# Spectrum of polarization bremsstrahlung in a solid near its absorption edge

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The parabolic energy band approximation is used in a calculation of the spectral dependence of the polarization bremsstrahlung emitted by a Born particle in a crystal. The calculation applies near the absorption edge. It is pointed out that a modulation method can be used to investigate experimentally such radiation.

The emission of the polarization bremsstrahlung by scattering of the self-field of an incident particle on target electrons, accompanied by the creation of a real photon, has been investigated for some time[1–4] in various physical situations using several different approaches. The only concept that has revealed the common nature of various emission mechanisms proposed earlier was formulated in Ref. 5. It became clear that such radiative processes as the emission from a particle in an inhomogeneous medium,[6] transition bremsstrahlung,[2] and dynamic or atomic bremsstrahlung[1,3,4] have the same physical basis. A self-consistent quantum-mechanical approach to a description of these phenomena, applied first to atomic targets, has subsequently been extended to a partly ionized plasma (Chap. 6 in Ref. 5) and has made it possible to supplement the results obtained earlier by classical analysis.

We are now faced with the important task of developing a theory of the polarization bremsstrahlung in a solid. This was first considered for crystals in Ref. 7 at the quantum microscopic level and a detailed study was made of the radiation generated as a result of the scattering of the self-field of an incident particle by a static electron charge screening nuclei in the lattice in the limit of high (compared with atomic) frequencies. On the other hand, at frequencies comparable with the atomic values the polarization bremsstrahlung spectrum should manifest the structure of the energy bands of matter. An important feature, which is novel compared with the radiation emitted by individual atoms, is the existence of a continuous part in the electron spectrum, which should modify the resonances of the polarization bremsstrahlung emitted by an atom (Chap. 4 in Ref. 5).

The aim of the present paper is to consider the polarization bremsstrahlung created as a result of virtual (without real population of intermediate levels) transitions of electrons from filled energy bands to vacant ones, which is particularly important near the optical absorption edge of matter. Naturally, the amplitude of the radiation should also include the static terms corresponding to the traditional bremsstrahlung in crystals considered in the monograph of Ter-Mikaelyan.[8] However, we shall not consider this static term since its contribution to the total amplitude amounts to $1/m_0$ ($m_0$ is the mass of the incident particle) and can be made small if $m_0 \gg m$, where $m$ is the electron mass.

We shall assume that a Born particle is scattered in a solid and emits a photon. For simplicity, we shall consider a cubic crystal with a primitive unit cell. In the second order of perturbation theory, in full analogy with the conclusions reached in Ref. 9 and in Chap. 5 of Ref. 5, the polarization bremsstrahlung intensity is given by

$$dI(\mathbf{e},\omega)=\int\frac{d\mathbf{q}\,d\mathbf{k}}{(2\pi)^4}\,|e_h A_l(q)\,\omega^2 c_{ii}^{hl}(k,q)|^2. \quad (1)$$

Here, $k=(\omega,\mathbf{k})$ is the 4-momentum of the emitted photon; $\mathbf{e}$ is a unit vector of its polarization; $q=(E_f-E_i\equiv q^0,\ \mathbf{P}_f-\mathbf{P}_i)$ is the change in the 4-momentum of the incident particle (assumed to be small compared with the 4-momentum itself); $\mathbf{A}(q)$ is the Fourier transform of the vector potential of the field of the incident particle with a charge $e_0$ traveling at a velocity $\mathbf{v}$:

$$\mathbf{A}(q)=\frac{4\pi e_0}{(q^0)^2}\,\frac{\mathbf{q}-q^0\mathbf{v}}{\mathbf{q}^2-(q^0)^2} \quad (2)$$

(the system of units used here is such that $\hbar=c=1$ and the gauge of the electromagnetic field has zero scalar potential); $c_{ii}^{hl}(k,q)$ is the tensor representing scattering of the electromagnetic field by a unit volume of the medium, generally given by the expression

$$c_{ii}^{hl}(k,q)=\frac{r_e}{\omega^2}\Big\{m\sum_s\Big[\frac{j_{is}^{h}(\mathbf{k})\,j_{si}^{l}(\mathbf{q})}{\omega_{is}+\omega+i\gamma_{is}/2}+\frac{j_{is}^{l}(\mathbf{q})\,j_{si}^{h}(\mathbf{k})}{\omega_{is}-\omega-i\gamma_{is}/2}\Big]-\delta^{hl}F(q_1)\Big\} \quad (3)$$

where $r_e=e^2/m$; $q_1=q+k$; $e$ is the electron charge; $\mathbf{j}_{is}(\mathbf{k})$, $\mathbf{j}_{is}(\mathbf{q})$ are the Fourier transforms of the matrix elements of the currents representing the transition; $\omega_{is}=\varepsilon_i-\varepsilon_s$ and $\gamma_{is}$ are the frequencies and damping constants of the transitions; $F(q_1)$ is the Fourier transform of the electron density operator; $i$ and $s$ are the indices indicating the initial and intermediate electron states; $\delta^{hl}$ is the Kronecker delta.

The last term in the braces of Eq. (3) substituted in Eq. (1) describes the process of scattering of the incident particle by the static distribution of the electron charge in the medium considered in Ref. 7. We shall be interested in the radiation described by summing over the intermediate states in Eq. (3) and resulting from virtual transitions from the valence ($v$) to the conduction ($c$) band, when the latter is assumed to be empty. We shall consider the case of a resonance when $\omega\sim\omega_{cv}$ and we can retain only one term representing this resonance in Eq. (3). The relevant part of the scattering tensor is

$$c_p^{hl}(k,q)=\int_{\Omega_B}\frac{d\varkappa}{(2\pi)^3}\,\frac{e^2}{\omega^2}\,\frac{j_{vc}^{h}(\mathbf{k},\varkappa)\,j_{cv}^{l}(\mathbf{q},\varkappa)}{\omega-\omega_{cv}(\varkappa)+i\gamma_{cv}/2}. \quad (4)$$

Here, integration is carried out with respect to the electron quasimomentum $\varkappa$ and the Brillouin zone $\Omega_B$ and

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$\omega_{cv}(\varkappa) = \varepsilon_c(\varkappa) - \varepsilon_v(\varkappa)$. We shall assume that the current $j_{vc}$ of the transition is independent of $\varkappa$. In general, it is necessary to integrate Eq. (4) allowing for the explicit form of the functions $\varepsilon_{v,c}(\varkappa)$. However, we shall consider the parabolic energy band approximation when $\varepsilon_{v,c}(\varkappa) = \varepsilon^0_{v,c} \mp \varkappa^2/2m_{v,c}$, where $m_{v,c}$ are the effective masses of electrons near the top of the valence band and near the bottom of the conduction band. Then, averaging over the photon polarizations, we find that Eqs. (1) and (4) yield

$$\frac{dI_p}{dk} = B(\omega)\,|J(\Delta)|^2, \tag{5}$$

where

$$B(\omega) = \int \frac{d\mathbf{q}}{(2\pi)^{10}} \mu^3 e^4 [\mathbf{n}\mathbf{j}_{vc}(\mathbf{k})]^2 (\mathbf{A}(q)\mathbf{j}_{cv}(\mathbf{q}))^2,$$

$$J(\Delta) = \begin{cases} 1 - \dfrac{\Delta^{1/2}}{a} \tan^{-1} \dfrac{a}{\Delta^{1/2}}, & \Delta > 0, \\[2ex] 1 + \dfrac{|\Delta|^{1/2}}{2a} \ln \left| \dfrac{a - |\Delta|^{1/2}}{a + |\Delta|^{1/2}} \right|, & \Delta < 0, \end{cases}$$

$$\Delta = \varepsilon_c^0 - \varepsilon_v^0 - \omega, \quad \mu^{-1} = m_c^{-1} + m_v^{-1}, \quad a \propto N_v^{1/3}/\mu^{1/2}, \quad \mathbf{n} = \mathbf{k}/|\mathbf{k}|,$$

$N_v$ is the density of the valence band electrons.

The function $B(\omega)$ has no resonance singularities for the case under discussion here: $\omega \ll v/d$ ($d$ is the lattice constant), so that the spectrum of the resonant polarization bremsstrahlung is described by the function $|J(\Delta)|^2$. It should be noted that in the limit $a^2 \ll |\Delta|$ we get in Eq. (5) a factor $|\omega_{cv}/\Delta|^2$ typical of the polarization bremsstrahlung emitted by a single atom (Chap. 4 in Ref. 5). We shall assume that $a^2 \neq |\Delta|$ (because otherwise we have to allow for $\gamma$). It should be stressed that if $\omega_{cv} > \omega$ and $|\Delta| > \gamma$, a multistage (cascade) process associated with real population of the intermediate state is impossible. It should be noted that since the intensity of the resonant polarization bremsstrahlung depends strongly on the quantity $\Delta$, such bremsstrahlung can be separated by a modulation method described in Ref. 10.

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