Theory of muon depolarization in a polycrystalline ceramic superconductor

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An expression is derived for the time evolution of the muon polarization in a polycrystalline, anisotropic, uniaxial type II superconductor.

1. Measurements of the London penetration depth of superconducting ceramics are typically made on polycrystalline samples. In this case the London depth is found by comparing experimental data with a theoretically predicted mean square width of the magnetic-field distribution.[1,2] However, this width makes a definite contribution to the depolarization of muons only very early in the process, while the polarization is more than 98% of its original value. The reason is that the moments M_{2n} in the expansion

$$P(t)=1+\sum_{n=1}^{\infty}\frac{(-1)^n M_{2n}t^{2n}}{(2n)!} \tag{1}$$

increase very rapidly with increasing n for a type II superconductor. As a result, only 2% of the experimental information is useful in a rigorous analysis. It is thus obvious that we need to work from a theoretical expression for the time evolution of the muon polarization which incorporates the distribution of the magnetic field, not simply its second moment.

Most generally, the expression for $P(t)$ in a transverse field can be written in the form

$$P(t)=R(t)\cos[\gamma Bt+\varphi(t)], \tag{2}$$

where $R(t)$ is the envelope of the precession curve, φ is an auxiliary phase which depends on the time, and B is the mean magnetic field at the muon.

Barford and Gunn[2] have recently pointed out that differences between the lower critical fields H_{c1} for the randomly oriented crystallites in a polycrystalline sample might give rise to a broadening of the field distribution and thus an additional depolarization.

The boundary conditions for an individual crystallite in a polycrystalline sample are exceedingly complex, and it is by no means obvious that the mean magnetic fields in the crystallites would be different. If they were, however, a determination of the London depth in polycrystalline samples by the muon method would become essentially impossible, because of the large uncertainty which would arise.

There is the possibility in principle of demonstrating experimentally that there is no dispersion of the mean field. Such a demonstration would require comparing the time evolution of the precession amplitude with that of the precession phase. As we will see below, such experiments would be difficult, requiring a high accuracy, since phase oscillations set in when the muon polarization becomes small. However, once it has been established that there is no dispersion of the mean fields determined from the various crystallites, one can find the London penetration depth from a single point of the $R(t)$ function, i.e., from the envelope of the precession curve on the part of this curve corresponding to early times, where the polarization is still more than 20% of the initial value.

2. The calculations below are based on solutions derived by Barford and Gunn[2] for the London equation with an anisotropic tensor Λ:

$$\Lambda_{ij}=\Lambda_\alpha\delta_{ij}+\Lambda_\beta n_i n_j, \tag{3}$$

where $\mathbf{n}$ is a unit vector along the c axis of the crystal. The quantity $\lambda_\perp=\Lambda_\alpha^{1/2}$ is usually adopted as the London depth. According to Ref. 2, the Fourier transform of the magnetic field is

$$\tilde{\mathbf{H}}(\mathbf{G})=(1+\Lambda_\alpha G^2)^{-1}\left(\mathbf{e}_z-\frac{\Lambda_\beta Q_z\mathbf{Q}}{1+\Lambda_\alpha G^2+\Lambda_\beta Q^2}\right)N\Phi_0. \tag{4}$$

Here $\mathbf{G}$ is a vector of the magnetic reciprocal lattice, $Q=[\mathbf{G}\times\mathbf{n}]$, N is the number of magnetic flux quanta through a unit area, and Φ_0 is the flux quantum. The magnetic field is directed along the z axis; $\mathbf{e}_z$ is a unit vector. Expression (4) corresponds to the region of intermediate magnetic fields, $H_{c1}\ll B\ll H_{c2}$, and to crystallite dimensions which are infinitely large in comparison with the magnetic lattice constant and in comparison with the magnetic-field penetration depth. Below we assume that there is no dispersion of the mean field.

The magnetic field inside the sample is

$$\mathbf{H}(\mathbf{r})=\sum_G\tilde{\mathbf{H}}(\mathbf{G})e^{i\mathbf{G}\mathbf{r}}, \tag{5}$$

and the deviation of the field from its mean value is given by

$$\mathbf{h}(r)=\mathbf{H}(\mathbf{r})-\mathbf{B}=\sum_{\mathbf{G}\neq 0}\tilde{\mathbf{H}}(\mathbf{G})e^{i\mathbf{G}\mathbf{r}}. \tag{6}$$

In the limit $r\to 0$, the sum in (6) diverges, but this is a well-known and integrable logarithmic divergence. We can restrict all the calculations to a region $r>r_0$ such that $h(r_0)\ll B$. The condition $h\ll B$ simplifies all the calculations substantially. The usual approach is to measure the projection of the polarization vector onto its original direction, which we take to be the x axis. We can then write

$$P_x(t)=\left\langle\frac{[(B+h_z)^2+h_y^2]^{1/2}}{(B+h)^2}\cos(|\mathbf{B}+\mathbf{h}|t\gamma+h_x)\right\rangle$$
$$\approx\langle\cos(|\mathbf{B}+\mathbf{h}|t\gamma)\rangle\approx\cos[t\gamma(B^2+2Bh_z)^{1/2}]. \tag{7}$$

Here γ is the gyromagnetic ratio of the muon. The last approximation in (7) is valid under the condition $t\ll B/(\gamma h^2)$. In sufficiently strong external fields, we thus need consider only the variations of the z component of the magnetic field. From expression (7) we find

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$$P_x(t)=\cos(Bt\gamma\{\langle\cos(h_z t\gamma)\rangle-\sin(Bt\gamma)\langle\sin(h_z t\gamma)\rangle, \quad (8)$$

which corresponds to (2), with an envelope

$$R(t)=[\langle\cos(h_z t\gamma)\rangle^2+\langle\sin(h_z t\gamma)\rangle^2]^{1/2}, \quad (9)$$

while the phase $\varphi(t)$ is determined by the conditions

$$\sin(\varphi(t))=\langle\sin(h_z t\gamma)\rangle/R(t),$$
$$\cos(\varphi(t))=\langle\cos(h_z t\gamma)\rangle/R(t). \quad (10)$$

We restrict the discussion below to the limiting case of an infinitely large anisotropy,

$$\Lambda_\alpha\ll\Lambda_\beta, \quad \Lambda_2=\Lambda_\alpha+\Lambda_\beta\approx\Lambda_\beta.$$

The basis vectors of the direct lattice then have the components

$$a_{1x}=a_1, \quad a_{1y}=0, \quad a_{2y}=\frac{3^{1/2}}{2}\frac{a_1}{\cos\theta}, \quad a_{2x}=\frac{a_1}{2}.$$

Here θ is the angle between the external magnetic field and the c axis of an individual crystallite. The components of the basis vectors of the reciprocal lattice are

$$b_{1x}=0, \quad b_{1y}=\frac{4\pi\cos\theta}{3^{1/2}a_1},$$
$$b_{2x}=\frac{2\pi}{a_1}, \quad b_{2y}=-\frac{2\pi\cos\theta}{3^{1/2}a_1}. \quad (11)$$

The number of magnetic flux quanta through a unit area is

$$N=\frac{2}{3^{1/2}}\frac{\cos\theta}{a_1^2}. \quad (12)$$

The reciprocal-lattice vector is

$$\mathbf{G}=k_1\mathbf{b}_1+k_2\mathbf{b}_2. \quad (13)$$

In very strong external fields we can assume that the condition $\Lambda_\alpha G^2\gg1$ holds for any $\mathbf{G}\neq0$. Using the condition $\Lambda_\alpha\ll\Lambda_\beta$, we can then replace (4) by

$$H_z(\mathbf{G})=\frac{N\Phi_0}{\lambda_\perp^2}\left(\frac{Q_x^2+Q_y^2}{Q^2}\right), \quad (14)$$

and from (11) and (13) we find

$$\frac{Q_x^2+Q_y^2}{Q^2}=1-\frac{(k_1-2k_2)^2\sin^2Q}{^3/_4k_1^2+(k_1-k_2)^2}. \quad (15)$$

The deviation of the z projection of the magnetic field from the mean value over the crystallite is then

$$h_z=\frac{\Phi_0\cos\theta}{3^{1/2}\cdot2\pi^2\lambda_\perp^2}\times\sum_{\substack{k_1,k_2\\k_1^2+k_2^2\neq0}}\left(1-\frac{(k_1-2k_2)^2\sin^2\theta}{3k_1^2+(k_1-2k_2)^2}\right)\frac{F}{k_1^2+{}^1/_3(2k_2-k_1^2)\cos^2\theta}, \quad (16)$$

where

$$F=\cos(k_1\mathbf{b}_1\mathbf{r})\cos(k_2\mathbf{b}_2\mathbf{r})-\sin(k_1\mathbf{b}_1\mathbf{r})\sin(k_2\mathbf{b}_2\mathbf{r}), \quad (17)$$

$$\mathbf{b}_1\mathbf{r}=2\pi\left(x-\frac{y\cos\theta}{3^{1/2}}\right), \quad \mathbf{b}_2\mathbf{r}=4\pi\frac{y}{3^{1/2}}\cos\theta. \quad (18)$$

Expressions (16)–(18) are independent not only of the magnetic lattice constant but also of the azimuthal angle, i.e., of the relative orientation of the initial polarization vector and the c axis of the crystallite. This circumstance goes a long way toward simplifying the process of taking an average of (8) over angles. By virtue of the symmetry of the direct lattice, it is sufficient to consider only the region

$$0<x\leqslant0{,}5, \quad 0<y\leqslant\frac{3^{1/2}}{4\cos\theta}.$$

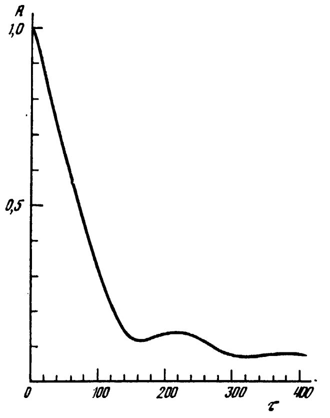

FIG. 1. The envelope of the polarization versus the dimensionless time $\tau=t\Phi_0\gamma/\lambda_\perp^2$.

Figures 1 and 2 show the envelope $R(t)$ and the phase $\varphi(t)$ for a polycrystalline sample. It follows from these figures that over the rather wide interval $t\Phi_0\gamma/\lambda_\perp^2<120$ the polarization is described by a damped cosine function, since the phase is a linear function of the time. The precession frequency is determined by the mean field B decreased by an amount $0.01\Phi_0/\lambda_\perp^2$. This frequency shift is totally unrelated to the lower critical field H_{c1}; it is due exclusively to the spectrum (or distribution) of magnetic fields, which is slightly asymmetric. In this region, the function $R(t)$ can be described somewhat crudely by a single exponential function:

$$R(t)\approx\exp(-0{,}01t\gamma\Phi_0/\lambda_\perp^2). \quad (19)$$

At $t\gamma\Phi_0/\lambda_\perp^2>130$, oscillations of both the phase and the envelope begin. We note in conclusion that a description of the oscillations of the envelope in the phase by means of the same parameter $\lambda_\perp$ may serve as a test of the absence of a

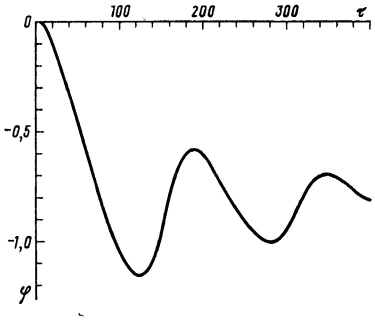

FIG. 2. The additional phase φ versus the dimensionless time τ.

dispersion of the mean field in an anisotropic polycrystalline superconductor—a necessary condition for the application of the muon method to polycrystalline samples.

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[2] W. Barford and J. M. F. Gunn, Physica C **156**, 515 (1988).

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