Bremsstrahlung from slow electrons scattered by ions in an external electromagnetic field

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We construct a theory of the bremsstrahlung emitted by slow electrons scattered by positively charged ions in a monochromatic external electromagnetic field of amplitude $f \ll \omega_f^{-5/3}$ ($\hbar = m_e = e = 1$). The theory enables us to describe the process at electron energies E_p that are of the same order as the energy ω_f carried by the field photons without invoking perturbation theory. We describe in some detail the resonant structure of bremsstrahlung spectra associated with the production of intermediate Rydberg atomic states. We show that the sharpest of these resonances occur in the spectral range delimited by the energy of the field photons, $0 < E_p < \omega_f$. The integrated (averaged over energy) characteristics of the process undergo changes in that energy range, the total output of bremsstrahlung photons with $0 \leqslant \omega \leqslant E_p$ ($E_p < \omega_f$) typically doubling, irrespective of the intensity of the inducing radiation (for the energy in which the field widths are greater than the natural widths). At energies $\omega > E_p$, bremsstrahlung is accompanied by absorption of field photons. The spectrum then contains, apart from lines corresponding to transitions to Rydberg levels (as for radiative capture when there is no external field), satellite lines due to field interactions with the Rydberg resonances.

1. INTRODUCTION

The radiative collisions involved in the elementary photonic processes that take place in an intense monochromatic radiation field are of interest in a variety of fields of modern physics, and have been carefully investigated both experimentally and theoretically. One of the more important of these areas of investigation has been laser-induced free-free transitions associated with the scattering of electrons by positively charged (single-atom) ions. Our objective in the present paper is to describe such transitions at low energies (of the same order as the photon energy of the external field), a subject that has received little theoretical attention even though it is extremely important to the spectroscopy of highly excited and resonant atomic states.

The induced field is assumed to be monochromatic and linearly polarized, and its interaction u^f with the electron + target system can be described by the time-periodic function

$$u^f = 2V^f \cos \omega_f t, \qquad V^f = \frac{\mathbf{D}f}{2}, \tag{1}$$

where **D** is the dipole moment operator, and f is the electric field amplitude ($f \ll 1$). Hereafter we employ the atomic system of units $\hbar = m_e = e = 1$. We also assume that f satisfies

$$\rho = \frac{pf}{\omega_f^2} \ll 1 \tag{2}$$

(p is the electron momentum), which means that the particle oscillation amplitude in the field is much smaller than its wavelength. We shall assume below that the external field does not alter the initial and final state of the target M^+.

A great deal of work has been devoted to the problem of spontaneous bremsstrahlung (see, e.g., Ref. 1 and the references therein). Induced bremsstrahlung—i.e., radiation at multiples of the external field frequency—has also been studied in detail.[2-5] One of the most recent theoretical achievements has been to demonstrate the feasibility of a semiclassical approximation that transcends (2).

Here we assume that (2) holds, but we allow the electron energy E_p and the frequency ω of the emitted photon to be of the order of the field frequency ω_f,

$$E_p, \ \omega \sim \omega_f. \tag{3}$$

In contrast to Refs. 2–5, therefore, here the semiclassical method (just as in the theory of induced photoassociation in a strong electromagnetic field[6]) is in principle not applicable. Furthermore, the spontaneous bremsstrahlung in an external field that we shall examine is a different physical phenomenon from induced bremsstrahlung considered in Refs. 2–5. A previous investigation[7] was confined to first-order perturbation theory in the interaction with the external field, but that approach (apart from its inability to describe those spectral ranges in which the induced field exerts its strongest influence) is unable to account for spontaneous transitions in which there is no energy exchange with the laser field (we shall be primarily concerned with the analysis of spectra produced by these transitions).

One feature of the multichannel quantum defect (MQD) method utilized below—and for the first time in the bremsstrahlung problem—is its ability to provide a formally exact description of the processes under consideration. Perturbation theory is only invoked here to calculate system parameters that are smooth functions of the energy and are insensitive to the details of the solutions obtained. There are thus no energy constraints on the functions derived below.

The features that come into play in the presence of a strong external electromagnetic field are graphically illustrated in Fig. 1, where we have depicted the energy levels of the $e^- + M^+$ system for $f = 0$ and $f \neq 0$. For $f = 0$ and a structureless ion, the free-free transition cross sections ought to exhibit a smooth energy dependence, since the continuum is homogeneous. For $f \neq 0$, however, the electron energy is only defined up to $k\omega_f$, so each of the continua k_0 contains discrete levels of states with $k > k_0$:

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$$E^{(k)}_{nl} = -\frac{1}{2(n-\mu_l)^2} + k\omega_f \quad (4)$$

(here n is the principal quantum number, and μ_l is the quantum defect for an electron with angular momentum l). Under those circumstances, scattering and bremsstrahlung can pass through a stage in which a Rydberg complex M** is formed—in other words, they are resonant processes.

We note here that specific theoretical problems involving the advent of laser-induced Rydberg resonances have been discussed in the literature. We may point, for example, to calculations of $e^- + H^+$ elastic scattering cross sections in an excimer laser field,[8] where the resonant structure of the scattering cross sections was elucidated by numerically integrating the strong-coupling equations. The behavior of a Rydberg electron in a laser field has also been considered in problems involving the photoionization and photodissociation of molecules,[9-11] and in the theory of radiative collisions of slow electrons with molecular ions.[12] In contrast to those processes, bremsstrahlung possesses a number of fundamentally new features that require special consideration, the reason being that a spontaneous radiative transition couples states of two continua, each of which has its own inherent resonant structure.

2. FUNDAMENTAL EQUATIONS OF THE THEORY

Condition (2) makes it possible to neglect the effect of the field on the state of a free electron (or to take it into account via perturbation theory). One is led to this conclusion by an analysis of the familiar expression for the wave function of a free electron in an electromagnetic field. The

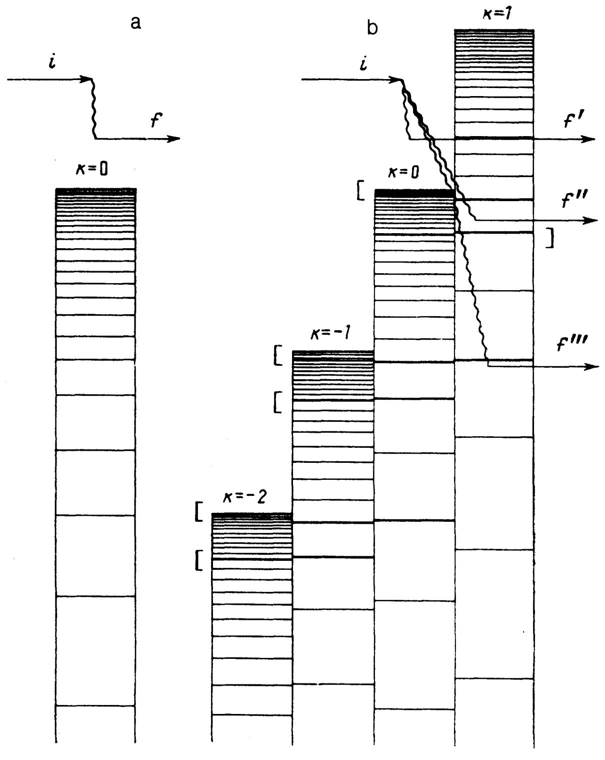

FIG. 1. Energy level diagram for the $e^- + M^+$ system in the absence (a) and presence (b) of an external monochromatic electromagnetic field. Those portions of the spectrum are shown for which, given the field strengths under consideration, there are significant interactions between Rydberg series with different photon occupation numbers.

results obtained in Refs. 2 and 3 are also relevant here; they show that electron scattering from "simple" targets that can be described in the Born approximation, in the presence of a field that satisfies (2), is essentially the same as in the field-free case. It should therefore be obvious that when (2) holds the possible appearance of strong field effects in bremsstrahlung from complex targets (for example, when the electron + target system has stationary or quasistationary states) has nothing to do with the motion of the electron in the asymptotic region $r \to \infty$ as with its behavior near the target—i.e., in a bounded spatial region.

This then enables us to solve for the wave functions of the $e^- + M^+$ system in a radiation field by invoking the formal mechanisms of quantum scattering theory (this was the first approach suggested in the theory of bremsstrahlung). Bremsstrahlung may then be most conveniently investigated via those formalisms that take advantage of the possibility of making a functional distinction between the contribution of direct processes (evolving against a smooth continuum background) and resonant processes (taking the effects of coupled states into account).

There are presently a number of approaches that put this idea into practice. Here we shall be following the technique proposed in Refs. 11–13, in which the separation between the resonant and background interactions is made using a rearrangement of the Lippmann-Schwinger equations for a set of continuum states perturbed by simultaneous interaction of an electron with the target and the radiation field.

The calculation of spontaneous bremsstrahlung reduces to the determination of the matrix elements for free-free transitions among the states $\Psi^{(0)}_p$ and $\Psi^{(k)}_{p'}$ of the $e^- + M^+$ system in the presence of a radiation field, which correlate with the given state of the free electron at infinity and with the given number of photons in the system N. Since $N \gg 1$ and $k = \Delta N \sim 1$, we shall, as usual, concern ourselves solely with the change k in the number of photons, using the superscript 0 to denote the initial wave function $\Psi^{(0)}_p$ and the superscript $k = 0, \pm 1, \pm 2, \ldots$ to denote the wave function of the final state. We incorporate into the symbol q all of the electron quantum numbers, including the angular momentum l and its projection m on the f axis (for simplicity, spin variables are omitted). The electron states $|q\rangle$ in the field of the ion M^+, which we treat in the "frozen-in core" approximation, will be assumed to be known. Using the general transformation approach outlined in Refs. 12 and 13 and the possibility of representing the Green's function for the $e^- + M^+$ system in the form[14-16]

$$G = \sum_q |q\rangle\langle q| \operatorname{ctg}\pi(\nu_q + \mu_l) + G_0, \quad (5)$$

where $|q\rangle$ is the electron wave function that is regular at the origin and G_0 is the part of the Green's function that is a smooth function of energy, we obtain for the multichannel scattering matrix T and the system wave function $\Psi^{(k)}_q$ in a radiation field

$$T^{k,k'}_{lm,l'm} = t^{k,k'}_{lm,l'm} + \sum_{l'',k''} t^{k,k''}_{lm,l''m} \operatorname{ctg}\pi(\nu_{k''} + \mu_{l''}) T^{k'',k'}_{l''m,l'm}, \quad (6)$$

$$\Psi^{(k)}_{lm} = \varphi^{(k)}_{lm} + \sum_{l',k'} T^{k,k'}_{lm,l'm} \operatorname{ctg}\pi(\nu_{k'} + \mu_{l'}) \varphi^{(k')}_{l'm}, \quad (7)$$

where $\nu_k = (-2\varepsilon_k)^{-1/2}$ and $\varepsilon_k = E - k\omega_f$ is the electron energy in channel k. Other quantities appearing in (6) and (7) are

$$t = V^f\Omega = V^f + V^f \sum_k |k\rangle G_0(\varepsilon_k)\langle k| V^f + O((V^f)^3), \quad (8)$$

where $|k\rangle$ is the photon part of the wave function, and

$$\varphi_q^{(k)} = \Omega|qk\rangle, \quad t_{qq'}^{kk'} = \langle qk|t|q'k'\rangle$$

(electron energies in state $|qk\rangle$ are those of channel k). The normalization condition for $|qk\rangle$ and $\varphi_q^{(k)}$ is

$$\langle\varphi|\varphi\rangle = \pi\delta(E-E')\delta_{kk'}.$$

Then t, q, and φ depend on the conserved quantum number m. The poles of $\operatorname{ctg}\pi(\nu_k + \mu_l) = \infty$ in (6) reproduce the position of the atomic Rydberg levels (5) unperturbed by the field in states with differing numbers of photons in the system. The operator t is constructed out of smooth continuum states. The matrix elements $t_{lm,l'm}^{kk'}$ and the functions $\varphi_q^{(k)}$ therefore depend smoothly on energy.

In the neighborhood of an isolated resonance, where

$$\operatorname{ctg}\pi(\nu_k + \mu_l) = \frac{1}{\pi n^3(\varepsilon_k - E_{nl})},$$

Eq. (6) may be used to derive the Breit-Wigner formula for scattering from such a resonance. It can also be shown that the representation (7) is consistent with the general precepts of the Fano theory:[17,18] the functions $\varphi_{lm}^{(k)}$ are analogous to the modified Fano states, and the solutions thus obtained (for the noninteracting resonances and noninteracting continua considered in Refs. 17 and 18) are structurally similar to the solutions obtained in Refs. 17 and 18.

Equations (6) and (7) place absolutely no constraints on the energy. We shall assume below, however, that the energies E_p and $E_{p'}$, as well as the frequencies ω_f and ω, are all small on an atomic scale, i.e.,

$$E_p,\ E_{p'},\ \omega_f,\ \omega \ll 1.$$

This condition enables us to make use of some reasonably simple quasiclassical representations for the functions $|q\rangle$ and G_0 in (8) (see Refs. 15, 16) and for the matrix elements $t_{qq'}^{kk'} = \langle qk|V^f|q'k'\rangle$ (see Ref. 1), assuming (as in Ref. 7) that $l \gg 1$ for the dominant angular momenta. Then

$$t_{qq'}^{kk'} = \langle lmk|V^f|l'mk'\rangle$$
$$= \frac{1}{2}\left(1 - \frac{m^2}{l^2}\right)^{1/2} \frac{fl^2}{2\cdot 3^{1/2}\omega_f}\left[K_{2/3}\left(\omega_f \frac{l^3}{3}\right) \pm K_{1/3}\left(\omega_f \frac{l^3}{3}\right)\right], \quad (9)$$

where $K_\nu(z)$ is the modified Bessel function of the second kind. The plus sign is used in (9) for $l' = l+1, k-1$ or $l' = l-1, k+1$, and the minus sign for $l' = l+1, k+1$ or $l' = l-1, k-1$.

There are in general a great many closed channels that need to be taken into account. There is in addition an important special case, in which laser-induced bremsstrahlung can be described by a rather simple set of analytic expressions. For Rydberg atoms (apart from hydrogen), there is no pure degeneracy, even in states of high angular momentum, where[19]

$$\mu_l = \frac{3}{4}\beta/l^5$$

(β is the dipole polarizability of the ionic core; typically, $\beta > 10$). Therefore, if

$$\mu_l \gg (fD)^2, \quad (10)$$

where $fD \sim f\omega^{-5/3} \ll 1$ characterizes the field widths and shifts of the Rydberg levels (proportional to $(fD)^2/n^3$), then as before, the kl and kl' series will not interact with one another in the presence of a field. At the field strengths and frequencies under consideration, $f \sim 10^{-7}$–10^{-6} and $\omega_f \sim 10^{-2}$–10^{-1}, condition (10) encompasses the range up to $l \sim 10$, which also suffices for calculating the total bremsstrahlung cross section. It can be shown that the bremsstrahlung cross section integrated over the photon and scattered electron angular variables may be expressed in terms of the matrix elements,

$$A_{lm,l'm'}^{s,k\to k'} = \langle \Psi_{lm}^{(k)} | r_s | \Psi_{l'm'}^{(k')} \rangle,$$

in a coordinate system in which the vector **f** defines the quantization axis:[1)]

$$\frac{d\sigma}{d\omega} = \frac{128\pi^2}{9}\left(\frac{\omega}{c}\right)^3 \frac{1}{p^2} \sum_{l'm',s,lm} |Y_{lm}(\mathbf{n}) A_{lm,l'm'}^{s,k\to k'}|^2. \quad (11)$$

Here c is the speed of light, $Y_{lm}(\mathbf{n})$ is a spherical harmonic, $r_s = rY_{1s}(\theta,\varphi)$ is the spherical component of the electron radius vector, and **n** is a unit vector directed along the momentum **p** in the coordinate system associated with **f**. As $V^f \to 0$, the wave functions $\Psi_{lm}^{(k)}$ turn into the basis functions $|lm,k\rangle$ of Eqs. (6) and (7), and are normalized in the same way as the latter.

3. RESONANCE STRUCTURE OF BREMSSTRAHLUNG SPECTRA: FIELD-INDUCED POPULATION OF ISOLATED SERIES OF RYDBERG STATES

An l electron of energy $E_p < \varphi_f$, with the z-component of its angular momentum equal to m (the z axis is parallel to **f**), can efficiently excite two series of Rydberg resonances—$k = 1, l+1$ and $k = 1, l-1$. The wave function of a system with $k = 0$ should therefore take the form

$$\Psi_{lm}^{(0)} = |lm,0\rangle + \eta_{0l}^{1,l+1}|l+1,m,1\rangle + \eta_{0l}^{1,l-1}|l-1,m,1\rangle$$
$$+ \eta_{0l}^{-1,l+1}|l+1,m,-1\rangle + \eta_{0l}^{-1,l-1}|l-1,m,-1\rangle, \quad (12)$$

where the η are coefficients obtained from the solution of (6). In addition to the states $|lm,0\rangle$ in (12), we have mixed in functions of the weak continua $|l+1,m,-1\rangle$ and $|l-1,m,-1\rangle$.[2)]

It can be shown that when

$$|t_{lm,l'm}^{0,\pm 1}| \ll 1$$

these coefficients are

$$\eta_{0l}^{1,l\pm 1} = \frac{t_{lm,l\pm 1m}^{01}}{z_{1,l\pm 1} + i\gamma_{l\pm 1}}, \quad \eta_{0l}^{-1,l\pm 1} = -it_{lm,l\pm 1m}^{0,-1},$$

where for convenience we have introduced the notation

$$z_{1,l} = \operatorname{tg}\pi\{[2(\omega_f - E)]^{-1/2} + \mu_l\};$$

here the γ_l are quantities associated with the field-induced halfwidths of the Rydberg levels, i.e.,

$$\Gamma_{nl}=\frac{\gamma_l}{\pi n^3},\qquad \gamma_l=(t^{01}_{lm,l+1m})^2+(t^{01}_{lm,l-1m})^2.$$

Making use of (12), in the energy range $E>\omega_f$, and neglecting two-photon transitions (in the t matrix), we have $\Psi^{(0)}_{lm}=|lm,0\rangle$.

Calculations of the amplitude of free-free transitions with and without changes in the number of field photons involve the wave functions $\Psi^{(0)\prime}_{lm}$ and $\Psi^{(k)\prime}_{lm}$, which are analogous to $\Psi^{(0)}_{lm}$, but which correspond to a finite electron state. According to (12), the field exerts its influence by mixing discrete states of the atom M into the free-electron wave function. These states are populated at some intermediate stage of the process, and quantum uncertainties prevent us from identifying the exact time at which they come into being. It is for this reason that the statement of the present problem and the need to take the interaction of discrete and continuum states into consideration to solve it differ from the requirements of more traditional problems involving the interaction of laser radiation with atoms and molecules (see, e.g., Ref. 20).

In the system at hand, the most typical conditions are those in which resonances corresponding to different types of transitions in the M** complex fail to overlap. Minor interference effects can therefore be neglected. Restricting our attention below to the case with **f** parallel to **p** and omitting details of the intermediate calculations, we present the final bremsstrahlung cross sections for a photon of frequency ω, summed over all quantum numbers.

Transitions within the energy band $0<E,E'<\omega_f$ involve a change of either one or three units of electron angular momentum. The corresponding bremsstrahlung cross section takes the form

$$\frac{d\sigma}{d\omega}=\sum_l\frac{d\sigma_l^{(0)}}{d\omega}\Bigg\{1+\frac{\xi^{(+)}_{l+1}\gamma_{l+1}}{\mathrm{tg}^2\pi(\nu_1+\mu_{l+1})+\gamma^2_{l+1}}$$

$$\times\left[\frac{\zeta^{(+)}_{l+1}\gamma_l}{\mathrm{tg}^2\pi(\nu_1'+\mu_l)+\gamma_l^2}+\frac{\zeta^{(-)}_{l+1}\gamma_{l+2}}{\mathrm{tg}^2\pi(\nu_1'+\mu_{l+2})+\gamma^2_{l+2}}\right]$$

$$+\frac{\xi^{(-)}_{l+1}\gamma_{l-1}(1-\delta_{l0})}{\mathrm{tg}^2\pi(\nu_1+\mu_{l-1})+\gamma^2_{l-1}}$$

$$\times\left[\frac{\zeta^{(-)}_{l-1}\gamma_l}{\mathrm{tg}^2\pi(\nu_1'+\mu_l)+\gamma_l^2}+\frac{\zeta^{(+)}_{l-1}\gamma_{l-2}(1-\delta_{l1})}{\mathrm{tg}^2\pi(\nu_1'+\mu_{l-2})+\gamma^2_{l-2}}\right]\Bigg\}. \qquad (13)$$

Here ν_1 and ν_1' are the effective principal quantum numbers,

$$\nu_1=[2(\omega_f-E)]^{-1/2},\qquad \nu_1'=[2(\omega_f-E')]^{-1/2};$$

$$\frac{d\sigma_l^{(0)}}{d\omega}=\frac{16l^5}{9}\frac{\omega}{c^3}\frac{1}{p^2}\left[K^2_{2/3}\left(\omega\frac{l^3}{3}\right)+K^2_{1/3}\left(\omega\frac{l^3}{3}\right)\right] \qquad (14)$$

characterizes the bremsstrahlung photons produced in the absence of a radiation field. The quantity

$$\gamma_l=\frac{f^2l^4}{24\omega_f^2}\left[K^2_{2/3}\left(\omega_f\frac{l^3}{3}\right)+K^2_{1/3}\left(\omega_f\frac{l^3}{3}\right)\right] \qquad (15)$$

is a smoothly varying (for $l\lesssim\omega_f^{-1/3}$) function of l related to the halfwidth Γ_{nl} of the Rydberg levels by $\Gamma_{nl}=\gamma_l/\pi n^3$;

$$\xi_l^{(\pm)}=\frac{\gamma_l^\pm}{\gamma_l}=\frac{[K_{2/3}(\omega_fl^3/3)\pm K_{1/3}(\omega_fl^3/3)]^2}{2[K^2_{2/3}(\omega_fl^3/3)+K^2_{1/3}(\omega_fl^3/3)]}$$

gives the relative partial halfwidth of a Rydberg complex, corresponding to decay with either a decrease ($\xi_l^{(+)}$) or increase ($\xi_l^{(-)}$) in electron angular momentum. Finally,

$$\zeta_l^{(\pm)}=\frac{[K_{2/3}(\omega l^3/3)\pm K_{1/3}(\omega l^3/3)]^2}{2[K^2_{2/3}(\omega l^3/3)+K^2_{1/3}(\omega l^3/3)]} \qquad (16)$$

is the relative partial probability of spontaneous emission, which is responsible for transitions between energy states E and $E'=E-\omega$, with an increase ($\zeta_l^{(+)}$) or decrease ($\zeta_l^{(-)}$) in angular momentum l.

In Eq. (13), the expression in curly brackets accounts for the change an external electromagnetic field induces in the nature of the process, the prime manifestation of which is the onset of resonance structure in the bremsstrahlung cross sections.

The maximum in the distribution function of detected bremsstrahlung photons occurs at

$$\omega_r=-\frac{1}{2(n-\mu_l)^2}+\frac{1}{2(n'-\mu_{l'})^2},$$

when the resonances at the initial and final electron energies match. In the resonance region, one then observes an abrupt rise in the intensity of spontaneous emission, just as in the absorption of light due to free-free transitions in resonant scattering of electrons by atoms[21] (here the effect is demonstrated by an e^-+H with Feshbach resonant states at energies of approximately 12 eV). Integrating over electron energy, which is specified by the function $F(E)$ with characteristic width ΔE much greater than the widths of the levels, we obtain

$$\left(\frac{\overline{d\sigma_l}}{d\omega}\right)_E=\frac{d\sigma_l^{(0)}}{d\omega}\Bigg\{1+\sum_{nn'}F(E_{nl})\frac{1}{(nn')^3}[\xi^{(+)}_{l+1}(\zeta^{(+)}_{l+1}f^{l+1,l}_{nn'}$$

$$\times(E_{nl+1}$$

$$-E_{n'l}-\omega)+\zeta^{(-)}_{l+1}f^{l+1,l+2}_{nn'}(E_{nl+1}-E_{n'l+2}-\omega))$$

$$+\xi^{(-)}_{l-1}(\zeta^{(-)}_{l-1}f^{l-1,l}_{nn'}(E_{nl-1}$$

$$-E_{n'l}-\omega)+\zeta^{(+)}_{l-1}f^{l-1,l-2}_{nn'}$$

$$\times(E_{nl-1}-E_{n'l-2}-\omega)(1-\delta_{l1}))(1-\delta_{l0})]\Bigg\}, \qquad (17)$$

where

$$E_{nl}=\omega_f-\frac{1}{2(n-\mu_l)^2},\qquad f^{ll'}_{nn'}(\varepsilon)=\frac{\Gamma_{nl}+\Gamma_{n'l'}}{\pi[\varepsilon^2+(\Gamma_{nl}+\Gamma_{n'l'})^2]}.$$

The quantity

$$\frac{1}{(nn')^3}\zeta^{(\pm)}_{l\pm1}\frac{d\sigma_l^{(0)}}{d\omega}$$

is the dipole cross section for a transition between Rydberg states nl and $n'l\pm1$.

Field effects show up in the line shapes—the stronger the field, the broader the lines. The field also determines the energy range $0<E<\omega_f$ in which the line structure of the spectrum appears, regardless of the field strength (a necessary condition here is that the induced broadening of the spectral linewidth Γ_{nl} be much greater than the natural linewidth $\Gamma_{nl}^{(r)} = \gamma^{(r)}/\pi n^3$). One important point is that the spectral line structure also shows up in quantities averaged over electron energy, which suggests that it may be possible to carry out high-precision measurements of the quantum defects of Rydberg atoms using nonmonochromatic electron beams.

It is not hard to show, using (17) and the fact that $\xi_l^{(+)} + \xi_l^{(-)} = 1$ and $\zeta_l^{(+)} + \zeta_l^{(-)} = 1$, that irrespective of the field strength, the total efflux of bremsstrahlung photons averaged over the Rydberg resonances differs by about a factor of two, depending on whether or not high-intensity radiation is present.

It can also be shown that when $\Gamma_{nl} \sim \Gamma_{nl}^{(r)}$, the γ_l in (13) and (17) turn out to be independent of the spontaneous decay rate $\gamma_l^{(r)}$, i.e., $\gamma_l = \gamma_l^+ + \gamma_l^- + \gamma_l^{(r)}$. Thus, when $\Gamma_{nl} \sim \Gamma_{nl}^{(r)}$ and the strength of the external field f decreases, its influence on spontaneous bremsstrahlung is also reduced, and in the limit $\gamma_l^{\pm} \ll \gamma_l^{(r)}$, we can trace out the transition to the field-free case.

Note that all of the foregoing features of spontaneous bremsstrahlung, which involves no energy exchange with the radiation field, appear solely within the energy range $0<E,\ E'<\omega_f$. Outside that range, for $E>\omega_f$ and derivatives $E'>0$, the cross section is

$$\frac{d\sigma_l}{d\omega} \approx \frac{d\sigma_l^{(0)}}{d\omega},$$

or in other words, it is the same as in the field-free case.

Bremsstrahlung accompanied by the absorption of field photons takes place either with no change in the electron angular momentum ($\Delta l = 0$) or with a change of two units ($\Delta l = \pm 2$). We may write the corresponding cross section in the form

$$\frac{d\sigma_l^{0\to-1}}{d\omega} = \overline{\left(\frac{d\sigma_l^{(0)}}{d\omega}\right)} \Big\{ \zeta_l^{(-)} \gamma_{l+1} \Big[1 + \frac{1}{\mathrm{tg}^2 \pi(\nu_0' + \mu_{l+1}) + \gamma_{l+1}^2} \Big]$$
$$+ \zeta_l^{(+)} \gamma_{l-1} \Big[1 + \frac{1}{\mathrm{tg}^2 \pi(\nu_0' + \mu_{l-1}) + \gamma_{l-1}^2} \Big] (1 - \delta_{l0}) \Big\},$$
$$\nu_0' = [2(\omega - E)]^{-1/2}. \tag{18}$$

Here $\overline{(d\sigma_l^{(0)}/d\omega)}$ is the radiative capture cross section ($\omega > E$) for Rydberg states in the absence of an external field, averaged over the energy interval $\Delta E \gg 1/n^3$. The overall picture is as follows. First, an electron is radiatively captured into a bound level with $k=0$, and then field ionization of the Rydberg complex M** takes place ($k=0, l\pm 1$). After emission of a bremsstrahlung photon, the electron acquires energy $E_{p''} = E_p + \omega_f - \omega$, the induced Rydberg resonances being substantial for arbitrary E_p (but with $E_{p'} < \omega_f$), while the dependence on the initial electron energy is smooth.

We can compare the cross section (18) with Eq. (20) of Ref. 7, where perturbation theory was employed to examine spontaneous transitions among continuum states. Indeed, for $\mu_l \to 0$, and noting that $\gamma_{l+1} \approx \gamma_{l-1} \approx \gamma$, Eq. (18) yields

$$\frac{d\sigma_l}{d\omega} \sim \Big[1 + \frac{1}{\mathrm{tg}^2 \pi \nu_0' + \gamma^2} \Big],$$

which in the wings of the resonance peaks gives a result consistent with Eq. (20) of Ref. 7,

$$\frac{d\sigma_l}{d\omega} \propto 1 + \mathrm{ctg}^2 \pi \nu_0'.$$

The radiative transition probabilities to the spectral range between ω and $\omega + \Delta\omega$, with $\Delta\omega \gg 1/n^3$ (as well as other functional dependences averaged over electron energy for a given photon frequency) contain no resonance features. The total energy-averaged cross section is then the same as for free-bound transitions in the absence of a field.

4. FIELD INTERACTION OF CLOSED-CHANNEL STATES

The field interaction of Rydberg states is significant when resonances belonging to different series fortuitously overlap; appropriate portions of the spectrum are marked with a bracket in Fig. 1. At $(k-1)\omega_f < E < k\omega_f$, the kl series has two continuum decays accessible, to $(k-1, l+1)$ and $(k-1, l-1)$, while the state $(k+1, l')$ cannot decay. We may therefore assume the $(k+1, l')$ level to have zero width. An admixture of closed-channel states can lead to bremsstrahlung involving the absorption of two photons from the external field.

Let us investigate this problem, given our intrinsic interest in the strong perturbation of a large set of Rydberg levels by a single level of a higher-lying Rydberg series. A level with $k=0$ and energy

$$E_{n_0 l} = -\frac{1}{2(n_0 - \mu_l)^2}$$

will be strongly perturbed, with

$$\frac{|t_{l,l\pm1}^{0,-1}|^2}{z_0} = \frac{|t_{l,l\pm1}^{0,-1}|^2}{\pi n_0^3 (E - E_{n_0 l})} \sim 1,$$

when that level is immediately juxtaposed with the continuum ($k=-1, l$), i.e., when $n_0 \ll n_{-1}$. Then $|t_{l,l\pm1}^{0,-1}| \ll 1$, and we still have $f\omega_f^{-5/3} \ll 1$. Since in the present case, as a result of field interaction, the state $E_{n_0 l}$ simultaneously perturbs two series of Rydberg resonances—$(k=-1, l+1)$ and $(k=-1, l-1)$—the equation for the energy eigenvalues, according to (6), takes the form

$$\tilde{z}\tilde{z}' - \varkappa' \tilde{z} - \varkappa \tilde{z}' = 0, \tag{19}$$
$$\tilde{z} = \mathrm{tg}\,\pi(\nu_{-1} + \mu_{l-1}) + i\gamma_{l-1}, \quad \tilde{z}' = \mathrm{tg}\,\pi(\nu_{-1} + \mu_{l+1}) + i\gamma_{l+1},$$
$$\varkappa = (t_{l,l+1}^{0,-1})^2/z_0, \quad \varkappa' = (t_{l,l-1}^{0,-1})^2/z_0.$$

For the sake of simplicity, we restrict attention to the case $|\mu_{l\pm1}| \ll 1$, obtaining

$$\mathrm{tg}\,\pi\nu_{-1} = -i\frac{\varkappa'\gamma_{l+1} + \varkappa\gamma_{l-1}}{\varkappa + \varkappa'} = -i\tilde{\gamma},$$
$$\mathrm{tg}\,\pi\nu_{-1}' = \varkappa + \varkappa' - i(\gamma_{l-1} + \gamma_{l+1}) + i\frac{\varkappa'\gamma_{l+1} + \varkappa\gamma_{l-1}}{\varkappa + \varkappa'} = \varkappa + \varkappa' - i\tilde{\gamma}'. \tag{20}$$

In the ν_{-1} series, all we see is a minor change in the widths of the levels, but there is a much more substantial realignment of the ν'_{-1} series:

$$E'_{n_{-1}}=-\omega_f-\frac{1}{2(n_{-1}-\mu)^2}-i\Gamma'_{n_{-1}},$$

and as a consequence of the appearance of the quantum defect

$$\mu=\frac{1}{\pi}\operatorname{arctg}[\varkappa+\varkappa']$$

there is a narrowing of the levels,

$$\Gamma'_{n_{-1}}=\frac{\tilde{\gamma}'}{\pi n_{-1}^3}\cos^2\pi\mu. \tag{21}$$

Under these circumstances, the interacting Rydberg states have no common decay continua, so in contrast to the effect examined in Ref. 22, the narrowing of the levels is related to the induced quantum defect in the decaying state.

The cross section for bremsstrahlung accompanied by two-photon absorption from the driving field then reproduces the structure of the perturbed levels of the $k=-1$ series adjacent to the spectral limit. We present below an expression for the bremsstrahlung cross section when the bremsstrahlung photon energy is

$$\omega=E+\frac{1}{2(n_0-\mu_l)^2},$$

so the spontaneous transition leads to a population of the level $n_0 l$, which strongly interacts with an infinite series of states $nl\pm 1$ (with $k=-1$) that decays to the $k=-2$ continuum:

$$\frac{d\sigma^{0\to-2}}{d\omega}=\overline{\left(\frac{d\sigma_l^{(0)}}{d\omega}\right)}\frac{\tilde{z}'^2\gamma_{l-1}\varkappa+\tilde{z}^2\gamma_{l+1}\varkappa'}{z_0|\tilde{z}\tilde{z}'-\varkappa'\tilde{z}-\varkappa\tilde{z}'|^2}, \tag{22}$$

where $\tilde{z},\tilde{z}'$ and $\varkappa,\varkappa'$ have been defined in (19), and $\overline{(d\sigma_l^{(0)}/d\omega)}$ in (18).

The denominator of (22), according to (20), reproduces two series of states with significantly different decay rates. It can be shown that the brightest satellite lines are produced by the series with the narrower resonance components.

We may write for the line intensity at a frequency $\omega=E+\omega_f+1/2n^2$ (as before, we assume $\mu_l\ll 1$, $l\gg 1$, and $\gamma_{l-1}\approx\gamma_{l+1}$)

$$\overline{(\sigma)}_\omega=\sigma_{n_0l}^{(0)}\frac{1}{\pi n^3}\frac{a_l}{(\Delta E)^2+a_l^2}, \tag{23}$$

where

$$\sigma_{n_0l}^{(0)}=\overline{\left(\frac{d\sigma_l^{(0)}}{d\omega}\right)}\frac{1}{n_0^3},$$

$$\Delta E=-\frac{1}{2n^2}+\frac{1}{2(n_0-\mu_l)^2}-\omega_f,\qquad a_l=\gamma_l/\pi n_0^3.$$

According to (23), the line satellites have a Lorentz distribution that depends on the distance $\Delta E=\Delta n/n^3$ from the level $n_0 l$. The intensity envelope then has the same shape as the isolated $n_0 l$ level would have against the $k=-1$ continuum. We also see from (23) that the total intensity (summed over Rydberg resonances n) equals $\sigma_{n_0l}^{(0)}$, corresponding to radiative capture into the $n_0 l$ level in the absence of a field.

5. CONCLUSION

We now summarize some of the results of our investigation into bremsstrahlung spectra in a high-intensity electromagnetic field. Above all, we note that according to (14), an analysis of the resonance structure of spectrum of the emitted photons makes it possible to measure directly the dependence of the quantum defect on the angular momentum l over a very wide range. In general, such measurements are not feasible using existing optical methods, such as multiphoton ionization or resonance fluorescence, owing to the small absorption cross sections involved. Our approach differs from the standard photoemission method[23] for the detection of laser-induced bremsstrahlung by relaxing the monochromaticity requirements on the incident electron beam. An important future application may be optical scanning of the electron distribution function $F(E)$ reproducible in the vicinity of the resonance levels E_{nl} over the energy range $0<E<\omega_f$. The determination of the electron distribution function in the vicinity of the long-wavelength wing is especially important in diagnostics of low-temperature plasmas.

One of the most interesting results is surely the possibility of field-induced stabilization of resonant states, which comes about through the appearance of an induced quantum defect in the decaying state at moderate external field strengths. Such states, known in atomic physics as BIC (bound in continuum[24]) states, have of late been under active study.

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[1)] If the incident beam is directed along **f**, then $Y_{lm}(\mathbf{n})=[(2l+1)/4\pi]^{1/2}\delta_{m0}$.

[2)] The last two terms in (12) make a second-order contribution, which is taken into account below only in Eq. (18) (for a more complete comparison with the results obtained in Ref. 7).

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