Dynamics of the propagation and interaction of electromagnetic pulses in two-level media

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The dynamics of the propagation and interaction of electromagnetic pulses in nonlinear media, when current generation can be described on the basis of a two-level quantum system (Maxwell-Bloch equations), is investigated. The collisional properties of stationary "half-wave" solutions are investigated for the case of an absorbing two-level medium.[9] It is shown that in the limit of long or short wave packets that are wide or narrow, compared with ω_0^{-1}, where ω_0 is the transition frequency, these solutions have soliton properties. In the case of an amplifying medium a new class of nonstationary nonlinear solutions in the form of wave packets, whose frequency shifts into the blue region of the spectrum as they are amplified, is found. The conditions under which the nonlinear optics of a two-level medium can be used to describe the field electrodynamics of superconducting (including high-T_c) planar structures are determined. The concept of "population inversion" of a Josephson superconducting structure is introduced on the basis of this analogy and an interpretation of the new concept is given.

1. INTRODUCTION

The development of methods for producing and shaping pulses of light of length $\sim 10^{-15}$ s has greatly increased interest in the physics of the propagation of short powerful wave packets in linear and nonlinear media. A general feature of the wave processes occurring here is that they cannot be described with the help of the apparatus of traditional nonlinear optics—the method of slowly varying amplitudes and phases (SVAP), which operates with quasimonochromatic fields. The large width of the spectrum of a pulse is, however, not the only factor limiting the applicability of the SVAP method. In the strong fields of an ultranarrow pulse the basic assumption of traditional nonlinear optics that the medium is weakly nonlinear and strongly dispersive breaks down. This assumption made it possible to confine attention to a finite number of nonlinearly interacting waves: on the one hand, the number of terms in the expansion of the polarization in powers of the field now becomes significant, while on the other hand in strong fields the condition of phase matching can be satisfied simultaneously for all harmonics.[1,2] These circumstances imply that an adequate description of wave processes on the femtosecond time scale is possible (and, as it happens, convenient) only in terms of the real field and real polarization induced by it.[3,4]

The character of the interaction of a light pulse with the medium depends on both the parameters of the pulse itself (spectrum and field intensity) and on the structure of the quantum levels of the material, and in the general case it can be very complicated. For this reason it is of interest to study the nonlinear dynamics of intense ultrashort electromagnetic pulses for the example of simple quantum systems, such as, for example, a two-level system, which reveals at least qualitatively the physics of the interaction of a powerful ultrashort electromagnetic pulse with the medium in the more general case also. Maxwell's equations together with the equations describing the interaction of the field with a medium consisting of two-level particles form the system of Maxwell-Bloch equations (MB).

At the present time the linear and nonlinear properties of the "truncated" MB equations, which are obtained from the exact equations in the approximation of resonant interaction of quasimonochromatic radiation with the medium and are formulated in terms of the SVAP of the field and the material variables of the medium, have been studied in greatest detail.[5] Phenomena such as self-induced transparency of $2\pi n$ pulses in absorbing media, π-pulse formation in amplifying media, and a number of other phenomena are described on the basis of the "truncated" MB equations (see the reviews Refs. 6, 7, and 8).

There are significantly fewer works concerning the exact MB equations. We call attention to Ref. 9, in which solutions in the form of solitary waves were found for the MB equations. This stimulated interest in the MB system from the viewpoint of the possibility of integrating it by the inverse scattering method.[10,11] In particular, in Ref. 10 it was shown that by means of the inverse scattering method an approximation of the MB system (the reduced MB equations) rather than the exact system can be integrated for the case of a medium with low density. We also call attention to Ref. 12, where the soliton properties of the solutions found in Ref. 9 were investigated by integrating numerically the exact equations and it was shown that the indicated solutions are not solitons, since they do not have the property of elastic scattering.

The purpose of this work is to investigate analytically and numerically the properties of solutions of the exact MB equations for the cases of both absorbing and amplifying media. By studying the collisions of pulses with a wide range of initial characteristics it was possible to find the range of parameters for which the interaction is of a quasisoliton character (Sec. 2). In the case of an amplifying medium (Sec. 3) a new class of nonstationary nonlinear solutions was found in the form of wave packets which shift into the blue region of the spectrum. The energy of these packets increases not as a result of the increase in the number of photons per pulse but rather as a result of the increase in the energy of each photon as the pulse propagates in the medium.

In Sec. 4 an analogy is drawn between the propagation of a pulse in a two-level medium and the evolution of the electromagnetic field in planar superconducting Josephson structures. This analogy makes it possible to transfer under appropriate conditions the results of Secs. 2 and 3 to the electrodynamics of layered high-T_c superconductors.

2. ABSORBING TWO-LAYER MEDIUM: QUASISOLITON BEHAVIOR OF THE EXACT SOLUTIONS OF THE MB EQUATIONS

We shall study the propagation of an electromagnetic plane wave with electric field intensity $\mathscr{E}$ in a medium of two-level particles, having transition frequency ω_0 and dipole moment μ. The self-consistent system of equations describing the propagation of a field pulse along the z axis (MB equations) includes the equations for the material variables of the medium

$$\frac{\partial p}{\partial t}=i\omega_0 p+i\frac{2\mu}{\hbar}\mathscr{E}n, \tag{1a}$$

$$\frac{\partial n}{\partial t}=-\frac{2\mu}{\hbar}\mathscr{E}\,\mathrm{Im}\,p, \tag{1b}$$

and the wave equation

$$\frac{\partial^2\mathscr{E}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\mathscr{E}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2}. \tag{2}$$

Here $p=2\rho_{12}$ is the polarization, $n=\rho_{22}-\rho_{11}$ is the difference of the populations of the levels of a separate particle, and ρ_{ij} is the density matrix of the two-level system. The macroscopic polarization of the medium P, appearing in the wave equation (2), is connected with the polarization of a separate particle p by the relation

$$P=N\mu\,\mathrm{Re}\,p, \tag{3}$$

where N is the particle number density. In the material equations (1) we dropped the relaxation terms, making the assumption that the pulses studied are significantly shorter than the longitudinal T_1 and transverse T_2 relaxation times of the medium.

In this section we shall study the properties of an absorbing medium, i.e., we shall assume that before the pulsed field appears (at $t=-\infty$) all particles of the medium are in the bottom level: $\rho_{11}(-\infty)=1$, $\rho_{22}(-\infty)=\rho_{12}(-\infty)=0$.

For the system of equations (1) and (2) we can introduce the dimensionless parameter

$$\alpha=8\pi\mu^2 N/\hbar\omega_0, \tag{4}$$

which characterizes the relation between the field of the electromagnetic wave and the two-level medium.[10] For low densities of the medium, such that $\alpha\ll 1$, the backscattered part of the pulse $\mathscr{E}(z,t)$ is negligibly small (in this case in the wave equation $P\ll\mathscr{E}$), and the wave equation (2) can be reduced to an equation describing the propagation of a pulse in one direction.[10]

We shall study the case $\alpha\sim 1$, when the presence of the reflected wave can significantly affect the evolution of the pulse and makes it necessary to investigate the complete system. According to Ref. 9, Eqs. (1) and (2) have exact solutions in the form of bell-shaped pulses:

$$\mathscr{E}(z,t)=\frac{\hbar}{\mu\tau_p}\,\mathrm{sech}\left\{\frac{t-z/v}{\tau_p}\right\}, \tag{5}$$

where τ_p is the parameter of the solution which determines the pulse width, and v is the velocity of the pulse and is connected with τ_p by the relation

$$\frac{1}{v^2}=\frac{1}{c^2}\left[1+\frac{\alpha(\omega_0\tau_p)^2}{1+(\omega_0\tau_p)^2}\right]. \tag{6}$$

Before investigating numerically a collision of the pulses (5), it is useful to study the analytical properties of the system of equations (1) and (2) in some limiting cases. Two parameters characterizing the interaction of an electromagnetic pulse with a medium can be identified in the material equations (1). The first parameter is the ratio of the characteristic time scale τ_p of the variation of the field in the pulse to the period ω_0^{-1} of the characteristic oscillations of the two-level system. We shall call a pulse long if $\tau_p\omega_0\gg 1$ and, correspondingly, short if $\tau_p\omega_0\ll 1$. It is convenient to characterize the magnitude of the field of the pulse by the parameter $\mathscr{E}/\mathscr{E}_0$, where $\mathscr{E}_0=\hbar\omega_0/2\mu$ is the saturation field strength of the two-level system.

1. We first study the interaction of a long pulse, $\tau_p\omega_0\gg 1$, with the medium. Since the material equations (1) in which the transition frequency ω_0 is replaced by detuning from resonance $\Delta\omega$ and the field $\mathscr{E}$ is replaced by the wave amplitude E become identical to the truncated equations of resonant interaction,[5] the case which we are studying is analogous to the adiabatic interaction of radiation with a medium when the polarization can adjust to the instantaneous value of the electromagnetic pulse. Following the method proposed for this case in Ref. 13, we shall represent the formal solution of Eqs. (1) written in the integral form

$$p(t)=-i\frac{2\mu}{\hbar}\int_0^{\infty}\mathscr{E}(t-s)\,n(t-s)\exp(i\omega_0 s)\,ds, \tag{7}$$

as a series in the small parameter $(\tau_p\omega_0)^{-1}$:

$$p(t)=-\frac{2\mu}{\hbar\omega_0}\sum_{n=0}^{\infty}\left(\frac{i}{\omega_0}\right)^n\frac{d^n}{dt^n}[\mathscr{E}(t)\,n(t)]. \tag{8}$$

In the linear case, when the change in the populations can be neglected ($n=-1$), according to Eq. (8) the polarization of the medium can be represented in the form

$$P=\frac{2\mu^2}{\hbar\omega_0}N\left(1-\frac{1}{\omega_0^2}\frac{d^2}{dt^2}-\ldots\right)\mathscr{E}. \tag{9}$$

The differential operator on the right-hand side of the expression (9) describes the linear dispersion of a two-level medium in the low-frequency limit $\omega\ll\omega_0$. This can be easily verified by calculating the response at the frequency ω corresponding to the polarization (9) which is simply the expansion of the linear susceptibility of the medium

$$\chi_L(\omega)=2N\mu^2\omega_0/\hbar(\omega_0^2-\omega^2)$$

in powers of ω/ω_0.

When the nonlinearity is taken into account the simultaneous solution of Eq. (8) with the equation for the populations (1b) gives terms in the expansion which are analogous to the terms obtained in Ref. 13 and which describe the saturation of the transition. These expressions, however, can be simplified for the following reason. From Eq. (5) for the exact solution of the MB equations it follows that for

$\tau_p\omega_0 \gg 1$ the maximum value of the field of the pulse satisfies $\mathscr{E}_{max} \sim \hbar/\mu\tau_p \ll \mathscr{E}_0$, i.e., far from the saturating field. Thus the first nonlinear correction to the linear polarization is proportional to $\mathscr{E}(\mathscr{E}/\mathscr{E}_0)^2$. Retaining in the polarization the first dispersive and the first nonlinear terms of the expansion we obtain

$$P=\frac{2\mu^2}{\hbar\omega_0}N\left[\mathscr{E}-\frac{1}{2}\mathscr{E}\left(\frac{\mathscr{E}}{\mathscr{E}_0}\right)^2-\frac{1}{\omega_0^2}\frac{\partial^2\mathscr{E}}{\partial t^2}\right]. \qquad (10)$$

The smallness of the nonlinear and dispersion corrections to the polarization makes it possible to go from the wave equation (2) to an equation describing the propagation of a pulse in one direction. This equation will have the following form:

$$\frac{\partial\mathscr{E}}{\partial z}+\frac{1}{v_0}\frac{\partial\mathscr{E}}{\partial t}+c_1\mathscr{E}^2\frac{\partial\mathscr{E}}{\partial t}-c_2\frac{\partial^3\mathscr{E}}{\partial t^3}=0, \qquad (11)$$

where

$$v_0=c[1+4\pi\chi_L(0)]^{-1/2},\quad c_1=6\pi c^{-2}v_0\chi_{NL}(0),$$
$$c_2=\pi c^{-2}v_0[\partial^2\chi_L/\partial\omega^2]_{\omega=0},\quad \chi_{NL}(0)=-{}^1/_2\mu N\mathscr{E}_0^{-3}.$$

Equation (11) is the modified Korteweig–de Vries equation, which has soliton solutions. The simplest one-soliton solution of Eq. (11) in the form of a hyperbolic secant (see, for example, Ref. 14) converges to Eq. (5) in the limit $\tau_p\omega_0 \gg 1$.

2. We now consider the case of the interaction of a short pulse, $\tau_p\omega_0 \ll 1$, with the medium. Since the pulse is shorter than the time interval ω_0^{-1} during which the response of the medium is established, the polarization at a given time is determined by the value of the field at all preceding times. In addition, it follows from the exact solution (5) that for a short pulse the intensity of the field at the maximum is significantly higher than the saturation intensity $\mathscr{E}_0$. Thus in the case at hand the medium is strongly nonlinear and strongly dispersive and it is impossible to make an expansion in some parameter. It can be seen, however, that for $\tau_p\omega_0 \ll 1$ the material equations can be solved for any form of the pulse $\mathscr{E}$. Neglecting at first in Eq. (1a) the term $\omega_0 p$ compared with $\partial p/\partial t$ we find that

$$\operatorname{Im} p=\sin\Psi(t),\quad n=-\cos\Psi(t), \qquad (12)$$

where

$$\Psi(t)=\frac{2\mu}{\hbar}\int_{-\infty}^{t}\mathscr{E}(t')\,dt',$$

is the phase of rotation of the material variables. Hence, according to Eq. (1a), we obtain for the real part of the polarization

$$\operatorname{Re} p=-\omega_0\int_{-\infty}^{t}dt'\sin\Psi(t'). \qquad (13)$$

Substituting Eq. (13) into the wave equation (2) we obtain

$$\frac{\partial^2\mathscr{E}}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\mathscr{E}}{\partial t^2}=\frac{4\pi N\mu\omega_0}{c^2}\frac{\partial}{\partial t}\sin\frac{2\mu}{\hbar}\int_{-\infty}^{t}\mathscr{E}(t')\,dt'. \qquad (14)$$

It is easy to see that Eq. (14) can be rewritten for the function Ψ in the form

$$\frac{\partial^2\Psi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}=\alpha\left(\frac{\omega_0}{c}\right)^2\sin\Psi, \qquad (15)$$

the sine-Gordon equation, which has soliton-like solutions.[14] The simplest one-soliton solution of Eq. (15) is identical to Eq. (5) in the limit $\omega_0\tau_p \ll 1$.

Thus in both limiting cases studied above—a long pulse (the case of a weakly nonlinear and weakly dispersive medium) and a short pulse (the case of a strongly nonlinear and strongly dispersive medium)—the system of MB equations reduces to equations which have soliton-like solutions. It can thus be expected that the exact solutions (5) in these cases have soliton properties.

In order to investigate the properties of the exact solutions we investigated numerically the collision of pulses (5) in a wide range of pulse lengths. In the calculations the behavior of the material variables of the medium was given in a self-consistent manner with the profile of the electric field:

$$p(z,t)=\frac{2}{1+(\omega_0\tau_p)^2}\operatorname{sech}\zeta\,(\omega_0\tau_p+i\,\operatorname{th}\zeta),$$
$$n(z,t)=-1+\frac{2}{1+(\omega_0\tau_p)^2}\operatorname{sech}^2\zeta,\quad \zeta=\left(t-\frac{z}{v}\right)\tau_p^{-1}. \qquad (16)$$

The distance between the colliding pulses was chosen from the condition that the results of a collision should not depend on the initial spatial separation of the pulses.

The most characteristic computational results are presented in Figs. 1–3. It was established, first of all, that the exact solutions of the MB equations are not solitons in the strict sense. In the region of pulse lengths τ_p of the order of the period of the characteristic oscillations ω_0^{-1} of the two-level system a collision of the pulses results in distortion of the initial profile of the pulses and energy loss (Fig. 1). An oscillatory structure of the field characteristically forms after a collision. In the case of a collision of pulses with dif-

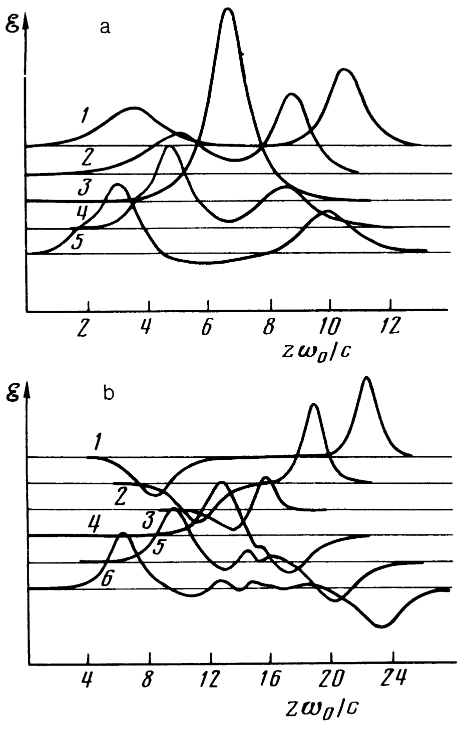

FIG. 1. The evolution of the spatial distribution of the field intensity $\mathscr{E}(z, t)$ of colliding pulses having the same (a) and opposite polarities (b) with $\tau_{p1}=2\omega_0^{-1}$, $\tau_{p2}=\omega_0^{-1}$, and $\alpha=4$: a) $\omega_0 t=0$ (1), 3.4 (2), 6.8 (3), 10.2 (4), and 13.6 (5); b) $\omega_0(t)=0$ (1), 5.6 (2), 11.2 (3), 16.9 (4), 22.4 (5), and 28.0 (6).

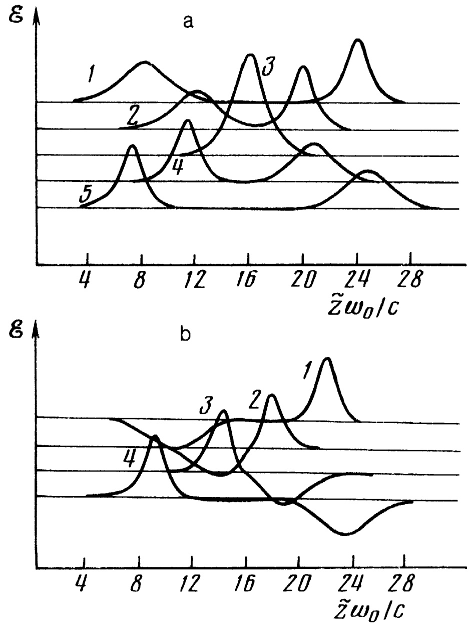

FIG. 2. The evolution of the spatial distribution of the field intensity $\mathscr{E}(z, t)$ of colliding pulses having the same (a) and opposite (b) polarities with $\tau_{p1} = 7\omega_0^{-1}$, $\tau_{p2} = 4\omega_0^{-1}$, and $\alpha = 4$ [$\tilde{z} = z - (v_1 + v_2)t/2$]: $\omega_0(t) = 0$ (1), 1500 (2), 3000 (3), 4500 (4), and 6000 (5).

ferent polarity, the decay of the pulses accelerates. These results agree with the numerical calculations performed in Ref. 12.

At the same time it was found that when the lengths of the colliding pulses are three to four times greater or less than $\tau_p \approx \omega_0^{-1}$ the "inelastic" effects are significantly weaker. Figure 2 shows the results of calculations of a collision of longer pulses ($\tau_{p1} = 7\omega_0^{-1}$ and $\tau_{p2} = 4\omega_0^{-1}$). It should be noted that because the medium is excited only weakly in the field of long pulses, the accumulated effects of

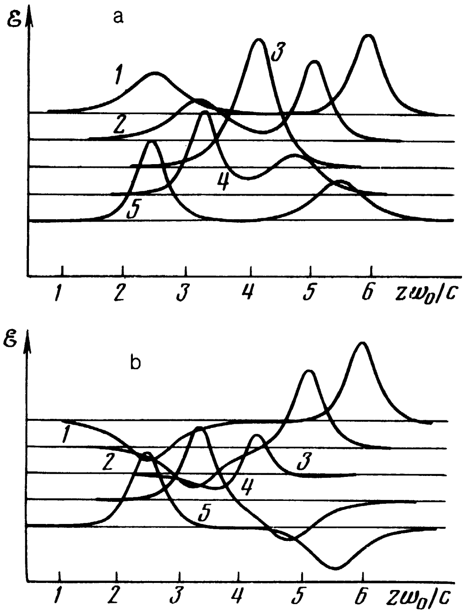

FIG. 3. The evolution of the spatial distribution of the field intensity $\mathscr{E}(z, t)$ of colliding pulses having the same (a) and opposite (b) polarities with $\tau_{p1} = 0.4\omega_0^{-1}$, $\tau_{p2} = 0.2\omega_0^{-1}$, and $\alpha = 4$: $\omega_0(t) = 0$ (1), 0.8 (2), 1.6 (3), 2.4 (4), and 3.2 (5).

the nonlinear interaction of the pulses are realized only when they propagate in tandem. One can see that the dynamics of a collision in this case is of a practically elastic character (Fig. 2). Figure 3 shows the results for the collision of relatively short pulses ($\tau_{p1} = 0.4\omega_0^{-1}$ and $\tau_{p2} = 0.2\omega_0^{-1}$). The calculations show that in this case also the interaction occurs essentially elastically, and in addition for both tandem and head-on collisions of pulses of both polarities.

Summarizing our analysis of the collisional properties of the exact solutions of the MB equations we can draw the conclusion that these solutions have a quasisoliton character in two ranges of pulse lengths: $\tau_p \gtrsim 3\omega_0^{-1}$ and $\tau_p \lesssim 0.3\omega_0^{-1}$. In the long-pulse limit $\tau_p \gg \omega_0^{-1}$ a collision of the solitons (5) can be described quite accurately by the Korteweg–de Vries equation (11) while in the short-pulse limit $\tau_p \ll \omega_0^{-1}$ the collision is described by the sine-Gordon equation (15).

3. AMPLIFYING TWO-LEVEL MEDIUM: NONSTATIONARY SOLUTIONS OF THE MB EQUATIONS IN THE FORM OF WAVE PACKETS SHIFTED INTO THE BLUE REGION OF THE SPECTRUM

In this section we shall study the characteristics of the propagation of an electromagnetic pulse in a medium of inverted two-level particles. Before the arrival of the pulse all particles are in the upper level and in Eqs. (1) we must set $\rho_{22}(-\infty) = 1$, $\rho_{11}(-\infty) = \rho_{12}(-\infty) = 0$.

The propagation of short laser pulses in amplifying two-level media is studied, as a rule, for the case when the field of the pulse is a quasimonochromatic wave in resonance with the transition frequency and the motion of the pulse is described in the SVAP approximation.[7,8,15,16] In a long amplifier a pulse whose envelope $E(z, t)$ behaves in a self-similar manner

$$E(z,t) \propto zF\left(z\left(t-\frac{z}{c}\right)\right), \tag{17}$$

where $F(x)$ is a sign-alternating function that decays as $|x| \to \infty$, is formed. The pulse described by such a solution consists of successive subpulses, the structure of each of which is close to that of a 2π pulse of self-induced transparency, and in addition the total area of the entire pulse is

$$\Psi(z,\infty) = \frac{\mu}{\hbar}\int_{-\infty}^{\infty} E(z,t)\,dt = \pi$$

(the so-called π-pulse of amplification). As it propagates through the medium a π-pulse removes all energy stored in the medium. The energy of the pulse increases in proportion to the traversed path z, and since the area remains constant the time scale of the variation of the envelope of the pulse decreases as $\propto z$. Over long amplification lengths the pulse length τ_p becomes of the order of the period of the characteristic oscillations of the two-level system ω_0^{-1} and the SVAP approximation can no longer be used to describe the dynamics of amplification of the pulse. Thus analysis of coherent amplification of a quasimonochromatic field in a two-level medium leads in a natural manner to the investigation of the exact MB equations.

Before analyzing the numerical results, we shall discuss some analytical properties of the solutions of the MB equations in the case of an amplifying medium, which are manifested on a long propagation paths. We assume (and we shall give a proof below) that in the process of amplification from arbitrary initial conditions the time scale of the variation of the field of the pulse becomes less than the period of the characteristic oscillations of the system ω_0^{-1}. Under this condition in Eqs. (1) and (2), and as was done in Sec. 2, the term $\omega_0 p$ can be neglected compared with $\partial p/\partial t$, and after the equations are solved we arrive at the equation

$$\frac{\partial^2\Psi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}=-\alpha\left(\frac{\omega_0}{c}\right)^2\sin\Psi, \qquad (18)$$

which differs from Eq. (15) by the minus sign on the right-hand side. The transformation of variables $\xi = t + z/c$ and $\eta = t - z/c$ transforms Eq. (18) into the following form:

$$\frac{\partial^2\Psi}{\partial\xi\partial\eta}=\Omega^2\sin\Psi, \quad \Omega^2=\frac{1}{4}\alpha\omega_0^2. \qquad (19)$$

It is not difficult to see that Eq. (19) agrees to within the notation with the fact that it describes the evolution of the envelope of a pulse in a resonantly amplifying medium [in the latter case the phase of rotation Ψ of the material variables is determined by the envelope of the pulse while in our case it is determined by the real field $\mathscr{E}$; see Eq. (12)]. Equation (19) has a solution with the self-similar variable $u = \Omega^2\xi\eta$.[16] Here $\Psi(u)$ is given by the equation

$$u\Psi''+\Psi-\sin\Psi=0, \qquad (20)$$

and the field intensity is connected with the function Ψ by the relation

$$\mathscr{E}(z,t)=\frac{\hbar\Omega^2}{\mu}t\Psi'(u). \qquad (21)$$

Equation (20) has solutions that are regular at $u = 0$.[15,16] These solutions are such that $\Psi(u)$ is a sign-alternating oscillating function of the wave-packet type, not equal to zero in the neighborhood of $u = 0$. For solutions propagating in the positive direction along the z-axis, i.e., setting $\eta \approx 0$, the expression (21) can be represented in the form

$$\mathscr{E}(z,t)=\frac{\hbar\Omega^2}{c\mu}z\Psi'\left(\frac{2\Omega^2}{c}z\left(t-\frac{z}{c}\right)\right). \qquad (22)$$

The evolution of the field intensity $\mathscr{E}$ of the pulse is mathematically very similar to that of the envelope E of the pulse. The field (22) consists of a sequence of subpulses with areas of the order of $\pm 2\pi$. The total area of the subpulses $\Psi(z, \infty)$ is conserved and is equal to π. The field removes all energy stored in the medium. However for conditions of propagation when the effect can be described only in terms of the total intensity and polarization the physics of the evolution of the field is significantly different. In the first case (envelope model) as it propagates in the medium the pulse is compressed but its frequency does not change. In the second case, when the pulse is quite short and powerful and the motion of the pulse cannot be described in the envelope model the pulse is not only compressed but its frequency, which according to Eq. (22) is equal

$$\omega(z)\approx\frac{2\Omega^2}{c}z, \qquad (23)$$

also changes and shifts into the blue region of the spectrum in proportion to the path length traversed by the pulse.

Then, as in the first case, the energy of the field grows as

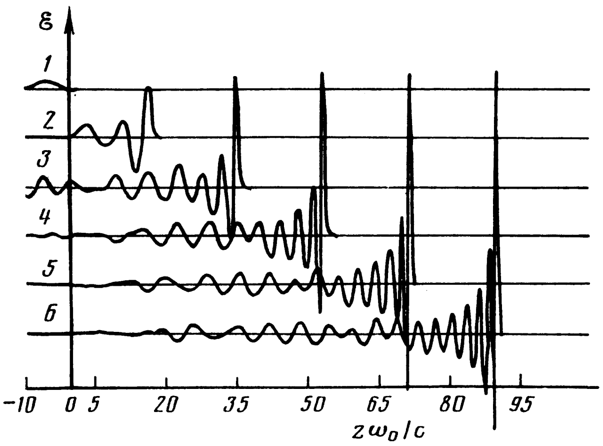

FIG. 4. The evolution of the field intensity of a pulse in an amplifying two-level medium ($\alpha = 0.5$): 1) pulse incident from vacuum on the half-space $z > 0$ filled with the active medium [$\omega_0(t) = 0$]; 2) $\omega_0(t) = 18.1$; 3) 36.2; 4) 54.3; 5) 72.4; 6) 90.5. The field in the region $z < 0$ at times $t > 0$ corresponds to a wave emitted by the active region into vacuum.

a result of the addition to the pulse of photons having the same frequency as the pulse. In the second case the number of photons in the pulse does not change; the energy of the pulse increases as a result of the addition to the instantaneous energy of the photons of the field an energy $\hbar\omega_0$, emitted by a particle in an induced transition between levels.

Figure 4 shows the typical results of the numerical investigation of the dynamics of the propagation of a pulse in an amplifying medium. The amplified pulse was incident from the vacuum on an inverted medium, filling the half-space $z > 0$. Irrespective of the initial profile of the amplified pulse, two stages can be traced in the dynamics of amplification. At the starting stage the spectral components of the pulse which are close to resonance with the characteristic transition frequency ω_0 are predominately amplified and the pulse is transformed into a wave packet whose average frequency is approximately equal to ω_0 (Fig. 4, $t = 18.1\,\omega_0^{-1}$). In the process of amplification the envelope of the wave packet is compressed in width, and the amplitude of the field in the envelope grows in accordance with the theory of coherent amplification in the SVAP approximation. When the time scale of the variation of the envelope becomes of the order of ω_0^{-1} (in this case the field of the pulse at maximum is $\mathscr{E}_{max} \sim \mathscr{E}_0$), the amplification process enters a qualitatively new stage. The spatial and temporal scale of not only the envelope but also the entire structure of the wave packet is reduced and at the same time the amplitude of the wave packet increases (see Fig. 4, $t \gtrsim 54.3\omega_0^{-1}$). The amplification of the pulse is essentially accompanied by an increase of the average frequency of the pulse. The dynamics of the "increasingly bluer" wave packet at this stage is close to self-similar and is described by the formula (22): The frequency and energy of the pulse increase in proportion to the distance traversed in the amplifying medium.

4. THE MB EQUATIONS AS A MODEL OF PROPATION OF AN ELECTROMAGNETIC PULSE IN PLANAR SUPERCONDUCTING STRUCTURES

We shall study the propagation of a electromagnetic pulse in a planar Josephson structure and in layered high-T_c superconductors, for example, crystals of the form

$Y_1Ba_2Cu_3O_{6.9}$, which essentially consist of a collection of parallel Josephson contacts with superconductors of atomic thickness.

In the first case the field propagates in the layer $|x| < d/2$ of a dielectric with thickness $d \sim 10$–50 Å. The region $|x| > d/2$ is filled with the superconducting medium. The vector $\vec{\mathscr{E}}(z,t) = \mathbf{n}\mathscr{E}(z,t)$ of the field intensity is directed along the normal to the planes at $x = \pm d/2$.

The field-induced Josephson current

$$\mathbf{j} = \mathbf{n} j_c \sin\varphi \tag{24}$$

and the free-energy density in the d-layer

$$F_v = \frac{\hbar j_c}{2ed}\,\frac{1-\cos\varphi}{2} \tag{25}$$

are determined by the phase difference

$$\varphi = \frac{2ed}{\hbar}\int_{-\infty}^{t} \mathscr{E}(z,t')\,dt' + \varphi_0 \tag{26}$$

of the wave functions of the edges of the planar structure and the critical current density j_c.[17]

The wave equation, containing the current (24), for $\varphi(z,t)$ is the sine-Gordon equation (15), and for this reason everything said above about the dynamics of an ultrashort pulse in a two-level medium should also be true here. The dynamics of the field in a superconducting structure can thus be described on the basis of the nontruncated MB equations (1) and (2) for particles with density N, transition frequency ω_0, and free-energy density[1)]

$$F_v = N\hbar\omega_0\,\frac{1-\cos\varphi}{2}.$$

When the field $\mathscr{E}(z,t)$ varies over the characteristic time scale τ_p the frequency ω_0 must satisfy the condition $\omega_0 \ll \tau_p^{-1}$, after which the particle density N will be determined by equating of the free energies:

$$\frac{\hbar j_c}{2ed} = N\hbar\omega_0. \tag{27}$$

We now note that the initial value of the phase φ_0 in Eq. (26) in the y direction, perpendicular to the direction z of propagation of the wave and to the polarization vector $\mathbf{n}$ of the field, can be changed, for example, with the help of a constant magnetic field H_z or a constant current j_y, which spatially modulate the wave functions of the superconducting electrons.[17] Then the range of values of y where $\varphi_0(y) \approx 0$ will correspond to the case of a noninverted medium and the region of y where $\varphi_0(y) \approx \pi$ will correspond to the case when the particles are in the upper level of the transition.

We shall now discuss the electrodynamics of layered high-T_c superconductors, which crystallize in the form of superconducting planes (with thickness $l \sim 2$–3 Å), separated by layers of dielectric ($d \sim 15$–20 Å). If the Landau-Ginzberg equation is taken here for the starting material equations

$$\frac{\hbar^2}{2m^*}\left[\left(-i\frac{\partial}{\partial\rho} - \frac{2e}{\hbar c}\mathbf{A}\right)^2 - |a| + |b|\,|\psi_n|\right]\psi_n$$

$$+|\varkappa|\,[2\psi_n - \psi_{n+1}\exp(-i\chi_n) - \psi_{n-1}\exp(i\chi_n)] = 0, \tag{28}$$

$$\chi_n = \frac{2e}{\hbar c}(\mathbf{An})d,$$

to treat the interaction of the wave functions of neighboring superconducting layers, then the current density between the nth and $(n+1)$st layers will be determined by the relation (see, for example, Refs. 18 and 19):

$$j \propto \psi_n\psi_{n+1}^*\exp(i\chi_n) - \psi_n^*\psi_{n+1}\exp(-i\chi_n). \tag{29}$$

In Eqs. (28) and (29) $\psi_n(\rho, t)$ is the order parameter of the nth superconducting layer with the coordinates $\rho = (x, y)$, $\mathbf{A}$ is the vector potential of the field of the pulse, and $\varkappa$ is a constant characterizing the coupling of the ψ_n functions of neighboring layers. The constants a and b determine the stationary value of the order parameter ψ_0 and the coherence length ξ_0 of the superconducting electrons:

$$\psi_0 = (|a|/|b|)^{1/2}, \quad \xi_0^2 = \hbar^2/2m^*|a|.$$

The quantity ξ_0 is given here as the characteristic distance over which a small perturbation $\delta\psi$ of the wave functions of the layers $\psi_n = \psi_0 + \delta\psi$ decays. In a layered superconductor, however, the coherence length can also be defined as the characteristic distance over which the functions

$$\psi_n = \psi_0 + \delta\psi\exp(in\pi)$$

decay to the value ψ_0—an idea that is impossible for a massive superconductor. This dependence of ψ_n on n corresponds to a lesser degree of modulation of the order parameter of the layers and a decrease of both the kinetic energy of the electrons of the superconductor and the coherence length ξ $[\xi^2 = \xi_0^2|a|/(|a| + 2|\varkappa|)]$.

We shall now discuss briefly the critical temperature of a superconductor. We are inclined to believe that, at least qualitatively, the temperature-dependent coherence length in the Ginzberg-Landau theory is related to the critical temperature T_c of the superconductor by the relation from the BCS theory, where these quantities are inversely proportional to one another. In this connection, $T_c \propto 1/\xi$ for the functions $\psi_n = \psi_0 + \delta\psi\exp(i\pi n)$ can significantly exceed $T_c \propto 1/\xi_0$ (according to, for example, Ref. 19 the constant $2|\varkappa|$ in ξ^2 can be appreciably greater than the constant $|a|$; see also Ref. 20).

The relations (24) and (26), which describe the electrodynamics, already studied above, of a pulse propagating in a planar Josephson structure or, under corresponding conditions, in a medium of two-level particles, follow from Eq. (29). We note that the experimental results on the reflection and scattering of light from a series of high-T_c crystals can be interpreted in a manner so that the coupling energy E_c of Cooper electron pairs in the x direction lies in the infrared frequency range.[2] It follows that at least up to pulse lengths $\tau_p \sim \hbar/E_c$ the electrodynamic processes in a high-T_c superconductor (reflection of a pulse, transmission, amplification, or decay in the linear and nonlinear in $\mathscr{E}$ cases) can be obtained from the well-known solutions of the MB equations.

CONCLUSIONS

In this paper we studied the propagation of electromagnetic pulses in nonlinear media for the example of an absorbing or amplifying medium of two-level particles which can be described by the MB equations. It was shown that for the case of an absorbing medium the solution of these equations in the form of stationary bell-shaped pulses in the limit of both large and small (compared with the period of characteristic oscillations of the two-level medium) pulse lengths have soliton properties. Thus the equations of nonlinear optics admit the existence and stable propagation of new wave objects—unipolar pulses, which by analogy to Cherenkov bipolar pulses (see Ref. 1) can be called half-wavelength pulses.

In the case of an amplifying medium we found a new class of nonstationary nonlinear solutions in the form of wave packets, whose energy increases not as a result of an increase in the number of photons per pulse but rather as a result of an increase in the energy of each photon as the pulse propagates in the medium.

An analogy was drawn between the propagation of a pulse in two-level media and the evolution of the electromagnetic field in planar superconducting (including high-T_c) Josephson structures. It was shown that the transition from equations governing a two-level medium to the equations of electrodynamics of a Josephson structure corresponds in the MB equations to the limit $\omega_0 \to 0$, $N \to \infty$, and $\omega_0 N = \text{const}$. On the basis of this analogy we introduced the concept of "inversion of a Josephson structure" and gave an interpretation of it. Thus the results on the propagation of pulses in absorbing and amplifying two-level media can be transferred to the electrodynamics of the field in superconductors.

In conclusion we shall briefly discuss the expression for the current induced by a medium of two-level particles in the case $\tau_p \ll \omega_0^{-1}$. According to Eq. (13), this current is equal to

$$j = \frac{\partial P}{\partial t} = j_c \sin \Psi, \quad j_c = -N\mu\omega_0. \tag{30}$$

The Josephson-type current (30) is unique in that it is multiplied by the frequency. For this reason it is interesting to indicate the efficiency with which harmonics are generated by the current which is excited by the field $\mathscr{E}(t) = E_0 \cos(\omega t - kz)$. The condition under which the formulas (30) are applicable for a periodic field evidently reduces to the requirement $\omega \gg \omega_0$ Then it follows from Eq. (30) that

$$j(t) = j_c \sin\left[\frac{2\mu}{\hbar\omega} E_0 \sin(\omega t - kz)\right] = j_c \sum_n J_n\left(\frac{2\mu E_0}{\hbar\omega_0}\right) \sin[n(\omega t - kz)]. \tag{31}$$

In contradistinction to the case of generation of harmonics under the conditions of applicability of the SVAP method, when the accumulated interactions can be realized for a fixed number of waves (usually not exceeding two or three), the current (31) can excite simultaneously $\sim 10^2$–10^3 harmonics with comparable amplitudes.[17] This follows because the coefficients in the expansion of j in harmonics are Bessel functions $J_n(s)$, and for certain values of the parameters n and $s = 2\mu E_0/\hbar\omega$ they decay very slowly. For example for $s \approx n \gg 1$, $J_n(s) \approx 0.67 n^{-1/3}$.

We shall now estimate the conditions under which a Josephson current is obtained in the optical range. Setting $\omega \sim 10^{15}$ rad/s, $\omega_0 \sim 10^{14}$ rad/s, and $\mu \approx 5 \cdot 10^{-18}$ cgs units, we obtain from the formula (31) that for radiation fluxes $I \sim 10^{13}$ W/cm^2 efficient generation of the tenth harmonic of the neodymium laser, i.e., radiation with photons $\hbar\omega \sim 10$ eV, becomes possible.

In the case of an amplifying two-level medium the characteristic distance z at which the frequency of the photons in a π-pulse increases by an amount equal to the transition frequency ω_0 is estimated from the condition

$$z \sim \frac{c\hbar}{4\pi N \mu^2}.$$

For $N \approx 10^{17}$ cm^{-3}, $\mu \approx 5 \cdot 10^{-18}$ cgs units, and $\omega_0 \sim 10^{14}$ rad/s this condition gives $z \sim 1$ cm. The intensity of a π pulse in this case is $I > \hbar^2\omega^2 c/4\pi\mu^2 \sim 10^{11}$ W/cm^2.

[1)] For this definition of F_v (or, which is the same thing, for the definition of a population inversion with the help of the equality $n = -\cos\varphi$) the angles $\varphi = 0$ and π correspond, respectively, to a noninverted medium and a completely inverted medium of particles.

[1] S. A. Akhmanov, V. A. Vysloukh, and A. S. Chirkin, *Optics of Femtosecond Laser Pulses* [in Russian], Nauka, Moscow (1988).
[2] S. A. Akhmanov and R. V. Khokhlov, *Problems of Nonlinear Optics*, Gordon and Breach, New York (1972).
[3] E. M. Belenov, P. G. Kryukov, A. V. Nazarkin *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 442 (1988) [JETP Lett. **47**, 523 (1988)].
[4] E. M. Belenov and A. V. Nazarkin, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 252 (1990) [JETP Lett. **47**, 523 (1990)].
[5] L. Allen and J. Eberly, *Optical Resonance and Two-Level Atoms*, Wiley, N.Y. 1975.
[6] I. A. Poluektov, Yu. M. Popov, and V. S. Roĭtberg, Usp. Fiz. Nauk **114**, 97 (1974) [Sov. Phys. Usp. **17**(5), 673 (1975)].
[7] P. G. Kryukov and V. S. Letokhov, Usp. Fiz. Nauk **99**, 169 (1969) [Sov. Phys. Usp. **12**(5), 641 (1970)].
[8] E. M. Belenov, P. G. Kryukov, A. V. Nazarkin *et al.*, J. Soc. Am. B **5**, 946 (1988).
[9] R. K. Bullough and F. Ahmad, Phys. Rev. Lett. **27**, 330 (1971).
[10] J. C. Eilbeck, J. D. Gibbon, P. J. Caudrey *et al.*, J. Phys. A **6**, 1337 (1973).
[11] A. I. Maĭmistov, Kvantovaya Elektron. **10**, 360 (1983) [Sov. J. Quantum Electron. **13**, 198 (1983)].
[12] P. J. Caudrey and J. C. Eilbeck, Phys. Lett. A **62**, 65 (1977).
[13] M. D Crisp, Phys. Rev. A **1**, 1604 (1970).
[14] S. P. Novikov, V. E. Zakharov, S. V. Manakov, and L. P. Pitaevskiĭ, *Theory of Solitons*, Consultants Bureau, New York, 1984.
[15] S. V. Manakov, Zh. Eksp. Teor. Fiz. **83**, 68 (1982) [Sov. Phys. JETP **56**, 37 (1982)].
[16] G. L. Lamb, Rev. Mod. Phys. **43**, 99 (1971).
[17] A. Barone and J. Paterno, *Physics and Applications of the Josephson Effect*, Wiley, New York, 1981.
[18] W. E. Lowrence and S. Doniach, *Proceedings of the 12th Conference on Low Temperature Physics (LT-12)*, Kyoto (1970), p. 361.
[19] L. N. Bulaevskii, Usp. Fiz. Nauk **116**, 449 (1975) [Sov. Phys. Usp. **18**, 514 (1975)]. L. N. Bulaevskiĭ, V. L. Ginzburg, and A. A. Subyanin, Zh. Eksp. Teor. Fiz. **94**, 355 (1988) [Sov. Phys. JETP **68**(1), 1499 (1988)].
[20] D. H. Lowndes, D. P. Norton, and J. D. Budai, Phys. Rev. Lett. **65**, 1160 (1990).
[21] E. V. Abel', V. S. Bagaev, D. N. Basov *et al.*, Kratkie soobshcheniya po fizike **8**, 31 (1989).

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