\mathbf{q},\Omega) = \pm\pi/2$ is satisfied for $m = 1,2$.

The one-dimensional spectrum of fluctuations $(\delta i^2)_\Omega$ of the photocurrent under conditions of stationary interference of light waves in the nonclassical state has been studied in many works. The three-dimensional frequency and spatial-frequency spectrum obtained above transforms into a one-dimensional spectrum if in Eq. (22) we pass to the limit $q\to 0$, since the temporal fluctuations of the photocurrent, summed over the detection surface, correspond to zero spatial frequency.

The symmetrical balance scheme for detecting nonclassical states of light is widely employed in experiments (see Refs. 10 and 11 and many other works), since in this scheme fluctuations of the reference wave are suppressed. In the balance scheme the spectrum of fluctuations of the difference of the photocurrents at the two outputs of the scheme is measured. We shall discuss the frequency and spatiotemporal spectrum of fluctuations in balance photodetection of light in a spatially multimodal squeezed state.

In the scheme shown in Fig. 1, where $c^2 = s^2 = 1/2$, a coherent reference wave without squeezing, $\beta_1 \neq 0$ and $r_1(\mathbf{q},\Omega) = 0$, is fed into the input $m = 1$. At the input $m = 2$ there is no reference wave, $\beta_2 = 0$, but there is spatially multimodal squeezing ("squeezed vacuum"). The spectrum of the difference of the current densities

$$\delta i_-(\boldsymbol{\rho},\ t)=\delta i_1(\boldsymbol{\rho},\ t)-\delta i_2(\boldsymbol{\rho},\ t)$$

is observed. A calculation analogous to that performed above gives

$$(\delta i_-^2)_{\mathbf{q},\Omega}=c\eta|\beta_1|^2\{1-\eta+\eta[\cos^2\theta_{12}(\mathbf{q},\ \Omega)\exp\{2r_2(\mathbf{q},\ \Omega)\} +\sin^2\theta_{12}(\mathbf{q},\ \Omega)\exp\{-2r_2(\mathbf{q},\ \Omega)\}]\}. \tag{24}$$

Fluctuations of the difference of the current densities are suppressed, if the phases of the reference wave and of the squeezed state at the frequencies of interest to us $\mathbf{q}$, Ω are matched so that the condition $\theta_{12}(\mathbf{q},\Omega) = \pm\pi/2$ is satisfied.

In the last few years a variant of the balance scheme in which a time delay is introduced at one of the outputs has been increasingly used in experiments.[14,15] The observed spectrum of fluctuations of the difference of the photocurrents in this case fluctuates between the level characteristic for the quadrature of the radiation under study separated by the reference wave at the input $m = 2$ and the Poisson level of the heterodyne noise; this is convenient for calibrating the observations. This same detection scheme can in principle also be used for balanced photodetection of spatially multimode squeezed states of light. Oscillations as a function of the frequencies $\mathbf{q}$, Ω with periods $\Delta q = 2\pi/\Delta\rho$ and $\Delta\Omega = s\pi/\Delta t$ arise in the spectrum of fluctuations of the difference of the current densities of the form

$$\delta i_1(\boldsymbol{\rho},\ t)-\delta i_2(\boldsymbol{\rho}+\Delta\boldsymbol{\rho},\ t+\Delta t),$$

where a shift is introduced in space and time.

We shall discuss the general case of high degree of spatial and temporal modulation of the scattering coefficients of the interference mixer. The results obtained will pertain not only to optical images with low quantum noise but also to only a high-degree of temporal modulation of "one-dimensional" light fluxes in a nonclassical state.

In order to achieve quantum-noise-free control of light at the output n it is sufficient to satisfy simultaneously the conditions $G^{(2)}_{nm}(\mathbf{q},\Omega) = 0$ for $m = 1,2$ in the required frequency range; see Eq. (11). The effective field of the image for both $m = 1$ and $m = 2$ will be concentrated in the quadrature 1. By controlling the phase of the "squeezed" light at the inputs the beats of the effective field of the image with "noisy" quadrature of the fluctuations can be eliminated.

The amplitude $B_{nm}(\boldsymbol{\rho},t)$ [see Eq. (14)] contains the contribution $p = m$ with a definite and constant phase, equal to $\arg(\beta_m)$, and the contribution $p\neq m$. A sufficient condition for both contributions to have the same definite and constant phase, irrespective of the spatial and temporal modulation, is

$$\arg\beta_1-\varkappa_1(\boldsymbol{\rho},\ t)=\arg\beta_2-\varkappa_2(\boldsymbol{\rho},\ t). \tag{25}$$

For an interference device the condition (25) means that the output phase increments $\varphi_1(\boldsymbol{\rho},t)$ and $\varphi_2(\boldsymbol{\rho},t)$ [see Eq. (6)] are arbitrary, while the constraint

$$\varkappa_1(\boldsymbol{\rho},\ t)-\varkappa_2(\boldsymbol{\rho},\ t)=\text{const}, \tag{26}$$

is imposed on the phase increments at the input. The latter condition is satisfied only for some physical methods of controlled interference mixing. If the condition (26) is satisfied and the difference of the phases of the reference waves is chosen on the basis of Eq. (25), then from the definition of the quadrature components of the effective field of the image (see Appendix) it follows that

$$\Phi_{nm}=\arg\beta_m,$$
$$G^{(1)}_{nm}(\mathbf{q},\Omega)=(|B_{nm}|^2)_{\mathbf{q},\Omega},\quad G^{(2)}_{nm}(\mathbf{q},\Omega)=0. \tag{27}$$

In the spectrum (11) the contribution of beats with quadrature 2 of the effective field of the image disappears. The quantum fluctuations in the optical image are suppressed in proportion to the squeezing at both inputs with optimal choice of phase of the "squeezed" input signals $m = 1,2$:

$$\theta_{nm}(\mathbf{q},\ \mathbf{q}',\ \Omega,\ \Omega')=\pm\pi/2. \tag{28}$$

It is easy to see that the matching of the phases (25) and (28) results in the suppression of noise in the two detectors simultaneously.

It is easy to illustrate these results graphically. Figure 2 shows in the plane of the quadrature components of the output amplitude $e_n(\boldsymbol{\rho},t)$ the contributions of the amplitudes of the reference waves and the regions of uncertainty of the "squeezed" fluctuations. Figure 2a pertains to the general case (there is no matching of the phases) and Fig. 2b pertains to suppression of quantum noise in the intensity. Under the conditions found above the spatial and temporal modulation of the scattering coefficients does not destroy the mutual matching of the phases of the contributions to the output field, shown in Fig. 2b.

We shall investigate the noise-free control of

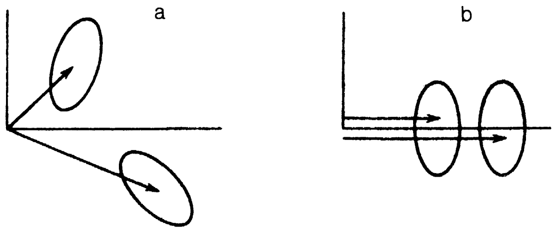

FIG. 2. Matching of the phases of the reference and "squeezed" waves arriving from two inputs on one of the detectors of the optical image: a) general case and b) fluctuations of the intensity are suppressed in proportion to squeezing.

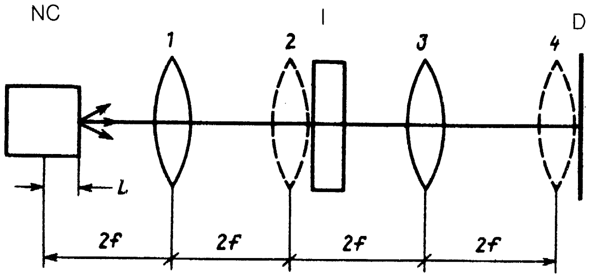

FIG. 3. Compensation of diffraction in the output layer of the nonlinear crystal and under conditions of free propagation with the help of focusing.

"squeezed" light with the help of controlled Faraday rotation or birefringence. Let the input light beams in the squeezed state be polarized linearly and orthogonal to one another. The analyzer PP_2 separates the scattered waves with the same states of polarization. The interference mixer contributes the externally controlled phase increments $\chi_1(\boldsymbol{\rho},t)$ and $\chi_2(\boldsymbol{\rho},t)$ of the waves with orthogonal circular polarizations (Faraday rotation). The transfer of the light amplitudes from the input to the output (6) arises in the form

$$\hat{R}(\boldsymbol{\rho},t) = \exp[i\chi'(\boldsymbol{\rho},t)]\begin{pmatrix} e^{i\Delta_1} & 0 \\ 0 & e^{i\Delta_2} \end{pmatrix}\begin{pmatrix} \cos\chi''(\boldsymbol{\rho},t) & \sin\chi''(\boldsymbol{\rho},t) \\ -\sin\chi''(\boldsymbol{\rho},t) & \cos\chi''(\boldsymbol{\rho},t) \end{pmatrix} \times\begin{pmatrix} e^{-i\Delta_1} & 0 \\ 0 & e^{-i\Delta_2} \end{pmatrix}, \tag{29}$$

where

$$\chi'(\boldsymbol{\rho},t)=[\chi_1(\boldsymbol{\rho},t)+\chi_2(\boldsymbol{\rho},t)]/2, \qquad \chi''(\boldsymbol{\rho},t)=[\chi_1(\boldsymbol{\rho},t)-\chi_2(\boldsymbol{\rho},t)]/2. \tag{30}$$

Here Δ_1 and Δ_2 are the constant phase shifts. If orthogonal circular polarizations are separated at the input and output and noncoincident controllable phase increments of the linearly polarized waves (birefringence) arise in the interference mixer, then a transformation of the form (29) likewise arises.

It is obvious that the condition (26) is satisfied for the transformation (29). With the help of Faraday rotation or birefringence it is possible to control the wavefronts of the "squeezed" radiation without introducing photon (shot) noise with any degree of modulation.

Simple arguments show that the largest "slope" of the control of the intensity with a small increment $\delta\chi''(\boldsymbol{\rho},t)$ to the angular parameter $\chi''(\boldsymbol{\rho},t)$ in Eq. (29) occurs when the intensities of the scattered waves at the outputs $n = 1,2$ are equal.

4. ROLE OF DIFFRACTION OF RADIATION IN THE SQUEEZED STATE

The effect of diffraction on the statistical properties of radiation in a spatially multimodal squeezed state was studied in detail in Refs. 6 and 7, where it was shown that diffraction becomes important if the mean free path of the light exceeds the parametric amplification length $l_{\rm amp}$ in the nonlinear crystal—the source of the "squeezed" light. Based on the results of these studies, we shall show that by focusing radiation in the optical scheme in Fig. 1 it is possible, first of all, to eliminate the undesirable effect of free diffraction and, second, to reduce to a minimum the area in the plane of detection of the optical image on which shot-noise-free photon collection is possible.

Let two thin lenses, centered on the optical axis ($\boldsymbol{\rho} = 0$) and having focal length f, be located at the points $x + 2f$ and $x + 4f - \delta x$, where $\delta x \to 0$. Neglecting the limited aperture of the lenses, the field is transferred from the plane x to the plane $x + 4f$ (see, for example, Ref. 16) according to the law

$$e(x+4f,\ \boldsymbol{\rho},\ t) = e(x,\ -\boldsymbol{\rho},\ t-4f/c). \tag{31}$$

Figure 3 shows one arm of an interference mixing scheme to which lenses have been added. The lenses 3 and 4 transfer the light field from the output of the interference mixer into the detection plane, introducing an unimportant delay and inversion $\boldsymbol{\rho} \to -\boldsymbol{\rho}$ in the transverse plane [see Eq. (31)]. The lenses 1 and 2 map onto the mixer some plane located in the nonlinear crystal at a distance L from the output face. In the process, the diffraction of the "squeezed" radiation accompanying propagation of the radiation not only in empty space but also in the nonlinear crystal—in the layer where the radiation is efficiently formed—is compensated. Arguing analogously to Refs. 6 and 7 it is easy to see that such focusing is described in the spectrum of fluctuations of the photocurrent by a change in the phase parameters of the wide-band squeezing $\Psi_m(\mathbf{q},\Omega)$, $\theta_{nm}(\mathbf{q},\mathbf{q}',\Omega,\Omega')$, and $\theta_{nm}(\mathbf{q},\Omega)$; see Eqs. (5), (20), and (23).

For the optimal choice of the length L, which is given in the papers cited, the rotation angle of the squeezing ellipse $\Psi_m(\mathbf{q},\Omega)$ no longer depends significantly on the spatial frequency $\mathbf{q}$. Since the squeezing phenomenon is the quantum analog of the modulation phenomenon,[17] this means that the "squeezed" radiation preserves, when the spatial frequency changes, a definite type of modulation at the level of quantum fluctuations of the field. The photon (shot) noise in the optical image, in this case, can be suppressed in the interval of spatial frequencies where squeezing is significant and strong (degenerate) parametric scattering occurs.

In conclusion we shall show that the optical scheme, shown in Fig. 3, for compensating the diffraction of the "squeezed" light can be simplified without violating the conditions, found above, for matching the phases of the incident and scattered waves. The lens 2 can be regarded as a part of an interference mixer that adds to the increments $\varkappa_1(\boldsymbol{\rho},t)$ and $\varkappa_2(\boldsymbol{\rho},t)$ to the input phases the same quantity $k_h\rho^2/2f$, in agreement with the properties of a thin lens. This phase correction is cancelled in the definition of the effective field of the image (14) and in the condition (25) for matching the phases. For this reason, the spectrum of fluctuations of the photocurrent density at the output of the mixer and [which is equivalent, by virtue of Eq. (31)] in the detection plane does not change when the lens 2 is removed. Since the lens 4, located directly in front of the detector, also gives an increment to the phase of the field and does not affect the intensity, all conclusions concerning suppression of fluctuation during photodetection of an optical image remain valid also when the lens 4 is removed.

It is not difficult to show that focusing of radiation in the spatially multimodal squeezed state in the scheme for obtaining optical images with suppression of quantum noise makes it possible not only to achieve optimal spatial resolution but also to change, to the required degree, the spatial scale of the image.

APPENDIX

For the physical interpretation of the spectrum (11) of fluctuations of the current density on photodetection of an optical image it was important that the correlation functions $G_{nm}^{(p)}(\mathbf{q},\Omega)$, where $p = 1,2$, correspond to the spectral power of the quadrature components of the effective field of the image $B_{nm}(\boldsymbol{\rho},t)$. We shall prove this explicitly. The quadrature components of the classical complex amplitude $B(\boldsymbol{\rho},t)$ are the real amplitudes of the motions along two Cartesian axes, occurring at arbitrary frequencies $\mathbf{q},\Omega$ (in this Appendix we drop the indices nm). By definition

$$B(\boldsymbol{\rho},t)=(2\pi)^{-3}\int dq\,d\Omega\exp[i\Phi(\mathbf{q},\Omega)]\{[B_c^{(1)}(\mathbf{q},\Omega)+iB_c^{(2)}(\mathbf{q},\Omega)]\cos(\Omega t-\mathbf{q}\boldsymbol{\rho})+[B_s^{(1)}(\mathbf{q},\Omega)+iB_s^{(2)}(\mathbf{q},\Omega)]\sin(\Omega t-\mathbf{q}\boldsymbol{\rho})\}.$$

Here the integration is performed over the half-space $(\mathbf{q},\Omega)$, for example, for $\Omega>0$ and arbitrary q. The indices c and s distinguish the slow motions according to the laws $\cos(\Omega t-\mathbf{q}\boldsymbol{\rho})$ and $\sin(\Omega t-\mathbf{q}\boldsymbol{\rho})$ and the upper indices 1 and 2 indicate the direction of motion in the complex plane, i.e., the fast (optical) phase, equal to $\Phi(\mathbf{q},\Omega)$ or $\Phi(\mathbf{q},\Omega)+\pi/2$. The phase angle $\Phi(\mathbf{q},\Omega)$ is not specified for the time being.

The quadratures are related as follows to the standard Fourier amplitudes $B(\mathbf{q},\Omega)$:

$$B(\mathbf{q},\Omega)\exp[-i\Phi(\mathbf{q},\Omega)]={}^1/_2\{B_c^{(1)}(\mathbf{q},\Omega)+iB_s^{(1)}(\mathbf{q},\Omega)+i[B_c^{(2)}(\mathbf{q},\Omega)+iB_s^{(2)}(\mathbf{q},\Omega)]\},$$

$$B^*(-\mathbf{q},-\Omega)\exp[i\Phi(\mathbf{q},\Omega)]={}^1/_2\{B_c^{(1)}(\mathbf{q},\Omega)+iB_s^{(1)}(\mathbf{q},\Omega)-i[B_c^{(2)}(\mathbf{q},\Omega)+iB_s^{(2)}(\mathbf{q},\Omega)]\}.$$

The spectral density of the form (18), in terms of which the correlation functions $G^{(p)}(\mathbf{q},\Omega)$ are written, in turn is related with the average Fourier amplitudes by the relation

$$(2\pi)^3\delta(\mathbf{p})\delta(\omega)(AB)_{\mathbf{q},\Omega}=\overline{A(\mathbf{p}/2-\mathbf{q},\ \omega/2-\Omega)B(\mathbf{p}/2+\mathbf{q},\ \omega/2+\Omega)},$$

which we shall write conventionally as

$$(AB)_{q,\Omega}\propto\overline{A(-\mathbf{q},-\Omega)B(\mathbf{q},\Omega)}.$$

We now choose the characteristic optical phase $\Phi(\mathbf{q},\Omega)$ in the definition of the quadratures from the condition

$$\Phi(\mathbf{q},\Omega)={}^1/_2\arg(B^2)_{\mathbf{q},\Omega}.$$

Using the relations derived above it is easy to relate the correlation functions (16) and (17) with the spectral power of the quadrature components of the effective field of the image:

$$G^{(p)}(\mathbf{q},\Omega)\propto{}^1/_4\{\overline{[B_c^{(p)}(\mathbf{q},\Omega)]^2}+\overline{[B_s^{(p)}(\mathbf{q},\Omega)]^2}\},\quad p=1,2,$$

$$G(\mathbf{q},\Omega)=\sum_{p=1}^{2}G^{(p)}(\mathbf{q},\Omega).$$

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