Fluctuation corrections to the index of refraction. Nematics

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The well-known solution of the electrostatic problem of finding the average dielectric constant in the presence of local fluctuations is extended in two directions: 1) in electrostatics—for an anisotropic medium with anisotropically distributed tensor fluctuations and 2) in electrodynamics—for the equally general problem of determining the correction to the effective dielectric constant ε. In the second case the imaginary part $\mathrm{Im}(\delta\varepsilon)$ describes the well-known scattering-induced attenuation of a wave. The correction to the real part of the effective dielectric constant is calculated in detail for ordinary waves in a uniaxial nematic. This correction causes the phase velocity of the ordinary wave to become direction dependent, which in turn results in spatial dispersion. The possibility of measuring $\mathrm{Re}(\delta\varepsilon)$ is discussed in connection with recent experiments on the passage of an ordinary wave through a nematic.

1. INTRODUCTION

Rayleigh scattering of light by fluctuations of the refractive index of air is perceived, even by an inexperienced observer, in two ways: first, as scattered light (i.e., waves propagating in directions which are different from their initial direction of propagation), e.g., the blue color of the sky, and second, according to the law of conservation of energy, as attenuation of the initial waves. The latter phenomenon is clearly seen at sunrise and sunset as reddening of the solar disk owing to the elimination of predominantly blue photons. Remarkably, however, the visible contour of the sun remains sharp, though it undergoes refraction in the vertically inhomogeneous atmosphere.

The attenuation of a beam passing undistorted through a medium can be described phenomenologically by inroducing a constant positive imaginary part in the effective dielectric constant. It is intuitively obvious that there are grounds for expecting also a correction to $\mathrm{Re}(\varepsilon)$, i.e., to the refractive index; however, because of spatial nonlocality the Kramers-Kronig relations are not directly applicable here. This question was studied in detail in Refs. 1 and 2 for the propagation of light in a turbulent atmosphere with scalar fluctuations $\delta\varepsilon$.

On the other hand, the solution of the problem of determining the effective dielectric constant of a mixture, i.e., once again a medium with fluctuations of ε, is well known in electrostatics. The solution is given in Sec. 9 of Ref. 3, and the answer has the form

$$\varepsilon^{\mathrm{eff}}=\bar{\varepsilon}-\langle(\delta\varepsilon)^2\rangle/3\bar{\varepsilon},\qquad \langle(\delta\varepsilon)^2\rangle=\langle\delta\varepsilon(\mathbf{r})\delta\varepsilon(\mathbf{r}+\boldsymbol{\rho})\rangle,\ \rho\to 0. \tag{1}$$

It is obvious that in the considered second-order approximation in $\delta\varepsilon$ the answer must be a linear functional of the correlation function of the fluctuations. However such a remarkably simple form of this functional is a consequence of three assumptions: 1) The spatial average of the dielectric constant is a scalar, $\langle\varepsilon_{ik}\rangle=\bar{\varepsilon}\delta_{ik}$; 2) the fluctuations are scalar, $\delta\varepsilon_{ik}(\mathbf{r})=\delta\varepsilon(\mathbf{r})\delta_{ik}$; and, 3) these fluctuations are isotropic, $\langle\delta\varepsilon(\mathbf{r})\delta\varepsilon(\mathbf{r}+\rho)\rangle=f(|\rho|)$. In Sec. 2 below we shall obtain a general quasistatic formula in the same second-order approximation in $\delta\varepsilon$ and we shall verify that if we forgo any of these assumptions the linear functional will become much more complicated.

It turns out that it is even more difficult to calculate $\mathrm{Re}\varepsilon^{\mathrm{eff}}+i\mathrm{Im}\varepsilon^{\mathrm{eff}}$ for electromagnetic waves in anisotropic media. The solution of this problem is given in Sec. 3 and incorporates the results of Sec. 2 as a special case.

At first glance, the calculation of $\mathrm{Re}(\varepsilon^{\mathrm{eff}})$ seems too be of little interest, since the corresponding integrals usually diverge for small values of ρ, i.e., for large Fourier vectors $\mathbf{q}$. Moreover, it is usually impossible to determine $\bar{\varepsilon}_{ik}$ separately, i.e., without corrections, from experiment.

In this respect nematic liquid crystals provide a rare possibility. First, it is well known that in nematics strong fluctuations of the orientation of the director are present and, as a consequence of this, strong scattering of light occurs. Second, and most important, neglecting the fluctuations, a nematic is an optically uniaxial uniform medium. In the latter medium, as is well known, the phase velocity of the ordinary wave (o-wave) does not depend on the angle between the axis $\mathbf{n}$ and the wave vector $\mathbf{k}_o$. This assertion is the basis of the previously predicted[4] and recently observed[5] effect in which an ordinary wave passes virtually undistorted, though with appreciable attenuation owing to scattering, through a thick (~ 5 mm) cell.

The absence of parasitic distortions suggests that it may be possible to measure the small difference of the refractive indices for different angles between $\mathbf{n}$ and $\mathbf{k}_o$; this difference should be due precisely to fluctuations. The calculation of $\delta\varepsilon$ for an ordinary wave in a nematic is presented in Sec. 4.

2. EFFECTIVE STATIC DIELECTRIC CONSTANT OF A MEDIUM IN THE PRESENCE OF LOCAL FLUCTUATIONS

We shall study the system of equations of electrostatics

$$\mathrm{div}\,\mathbf{D}(\mathbf{r})=0, \tag{2a}$$

$$\mathrm{rot}\,\mathbf{E}(\mathbf{r})=0, \tag{2b}$$

$$D_i(\mathbf{r})=\varepsilon_{ik}(\mathbf{r})E_k(\mathbf{r}) \tag{2c}$$

for a medium with nonuniform and anisotropic local permittivity $\varepsilon_{ik}(\mathbf{r})$. We shall represent the intensity of the electric field $\mathbf{E}$ and the induction $\mathbf{D}$ in the form

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$$E_i(\mathbf{r})=E_i+A_i(\mathbf{r}),\quad D_i(\mathbf{r})=D_i+B_i(\mathbf{r}),$$
$$\varepsilon_{ik}(\mathbf{r})=\varepsilon_{ik}+v_{ik}(\mathbf{r}),\qquad (3)$$

where quantities without an argument **r** are the spatially averaged values. The problem is to determine $\varepsilon_{ik}^{\text{eff}}$, i.e., the relation between **D** and **E** with accuracy up to terms $\sim v^2$ inclusively, by iterating Eqs. (2) with respect to $v_{ik}(\mathbf{r})$. We change to the Fourier representation

$$f(\mathbf{r})=\int \tilde{f}(\mathbf{k})\exp(i\mathbf{kr})\,d^3\mathbf{k}.$$

Then

$$\{\tilde{E}_i(\mathbf{k}),\ \tilde{D}_i(\mathbf{k}),$$
$$\tilde{\varepsilon}_{ik}(\mathbf{k})\}=\{E_i, D_i, \varepsilon_i\}\delta^{(3)}(\mathbf{k})+\{\tilde{A}_i(\mathbf{k}), \tilde{B}_i(\mathbf{k}), \tilde{v}_{ik}(\mathbf{k})\},\quad (4)$$

where the fluctuations $\tilde{A}$, $\tilde{B}$, and $\tilde{v}$ are of the same order of magnitude in v and they do not have a δ-function singularity at $k=0$. To satisfy the equation curl $\mathbf{E}=0$ it is sufficient to introduce the potential $\mathbf{E}(\mathbf{r})=-\nabla\varphi(\mathbf{r})$. Then $\tilde{E}_i(\mathbf{k})=-k_i\tilde{\varphi}(\mathbf{k})$. The equation div $\mathbf{D}(\mathbf{r})=0$ becomes $k_i\tilde{D}_i(\mathbf{k})=0$, which, using the constitutive equation (2c) at $\mathbf{k}=0$, gives

$$-ik_i\varepsilon_{ij}k_j\tilde{\varphi}(\mathbf{k})+k_i\tilde{v}_{ij}(\mathbf{k})E_j=k_i\tilde{D}_i(\mathbf{k})=0.\qquad (5)$$

The transform $\tilde{\varphi}(\mathbf{k})$ is determined from here and must be substituted again into the constitutive equation (2c). Because of the spatial uniformity of the averages the correlation function of the fluctuations depends only on the distance ρ:

$$\langle v_{ik}(\mathbf{r})v_{lm}(\mathbf{r}+\boldsymbol{\rho})\rangle=\int T_{iklm}(\mathbf{k})\exp(i\mathbf{k}\boldsymbol{\rho})\,d^3\mathbf{k},$$
$$\langle\tilde{v}_{ik}(\mathbf{k}_1)\tilde{v}_{lm}(\mathbf{k}_2)\rangle=\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2)T_{iklm}(\mathbf{k}_2).\qquad (6)$$

As a result, the relation between the average values of **D** and **E** assumes the form $D_i=\varepsilon_{ij}^{\text{eff}}E_j$, where

$$\varepsilon_{ij}^{\text{eff}}=\varepsilon_{ij}-\int T_{ipqj}(\mathbf{k})k_pk_q(k_s\varepsilon_{st}k_t)^{-1}\,d^3\mathbf{k}.\qquad (7)$$

The formula (7) solves the problem. If all three assumptions made above are made, i.e., 1) $\varepsilon_{ik}=\bar{\varepsilon}\delta_{ik}$, 2) $T_{ipqj}(\mathbf{k})=\delta_{ip}\delta_{qj}T(\mathbf{k})$ and 3) $T(\mathbf{k})=T(|\mathbf{k}|)$, then the wave vector vanishes from the integrand in Eq. (7) and we arrive at the well-known formula (1). This calculation is conceptually similar to the "distributed dipole" approach to the description of spatial dispersion in the theory of the scattering of light[6] and to the calculation of the phase velocity of waves in spatially nonuniform media.[7,8]

If the medium is isotropic on the average but the fluctuations $v_{ik}(\mathbf{r})$ are not scalar, then the phenomenological expression derived in Ref. 9 can be employed for the tensor $T_{iklm}(\mathbf{k})$. Although in this case $\varepsilon_{ij}^{\Phi}\propto\delta_{ij}$, the relation (1) is no longer applicable.

Media in which the fluctuations of the chemical composition are frozen, for example glass, where it can be assumed that $v_{ik}(\mathbf{r})=\delta_{ik}v(\mathbf{r})$, are of great interest. Suppose that glass has been heated, strongly deformed at constant volume (as done in the drawing of lightguides), and annealed in order to relieve the stresses. It can then be expected that $v_{\text{new}}(\mathbf{r})_{ik}=v(\hat{C}\mathbf{r})\delta_{ik}$, where $\hat{C}$ is the matrix of an affine transformation, so that the isotropic correlation function will become anisotropic at the outset. In particular, in the case when the fiber is drawn along the z axis the fluctuation contribution will cause $\varepsilon_{zz}^{\text{eff}}$ to become somewhat greater than $\varepsilon_{xx}^{\text{eff}}=\varepsilon_{yy}^{\text{eff}}$.

Some conclusions regarding the properties of the fluctuations $v(r)$ can be drawn by measuring the birefringence. These properties are also important in connection with Rayleigh scattering of light in fiber-optic waveguides and minimizing the losses in them.

3. PROPAGATION OF LIGHT IN AN ANISOTROPIC MEDIUM WITH FLUCTUATIONS

We shall write Maxwell's equations for the complex amplitude of a monochromatic field $\mathbf{E}(\mathbf{r})e^{-i\omega t}$ in the form

$$(\text{rot rot}\,\mathbf{E})_i-\frac{\omega^2}{c^2}\varepsilon_{ik}(\mathbf{r})\,(\mathbf{E}(\mathbf{r}))_k=0.\qquad (8)$$

It is assumed here that the fluctuations of the tensor $\hat{\varepsilon}$ change over time intervals that are many orders of magnitude longer than the period of the light oscillations. We shall seek the solution of Eq. (8) in the form of a plane wave with a wave vector **k** and a field **A** scattered by the permittivity fluctuations $\hat{v}(\mathbf{r})$; see Eq. (3). We shall find the fluctuation correction to the propagation constant or, which is approximately the same thing, to the effective dielectric constant.

As was done for the quasistatic problem we shall designate by a vector without the argument **E** the coefficient of the exponential function:

$$\mathbf{E}(\mathbf{r})=\mathbf{E}\exp(i\mathbf{kr})+\mathbf{A}(\mathbf{r}).\qquad (9)$$

The polarization of the wave **E** and the length of the wave vector **k** in a given direction are strictly determined by the crystal and correspond to one of the two solutions of the unperturbed eigenvalue equation. Taking into account the scattered wave **A** (both real and virtual) and the perturbations $\hat{v}$, this equation assumes the form

$$\left(\mathbf{k}^2\delta_{ik}-k_ik_k-\frac{\omega^2}{c^2}\varepsilon_{ik}\right)E_k-\frac{\omega^2}{c^2}\langle v_{ik}(\mathbf{r})e^{-i\mathbf{kr}}A_k(\mathbf{r})\rangle=0,\qquad (10)$$

where the angle brackets $\langle\ldots\rangle$ once again denote averaging over an ensemble of fluctuations. As in the case of a static field, it is convenient to solve the equation for the correction to the field

$$\text{rot rot}\,\mathbf{A}-\frac{\omega^2}{c^2}\hat{\varepsilon}\mathbf{A}(\mathbf{r})=\frac{\omega^2}{c^2}\hat{v}(\mathbf{r})\mathbf{E}e^{i\mathbf{kr}}\qquad (11)$$

in the Fourier representation

$$\tilde{A}_i(\mathbf{p})=G_{ik}(\mathbf{p})\tilde{v}_{kj}(\mathbf{p}-\mathbf{k})E_j,\qquad (12)$$

where

$$(\hat{G}^{-1}(\mathbf{p}))_{ik}=-\varepsilon_{ik}+\frac{c^2}{\omega^2}(\mathbf{p}^2\delta_{ik}-p_ip_k).\qquad (13)$$

Substitution of Eqs. (12) and (13) into Eq. (10) gives the correction $\delta\varepsilon_i^{\text{eff}}(\mathbf{k})$ in the form

$$\delta\varepsilon_{ij}^{\text{eff}}(\mathbf{k})=\int d^3\mathbf{q}\,G_{st}(\mathbf{k}+\mathbf{q})T_{istj}(\mathbf{q}).\qquad (14)$$

This formula gives the solution of our problem. If **p** is assumed to be fixed and (ω/c) is made to approach zero, then the expression for $\hat{G}(\mathbf{p})$ assumes the form

$$G_{ik}(\mathbf{p})=-\frac{p_ip_k}{(\mathbf{p}\hat{\varepsilon}\mathbf{p})}+O(\omega/c),$$

whence the quasistatic expression (7) of the preceding section is obtained. The expression for $\hat{G}$ contains two poles, corresponding to the contributions of waves which are actually scattered and pass to infinity. These poles must be bypassed according to Sommerfeld's radiation principle, i.e., by making the substitutions $\omega\to\omega+i\gamma$ and $\gamma\to+0$. The contribution of a pole corresponds to to the imaginary (more precisely, antihermitian) part of $\delta\hat{\varepsilon}^{\rm eff}$ and describes the scattering-induced attenuation of the starting wave. The extinction coefficient could be obtained by calculating the scattered field $\mathbf{A}(\mathbf{r})$ in the far zone of the coordinate representation and integrating over solid angles the corresponding Poynting vector. The method of calculation based on the treatment of the pole in Eq. (14) is more convenient in the case of strongly anisotropic media, since in this method all effects such as the mismatch of the group velocity and the phase velocity, etc., are taken into account automatically.

The real (i.e., Hermitian) part of $\delta\varepsilon_{ik}$ from Eq. (14) is given by a principal-value integral and corresponds to the contribution of virtual-scattering processes, i.e., excitation of waves $\tilde{A}(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{r}}$ with the Fourier argument $\mathbf{p}$ which does not satisfy the wave equation.

The contribution of fluctuations corresponding to $|\mathbf{p}|\gg\omega/c$ is described by the quasistatic expression. The corresponding part of the integral over $d^3\mathbf{p}$ in most physical models of media diverges as $|\mathbf{p}|\to\infty$. It is natural to include this quasistatic part in $\langle\varepsilon_{ik}\rangle$ by redefining or renormalizing the latter. One can hope that the remaining part of the contribution from fluctuations with moderate values of $|\mathbf{q}|=|\mathbf{p}-\mathbf{k}|\sim\omega/c$ will be given by a converging integral and depends on the magnitude and direction of the vector $\mathbf{k}$. In this manner the contribution of permittivity fluctuations to spatial dispersion will be calculated in the distributed-dipole approach.[6] Specific calculations for nematic liquid crystals will be performed below in Sec. 4.

4. FLUCTUATION CONTRIBUTION TO $\delta\varepsilon$ FOR NEMATICS

We shall study a nematic liquid crystal, which, neglecting equilibrium thermodynamic fluctuations, we shall assume is uniform and has an undisturbed director orientation $\mathbf{n}^0$. We shall write the permittivity tensor at the frequency of light in the form

$$\varepsilon_{ik}=\varepsilon_\perp\delta_{ik}+\varepsilon_a n_i^0(\mathbf{r})n_k^0(\mathbf{r}). \tag{15}$$

Here $\varepsilon_a=\varepsilon_\parallel-\varepsilon_\perp$ is the difference of the two principal values of the tensor parallel and transverse to the axis. The unperturbed nematic is thereby an optically uniaxial medium. We shall represent the fluctuations of the permittivity tensor in the form

$$v_{ik}(\mathbf{r})=\varepsilon_a[n_i^0\delta n_k(\mathbf{r})+n_k^0\delta n_i(\mathbf{r})]. \tag{16}$$

We note that the term $\varepsilon_a\delta n_i\delta n_k$ contains fluctuations of the director at one point. Its average value can be included in the quasistatic part of $\delta\hat{\varepsilon}$, and we shall neglect its fluctuations as higher order infinitesimals.

Specific calculations for a nematic are based on two considerations. First, the explicit form of the matrix $\hat{G}(\mathbf{p})$ can be found for a uniaxial unperturbed medium:

$$G_{ik}(\mathbf{p})=\frac{\omega^2}{c^2}D_1^{-1}D_2^{-1}\Big\{\delta_{ik}D_2+\Big(\varepsilon_\parallel-\frac{c^2}{\omega^2}\mathbf{p}^2\Big)p_ip_k+\varepsilon_a\frac{\omega^2}{c^2}\varepsilon_\perp n_in_k-\varepsilon_a(\mathbf{p}\mathbf{n}^0)(p_in_k^0+p_kn_i^0)\Big\}, \tag{17}$$

$$D_1=\mathbf{p}^2-\frac{\omega^2}{c^2}\varepsilon_\perp,\qquad D_2=\mathbf{p}^2\varepsilon_\perp+\varepsilon_a(\mathbf{p}\mathbf{n}^0)^2-\varepsilon_\perp\varepsilon_\parallel\frac{\omega^2}{c^2}. \tag{18}$$

Second, the equilibrium thermodynamic fluctuations of the director in a nematic liquid crystal are determined by its resistance to strain. If $K_F=K_1=K_2=K_3$ are the Frank constants (in ergs/cm), then in the single-constant approximation (see Ref. 10) we have

$$\langle\delta n_i(\mathbf{r})\delta n_k(\mathbf{r}+\boldsymbol{\rho})\rangle=\int B_{ik}(\mathbf{q})e^{i\mathbf{q}\boldsymbol{\rho}}d^3\mathbf{q}, \tag{19}$$

$$B_{ik}(\mathbf{q})=\frac{1}{(2\pi)^3}\frac{k_BT}{K_F\mathbf{q}^2}(\delta_{ik}-n_i^0n_k^0), \tag{20}$$

where k_BT is the temperature in energy units, whence we obtain without difficulty an expression for the correlation function $\langle\hat{v}\hat{v}\rangle$ of the fluctuations. We thus obtain a result for $\mathrm{Re}(\delta\varepsilon_{ik}^{\rm eff}(\mathbf{k}))$ in the form of a principal-value integral over $d^3\mathbf{p}$, and an expression for $\mathrm{Im}(\delta\varepsilon_{ik}^{\rm eff}(\mathbf{k}))$ in the form of an integral over the solid angle.

Extinction, i.e., real scattering of light in nematics by equilibrium fluctuations, was calculated in detail in Refs. 10 and 11, and for this reason we shall cite the corresponding results here only in order to compare $\mathrm{Re}(\delta\varepsilon)$ and $\mathrm{Im}(\delta\varepsilon)$.

We shall be interested primarily in the corrections to the propagation of an ordinary wave. Real scattering from an ordinary wave occurs only into an extraordinary (e) wave. This can be easily seen from the expression (15) for $v_{ik}(\mathbf{r})$ and the expression $\mathbf{e}_o=[\mathbf{k}\mathbf{n}^0]/|[\mathbf{k}\mathbf{n}^0]|$ for the unit polarization vector $\mathbf{e}_o$. Since fluctuations in a nematic are particularly large for small values of $\mathbf{q}$, scattering is strongest in the forward direction. Let the wave vector $\mathbf{k}_o$ of the incident ordinary wave make an angle α with the director (Fig. 1). In most nematics the anisotropy much smaller than unity, i.e., $n_a=\varepsilon_\parallel^{1/2}-\varepsilon_\perp^{1/2}<0.25$. For this reason for the extraordinary wave the direction of the group velocity is close to that of the phase velocity. For ordinary and extraordinary waves propagating in the same direction the difference of their wave vectors is $\Delta k^0\approx(\omega/c)n_a\sin^2\alpha$. For this reason, for $o\to e$ scattering through an angle β the quantity

$$\mathbf{q}^2=(\Delta k^0)^2+(\omega_2/c^2)n_e^2\beta^2$$

is doubled and the deflection is given by

$$|\Delta\beta|=|n_a/n|\sin^2\alpha.$$

On the other hand, the amplitude of $o\to e$ scattering through a small angle contains the polarization factor $\sin\alpha$, and the intensity therefore contains the factor $\sin^2\alpha$. As a result, the differential extinction coefficient near the forward direction is equal to

$$\frac{dR(\beta=0)}{do}(\mathrm{cm}^{-1}\cdot\mathrm{sr}^{-1})=\frac{1}{(2\pi)^2}\frac{\omega^2}{c^2}n^2\frac{k_BT}{K_F}\frac{1}{\sin^2\alpha}, \tag{21}$$

i.e., as $\alpha\to0$ it increases as $(\sin\alpha)^{-2}$, and the integral over the angles decreases approximately as $[dR(\beta=0)/do]|\Delta\beta|^2\propto\sin^2\alpha$. An approximate analytic expression can be obtained by integrating over the scattering angles,

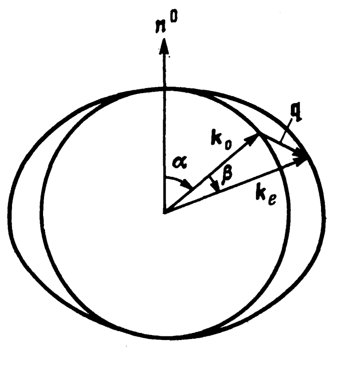

FIG. 1. The geometry of real scattering of an incident ordinary wave into an extraordinary wave; $\mathbf{n}^0$ is the director vector, which is also the optic axis; the circle and the ellipse are the surfaces of the wave vectors; $\mathbf{q} = \mathbf{k}_e - \mathbf{k}_o$ is the wave vector of the scattering fluctuation; β is the scattering angle.

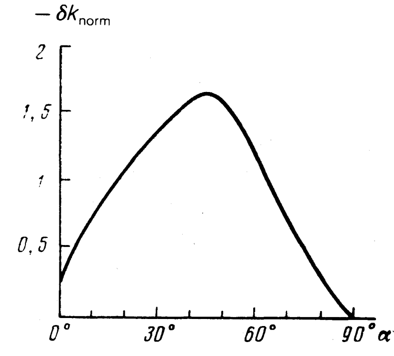

FIG. 3. The correction to the real part of the wave vector of an ordinary wave versus the angle α between $\mathbf{k}_0$ and the director for the same conditions as in Fig. 2. The quantity δk (cm^{-1}) is obtained by multiplying $\delta k_{\rm norm}$ from the figure by the factor $(\lambda^2 K_F)^{-1}$, where λ is given in microns and K_F in units of 10^{-6} dynes.

$$R\,(\text{cm}^{-1}) \approx \frac{1}{2\pi}\frac{\omega^2}{c^2}\frac{k_B T}{K_F} n_a^2 \sin^2\alpha \ln(n_a^{-2}\sin^{-2}\alpha). \qquad (22)$$

It corresponds to the well-known result that light scattering by molecules in a nematic crystal is weaker when the light propagates along the axis of the crystal.

More accurate results can be obtained by numerical integration. Figure 2 shows the function $R(\alpha)$ in the one-constant approximation for the nematic 5CB with $n_\parallel = 1.71$ and $n_\perp = 1.526$.

The calculation of $\text{Re}(\delta\varepsilon^{\text{eff}})$ is technically more difficult. First of all, the integral for $\text{Re}(\delta\varepsilon^{\text{eff}})$ diverges as $q_{\max}$ for large $|\mathbf{q}|$. The difference $\text{Re}[\delta\varepsilon(\mathbf{k}, \omega)] - \text{Re}[\delta\varepsilon(\mathbf{k}, \omega \to 0)]$ between the quantity we require and the quasistatic expression is given by an integral that diverges as $\ln q_{\max}$, but with a coefficient that vanishes on integrating over the angles $\mathbf{q}/q$. Finally, when the integral is calculated numerically it is necessary to add to the integrand a singular function whose principal-value integral over the required region vanishes identically while the singularity cancels the singularity for the initial expression near the pole.

The qualitative result is as follows. If the correction to $\text{Re}(\delta\varepsilon)$ for the ordinary wave is measured relative to the same for $\text{Re}(\delta\varepsilon)$ in the case of propagation transverse to the axis ($\alpha = 90°$), then as α decreases from 90° to 0° the quantity $\text{Re}(\delta\varepsilon)$ becomes increasingly more negative. This corresponds to the contribution of the fluctuations with the longest wavelengths ($\mathbf{q} \to 0$). The extraordinary-wave photon virtually admixed by fluctuations would have for the vector $\mathbf{p} = \mathbf{k}_o + \mathbf{q} \approx \mathbf{k}_o$ the frequency $\omega_e = \omega n_\perp / n_e$. This situation is analogous to the interaction of light with a resonance atomic transition ω_{ab} under conditions when $\omega_{ab} < \omega$ and then $\delta\varepsilon < 0$. Compared with extinction, $\text{Re}(\delta\varepsilon)$ contains an extra power of $(k_o - k_e)$ in the denominator, and the effect of $\delta\varepsilon < 0$ becomes stronger as α approaches zero.

This circumstance can also be interpreted differently. Thanks to the strong excitation of long-wavelength fluctuations by thermal impulses, $\text{Re}(\delta\varepsilon)$ is determined primarily by the interaction of the ordinary wave with the nearest extraordinary wave. It is well known from quantum mechanics that in second-order perturbation theory interacting terms repel one another. This means that the refractive index decreases for the ordinary wave (it was already lower, since $n_a = n_\parallel - n_\perp > 0$) and increases for the extraordinary wave. The role of small values of $|\mathbf{q}|$ is to enhance this effect for small values of α as compared with $\alpha = 90°$.

Finally, for very small values of α the difference between the ordinary and extraordinary waves is very small and the absolute magnitude of the contribution to $\text{Re}(\delta\varepsilon)$ once again decreases.

Figure 3 shows the results of the calculation of the three-dimensional singular integral for $\text{Re}(\delta k) = (\omega/c)\,\text{Re}(\delta\varepsilon^{\text{eff}})/2\varepsilon_\perp^{1/2}$ as a function of the angle α between the direction of propagation $\mathbf{k}_o$ and the director $\mathbf{n}^0$.

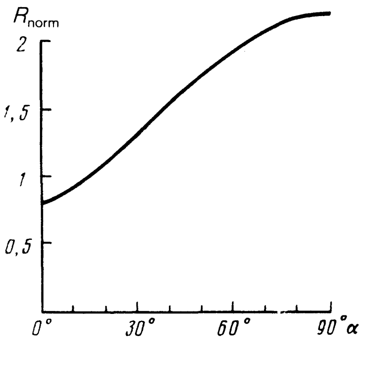

FIG. 2. The ordinary-wave extinction coefficient versus the angle α between the direction of propagation of the wave and the optic axis of the liquid crystal. The value of R (cm^{-1}, with respect to intensity) is obtained by multiplying $R_{\rm norm}$ from the figure by the factor $(\lambda^2 K_F)^{-1}$, where λ is the wavelength of light in microns and K_F is Frank's constant in units of 10^{-6} dynes.

5. DISCUSSION

We note first of all that the experimentally accessible cell thickness L must not be too large, so that the intensity of the transmitted coherent ordinary wave $I(L) = I_0 \exp(-RL)$ would make it possible to record the correction $\delta\varphi = \delta k L$ to the phase. Taking $I(L)/I_0 \approx e^{-4} = 1.8\%$, we obtain $\delta\varphi = 4\delta k / R$. This raises the question of the choice of medium for which the quantity $\delta k / R$ is as large as possible. The foregoing analysis shows that $R \sim n_a^2$ and $\delta k \sim n_a$, so that it is best to use a medium with a moderate value of n_a (for example, $n_a \approx 0.01$–0.05). Extremely small values of n_a could be dangerous in that for them the conditions of adiabatic trapping of the polarization of the ordinary wave by the director may not be satisfied.

We shall now make some numerical estimates. Even for the unfavorable case of the nematic 5CB, for which $n_a = 0.25$, for $\alpha = 90°$ we have $R = 5.48$ cm^{-1} and $\delta k = 0$ (the latter, by definition). For $\alpha = 30°$ in the medium we have $R = 3.23$ cm^{-1} and $\delta k = 3.5$ cm^{-1}. Thus it may be possible to observe the effect even in this unfavorable case. Measuring the effect will make it possible to understand better the properties of the an ordinary wave, propagating just as well and virtually undistorted in nonuniform nematics.

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[1] V. I. Tatarskiĭ, *Wave Propagation in a Turbulent Atmosphere* [in Russian], Nauka, Moscow (1967).
[2] S. M. Rytov, Yu. A. Kravtsov, and V. I. Tatarskiĭ, *Introduction to Statistical Radio Physics* [in Russian], Nauka, Moscow (1978).
[3] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon Press, N.Y. [Russian original, Nauka, Moscow (1982)].
[4] N. B. Baranova and B. Ya. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 636 (1980) [JETP Lett. **32**, 622 (1980)].
[5] N. B. Baranova, I. V. Gusev, B. Ya. Zel'dovich, and V. A. Krivoshchekov, Pis'ma Zh. Eksp. Teor. Fiz. **52**, 745 (1990) [JETP Lett. **52**, 106 (1990)].
[6] N. B. Baranova and B. Ya. Zel'dovich, Usp. Fiz. Nauk **127**, 421 (1979) [Sov. Phys. Usp. **22**, 143 (1979)].
[7] E. I. Kats, Zh. Eksp. Teor. Fiz. **59**, 1854 (1970) [Sov. Phys. JETP **32**, 1004 (1971)].
[8] R. S. Akopyan, B. Ya. Zel'dovich, and N. V. Tabiryan, Zh. Eksp. Teor. Fiz. **83**, 1770 (1982) [Sov. Phys. JETP **56**, 1024 (1982)].
[9] B. Ya. Zel'dovich, Zh. Eksp. Teor. Fiz. **63**, 75 (1972) [Sov. Phys. JETP **36**, 39 (1973)].
[10] P. de Gennes, *The Physics of Liquid Crystals*, Clarendon Press, Oxford, 1974. [Russian translation, Mir, Moscow (1977)].
[11] D. Langevin and M. Bouchiat, J. Phys. Paris **36**, Suppl. C1, 197 (1975).

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