Calculation of correlators of the chromoelectric hadron string

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The concluding stage in the calculation of hadronic correlators in the leading order of the $1/N$ expansion in strong-coupling $SU(N)$ gauge theory is presented. In the scalar quark approximation these quantities reduce in our approach to the correlators of the chromoelectric string with dynamic quarks at its ends. The calculation of these correlators leads to the Koba–Nielsen amplitudes. Pronounced differences between the chromoelectric string and the standard model appear mainly in the calculation of the partition function and, as it turns out, disappear from the expressions for the correlators.

1. INTRODUCTION

The chromoelectric string, that binds quarks in a hadron, arises as a theoretical object in the nonperturbative calculation of hadronic field correlators[1]

$$\langle \Psi^+(x_1)\Psi(x_1)\ldots\Psi^+(x_n)\Psi(x_n)\rangle \qquad (1)$$

in the framework of the $SU(N)$ gauge theory in the limit $N \gg 1$. Here

$$\Psi^+(x)\Psi(x) = \sum_{c=1}^{N}\Psi_c^+(x)\Psi_c(x)$$

is the color singlet operator of the composite meson, and $\Psi_c(x)$ is a scalar (for the sake of simplicity) quark field. In the calculation of the correlators (1) the quasiclassical expansion in the $1/N$ parameter is utilized in the strong coupling regime $e^2N \gtrsim 1$ (here e is the gauge coupling constant). For the quasiclassical approximation to be applicable it is necessary, in particular, to have a stable extremum. It has been established previously, on the basis of topological arguments,[2] that such an extremum exists not in terms of the original dynamic variables $A_\mu^a(x)$ (four-dimensional gauge fields) but rather in terms of new variables, namely two-dimensional gauge fields "living" on surfaces Σ imbedded into the flat space R^4 according to equations of the form

$$x_\mu = x_\mu(z^i); \quad \mu = 1, 2, 3, 4; \quad i = 1, 2.$$

Specifically, the stable extremum belongs to the subclass of quasi-two-dimensional fields of the following form:[2]

$$[A_\mu{}^a(x(z))]_\Sigma = \left[\frac{\partial x_\mu}{\partial z^i}A^{a,i}(z)\right]_\Sigma, \quad a = 1,\ldots,N^2-1.$$

In view of the given circumstances this subclass of fields provides the main contribution to (1) in the quasiclassical $1/N$ calculation of the correlator. The selection of the given subclass is carried out by inserting under the functional integration sign $\int DA_\mu^a(x)\ldots$ in (1) the corresponding projection operator

$$\Pi = \int D[A_\mu{}^a(x(z))]_\Sigma \delta\{[A_\mu{}^a(x(z))]_\Sigma - A_\mu{}^a(x)\}$$
$$= \int D[x_\mu(z)]_\Sigma \int D[A_i{}^a(z)]_\Sigma J_\Sigma[A(z), x(z)]\delta$$
$$\times\left\{\left[\frac{\partial x_\mu}{\partial z^i}A^{a,i}(z)\right]_\Sigma - A_\mu{}^a(x)\right\}.$$

Here $J_\Sigma[A,x]$ is the Jacobian of the transformation to the new variables $A_i^a(z)$, $x_\mu(z)$. Integration over the original fields $A_\mu^a(x)$ with the help of the δ-functions, contained in Π, reduces the calculation of the vacuum expectation value in (1) to integration over the two-dimensional field $[A_i^a(z)]\Sigma$ for fixed surface Σ and subsequent summation over the surfaces Σ. [We assume here the use of the standard measure for integration over $x_\mu(z)$ and $A_i^a(z)$]. As a result of the transition to the new fields there arises on each surface the two-dimensional Yang–Mills action, which reduces on the extremum to the strong action of the Nambu type.[1]

It should be, however, emphasized that the topologically nontrivial (i.e., stable) configuration of the Yang–Mills field, connected through the corresponding boundary condition to the quark flux flowing through the surface boundary $\partial\Sigma$, does not arise on any arbitrary surface Σ. The selection of such surfaces is ensured by the condition that the internal geometry on Σ be determined by the dynamics of the Yang–Mills field on that same surface. By this we mean here the identification of the spin connection on the surface Σ with the vector potential of the gauge field, or, what is equivalent, the identification of the curvature tensor with the Yang–Mills field intensity on Σ (see the second paper in Ref. 1). In that case the stable saddle configuration of the field forms a Nambu-type string with quarks at its ends. {Calculation of the Jacobian $J_\Sigma[A,x]$ to leading order in $1/N$ gives rise to an infinite constant, which can be eliminated by a suitable choice of normalization (see the second paper in Ref. 1)}. As a result of the above identification of the connections the world sheet of the chromoelectric string has constant scalar curvature R. Thus, in our approximation, summation should be carried out only over surfaces Σ selected by the condition

$$R = \text{const}. \qquad (2)$$

(We recall that R is invariant with respect to bending of the surface Σ.) This condition makes the chromoelectric hadronic string pronouncedly different from the standard model.[3]

Using these approximations we obtained the following expression for the action for the chromoelectric string:[1]

$$S[x,\lambda] = k_0\int_\Sigma d^2z\, h^{1/2}(x(z)) + \frac{1}{2}\oint_{\Gamma\in\partial\Sigma} d\gamma[\dot{x}^2(z(\gamma))/\lambda(\gamma) + m_0{}^2\lambda(\gamma)], \qquad (3)$$

$$h = \det h_{ab}, \quad h_{ab} = \frac{\partial x_\mu}{\partial z^a}\frac{\partial x_\mu}{\partial z^b}; \quad a, b = 1, 2.$$

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Here γ is a parametrization of the contour Γ, m_0 is the bare mass of the quark, $\lambda(\gamma)$ is a one-dimensional metric on Γ, and

$$\dot{x}_\mu=\frac{\partial x_\mu}{d\gamma}=\frac{\partial x_\mu}{\partial z^a}\frac{dz^a}{d\gamma}.$$

The bare string tension coefficient k_0 is equal to

$$k_0=\frac{e^2}{2\delta^2}\left(\frac{N^2-1}{2N}\right),\qquad e^2=e_0{}^2/N, \tag{4}$$

where δ is the ultraviolet regulator with dimensions of length. (In the process of the renormalization of the tension k resulting from subsequent summation over surfaces the parameter δ should tend to zero. This is equivalent to the removal of the cutoff and the introduction of a normalization point.[5]) The coefficient k_0 coincides exactly with the expression obtained previously for this quantity in leading order of the strong-coupling approximation in the Hamiltonian formulation of lattice gauge theory.[6]

Another difference between the chromoelectric string and the standard model is that its action is quantized:

$$k_0\int_\Sigma d^2zh^{1/2}=\pi|Q|,\qquad Q=\pm1,\ \pm2\ldots \tag{5}$$

This property is a consequence of the quantization of the chromoelectric flux on the string world sheet.[7] The condition (5) means that the area $A(\Sigma)$ of the world sheet does not change continuously, as is the case in the standard approach to string theory, but in discrete steps. The conditions (2) and (5) significantly restrict the space of quantum states of the hadronic string. As a consequence of the quantization condition (5) the partition function

$$Z=\int Dx_\mu(z)D\lambda(\gamma)\exp\{-S[x(z),\lambda(\gamma)]\} \tag{6}$$

breaks up into a sum of contributions from various topological sectors:

$$Z=\sum_{|Q|=1}^{\infty}Z_{|Q|}=\sum_{|Q|}(Z_{Q=+|Q|}+Z_{Q=-|Q|})\equiv\sum_{|Q|}(Z_{Q^+}+Z_{Q^-}). \tag{7}$$

Evaluation of the partition function (6), (7) shows[5] that the string with restricted configuration space is free from several defects present in the standard model,[8] namely,

1) the limit $\mathscr{D}\to-\infty$ becomes superfluous ($\mathscr{D}$ is the dimension of the imbedding space $R^{\mathscr{D}}$);

2) the metric $\hat{g}$ on the world sheet, that extremizes the effective action $F(g)=-\ln Z(g)$, has no singularities on the sheet boundary $\partial\Sigma$. The partition function Z is expressed in terms of just the Euler characteristic χ of the world sheet, which is, as is well-known, a topological invariant:

$$Z_{|Q|}\propto2\exp\left[-\mathscr{D}(\chi/6+{}^1/_3)|Q|\right]. \tag{8}$$

This means that in our approximation the $SU(N)$ gauge theory reduces to topological field theory. [For the convergence of the sum (7) it is necessary that the curvature R be negative.]

The procedure for the renormalization of the string tension coefficient k and the masses of the scalar quarks at the ends of the string has also been determined. The resultant Gell-Mann–Low function coincides with the leading term in the expansion of the β-function in the strong-coupling approximation for lattice field theory,[6] which confirms the correctness of our approach.

Despite the indicated differences from the standard model the final calculation of the correlators (1) in the momentum representation, as will be shown below, leads to the Koba–Nielsen formula for dual resonance amplitudes. All the above differences appear basically in the partition function and disappear from the expressions for the correlators.

2. CALCULATION OF THE CORRELATORS

We begin with the calculation of the correlator

$$K(p_1,\ldots,p_n)=\left\langle\oint_{\partial\Sigma}\prod_{k=1}^{n}\frac{ds_k}{n!}(2\pi)^{-\mathscr{D}n/2}:\exp[(-ip_k\cdot x(z(s_k))]:\right\rangle, \tag{9}$$

which determines the amplitude for the scattering of mesons—the ground states of the chromoelectric string.[5] Here s is the natural parameter on the boundary $\partial\Sigma$ of the string world sheet:

$$ds=(h_{ab}\dot{z}^a\dot{z}^b)^{1/2}d\gamma=m_0\lambda(\gamma)d\gamma. \tag{10}$$

The last equality in (10) reflects the condition for consistency of the metrics on Σ and $\partial\Sigma$.[5] The factor $n!$ in (9) is introduced to take into account that the scattering particles are identical. The vacuum expectation value is obtained making use of the action (3) with the additional conditions (2) and (5) taken into account.

We evaluate the integral over surfaces in (9) similarly to what was done in Ref. 5, where the partition function (6) was evaluated. According to that work, the evaluation of (9) can be carried out by going over to the analog of the second-order formalism,[3] introducing an additional integration over the auxiliary internal metric $g_{ab}(z)$, which in our case should be done only in leading order of the steepest-descent approximation. This restriction is connected with the fact that in contrast to the standard model[3] the expression (3) contains a Nambu-type action, which coincides with the model action[3] only on the extremum $\hat{g}_{ab}$. The formula for the correlator (9) takes the form

$$K(p_1,\ldots,p_n)\mathrel{\overset{\circ}{=}}(2\pi)^{-\mathscr{D}n/2}Z^{-1}\oint_{\partial\Sigma}\prod_{k=1}^{n}\frac{ds_k}{n!}Dg_{ab}(z)Dx_\mu(z)$$
$$\times\exp\left\{-\frac{k_0}{2}\int_\Sigma d^2zg^{1/2}g^{ab}\,\partial_ax_\mu\,\partial_bx_\mu-ip_k\cdot x(z(s_k))\right.$$
$$\left.-\frac{m_0}{2}\oint_{\partial\Sigma}ds\left[\left(\frac{dx_\mu(z(s))}{ds}\right)^2+1\right]+\alpha f_1{}^{\pm}[g]+\beta f_2[g]\right\}, \tag{11}$$

where f_1 and f_2 are the constraints that take into account the additional conditions (5) and (2), and α and β are Lagrange multipliers. The symbol $\overset{\circ}{=}$ indicates that the integral over g_{ab} is calculated in leading order of the steepest-descent approximation, i.e., that the integral is equal to the integrand evaluated at the saddle point.

The constraint $f_1[g]$ is of the form

$$f_1{}^{\pm}[g]=\pm\pi Q-k_0\int_\Sigma d^2z\,g^{1/2}. \tag{12}$$

If we apply the Gauss-Bonnet theorem

$$\frac{1}{2}\int_\Sigma d^2z\, g^{1/2}R+\oint_{\partial\Sigma} ds\, \varkappa_g=2\pi\chi, \tag{13}$$

the condition of constant curvature (2) for fixed winding number Q can be taken into account[5] by specifying the constraint f_2 in the form

$$f_2[g]=\oint_{\partial\Sigma} ds\varkappa_g-q, \tag{14}$$

where $q=\text{const}$ and $\varkappa_g$ is the geodesic curvature of the boundary $\partial\Sigma$.

Let us consider the integral over the insertions $x_\mu(z)$ connected with (11):

$$J=\int Dx_\mu(z)\exp\Big\{\frac{k_0}{2}\int_\Sigma d^2z\, g^{1/2}x_\mu(z)\Delta_g x_\mu(z)$$
$$-\frac{k_0}{2}\oint_{\partial\Sigma} ds\, x_\mu(z)n^a\partial_a x_\mu(z)$$
$$-\frac{m_0}{2}\oint_{\partial\Sigma} ds\left(\frac{dx_\mu}{ds}\right)^2-ip_\mu x_\mu(z(s))-\frac{m_0}{2}\oint_{\partial\Sigma} ds\Big\}, \tag{15}$$

where $\Delta g=g^{-1/2}\partial_a(g^{1/2}g^{ab}\partial_b)$ is the Laplace–Beltrami operator and n_a is the outward normal to the boundary $\partial\Sigma$. Elimination of the divergences arising in (15) is accompanied by the renormalization of the quantities $k_0\to k$, $m_0\to m$.[3,5,8] This renormalization is connected to the fact that the divergent terms in (15), proportional to the area and perimeter of the world sheet, are defined accurate up to finite contributions having the same geometric structure. Consistent renormalization is achieved in the limit $\delta\to 0$, $k_0\propto\delta^{-2}$ [see (4)], where for consistency one should also set $m_0\propto 1/\delta$.[5]

Evaluation of the partition function Z shows that in the end the contribution of the third term in (15) cancels after regularization against the contribution of the zero mode.[5] It is therefore convenient to eliminate that term in the integral (11) from the very beginning. To this end we introduce dimensionless coordinates and momenta

$$x_\mu\to y_\mu\equiv k_0^{1/2}x_\mu,\quad p_\mu\to p_\mu/k_0^{1/2}. \tag{16}$$

Then the integral (15) takes on the form

$$J\propto\int Dy_\mu\exp\Big\{\frac{1}{2}\int_\Sigma d^2z\, g^{1/2}y_\mu\Delta_g y_\mu-\frac{1}{2}\oint_{\partial\Sigma} ds\, y_\mu n^a\partial_a y_\mu$$
$$-\frac{m_0}{2k_0}\oint_{\partial\Sigma} ds\left(\frac{dy_\mu}{ds}\right)^2-\frac{m_0}{2}\oint_{\partial\Sigma} ds-ip_\mu y_\mu(z(s))\Big\}. \tag{17}$$

For $\delta\to 0$ we have $m_0/k_0\propto\delta\to 0$; in that limit the third term in (17) dies out in comparison with the remaining terms in the braces. For this reason we omit it in Eq. (11). Performing the integration over $y_\mu(z)$ with the Neumann boundary conditions

$$\partial_n y_\mu(z)=n^a\partial_a y_\mu(z)=0,\quad z\in\partial\Sigma, \tag{18}$$

we obtain:

$$\Big\langle\frac{1}{n!}\oint_{\partial\Sigma}\prod_{k=1}^n\frac{ds_k}{(2\pi)^{\mathscr{D}/2}}:\exp[-ip_k\cdot x(z(s_k))]:\Big\rangle_{Q\pm}$$
$$\stackrel{\circ}{=}\frac{1}{Z(2\pi)^{\mathscr{D}n/2}n!}$$
$$\times\int Dg_{ab}(z)\oint_{\partial\Sigma}\prod_{k=1}^n ds_k\int Dy_\mu(z)\exp\Big\{\frac{1}{2}\int_\Sigma d^2z g^{1/2}y_\mu(z)\Delta_g y_\mu(z)$$
$$-\frac{1}{2}\oint_{\partial\Sigma} ds y_\mu(z)\partial_n y_\mu(z)-ip_k\cdot y(z(s_k))$$
$$-\frac{m_0}{2}\oint_{\partial\Sigma} ds+\alpha f_1^\pm[g]+\beta f_2[g]\Big\}$$
$$\stackrel{\circ}{=}\frac{(2\pi)^{\mathscr{D}}\delta^{\mathscr{D}}\left(\sum_i p_i\right)}{Z(2\pi)^{\mathscr{D}n/2}n!}\int Dg_{ab}(z)\oint_{\partial\Sigma}\prod_{j=1}^n ds_j$$
$$\times\exp\Big\{-\frac{1}{2}\sum_{i,j=1}^n p_ip_jG(z(s_i),z(s_j);g)$$
$$-F(g)\mp\alpha\pi Q-\beta q\Big\}. \tag{19}$$

Here $G(z,z';g)$ is the Green's function with the appropriate boundary conditions (see Sec. 3). In the conformal gauge where

$$g_{ab}=e^\varphi\delta_{ab},\quad ds_i=\exp[\varphi(z_i)/2]\,dz,\quad dz=(dz^adz^a)^{1/2}, \tag{20}$$

the effective action is equal to[3,5,8]

$$F(\varphi)=-\frac{\mathscr{D}}{48\pi}\int_\Sigma d^2z\left[\frac{1}{2}(\partial_a\varphi)^2-\alpha ke^\varphi\right]+\frac{\mathscr{D}m}{48\pi}\oint_{\partial\Sigma} dze^{\varphi/2}$$
$$-\frac{\mathscr{D}}{24\pi}\oint_{\partial\Sigma} dz\varphi\varkappa_f+\left(\frac{\mathscr{D}}{8\pi}-\beta\right)\oint_{\partial\Sigma} dze^{\varphi/2}\varkappa_g. \tag{21}$$

Here $\varkappa_f$ is the flat curvature of the boundary of the integration region in the space of the parameters z^a on the world sheet, and k and m are the quantities renormalized according to the procedure described in Ref. 5.

The dependence on the conformal factor $\varphi(z)$ along with (21) is contained in ds_j (20) and in the Green's function

$$G(z,z';\varphi)=G(z,z';\varphi=0)+f_\varphi(z)+f_\varphi(z'),\quad z\neq z'. \tag{22}$$

Making use of the condition of conservation of the total momentum

$$\sum_{i=1}^n p_i=0,$$

which results from integrating (19) over the zero mode, we can exclude the dependence of the correlator (19) on $f_\varphi(z_i)$. After that the φ-dependence of the Green's function appears only for coincident points $z=z'$:

$$G(z,z;\varphi)=-\frac{1}{2\pi}[\ln\varepsilon-\varphi(z)], \tag{23}$$

where ε is the invariant ultraviolet cutoff.[3]

Including (20) and (23) we have ($\mathscr{D}=4$)

$$K_Q(p_1,\ldots,p_n) \overset{\circ}{=} \frac{(2\pi)^{4-2n}\delta^4\left(\sum_i p_i\right)}{Zn!}\exp(-\alpha\pi|Q|-\beta q)$$
$$\times\int D\varphi \oint \prod_{j=1}^{n} dz_j \exp\left[\frac{\varphi(z_j)}{2}(1-p_j^2/2\pi)\right]$$
$$\times\exp\left\{-F(\varphi)-\sum_{j<i=1}^{n} p_ip_jG(z_i,z_j)\right\}. \quad (24)$$

(We do not write out the factors containing the cutoff ε since they can be absorbed by an appropriate definition of normal ordering in the operators: e^{-ipx}:.)

Variation of the integrand in (24) with respect to φ leads to the same results as in the calculation of the partition function: we obtain the Liouville equation with definite boundary conditions on $\partial\Sigma$.[5] The sole difference consists of additional factors $\exp\{\varphi(z_j)(1-p_j^2/2\pi)/2\}$, which are connected only with discrete points z_j on the boundary. Variation with respect to φ at these points gives rise to the following condition on the masses of the external particles (in dimensionless units):

$$p_j^2=2\pi. \quad (25)$$

The solution of the Liouville equation is the metric with constant curvature

$$R=-\alpha|k|\equiv -2/a^2. \quad (26)$$

which in the Klein model has the form

$$\hat{g}_{ab}(z)=\exp[\hat{\varphi}(z)]\delta_{ab}=\frac{a^2}{y^2}\delta_{ab}, \quad z=x+iy, \quad y>0. \quad (27)$$

As a result of the boundary conditions on $\partial\Sigma$ it is also true that[5]

$$\varkappa_g^2=-R/2, \quad (28)$$

which is characteristic of the Beltrami pseudo-sphere. This means that at the point of extremum $\hat{\varphi}$ integration over d^2z in the effective action $F(\hat{\varphi})$ (21) proceeds over a surface isometric to the pseudo-sphere (about that surface see, for example, Ref. 9). This circumstance permits making the region of definition of the parameters $z=x+iy$ in (27) more precise. For the winding number $Q=1$ it follows from the properties of the pseudo-sphere that:

$$0\leqslant x\leqslant 2\pi a, \quad a\leqslant y<\infty.$$

If the pseudo-sphere is wound over itself Q times then

$$0\leqslant x\leqslant 2\pi a|Q|, \quad a\leqslant y<\infty.$$

Since the region of integration over z represents only a part of the Lobachevskii plane, the metric $\hat{g}_{ab}$ (27) has no singularities on the boundary, in contrast to the results of Ref. 8. The parameters α, β, q contained in expressions (19) and (21) were found in Ref. 5:

$$\alpha=\frac{\mathscr{D}}{12\pi}=\frac{1}{3\pi}, \quad \beta=\frac{\mathscr{D}}{8\pi}=\frac{1}{2\pi}, \quad q=2\pi|Q|. \quad (29)$$

Substitution of the metric $\hat{g}$ (27) into $F(g)$ gives

$$F(\hat{g})=\mathscr{D}\chi/6. \quad (30)$$

Taking into account that for the Beltrami pseudo-sphere $\chi=0$ hold, we obtain the following expression for the correlator (24):

$$K_Q(p_1,\ldots,p_n)$$
$$=\frac{\delta^4\left(\sum_{i=1}^{n} p_i\right)e^{-4|Q|/3}}{Z(2\pi)^{2n-4}n!}\oint_{\partial\Sigma}\prod_{i=1}^{n} dz_i\exp\left\{-\sum_{j>i=1}^{n} p_ip_jG_Q(z_i,z_j)\right\}. \quad (31)$$

3. THE GREEN'S FUNCTION AND THE SCATTERING AMPLITUDE

To proceed with the evaluation of the correlator (31) we must have explicitly the Green's function $G_Q(z_i, z_j)$, which is defined by the equation

$$\Delta_{\hat{g}}G(z,z')=-\frac{\delta^2(z-z')}{[\hat{g}(z)]^{1/2}}+1\Big/\int_\Sigma d^2z\hat{g}^{1/2} \quad (32)$$

with the Neumann boundary condition [see (18)]

$$\partial_nG(z,z')=0, \quad (33)$$

for $z=x+iy\in\partial\Sigma$, i.e., when $y=a$. The last term on the right-hand side of (32) is connected with the existence of a zero mode of the operator Δg with the boundary condition (18); we have

$$A(\Sigma)=\int_\Sigma d^2z\,\hat{g}^{1/2}=\int_0^{2\pi a|Q|}dx\int_a^\infty dy\,\hat{g}^{1/2}=2\pi a^2|Q|. \quad (34)$$

Further, since the world sheet Σ is isometrically similar to a pseudo-sphere we should impose the condition of periodicity on the coordinate $x=\operatorname{Re} z$:

$$G(z,z')=G(z+2\pi a|Q|,z'). \quad (35)$$

The solution of Eq. (32) with the conditions (33) and (35) can be found by the method of images. It has the following form (for $z=z'$):

$$G_Q(z,z')=-\frac{1}{4\pi}\left\{\ln\left|\sin\left(\frac{z-z'}{2aQ}\right)\right|^2+\ln\left|\sin\left(\frac{z-z'^*}{2aQ}\right)\right|^2\right.$$
$$\left.+2f(z)+f(z')+f(z'^*)+\text{const}\right\},$$
$$f(z)=f(y)=\left(\ln\frac{y}{a}-\frac{y}{a}\right)\frac{1}{|Q|}, \quad (36)$$

$z^*=x-iy+2ia$ is the image of the point z with respect to the boundary $y=a$.

For $z=z'$ and $z\in\partial\Sigma$ we have $z^*=z$ and

$$G(z,z)=-\frac{1}{2\pi}(\ln\varepsilon-\hat{\varphi}(z)).$$

As was already remarked, the function $f(z)$ makes no contribution to the correlator (31) as a consequence of conservation of the total momentum.

Let us go over in the expression (31) to dimensional momenta, taking for the measurement scale the renormalized tension coefficient $k\equiv 1/2\pi\alpha'$; then

$$p_i^2=1/\alpha', \quad (37)$$

$$\sum_{j>i}p_ip_jG(z_i,z_j)\to 2\pi\alpha'\sum_{j>i}p_ip_jG(z_i,z_j). \quad (38)$$

Substituting the explicit form (36) into Eq. (31) we obtain

$$K_Q(p_1,\dots,p_n)=\frac{\delta^4\left(\sum_i p_i\right)\exp\left(-\frac{4}{3}|Q|\right)}{(2\pi)^{2n-4}Zn!}\prod_{i=1}^{n}\int_0^{2\pi a|Q|}dx_i$$

$$\times\prod_{j>i=1}^{n}\left|\sin\left(\frac{x_i-x_j}{2aQ}\right)\right|^{2\alpha' p_i p_j}. \qquad (39)$$

After we make the change of variables

$\zeta_j=e^{i\omega_j}$, где $\omega_j=x_j/a|Q|$, $0\leqslant\omega_j\leqslant2\pi$,

we obtain

$$\left|\sin\left(\frac{x_i-x_j}{2aQ}\right)\right|^2=\left|\sin\left(\frac{\omega_i-\omega_j}{2}\right)\right|^2=\frac{1}{2}[1-\cos(\omega_i-\omega_j)]$$

$$=\frac{1}{4}|\zeta_i-\zeta_j|^2;$$

$$K_Q(p_1,\dots,p_n)=\frac{\delta^4\left(\sum_i p_i\right)\exp\left(-\frac{4}{3}|Q|\right)(4a|Q|)^n}{(2\pi)^{2n-4}Zn!}$$

$$\times\oint_{|\zeta|=1}\prod_{i=1}^{n}\frac{d\zeta_i}{\zeta_i}\prod_{j>i}^{n}|\zeta_i-\zeta_j|^{2\alpha' p_i p_j}. \qquad (40)$$

The full n-particle interaction amplitude is obtained as a result of summing (40) over all topological sectors Q:

$$K(p_1,\dots,p_n)=\sum_{Q\neq0}^{\infty}K_Q(p_1,\dots,p_n)=2\sum_{|Q|=1}^{\infty}K_Q(p_1,\dots,p_n).$$

Taking into account that

$$Z=\sum_{Q\neq0}Z_Q=2\sum_{|Q|=1}^{\infty}\exp\left(-\frac{4}{3}|Q|\right)=2/(e^{4/3}-1),$$

$$\sum_{|Q|=1}^{\infty}|Q|^n\exp\left(-\frac{4}{3}|Q|\right)\approx\frac{n!}{(4/3)^{n+1}},$$

we obtain the following expression for the dimensionless correlator:

$$K(p_1,\dots,p_n)=\text{const}\cdot(2\pi)^4\delta^4\left(\sum_i p_i\right)\lambda^n A_n(p_1,\dots,p_n), \qquad (41)$$

where

$$A_n(p_1,\dots,p_n)=\oint_{|\zeta|=1}\prod_{i=1}^{n}\frac{d\zeta_i}{\zeta_i}\prod_{j>i=1}^{n}|\zeta_i-\zeta_j|^{2\alpha' p_i p_j}, \qquad (42)$$

$$\lambda\approx\frac{3}{4\pi^2}\ll1. \qquad (43)$$

The condition (37) means for Euclidean momenta that the parameter α_0 of the Regge trajectory $\alpha(s)=\alpha_0+\alpha's$ is equal to unity:

$$\alpha_0=1. \qquad (44)$$

Therefore the amplitude (42) is invariant under cyclic permutations of the momenta p_i. Representing it as a sum over noncyclic permutations we arrive at the standard Koba-Nielsen formula for each individual term in that sum:

$$A_n(p_1,\dots,p_n)$$

$$=\frac{1}{V}\oint_{|\zeta|=1}\prod_{i=1}^{n}\frac{d\zeta_i}{\zeta_i}\theta(\arg\zeta_{i+1}-\arg\zeta_i)\prod_{j>i}|\zeta_i-\zeta_j|^{2\alpha' p_i p_j}, \qquad (45)$$

where V is the invariant volume of the group $SU(1,1)$.

4. CONCLUSION

The present work completes the series of papers by the author[1,2,5,7] devoted to nonperturbative calculation of hadronic field correlators in the framework of $SU(N)$ gauge field theory. In the strong coupling approximation for scalar quarks a quasiclassical expansion in the inverse of the number of colors $1/N$ was developed and the topologically nontrivial configuration of the gauge field that forms the chromoelectric string and binds the quarks in the hadron was found. The phenomenon of quantization of the chromoelectric flux on the world sheet of the string was discovered.[7] It was shown that the mechanism for string formation is similar to the phase transition of the II kind in statistical physics, where the number of colors N plays the role of an inverse dimensionless temperature $t=T/T_{\rm cr}$, with $N_{\rm cr}=1$ (see the first paper in Ref. 1). The partition function for the string was calculated and found to be expressed in terms of the Euler characteristic of the world sheet of the string,[5] indicating a connection to topological field theory.

A renormalization procedure for the string tension coefficient was determined;[5] it was shown that the Gell-Mann–Low function for the nonperturbative phase of quantum chromodynamics coincides with the leading term in the expansion of the β-function in the strong-coupling approximation for lattice field theory.[6] The string configuration of the gauge field dominates in the correlators and the partition function in leading order of expansion in the $1/N$ parameter, which ensures the phenomenon of quark confinement in the hadron in the limit $N\gg1$.

In the present paper we have obtained the final expressions (41)–(45) for the correlators (1). Although the chromoelectric string has a narrower space of quantum states [as a consequence of the restrictions (2) and (5)], which gives rise to a different Green's function (36) and to freedom from a number of deficiencies of the standard model[8] [see Sec. 1, the text that follows formula (7)], we have nevertheless arrived in the final answer at the Koba–Nielsen amplitudes. It is known that these amplitudes, in spite of a number of remarkable properties, contain difficulties connected with the unitarity problem at the tree level and the presence of tachyons in the mass spectrum of the particles. There are serious reasons to suppose that it will be necessary for the exclusion of tachyons to repeat anew the entire program of our calculations with the spin (and flavor) degrees of freedom of the quarks taken into account. This is suggested by the results obtained previously in the Neveu–Schwarz model, as well as by the conclusions of Ref. 10, where the question of inclusion of quark spin into the dual resonance model on the basis of the Bargmann–Wigner wave functions was discussed. In addition it should be remembered that, of course, there are a large number of corrections in next order in $1/N$ (see the second paper in Ref. 1), which could turn out to be quite relevant for $N=3$.

Another possible point of view on the problem of the appearance of tachyons is as follows. The effective string description of quantum chromodynamics is restricted by the strong-coupling approximation, and therefore it is impossible in principle to have a string theory which reproduces quantum chromodynamics at all distances. On the other hand the tachyon is a bound quark-antiquark state, in which the distance between them tends to zero, which corresponds to weak coupling. Therefore the appearance of the tachyon is a natural consequence of going beyond the framework of the allowed approximation. This is no tragedy as long as the string theory does not pretend to give a description of physical phenomena at all scales (in this connection see also Ref. 11).

In conclusion we emphasize the status of the dual-resonance model, which follows from our work. The model describes the interaction amplitude of composite mesons (ground states), calculated in the strong coupling regime to leading order in the $1/N$ expansion, in the case when the real quarks that form hadrons are replaced by scalar particles. The model is directly connected with the quark confinement phenomenon. Although these assertions do not appear to be absolutely new, it is important that they have been obtained for the first time starting from first principles on the basis of consistent nonperturbative calculations of hadronic correlators in the framework of $SU(N)$ gauge field theory.

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