Radiation by a nonrelativistic charge in a medium: orientation dependence of the emission spectrum in a crystal

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The total radiation emitted by a nonrelativistic charge in a medium, consisting of the bremsstrahlung plus the transition radiation, is analyzed. The interference between these radiation components is calculated. The range of applicability of macroscopic electrodynamics for the transition radiation emitted by a nonrelativistic charge is found. The dependence of the radiation spectrum emitted by a nonrelativistic charge on the direction in which the charge enters the crystal is predicted. Specific calculations are carried out for a cubic crystal.

1. INTRODUCTION

The radiation emitted by a charge in a medium results from a transfer of momentum to the medium due to interaction with an electron (bremsstrahlung) or with an emitted photon (transition radiation, in the broad sense of the term). These two emission mechanisms are usually examined separately, so the role played by interference between them cannot be evaluated. However, while a joint analysis of bremsstrahlung and transition radiation is a complicated matter in the general case, it becomes a comparatively straightforward matter in the case of a nonrelativistic charge. It is worthwhile to derive a general expression for the angular and frequency distribution of the radiation for a fixed law of motion of a nonrelativistic charge in a medium, with both bremsstrahlung and transition radiation being taken into account.

When this approach is taken, it is found that the length scale for the excitation of the transition radiation is not the wavelength λ of the photon, but the far shorter length $\lambda v/c$, where v is the velocity of the charged particle. This circumstance has some nontrivial consequences in the transition radiation emitted by a nonrelativistic particle. In particular, in a crystal the transition-radiation spectrum (the intensity integrated over all directions) in the optical frequency range, for which the crystal is always assumed to be homogeneous, turns out to depend on the direction in which the charge enters the crystal. A similar orientation dependence of the emission spectrum, but caused by other physical factors, has been predicted theoretically[1] and observed experimentally[2] for ultrarelativistic particles. The transition radiation emitted by relativistic particles was studied in Refs. 3–5.

Let us show that longitudinal distances ("longitudinal" meaning parallel to the velocity) on the order of $\lambda v/c$ are important for the excitation of transition radiation with a wavelength λ by a nonrelativistic charge in a medium. Since the change in the velocity of the charge over a distance $\lambda v/c$ can be ignored, it is sufficient to prove this point for a charge in uniform motion.

Transition radiation results from scattering of the field of the charge by the atoms of the medium. In a stationary medium the frequency of the field does not change, so the frequency ω of the emitted photon is equal to the "frequency" $\mathbf{q}\cdot\mathbf{v}$ of the Fourier component of the field of the charge which is responsible for the emission. The Fourier transform of this field can be written $(x\|\mathbf{v})$

$$\mathbf{E}_0(\mathbf{r},\omega)=\int\frac{dt}{2\pi}\mathbf{E}_0\left(y,z,t-\frac{x}{v}\right)\exp(i\omega t)$$
$$=\exp\left(\frac{i\omega x}{v}\right)\int\frac{dt'}{2\pi}\mathbf{E}_0(y,z,t')\exp(i\omega t'),\tag{1}$$

since the field of the charge depends on the difference between the radius vectors of the charge, $\mathbf{v}t$, and of the observation point, $\mathbf{r}$. The factor $\exp(i\omega x/v)$ shows that $\mathbf{E}_0$ varies substantially over distances $\sim\lambda v/c$.

One consequence of this circumstance is that a macroscopic description of the transition radiation emitted by a charge crossing an interface between two media is valid only under the inequality

$$\lambda(v/c)\gg a,\tag{2}$$

where a is a distance on the order of an atomic distance. This inequality is considerably more stringent than the condition for the applicability of a macroscopic description of the propagation of a field through a medium. The latter condition is

$$\lambda\gg a,\tag{3}$$

as was pointed out in Ref. 6.

Below we consider the formation of transition radiation with frequencies higher than atomic frequencies, in which case we can analyze the electromagnetic field in the medium by a method similar to that used in the theory of x-ray diffraction in crystals.[7] The following inequality must hold:

$$h\omega\gg mc^2(e^2/hc)^2.\tag{4}$$

We also note that in region (4) there is a narrower region,

$$mc^2\left(\frac{e^2}{hc}\right)\gg h\omega\gg mc^2\left(\frac{e^2}{hc}\right)^2,\tag{5}$$

in which the frequency is in fact high in comparison with atomic frequencies, while the wavelength is still large in comparison with atomic dimensions.

2. ANGULAR AND FREQUENCY DISTRIBUTION OF THE RADIATION EMITTED BY A NONRELATIVISTIC CHARGE IN A MEDIUM

At frequencies higher than atomic frequencies, the electromagnetic field in a medium satisfies the equation

$$\operatorname{rot}\mathbf{H}(\mathbf{r},\omega)=\frac{4\pi}{c}\mathbf{j}(\mathbf{r},\omega)-\frac{i\omega}{c}[\varepsilon_0-\chi(\mathbf{r})]\mathbf{E}(\mathbf{r},\omega),\tag{6}$$

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where

$$\varepsilon_0=1-\frac{4\pi n_0}{m\omega^2},\quad \chi(\mathbf{r})=\frac{4\pi\delta n(\mathbf{r})}{m\omega^2}, \tag{7}$$

n_0 is the number density of electrons, averaged over the volume of the medium, and $n_0+\delta n(\mathbf{r})$ is the microscopic number density of electrons in the medium, averaged over the quantum-mechanical distribution of electrons in the atoms and over the thermal vibrations of the atoms (no average is taken over volume in this case).

In this case we have $\chi\ll 1$, and we can use perturbation theory in χ. In the zeroth approximation we can set $\chi=0$. Equation (6) then becomes the same as the equation of macroscopic electrodynamics, and the expressions for the fields become

$$\begin{aligned}
\mathbf{E}_0(r,\omega)&=\int d^3q\,\mathbf{E}_0(\mathbf{q},\omega)\exp(i\mathbf{qr}),\\
\mathbf{H}_0(\mathbf{r},\omega)&=\int d^3q\,\mathbf{H}_0(\mathbf{q},\omega)\exp(i\mathbf{qr}),\\
\mathbf{E}_0(\mathbf{q},\omega)&=\frac{4\pi i}{\omega\varepsilon_0}\frac{\omega^2\mathbf{j}(\mathbf{q},\omega)-c^2\mathbf{q}(\mathbf{qj}(\mathbf{q},\omega))}{q^2c^2-\omega^2\varepsilon_0}\\
\mathbf{H}_0(\mathbf{q},\omega)&=-\frac{4\pi ic[\mathbf{qj}(\mathbf{q},\omega)]}{q^2c^2-\omega^2\varepsilon_0}.
\end{aligned}\tag{8}$$

The equations for the first-approximation fields,

$$\delta\mathbf{E}=\mathbf{E}-\mathbf{E}_0,\ \delta\mathbf{H}=\mathbf{H}-\mathbf{H}_0, \tag{9}$$

are the same as the macroscopic Maxwell's equations in a homogeneous medium with permittivity ε_0 and a given current density

$$\delta\mathbf{j}(r,\omega)=(i\omega/4\pi)\chi(\mathbf{r})\mathbf{E}_0(\mathbf{r},\omega). \tag{10}$$

A solution of the equation for the first-approximation fields thus differs from (8) by the substitution $\mathbf{H}_0\to\delta\mathbf{H}$, $\mathbf{E}_0\to\delta\mathbf{E}$, $\mathbf{j}_0\to\delta\mathbf{j}$, so that, for example we have

$$\delta\mathbf{H}(\mathbf{r},\omega)=\int d^3q\,\delta\mathbf{H}(\mathbf{q},\omega)\exp(i\mathbf{qr}), \tag{11}$$

$$\delta\mathbf{H}(\mathbf{q},\omega)=-4\pi ic\frac{[\mathbf{q}[\mathbf{q}\delta\mathbf{j}(\mathbf{q},\omega)]]}{q^2c^2-\omega^2\varepsilon_0}, \tag{12}$$

where

$$\chi(\mathbf{q})=(2\pi)^{-3}\int d^3r\,\chi(\mathbf{r})\exp(-i\mathbf{qr}), \tag{13}$$

$$\mathbf{E}_0(\mathbf{q},\omega)=(2\pi)^{-3}\int d^3r\,\mathbf{E}_0(\mathbf{r},\omega)\exp(-i\mathbf{qr}). \tag{14}$$

The angular and frequency distribution of the radiated energy is given by $(r\to\infty)$

$$d^2E(\mathbf{n},\omega)=(cr^2/\varepsilon_0^{1/2})\,|\mathbf{H}_0(\mathbf{r},\omega)+\delta\mathbf{H}(\mathbf{r},\omega)|^2d\omega d\Omega\quad(\mathbf{n}\equiv\mathbf{r}/r). \tag{15}$$

In the particular case in which there is no radiation in the zeroth approximation, $\mathbf{H}_0(\mathbf{r},\omega)$ falls off more rapidly than $1/r$ at large r, and as $r\to\infty$ we are left with only $\delta\mathbf{H}(\mathbf{r},\omega)$ in (15). To determine the asymptotic behavior of $\delta\mathbf{H}(\mathbf{r},\omega)$ at large distances, it is convenient to work from the known asymptotic equation

$$\int d^3q\frac{f(\mathbf{q})\exp(i\mathbf{qr})}{(k+q+i\delta)(k-q+i\delta)}=-\frac{2\pi^2}{r}f(k\mathbf{n})\exp(ikr), \tag{16}$$

which is valid at $kr\gg 1$ for functions $f(\mathbf{q})$ which have no singularities. The use of (16) requires consideration of an infinitely small positive imaginary part of ε_0. It then follows from (16) and (12) that at $kr\gg 1$ ($\mathbf{k}\equiv k\mathbf{n}$) we have

$$\delta\mathbf{H}(\mathbf{r},\omega)=\frac{2\pi^2\omega}{cr}\exp(ikr)\int d^3l\,\chi(\mathbf{k}-\mathbf{l})\,[\mathbf{k}\mathbf{E}_0(\mathbf{l},\omega)]. \tag{17}$$

If we are interested in the problem in which there is radiation even in the zeroth (macroscopic) approximation, then the quantity $\delta\mathbf{H}(\mathbf{r},\omega)$ receives contributions both from the scattering of the field of the particle, accompanied by the formation of transverse waves, and from the scattering of the radiation present in the zeroth approximation. The contribution from the scattering of the radiation can be ignored, since it leads to small corrections on the order of $a/\lambda\ll 1$.

When we substitute $\mathbf{E}_0(\mathbf{l},\omega)$ into (17), however, we need consider in $\mathbf{E}_0(\mathbf{l},\omega)$ only the field of the charge, for which we can assume $qc\gg\omega$ in (8). We thus find

$$\mathbf{E}_0(\mathbf{q},\omega)=-(4\pi i/\omega\varepsilon_0q^2)\mathbf{q}(\mathbf{qj}_0(\mathbf{q},\omega)) \tag{18}$$

and

$$\delta H(r,\omega)=-\frac{8\pi^3}{c\varepsilon_0r}\exp(ikr)\int d^3l\frac{[\mathbf{kl}]}{l^2}(\mathbf{lj}_0(\mathbf{l},\omega)). \tag{19}$$

On the other hand, writing $\mathbf{H}_0(r,\omega)$ in accordance with (8) as

$$\mathbf{H}_0(\mathbf{r},\omega)=-\frac{4\pi i}{c}\int d^3q\frac{[\mathbf{qj}_0(\mathbf{q},\omega)]}{q^2-k^2}\exp(i\mathbf{qr}),$$

and again using the asymptotic equation (16), we easily find the following result in the limit $r\to\infty$:

$$\mathbf{H}_0(\mathbf{r},\omega)=(8\pi i/cr)\exp(ikr)\,[\mathbf{kj}_0(\mathbf{k},\omega)]. \tag{20}$$

From (15), (19), and (20) we find

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega\,d\Omega}=\frac{(2\pi)^6}{c\varepsilon_0^{1/2}}\left|[\mathbf{kj}_0(\mathbf{k},\omega)]-\int d^3l\frac{[\mathbf{kl}]}{\varepsilon_0l^2}\chi(\mathbf{k}-\mathbf{l})(\mathbf{lj}_0(\mathbf{l},\omega))\right|^2. \tag{21}$$

Since the functional dependence $\chi(\mathbf{r})$ is determined by irregularities in the electron distribution in the atoms, the effective values of $\mathbf{k}-\mathbf{l}$ in $\chi(\mathbf{k}-\mathbf{l})$—for which $\chi(\mathbf{k}-\mathbf{l})$ is nonzero—are equal in order of magnitude to $1/a$: $|\mathbf{k}-\mathbf{l}|_{\text{eff}}\sim 1/a\gg k$. The effective values of l in the integral in (21) are thus large in comparison with k. We can thus rewrite (21) as

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega\,d\Omega}=\frac{(2\pi)^6}{c\varepsilon_0^{1/2}}\left|[\mathbf{kj}_0(\mathbf{k},\omega)]-\int d^3l\frac{[\mathbf{kl}]}{\varepsilon_0l^2}\chi(-\mathbf{l})(\mathbf{lj}_0(\mathbf{l},\omega))\right|^2. \tag{22}$$

Since we have

$$[\mathbf{kj}_0(\mathbf{k},\omega)]=(-i\omega/8\pi^3)\,[\mathbf{kd}(\omega)] \tag{23}$$

for a nonrelativistic particle, where

$$\mathbf{d}(\omega)=(2\pi)^{-1}\int dt\,e\mathbf{r}_a(t)\exp(i\omega t) \tag{24}$$

is the Fourier transform of the dipole moment of a charge moving in accordance with the law $\mathbf{r}_a(t)$, we can transform (22) to

$$d^2E(\mathbf{n},\omega)=\frac{e^2\omega^4\varepsilon_0^{1/2}}{4\pi^2c^3}\,|\,[\mathbf{n}(\mathbf{d}(\omega)+\mathbf{Q}(\omega))]\,|^2\,d\omega\,d\Omega,\quad (25)$$

where

$$\mathbf{Q}(\omega)=i\int\frac{dt}{2\pi}\int\frac{d^3l\,\chi(-\mathbf{l})}{l^2\omega\varepsilon_0}\mathbf{l}(\mathbf{l}\mathbf{v}_0(t))\exp[i\omega t-i\mathbf{l}\mathbf{r}_0(t)].\quad (26)$$

Here, in contrast with (23), we cannot ignore the quantity $i\mathbf{l}\cdot\mathbf{r}_0$ in the argument of the exponential function, since we have $l\sim\omega/v$ and $\omega t\sim\mathbf{l}\cdot\mathbf{r}_0$.

In the limit $\mathbf{Q}\to 0$, expression (25) becomes the usual formula for dipole radiation. The quantity $\mathbf{Q}(\omega)$ can thus be thought of as an effective correction to the Fourier transform of the dipole moment of a charge which is made to reflect the presence of irregularities in the medium. An integration over the emission direction $\mathbf{n}$ leads to the emission spectrum

$$\frac{dE}{d\omega}=\frac{2e^2\omega^4\varepsilon_0^{1/2}}{3\pi c^3}|\mathbf{d}(\omega)+\mathbf{Q}(\omega)|^2.\quad (27)$$

3. RADIATION BY A CHARGE IN UNIFORM MOTION IN AN AMORPHOUS MEDIUM

For a charge moving at a constant velocity $\mathbf{v}_0$ ($\mathbf{r}_0=\mathbf{v}_0t$), the Fourier transform $\mathbf{d}(\omega)$ vanishes at nonzero frequencies, so we have

$$\mathbf{Q}(\omega)=\frac{i}{v}\int d^2l_\perp\chi\left(-\mathbf{l}_\perp-\frac{\mathbf{v}\omega}{v^2}\right)\frac{\mathbf{l}_\perp+\mathbf{v}\omega/v^2}{l_\perp^2+(\omega/v)^2},$$

where $\mathbf{l}_\perp\equiv\mathbf{l}-\mathbf{v}(\mathbf{v}\mathbf{l})/v^2$, and (25) becomes

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega\,d\Omega}=\frac{e^2\omega^4}{4\pi^2c^3v^2\varepsilon_0^{3/2}}\left|\int\frac{d^2l_\perp\chi[-\mathbf{l}_\perp-\mathbf{v}\omega/v^2]}{l_\perp^2+(\omega/v)^2}\times\left([\mathbf{n}\mathbf{l}_\perp]+[\mathbf{n}\mathbf{v}]\frac{\omega}{v^2}\right)\right|^2.\quad (28)$$

We consider an amorphous medium, and we assume that the atomic shells of neighboring atoms do not overlap. (Even in condensed media, the interatomic distances are usually larger than the atomic diameters by a factor of 3 to 5.) If only a small fraction of the total number of electrons participate in valence bonds, this is a reasonable approximation. For many-electron atoms, these assumptions hold, so the number density of electrons in the medium can be written

$$n^e(\mathbf{r})=\sum_a F(\mathbf{r}-\mathbf{R}_a),\quad (29)$$

where $F(\mathbf{r}-\mathbf{R}_a)$ is the number density of electrons in an atom whose center of mass is at the point $\mathbf{R}_a$, and $F(\mathbf{r}-\mathbf{R}_a)$ is found by taking an average over the quantum-mechanical state of the atomic electrons and over the thermal vibrations of the atom. The number of electrons in an atom is evidently

$$Z=\int d^3r\,F(\mathbf{r}-\mathbf{R}_a).\quad (30)$$

It is convenient to introduce

$$f(\mathbf{k})=\int d^3r\,F(\mathbf{r})\exp(-i\mathbf{k}\mathbf{r}),\quad (31)$$

which differs from the Fourier transform F by a factor of $(2\pi)^3$. From (30) we have $f(0)=Z$. Then from (7) and (29) we find

$$\chi(\mathbf{r})=\frac{4\pi e^2}{m\omega^2}\left[n_0Z-\sum_a F(\mathbf{r}-\mathbf{R}_a)\right],\quad (32)$$

and, by virtue of (24),

$$\chi(\mathbf{q})=\frac{4\pi e^2}{m\omega^2}\left[n_0Z\delta(\mathbf{q})-(2\pi)^{-3}\sum_a f(\mathbf{q})\exp(-i\mathbf{q}\mathbf{R}_a)\right].\quad (33)$$

The quantity $\chi(\mathbf{q})$, and correspondingly, the radiation energy (28) depend on the coordinates of the atoms, $\mathbf{R}_a$. Since the specific coordinate values of the atoms of an amorphous substance are unknown—all that is known is that they are distributed uniformly in space on the average—we can average this expression over the positions of the atoms for the calculation in (28).

Assuming that the coordinates of different atoms are independent, we can average over the coordinates of the atoms in the following way:

$$\left\langle\sum_a\exp(-i\mathbf{q}\mathbf{R}_a)\right\rangle=n_0(2\pi)^3\delta(\mathbf{q})\,.\quad (34)$$

It follows that $\langle\chi(\mathbf{q})\rangle=0$.

Substituting in (33) and (28), we find the appearance of double sum over atoms, in which the diagonal and off-diagonal terms of the sum should be averaged separately:

$$\left\langle\sum_a\sum_b\exp(i\mathbf{q}\mathbf{R}_a+i\mathbf{q}'\mathbf{R}_b)\right\rangle=n_0(2\pi)^3\delta(\mathbf{q}+\mathbf{q}')+n_0^2(2\pi)^6\delta(\mathbf{q})\delta(\mathbf{q}').\quad (35)$$

We thus find

$$\langle\chi(\mathbf{q})\chi^*(\mathbf{q}')\rangle=n_0(4\pi e^2/m\omega^2)^2(2\pi)^{-3}\delta(\mathbf{q}-\mathbf{q}').\quad (36)$$

The substitution gives rise to a factor

$$\delta\left(\frac{v\omega}{v^2}-\frac{v\omega}{v^2}\right)\equiv\delta(0)=\frac{vT}{2\pi},$$

where T is the total observation time. We then have

$$\langle d^2E(\mathbf{n},\omega)/d\omega\,d\Omega\rangle=T\frac{n_0e^6}{4\pi^4m^2c^3v\varepsilon_0^{3/2}}\times\int\frac{d^2l_\perp}{[l_\perp^2+(\omega/v)^2]^2}\left\{[\mathbf{n}\mathbf{l}_\perp]+[\mathbf{n}\mathbf{v}]\frac{\omega}{v^2}\right\}^2\times\left|f\left(\mathbf{l}_\perp+\frac{\mathbf{v}\omega}{v^2}\right)\right|^2,\quad (37)$$

An integration over the emission angle yields the radiation spectrum

$$\left\langle\frac{dE(\omega)}{d\omega}\right\rangle=T\frac{2n_0e^6}{3\pi^3m^2c^3v\varepsilon_0^{3/2}}\times\int\frac{d^2l_\perp}{[l_\perp^2+(\omega/v)^2]^2}\left|f\left(\mathbf{l}_\perp+\frac{\mathbf{v}\omega}{v^2}\right)\right|^2.\quad (38)$$

We assume that the distribution of electrons in an atom is spherically symmetric. Then $f(\mathbf{q})$ does not depend on the direction of $\mathbf{q}$; i.e., $f(\mathbf{q})\equiv\Phi(q^2)$ or

$$f\left(\mathbf{l}_\perp+\frac{\mathbf{v}\omega}{v^2}\right)\equiv\Phi\left(l_\perp^2+\left(\frac{\omega}{v}\right)^2\right).$$

Integrating over the directions $\mathbf{l}_\perp$, and introducing the new integration variable $u=l_\perp^2+(\omega/v)^2$, we can put (38) in the form

$$\left\langle \frac{dE(\omega)}{d\omega} \right\rangle = T \frac{2n_0 e^6}{3\pi^2 m^2 c^3 v \varepsilon_0^{3/2}} \int_{(\omega/v)^2}^{\infty} \frac{du}{u} |\Phi(u)|^2. \quad (39)$$

The quantity $f(\mathbf{q})$ is nonzero in the region $q \lesssim 1/a$. The integral is thus negligible for $\omega/v > 1/a$. Since we have $f(\mathbf{q}) \to Z$ in the limit $q \to 0$, we can assume, for an order-of-magnitude estimate of the integral, that we have $\Phi(u) = Z$ for $u < (1/a)^2$ and $\Phi(u) = 0$ for $u > (1/a)^2$. The spectrum

$$\left\langle \frac{dE(\omega)}{d\omega} \right\rangle \sim T \frac{n_0 Z^2 e^6}{m^2 \pi^2 c^3 \varepsilon_0^{3/2} v} \ln\left(\frac{v}{a\omega}\right) \quad (40)$$

then vanishes for $v < a\omega$.

4. RADIATION BY A CHARGE MOVING THROUGH A CUBIC SINGLE CRYSTAL

In a single crystal, the number density of electrons, $n^e(\mathbf{r})$ as well as $\chi(\mathbf{r})$ are periodic functions of the coordinates, so we should replace (13) by the expansion

$$\chi(\mathbf{r}) = \sum_{\mathbf{g}} \chi_{\mathbf{g}} \exp(i\mathbf{g}\mathbf{r}), \quad (41)$$

where $\mathbf{g}(N_x \cdot 2\pi/b_x, N_y \cdot 2\pi/b_y, N_z \cdot 2\pi/b_z)$ is a system of reciprocal-lattice vectors; b_x, b_y, b_z are the periods of the lattice along directions x, y, z; and N_x, N_y, N_z are arbitrary integers. The substitution

$$\chi(\mathbf{q}) \to \sum_{\mathbf{g}} \chi_{\mathbf{g}} \delta(\mathbf{q}-\mathbf{g})$$

allows us to easily find the following result from (26):

$$\mathbf{Q}(\omega) = i \int \frac{dt}{2\pi} \sum_{\mathbf{g}} \chi_{\mathbf{g}} \frac{\mathbf{l}(\mathbf{l}\mathbf{v}_0(t))}{l^2 \omega \varepsilon_0} \exp[i\omega t - i\mathbf{l}\mathbf{r}_0(t)]. \quad (42)$$

For a charge in uniform motion we have

$$\mathbf{Q}(\omega) = i \sum_{\mathbf{g}} \chi_{\mathbf{g}} \frac{\mathbf{g}}{\varepsilon_0 g^2} \delta(\omega - \mathbf{g}\mathbf{v}_0). \quad (43)$$

The δ-function in (43) means that for a given direction of the velocity of the charge emission is possible only for discrete values of the frequency:

$$\omega_N = \frac{2\pi v_x}{b_x} N_x + \frac{2\pi v_y}{b_y} N_y + \frac{2\pi v_z}{b_z} N_z. \quad (44)$$

If the velocity is directed along one of the crystallographic axes, e.g., the x axis, we would have

$$\omega_N = \frac{2\pi v}{b_x} N. \quad (45)$$

If this frequency is to lie in the interval under consideration, (5), we must impose the inequalities

$$(e^2/hc) \ll N(v/c) \ll 1. \quad (46)$$

Substitution of (43) into (25) leads to the following result for the radiation emitted by a nonrelativistic charge in uniform motion in a crystal:

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega\, d\Omega} = T \frac{e^4 \omega^4}{(2\pi)^3 c^3 \varepsilon_0^{3/2}} \sum_{\mathbf{g}} |\chi_{\mathbf{g}}|^2 \frac{[\mathbf{n}\mathbf{g}]^2}{g^4} \delta(\omega - \mathbf{g}\mathbf{v}).$$

Noting that for $\mathbf{g} \neq 0$ we have

$$\chi_{\mathbf{g}} = (4\pi e^2/m\omega^2) n_{\mathbf{g}}^e,$$

we find

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega\, d\Omega} = T \frac{2e^6}{\pi m^2 c^3 \varepsilon_0^{3/2}} \sum_{\mathbf{g}} |n_{\mathbf{g}}^e|^2 \frac{[\mathbf{n}\mathbf{g}]^2}{g^4} \delta(\omega - \mathbf{g}\mathbf{v}). \quad (47)$$

An integration over angle leads to the emission spectrum

$$\frac{dE(\omega)}{d\omega} = T \frac{16 e^6}{3 m^2 c^3 \varepsilon_0^{3/2}} \sum_{\mathbf{g}} |n_{\mathbf{g}}^e|^2 \frac{1}{g^2} \delta(\omega - \mathbf{g}\mathbf{v}). \quad (48)$$

It can be seen from (48) that the emission spectrum varies with the direction of the initial velocity of the charge with respect to the crystal. This change also occurs in the frequency interval (5) if the wavelength of the radiation is large in comparison with the lattice constant, and the crystal can be regarded as homogeneous for the propagation of such radiation through the crystal the crystal. Expression (48) thus describes a new, previously unrecognized type of orientation effect for the radiation spectrum emitted by a nonrelativistic charge in a crystal.

5. EMISSION BY A NONRELATIVISTIC CHANNELED ION

As an example of the transition radiation emitted by a charge in nonuniform motion, we consider the radiation emitted by an ion in a regime of planar channeling with a nonrelativistic velocity (this case is possible if the wavelength of the ion is short in comparison with the lattice constant b). For a heavy ion, the transverse motion in the average lattice potential can be treated as a classical motion in a parabolic $2D$ potential well.[8] The transverse motion then constitutes a harmonic oscillation, and the law of motion of the charge is

$$\mathbf{r}_0(t) = \mathbf{v}t + \mathbf{a} \cos \omega_0 t, \quad (49)$$

where the vector $\mathbf{v}$ is parallel to the crystallographic planes forming the channel, and the vector $\mathbf{a}$ is perpendicular to $\mathbf{v}$ and comparable in order of magnitude to the lattice constant b.

For a motion of this type, the bremsstrahlung can be treated in the dipole approximation. The frequency of the radiation will be the same as the frequency (ω_0) of the transverse oscillations of the ion in the channel. The bremsstrahlung component of the total radiation is described by the term $\mathbf{d}(\omega)$ inside the absolute-value sign in (25).

To estimate the transition-radiation component, we need to find $\mathbf{Q}(\omega)$. Substitution of the law of motion (49) into (42) leads to

$$\mathbf{Q}(\omega) = \frac{i}{\omega \varepsilon_0} \sum_{s=-\infty}^{s=\infty} (-i)^s \times \sum_{\mathbf{g}} \chi_{-\mathbf{g}} \frac{\mathbf{g}}{g^2} (\mathbf{g}\mathbf{v} - s\omega_0 \mathbf{g}\mathbf{a}) J_s(\mathbf{g}\mathbf{a}) \delta(\omega - \mathbf{g}\mathbf{v} - s\omega_0). \quad (50)$$

where we have used the well-known formula for the expansion of an exponential function in Bessel functions J_s:

$$\exp(-iu \cos \varphi) = \sum_{s=-\infty}^{s=\infty} (-i)^s J_s(u) \exp(-is\varphi). \quad (51)$$

The value of the reciprocal-lattice vector $\mathbf{g}$ in (50) is unrelated to the channeling of the ion; it is related to the interaction of the field of the charge with the lattice. The direction of $\mathbf{g}$ is thus not associated with the orientation of the channel. The argument of the Bessel function in (50), however, depends on the orientation of **g** with respect to **a**. If $\mathbf{g}\cdot\mathbf{a} \ll 1$, all the Bessel functions with $s \neq 0$ are small in comparison with $J_0(\mathbf{g}\cdot\mathbf{a})$, and the radiation becomes monochromatic with the frequency $\omega = \omega_0$, like the bremsstrahlung. In the case $\mathbf{g} \| \mathbf{a}$ the minimum value of the argument of the Bessel function, $\mathbf{g}\cdot\mathbf{a} \sim 2\pi(a/b)$, is of order unity, since the amplitude a of the transverse oscillations of the ion in the channel is comparable in magnitude to the lattice constant and to the width of the channel. In this case we cannot ignore the value of J_s in comparison with J_0, and, in contrast with the bremsstrahlung case, there is a substantial emission of higher harmonics with frequencies $\omega = s\omega_0$.

A qualitative result of incorporating the transition radiation is thus the appearance of radiation at higher harmonics. In examining the harmonics we can completely ignore the bremsstrahlung and consider only those vectors $\mathbf{g} \| \mathbf{a}$ which make the greatest contribution. For values $\mathbf{g}\cdot\mathbf{a} \gg 1$, the Bessel functions become rapidly oscillating functions, contributing little in the integration over the frequency interval $d\omega$. In evaluating the intensities of the harmonics, it is thus sufficient to consider only the terms with $\mathbf{g}\cdot\mathbf{a} \sim 1$ near the minimum values of **g**. It thus becomes a straightforward matter to derive the angular and frequency distribution of the energy of the transition radiation emitted by a nonrelativistic channeled ion from (25):

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega\, d\Omega} = T\frac{e^4\omega^4}{(2\pi)^3c^2\varepsilon_0^{3/2}}[\mathbf{na}]^2 \times \sum_{s=-\infty}^{s=\infty} \Big|\sum_{\mathbf{g}} \chi_{\mathbf{g}} J_s(\mathbf{ga})\Big|^2 \delta(\omega - s\omega_0), \quad (52)$$

where T is the total observation time, and $\mathbf{v} \perp \mathbf{g} \| \mathbf{a}$. We need to stress that at frequencies $\omega \neq \omega_0$ expression (52) gives the total radiation emitted by a channeled ion. An integration over angle gives us the spectrum of the transition radiation,

$$\frac{dE(\omega)}{d\omega} = T\frac{e^4\omega^4a^2}{3\pi^2c^3\varepsilon_0^{1/2}}\sum_{s=-\infty}^{s=\infty}\Big|\sum_{\mathbf{g}}\chi_{\mathbf{g}}J_s(\mathbf{ga})\Big|^2\delta(\omega - s\omega_0), \quad (53)$$

which is the same as the total radiation spectrum for frequencies $\omega \neq \omega_0$.

6. DISCUSSION OF RESULTS

The most interesting result of this study is the prediction of a new orientation effect: a dependence of the radiation spectrum emitted by a nonrelativistic charge in a crystal on the orientation of the initial velocity of the charge with respect to the crystal. When photons whose wavelength λ is much greater than the lattice constant b are emitted, it is usually assumed that orientation effects are proportional to the ratio b/λ and thus negligible. In the process of the transition-radiation emission of such waves, however, the field of the charge is involved, and the "wavelength" of this field is on the order of $\lambda v/c$, i.e., much shorter than λ. This interaction of the field of the charge with the crystal depends on the orientation of the crystal, and this dependence becomes important at $\lambda v/c \sim b$. The latter circumstance has the consequence that the orientation effect is determined not by the small parameter b/λ but by the large parameter $bc/v\lambda$, so the orientation effect becomes appreciable. The existence of this effect has been demonstrated only in frequency interval (5), but the general nature of the effect leads us to expect that it would also occur at lower frequencies (although it would of course be seen at lower velocities). Since this effect is associated with transition radiation, the best choice for a study of this effect would be heavy particles, for which bremsstrahlung is slight.

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