Stimulated scattering of sound in weakly inhomogeneous media

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Induced scattering of sound in a medium with smooth inhomogeneities is investigated. It is shown that the most effective mechanism is a unique two-step scattering channel inactive in scattering of light and not considered in previously studied mechanisms of sound scattering. The scattering thresholds and the amplification of a weak scattered wave are obtained.

1. INTRODUCTION

Nonlinear interaction of acoustic waves with inelastic perturbations of a medium have been far less investigated than the sound–sound interaction familiar in nonlinear acoustics. Attention has been heretofore paid mostly to self-focusing[1] and stimulated scattering of sound,[2-4] which are analogous to self-focusing and stimulated temperature scattering of light.[5,6] Most investigated effects are analyzed by a procedure analogous to that used for propagation of light.

At the same time, effects relatively easy to observe in light waves are difficult to produce in sound waves, and some have not been produced to this day. Additional difficulties encountered in the acoustic case are due to the weak dispersion of sound waves, which leads to an abundance of competing nonlinear processes. For example, weakly stimulated scattering of sound can hardly compete with zero-threshold generation of harmonics.[4] At high intensities, however, propagation of the pump wave leads to generation of large-scale acoustic flows that disturb the resonant character of the scattering and greatly reduces its effectiveness.

We propose here a mechanism for effective interaction of acoustic waves with inelastic perturbations produced in a medium whose thermodynamic parameters vary smoothly. We show that if the characteristic length of the inhomogeneity (of the sound refraction) is less than the damping length, the sound is scattered most effectively by a two-step mechanism having no analog in light waves. During the first step, the interaction of the incident and of the scattered sound waves with the initial inhomogeneity of the medium excites a rotational wave which excites in turn, by convection, a scattering grating (e.g., a thermal wave). The resonant character of the interaction during the two steps makes the excited grating much larger than in one-step mechanisms.

It must be emphasized that in all the cases considered below the inhomogeneity of the medium is weak ($L_{\rm in} k \gg 1$), and the total temperature change in the entire interaction region is relatively small ($\delta T/T \ll 1$). As a result, first, in the linear approximation the sound is only refracted and its amplitude changes gradually; second, the resonance condition is not violated in the entire interaction region. Yet the stimulated-scattering mechanism described below is effective enough, even under these conditions, to cause significant growth of the scattered wave.

2. BASIC EQUATIONS

For the sake of clarity, we consider a medium described by the standard hydrodynamic equations

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0, \tag{1}$$

$$\rho \frac{\partial u_i}{\partial t} + \rho (\mathbf{u}, \nabla) u_i = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - {}^2/_3 \delta_{ij} \operatorname{div} \mathbf{u}\right)\right], \tag{2}$$

$$\rho T \frac{\partial S}{\partial t} + \rho T (\mathbf{u}, \nabla) S = \operatorname{div}(\varkappa \nabla T) + q \tag{3}$$

and by an equation of state of general form. All the results can be easily reformulated to cover more complicated cases. Here ρ, T, P, $\mathbf{u}$, and S are respectively the density, temperature, pressure, velocity, and entropy per unit mass, $\varkappa$ and η are the thermal-conductivity and dynamic-viscosity coefficients, and q represents the acting heat sources and sinks.

Consider sound propagation in a medium enclosed in a waveguide having rigid heat-conducting walls. The medium is made inhomogeneous either by bulk heat sources or by keeping opposite waveguide walls at different temperatures. In the undisturbed state the medium is immobile and the pressure in it is constant. The distribution of the temperature, density, and entropy is stationary, and their gradients are connected by the linear relations

$$\frac{\nabla T}{T} = \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_p \frac{\nabla \rho}{\rho} = \frac{\nabla S}{C_p},$$

where C_p is the isobaric heat capacity per unit mass.

We assume that the characteristic length $L_{\rm in} = T/|\nabla T|$ of the inhomogeneity exceeds the wavelength of the sound, but is much shorter than the bulk damping length.

The dynamics of the high-frequency (acoustic) and low-frequency (elastic) perturbations of the medium are determined by different processes. The relative values of the individual terms in Eqs. (1)–(3) differ correspondingly. The main perturbations in sound waves occur in the pressure, density, and the potential part of the velocity. The entropy and vorticity perturbations are relatively small. Dissipation produces acoustic boundary layers and relatively strong sound damping.[7] Outside these layers, however, dissipation plays a minor role compared with linear refraction and with effects due to nonlinear interaction. The dissipative terms in the equations for the sound waves can therefore be neglected, and the wall damping can be taken into account separately in the determination of the scattering threshold. In inelastic quasistationary waves, on the contrary, the entropy and vorticity are strongly perturbed. Allowance for the dissipative processes is therefore fundamental, since it is they which limit these perturbations. The pressure perturba-

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tion is relatively small and requires only insignificant nonlinear corrections to the sound dispersion. These corrections will be disregarded below.

In view of the foregoing, it is expedient to separate in the system (1)–(3) the high-frequency (ω) and the low-frequency (Ω) parts. For the acoustic perturbations we obtain

$$-c^{-2}\frac{\partial^2 \tilde{P}_\omega}{\partial t^2}+\Delta\tilde{P}_\omega =\rho^{-1}\nabla\tilde{P}_\omega\nabla\tilde{\rho}_\Omega+\frac{\partial^2}{\partial t^2}[\tilde{P}_\omega(\widetilde{c^{-2}})_\Omega]-2\rho\frac{\partial^2 f_{\Omega i}u_{\omega j}}{\partial x_i\,\partial x_j}, \tag{4}$$

$$\tilde{\rho}_\omega=\frac{\tilde{P}_\omega}{\rho c^2},\qquad \frac{\partial \mathbf{u}_\omega}{\partial t}=-\frac{1}{\rho}\nabla\tilde{P}_\omega,\qquad \frac{\partial\tilde{S}_\omega}{\partial t}=-\mathbf{u}_\omega\nabla\tilde{S}_\Omega,$$

$$\frac{\partial \mathbf{w}_\omega}{\partial t}=\mathrm{rot}[\mathbf{u}_\omega,\tilde{\mathbf{w}}_\Omega]+[\nabla\tilde{P}_\omega,\nabla\tilde{V}_\Omega], \tag{5}$$

$$\left.\frac{\partial\tilde{P}_\omega}{\partial \mathbf{n}}\right|_G=0. \tag{6}$$

Here the tilde marks a deviation from a homogeneous equilibrium value, $\tilde{\mathbf{w}}_\omega=\mathrm{rot}\,\tilde{\mathbf{u}}_\omega$ is the vorticity, c is the speed of sound, $V=1/\rho$ is the specific volume, and $\tilde{\mathbf{f}}_\Omega=V(\rho,\tilde{\mathbf{u}})_\Omega$ (for low-frequency perturbations it is convenient to use the mass-transport velocity directly).

The three terms in the right-hand side of (4) correspond to different scattering channel. The first accounts for the transport of the entropy wave by the sound waves, the second is due to modulation of the sound velocity in the quasistationary wave, and the third describes scattering by the vortex waves.

The equations for the low-frequency perturbations are

$$\frac{\partial\tilde{\mathbf{w}}_\Omega}{\partial t}-\nu\Delta\tilde{\mathbf{w}}_\Omega=-\mathrm{rot}\left(\tilde{V}_\Omega\nabla\left(\tilde{P}_\Omega-\tilde{P}_R+\frac{\rho\tilde{\mathbf{u}}_\omega^2}{2}\right)+\frac{(\widetilde{\rho c^2})_\Omega}{2\rho^3c^4}\nabla\tilde{P}_\omega^2\right), \tag{7}$$

$$\frac{\partial\tilde{S}_\Omega}{\partial t}-\chi\Delta\tilde{S}_\Omega=-\tilde{\mathbf{f}}_\Omega\nabla\tilde{S}_\Omega-\mathbf{u}_\omega\nabla\tilde{S}_\omega, \tag{8}$$

$$\mathrm{div}\,\tilde{\mathbf{f}}_\Omega=0,\qquad \Delta(\tilde{P}_\Omega-\tilde{P}_R)=0, \tag{9}$$

$$\tilde{\mathbf{f}}_\Omega|_G=0,\qquad \tilde{S}_\Omega|_G=0,$$

where

$$\tilde{P}_R=\tilde{P}_\omega^2/2\rho c^2-\rho\tilde{\mathbf{u}}_\omega^2/2$$

is the sound radiation pressure;[8] χ and ν are the thermal-diffusivity and kinematic-viscosity coefficients. We have also left out the terms describing the previously investigated quasi-stationary-wave buildup mechanisms which are parallel to the investigated one.

The size of the vorticity source [right-hand side of Eq. (7)] calls for special notice. To obtain its correct value it is necessary to take into account not only the radiation-pressure gradient but also the transport of weak intrinsic vorticity in the sound waves, and the third-order linear terms. The system (4)–(10) is closed by adding the continuity equation and the equation of state.

3. WAVE INTERACTION

We use Eqs. (4)–(10) to determine the gain of a weak scattered wave in the given-pump approximation. In this case we can neglect the self-action of the weak quasistationary waves. We assume in addition that the pump intensity is low enough to be able to neglect the large-scale flows it excites.

The modes of a plane waveguide can be represented as combinations of plane waves. We start therefore by considering the interaction of three waves: a strong acoustic pump wave "1" of frequency ω_1, propagating in the direction $\mathbf{n}_1$, a weak scattered sound wave "2" having a close frequency ω_2 and propagating parallel to $\mathbf{n}_2$, and a quasistationary scattering interference wave "3" having a wave vector $\mathbf{k}_3\approx(\mathbf{n}_1+\mathbf{n}_2)\omega/c$. The sound-wave interaction that excites wave "3" is small to the extent that the initial gradient $\nabla T/Tk$ is small. A major role in its formation is played by small violations of the adiabatic and potential (irrotational) character of the sound waves. A noticeable effect is produced by the nonstationary character of the scattering wave (the smallness of the quantities $\nu k_3^2,\chi k_3^2$, $\omega_1-\omega_2=\Omega$ compared with the sound frequency).

The ratios of the oscillation amplitudes of the different parameters in the sound waves have the standard geometric-acoustics forms.[9]

For wave "1" with amplitude a_1, the perturbations are $\tilde{P}_1=a_1\rho c^2$ for the pressure, $\tilde{\rho}_1=a_1\rho$ for the density, $\tilde{\mathbf{v}}_1=a_1\mathbf{n}_1c$ for the velocity, $\tilde{S}_1=-ia_1(\mathbf{n}_1\nabla S)c/\omega_1$ for the entropy, and $\mathbf{w}_1=a_1[\mathbf{n}_1,\nabla\rho]c/\rho$ for the vorticity. All the quantities are proportional to

$$\exp\left[-i\omega_1t+\int i\frac{\omega_1}{c}(\mathbf{n}_1\,dx)\right].$$

The relations for wave "2" are obtained by replacing the subscript "1" by "2." Account must also be taken of the nonlinear contribution made to the perturbation by the transport of the entropy component of wave "3" to wave "1" and responsible for one of the scattering channels [see Eq. (4)]:

$$\tilde{S}_2=-ia_2(\mathbf{n}_2\nabla S)c/\omega_2-(1-\mathbf{n}_1\mathbf{n}_2)a_1\tilde{S}_3.$$

Except where necessary, the subscripts of $\omega_{1,2}\approx\omega$ are omitted below.

Consider the steady-state picture of the interaction. Neglect of the influence of the boundary conditions on a quasi-stationary wave, permissible when the waveguide transverse dimension is significantly larger than $1/k_3$, makes Eqs. (7) and (8) algebraic in the perturbation amplitudes of the entropy and vorticity in the quasistationary wave "3":

$$\tilde{S}_3=\frac{a_1a_2^*\mathbf{n}_1\mathbf{n}_2c((\mathbf{n}_1+\mathbf{n}_2)\nabla S)}{\chi k_3^2-i\Omega}-\frac{ic(\nabla S,(\mathbf{n}_1-\mathbf{n}_2),\tilde{\mathbf{w}}_3)}{2(1-\mathbf{n}_1\mathbf{n}_2)\omega(\chi k_3^2-i\Omega)}, \tag{11}$$

$$\tilde{\mathbf{w}}_3=\frac{i\omega a_1a_2^*c[(\mathbf{n}_1-\mathbf{n}_2)\nabla\ln\rho]}{\nu k_3^2-i\Omega}\left(1-\mathbf{n}_1\mathbf{n}_2+\left(\frac{\partial\ln c^2}{\partial\ln\rho}\right)_p\right). \tag{12}$$

For the energy density $\tilde{\varepsilon}_2=|\tilde{P}_2|^2/\rho c^2$ in wave "2" we obtain the equation

$$\mathrm{div}(c\mathbf{n}_2\tilde{\varepsilon}_2)=(\Gamma_{N1}+\Gamma_{N2})c\tilde{\varepsilon}_2, \tag{13}$$

where Γ_{N1} is the nonlinear growth rate connected with single-step scattering channels

$$\Gamma_{N1}=\mathbf{n}_1\mathbf{n}_2\left(1-\mathbf{n}_1\mathbf{n}_2+\left(\frac{\partial \ln c^2}{\partial \ln \rho}\right)_p\right)$$

$$\times|a_1|^2\frac{\omega\Omega(\nu^2-\chi^2)k_3^4((\mathbf{n}_1-\mathbf{n}_2)\nabla \ln \rho)}{(\Omega^2+\chi^2k_3^4)(\Omega^2+\nu^2k_3^4)}, \qquad (14)$$

and Γ_{N2} is the nonlinear growth rate connected with the two-step channel:

$$\Gamma_{N2}=\left(1-\mathbf{n}_1\mathbf{n}_2+\left(\frac{\partial \ln c^2}{\partial \ln \rho}\right)_p\right)^2$$

$$\times|a_1|^2\frac{\omega\Omega(\nu+\chi)k_3^2[(\mathbf{n}_1-\mathbf{n}_2)\nabla \ln \rho]^2c}{(\Omega^2+\chi^2k_3^4)(\Omega^2+\nu^2k_3^4)}. \qquad (15)$$

Single-step scattering channels, in which the scattering wave is excited directly by interaction of an incident and a scattered wave, is described by the first terms in expressions (11) and (12). The only difference between this scattering mechanism and those studied earlier[2-4] is that inelastic scattering waves are excited not by damping but by refraction of sound waves. If, however, the bulk damping length L_3 exceeds the characteristic inhomogeneity length L_{in}, the scattering is mainly in two steps: the incident and scattered sound waves excite a rotational wave, the resultant convection excites an entropy wave, and the interaction between the latter and the incident wave amplifies the scattered acoustic wave. This channel is effective because resonant excitation of a quasistationary wave takes place in each step, so that each step enhances the effect. As a result, the characteristic value of the nonlinear growth rate is $(L_3/L_{\text{in}})^2$ times larger than the growth rate in the preceding scattering mechanisms.

The above hydrodynamic system has only one two-step scattering channel, because there is only one source of inhomogeneity. If, however, in general there are several such sources and accordingly additional degrees of freedom, additional two-step scattering channels appear. On the whole, the presence of such channels is one of the interesting features of this scattering.

Let us investigate in greater detail the possibility of observing scattering in the steady state. To be specific, we assume that the thermal-diffusivity is of the same order or smaller than the kinematic viscosity, as is the case in gases and liquids. The estimates that follow can obviously be generalized to other cases.

4. LONGITUDINAL INHOMOGENEITY IN WAVEGUIDE

If the parameters of the medium vary only along the waveguide, its modes constitute combinations of two plane waves whose transverse wave-vector components are oppositely directed and the longitudinal components are equal and vary smoothly along the waveguide.

To be specific, we assume henceforth that the pump wave is the zeroth mode of the waveguide (a plane wave propagating along its axis). The results for the other cases are essentially the same.

If the waveguide width exceeds the sound wavelength, we obtain directly from (13)

$$\frac{\partial}{\partial z}(\cos\theta c\tilde{\varepsilon}_2)=\Gamma_N c\tilde{\varepsilon}_2, \qquad (16)$$

where z is the propagation direction of the incident wave, 2θ is the angle between the components of the scattered wave, $k_3=2\omega(1-\cos\theta)/c$, and

$$\Gamma_N=\left(1-\cos\theta+\left(\frac{\partial \ln c^2}{\partial \ln \rho}\right)_p\right)^2$$

$$\times\frac{|a_1|^2c\omega\Omega(\nu+\chi)k_3^2|\nabla \ln \rho|^2\sin^2\theta}{(\Omega^2+\chi^2k_3^4)(\Omega^2+\nu^2k_3^4)}. \qquad (17)$$

The mutual influence of the scattered-wave components is small enough to be neglected. Expressions (16) and (17) are not valid for angles $\theta\approx\pi/2$ and $\theta\approx 0$.

The effective gain length of the scattered wave is limited in this case by the inhomogeneity length, so that the condition for observing scattering at angles $\theta\approx 1$ takes the form

$$|a_1|^2>10^2LT^2\nu\chi\omega^3/(\delta T)^2c^5\approx(\chi\omega/c^2)[10^2L_H/L_3]. \qquad (18)$$

Here L is the waveguide length, T is the temperature difference between its ends, and the observation condition is assumed to be $\Gamma_N L>10$.

The situation is unusual because the gain increases and the threshold is lowered when the waveguide is shortened. Under the approximations assumed, the lower bound of the waveguide length is of the order of the sound wavelength. The threshold intensity in this scattering mechanism is lower than in the preceding cases by a factor $\sim 10^3\lambda/L_3$ [the coefficient in the square brackets of (18)]. This factor is smaller than 0.1 for air at $\lambda>2$ cm ($\omega<10^5$ Hz), and for water at $\lambda>3\cdot 10^{-2}$ cm ($\omega<3\cdot 10^7$ Hz). This excess above threshold can be of fundamental significance, since the suppression of the scattering by acoustic flow provides an upper bound on the permissible amplitude of the incident wave.[10]

5. TRANSVERSE INHOMOGENEITY IN WAVEGUIDE

Producing a temperature drop in the transverse direction has the advantage that the resonant-interaction length along the waveguide axis is not limited in this case by the inhomogeneity length. This further increases the scattering efficiency.

Consider backscattering of a sound wave. If the waveguide is wider than the wavelength of sound, reflection is the most rapidly evolving scattering process. When, however, the waveguide width becomes less than half the wavelength, reflection is the only possible resonant scattering process.

Given the temperature difference between the walls, the gain of the scattered wave increases with decrease of the waveguide width. If the latter becomes less than the wavelength, account must be taken of the effect of the walls on the quasistationary wave.

Assume that the waveguide is planar and of width $2H$, with the x axis directed across the waveguide, $-H\leqslant x\leqslant H$. Then

$$\tilde{T}_0=\delta T\frac{x}{H},$$

where $\delta T=T_{+H}-T_{-H}\ll T$ is the temperature difference between the center and the wall. The equations describing the change of the perturbations across the waveguide and the smooth variation of the sound waves along the waveguide axis are

Pump wave:

$$2ik\frac{\partial a_1}{\partial z}+\Delta_\perp(a_1)=2k^2a_1\tilde{\tau}_0\left(\frac{\partial \ln c}{\partial \ln T}\right)_p, \qquad (19)$$

$$\tilde{\tau}_0=\frac{T_0}{T},\quad \mathbf{n}_1=\mathbf{n}_z,\quad k=\frac{\omega}{c},\quad \left.\frac{\partial a_1}{\partial x}\right|_G=0,$$

Reflected wave:

$$-2ik\frac{\partial a_2}{\partial z}+\Delta_\perp(a_2)$$
$$=2k^2a_2\tilde{\tau}_0\left(\frac{\partial \ln c}{\partial \ln T}\right)_p+2a_1k^2\left[\frac{\tilde{S}_3}{\rho c C_p}\left(\frac{\partial(\rho c)}{\partial \ln T}\right)_p-\frac{\tilde{f}_{3z}}{c}\right],$$

$$\mathbf{n}_2=-\mathbf{n}_z,\quad \left.\frac{\partial a_2}{\partial x}\right|_G=0. \tag{20}$$

Quasistationary wave:

$$(4\nu k^2-i\Omega)\tilde{\mathbf{w}}_3-\nu\Delta_\perp(\tilde{\mathbf{w}}_3)=-i\frac{4kc}{\rho}a_1a_2^*[\nabla(\rho c),\mathbf{n}_z], \tag{21}$$

$$(4\chi k^2-i\Omega)\tilde{S}_3-\chi\Delta_\perp(\tilde{S}_3)=-C_p(\tilde{\mathbf{f}}_3\nabla\tilde{\tau}_0), \tag{22}$$

$$\operatorname{div}\tilde{\mathbf{f}}_3=0,\quad \tilde{S}_3|_G=\tilde{\mathbf{f}}_3|_G=0. \tag{23}$$

The equations for the sound waves can be solved by perturbation theory, using the analogy between the wave equation in the parabolic approximation and the Schrödinger equation.[11] The right-hand sides of these equations can be regarded here as the result of the action of a linear operator L on the corresponding function a. The unperturbed basis will be a set of functions φ_n which are solutions of the equation

$$E_n\varphi_n+\Delta_\perp(\varphi_n)=0$$

with boundary conditions

$$\left.\frac{\partial\varphi_n}{\partial \mathbf{n}}\right|_G=0.$$

In the zeroth approximation the parameters of the sound are homogeneous across the waveguide. The first-order correction to the wave number is determined by the diagonal matrix element of the operator L. To calculate it we must accordingly determine the quasistationary waves for sound amplitudes that are independent of the transverse coordinates, and then average them over the cross section.

Taking all the above into account, we obtain the correction to the wave vector of the scattered wave:

$$\delta k_2=-\frac{(\delta T)^2c^2|a_1^2|}{2\nu\chi k T^2}\left(\frac{\partial\ln(\rho c)}{\partial\ln T}\right)^2\mathfrak{F}. \tag{24}$$

Here $\mathfrak{F}$ is the entropy-scattering form factor. The rotational waves make no contribution to the sound reflection. The form factor $\mathfrak{F}$ depends on the method used to make the temperature inhomogeneous.

If the inhomogeneity is due to the difference between the wall temperatures, then

$$\mathfrak{F}=\frac{1}{\mathcal{H}^2\mu^2\sigma^2}\left[1-\frac{\operatorname{th}\mu\mathcal{H}}{\mu\mathcal{H}}-\frac{\operatorname{th}\sigma\mathcal{H}}{\sigma\operatorname{th}\sigma\mathcal{H}-\operatorname{th}\mathcal{H}}\frac{\mu^2}{\mu^2-1}\right.$$
$$\times\left(\frac{\operatorname{th}\mathcal{H}}{\mathcal{H}}-\frac{\operatorname{th}\mu\mathcal{H}}{\mu\mathcal{H}}\right)$$
$$\left.+\frac{\operatorname{th}\mathcal{H}}{\sigma\operatorname{th}\sigma\mathcal{H}-\operatorname{th}\mathcal{H}}\frac{\mu^2}{\mu^2-\sigma^2}\left(\frac{\operatorname{th}\sigma\mathcal{H}}{\sigma\mathcal{H}}-\frac{\operatorname{th}\mu\mathcal{H}}{\mu\mathcal{H}}\right)\right]. \tag{25}$$

Here

$$\mu^2=1+s,\quad \sigma^2=1+\chi s/\nu,\quad s=i\Omega/(4\chi k^2),\quad \mathcal{H}=2kH.$$

When the cavity width is large ($\mathcal{H}\to\infty$) the expression in the square brackets tends to unity. In the opposite limit $\mathcal{H}\to 0$ we have $\mathfrak{F}(s=0)\to\mathcal{H}^4/105$.

The gain of the reflected wave is equal to the imaginary part of the correction δk_2 to the wave vector. The gain is an odd function of the detuning of the reflected wave from the incident one. The gain reaches a maximum at a certain value of this detuning. The characteristic value of the optimal detuning is determined by the damping of the quasistationary waves. $\Omega_{\rm opt}\approx 4\chi k^2$ if $\mathcal{H}>1$ and $\Omega_{\rm opt}\approx\chi/H^2$ if $\mathcal{H}<1$.

The dependence of the scattering increment on the waveguide width is determined by the experimental conditions. If the temperature difference and the pump amplitude are small, the waveguide width can be made optimal. This width and the corresponding value of the form factor are practically independent of the properties of the medium.

For air, for example, $\nu/\chi=0.73$, the increment is a maximum at $\mathcal{H}_0=2.9$ and $\mathfrak{F}(\mathcal{H}_0)=2\cdot 10^{-2}$, while for water $\nu/\chi=7$ we have $\mathcal{H}_0=2.8$, and $\mathfrak{F}(\mathcal{H}_0)=1.9\cdot 10^{-2}$. At larger temperature differences and pump amplitudes, account must be taken of the restrictions imposed by the maximum permissible distortions of the incident wave:

$$\alpha=\delta T\left(\frac{\partial\ln c}{\partial\ln T}\right)_p\mathcal{H}^2(T\pi^2)^{-1}<\alpha_{\rm np}, \tag{26}$$

and by the conditions for neglecting the excited acoustic flows:

$$ku_a\approx\alpha\mathcal{H}c\,|a|^2_{\rm lim}(kc/\nu)^{1/2}<\begin{cases}4\chi k^2, & \mathcal{H}>1,\\ \chi/H^2, & \mathcal{H}<1.\end{cases} \tag{27}$$

This shifts the optimum into the region of small $\mathcal{H}$. The increment limits can then reach the value of the sound wave vector.

The only process that competes with scattering when sound propagates in a narrow waveguide is damping of the sound at the walls.[7] The condition for exceeding the threshold is

$$|a|^2>|a|^2_{\rm thr}\approx\frac{\chi k}{c}\left[\left(\frac{\partial\ln(\rho c)}{\partial\ln T}\right)_p^{-2}\mathcal{H}T^2(\delta T)^{-2}\mathfrak{F}^{-1}\left(\frac{\nu k}{c}\right)^{1/2}\right]. \tag{28}$$

Scattering is possible under the condition $|a|^2_{\rm lim}>|a|^2_{\rm thr}$. This corresponds to an upper bound on the sound frequency:

$$k<\frac{\pi^2\delta T c}{\nu T\mathcal{H}^2}\left(\frac{\partial\ln(\rho c)}{\partial\ln T}\right)_p^2\mathfrak{F}. \tag{29}$$

For example, for $\delta T/T\leqslant 0.1$, $\mathcal{H}=\mathcal{H}_0$, $\alpha_{\rm lim}=0.1$ the condition (29) yields for air $\omega<\omega_{\rm lim}=3\cdot 10^6$ Hz ($\lambda>5\cdot 10^{-2}$ cm) and for water $\omega<\omega_{\rm lim}=2\cdot 10^{10}$ Hz ($\lambda>4\cdot 10^{-3}$ cm). When these conditions are met, the permissible excess above threshold is $|a|^2_{\rm thr}\omega_{\rm lim}/\omega$.

By way of illustration, we present estimates of the possible growth rates and threshold intensities for scattering of sound of wavelength $\lambda=1$ cm.

Using the data for the corresponding quantities[12] and expressions (24) and (28), we obtain

the threshold pump amplitude:

for water—$a_{\rm thr}\approx 2.2\cdot 10^{-7}$, for air—$a_{\rm thr}\approx 4\cdot 10^{-4}$;

the threshold intensity:

for water—$I_{\rm thr}\approx 1.6\cdot 10^{-5}$ W/cm^2, for air—$I_{thr} \approx 7 \cdot 10^{-4}$ W/cm^2;

the scattering length at near-threshold amplitudes:
for water—$L_N \approx 185$ cm, for air—$L_N \approx 23$ cm;

the scattering length at nearly limiting amplitudes:
for water—$L_N \approx 2.3$ cm, for air—$L_N < 1$ cm.

The limiting intensities are lower than the characteristic value of the threshold in previously investigated[3-5] types of scattering.

Thus, by varying the width of the waveguide and the transverse temperature difference we can significantly alter the characteristics of the nonlinear scattering without lowering its effectiveness.

6. CONCLUSION

Our analysis shows, as follows from the proposed mechanism, that even in the case of weak inhomogeneity stimulated scattering of sound has a rather low threshold. At moderate pump intensities the characteristic scattering length is of the order of several wavelengths and decreases when the inhomogeneity of the medium increases. This raises hopes of observing it in experiment. This possibility is indirectly confirmed by known experiments on excitation of a density wave by a standing sound wave in a transversely inhomogeneous waveguide.[13]

The scattering investigated must also be taken into account as a possible parasitic effect in propagation of strong sound. The medium can be made inhomogeneous enough as it becomes heated by the sound damping.

We have confined ourselves here to the initial scattering stage, when the scattered-wave amplitude is small. We have therefore neglected nonlinear self-action processes excited by quasistationary waves. The results of Ref. 13 show that these processes become significant when the low-frequency perturbations increase, and this can apparently limit the attainable coefficient for conversion of an incident wave into a scattered one.

We conclude by noting some possibilities of additionally increasing the efficiency of this type of scattering.

The scattering increment is quadratic in the quantity $(\partial \ln c^2/\partial \ln T)_p$, indicative of the relative change of the sound velocity with change of temperature. This quantity is equal to 1 in the media considered above. In media where it is large, however, this scattering efficiency will be higher.

The boundary conditions we investigated for quasistationary waves (ideally heat-conducting walls) can be regarded as the least favorable, since they damp these perturbations strongly near the walls. Thermal insulation of the walls or the use of inelastic modes of the medium without damping at the walls, for example density waves, also increases the scattering effectiveness.

In our system there was no reaction of the entropy wave on the rotational ones. In general the feedback can be negative as well as positive. If the force of gravity is taken into account, the scattering increment will increase when convective instability (with respect to the Rayleigh number) is approached. This is one more possibility of increasing the scattering efficiency and smoothly varying its characteristics.

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