

Dynamics of self-action of a two-dimensional (gap) wave beam in a medium with stricitive nonlinearity

L. M. Gorbunov and S. V. Tarakanov

Lebedev Physics Institute, USSR Academy of Sciences

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The initial stage of self-action of a two-dimensional (gap) wave beam in a medium with stricitive nonlinearity is investigated. It is shown that an intensity maximum sets at the boundary of the medium, increases, and moves into the medium. The establishment of a quasistationary state is investigated.

It is customarily assumed in the theory of stationary or quasistationary self-action of wave beams that the nonlinear response of a medium to the action of an electromagnetic field sets in more rapidly than the change of the incident-radiation intensity (see, e.g., Refs. 1 and 2). This assumption is fully justified in those cases when the processes that determine the nonlinear response are quite rapid (Kerr nonlinearity in a liquid, relativistic nonlinearity in a plasma). In many cases, however, the nonlinear response is produced by relatively slow processes (thermal, ionization, or stricitive interaction) that evolve over times exceeding the time in which the intensity changes. The self-action dynamics is determined under these conditions by the formation of the nonlinear response.

A nonstationary problem of this type was first considered in Refs. 3–6 for the propagation of a rectangular laser pulse in a medium with relaxational nonlinearity (see also Refs. 7 and 8). It was shown that the nonlinearity of the medium narrows down the trailing part of the pulse, where the waveguide propagation regime is formed, whereas the leading front is broadened by diffusion.

Ray deflection due to development of stricitive nonlinearity was considered in Ref. 9. It was found that a focus is produced initially in the interior of the medium and moves in the course of time towards the boundary. It was assumed that at the initial instant of time, before the nonlinear response could be produced, the diffraction is insignificant and the rays are straight lines. A similar result was obtained in an investigation,¹⁰ with neglect of thermal pressure (known as the “supersonic” limit), of a self-similar solution describing the dynamics of the self-action of a three-dimensional wave beam when stricitive nonlinearity evolves in a plasma.

We consider in the present paper the initial stage of the self-action of a two-dimensional (gap) wave beam in the case of stricitive nonlinearity. It is possible in this case to treat analytically the dynamics of extrusion of matter from the beam region by high-frequency pressure forces, and the evolution of the field. With a Gaussian beam as the example, it is shown that as the density of the medium becomes redistributed the nonlinear refraction compensates for the diffractive divergence of the beam, primarily near the boundary of the medium. It is just there that an intensity maximum appears on the beam axis at a definite instant of time. The maximum increases and shifts into the interior of the medium. Reaching the maximum value and the maximum distance from the boundary, it stops and begins to return to the boundary. Depending on the incident-radiation intensity, the maximum either returns to the boundary and vanishes, or stops without reaching the boundary and remains subse-

quently unchanged. This dynamics of the intensity maximum differs qualitatively from the focus dynamics considered in Refs. 9 and 10. The cause of the difference is that we take into account the initial diffractive broadening of the beam. As a result of depth and width of the produced “channel” in the medium depends substantially on the longitudinal coordinate. Nonlinear refraction turns out here to be more substantial near the boundary of the medium, and not in the interior.

1. FORMULATION AND GENERAL SOLUTION OF PROBLEM

Consider a semi-infinite initially homogeneous medium occupying the region $x > 0$. A wave beam, linearly polarized along the y axis and bounded in the z direction, with frequency ω and wave number k , is normally incident on the boundary $x = 0$ starting with the instant $t = 0$. The wave propagation in the medium, with allowance for the ponderomotive density perturbation $\delta\rho$ it produces, will be described in the quasioptic approximation by the system of equations (see, e.g., Refs. 10 and 11)

$$4i \frac{\partial U}{\partial x} + \frac{\partial^2 U}{\partial \eta^2} = \alpha \rho U, \quad (1)$$

$$\frac{\partial^2 \rho}{\partial \theta^2} - \frac{\partial^2 \rho}{\partial \eta^2} = \beta \frac{\partial^2 |U|^2}{\partial \eta^2}, \quad (2)$$

where $\rho = \delta\rho/\rho_0$ are the relative perturbations of the density of the medium and are assumed to be small, $U = E(x, \eta, \theta)/|E(x=0, \eta=0)|$ is the amplitude of the electric-field intensity referred to the limiting amplitude on the beam axis, $x = 2x/kz_0^2$ is the dimensionless longitudinal coordinate, z_0 is the characteristic dimension of the beam at the boundary; $\eta = z/z_0$ is the transverse coordinate, $\theta = st/z_0$ is the time, and s is the speed of sound. The constants α and β for the liquid are equal to

$$\alpha = \rho_0 (\partial \epsilon' / \partial \rho)_s (z_0 \omega / c)^2,$$

$$\beta = (1/16\pi) (\partial \epsilon' / \partial \rho)_s |E(x=0, \eta=0)|^2 / (\partial p / \partial \rho)_s,$$

where $(\partial \epsilon' / \partial \rho)_s$ and $(\partial p / \partial \rho)_s$ are the derivatives of the dielectric constant and of the pressure with respect to density, taken at constant entropy, and $s = (\partial p / \partial \rho)_s^{1/2}$. For a plasma, the corresponding coefficients in Eqs. (1) and (2) are

$$\alpha = (\omega_p z_0 / c)^2, \quad \beta = [e |E(x=0, \eta=0)| / (2v_T m \omega)]^2,$$

where ω_p and v_T are respectively the plasma frequency and the thermal velocity of the electrons.

We have neglected in (1) terms with a time derivative of U , a procedure justified when the inequality $(2\omega z_0 s / c^2) \ll 1$ is satisfied, and corresponding to an assumed

infinite propagation velocity of the electromagnetic radiation. In accord with these assumptions, a wave beam incident at $\theta = 0$ on the boundary $\kappa = 0$ penetrates immediately into the medium and has no time to produce density perturbations ($\rho = 0$). The field of the beam at the initial instant of time has, according to Eq. (1), the form²

$$U_0(\eta, \kappa) = \int_{-\infty}^{\infty} d\eta' f(\eta') G(\eta - \eta', \kappa), \quad (3)$$

where $G(\eta - \eta', \kappa) = (i\pi\kappa)^{-1/2} \exp[i(\eta - \eta')^2/\kappa]$ and $f(\eta) = U_0(\eta, \kappa = 0)$ is the beam field specified on the boundary and assumed independent of time. This assumption allows us to separate effects connected with the inertia of the nonlinear response from effects investigated in the theory of stationary self-focusing^{8,12-14} and connected with the time dependence of the intensity of the incident radiation.

It is natural to assume that the perturbation-density dynamics is determined for some time by the field (3). Substituting it in the right-hand side of (2) and using the initial conditions $\rho(\theta = 0) = (\partial\rho/\partial\theta)_{\theta=0} = 0$, we get

$$\rho(\eta, \kappa, \theta) = -\frac{\beta}{2} [2|U_0(\eta, \kappa)|^2 - |U_0(\eta + \theta, \kappa)|^2 - |U_0(\eta - \theta, \kappa)|^2]. \quad (4)$$

The first term in the right-hand side of (4) corresponds to a stationary solution of Eq. (2). The remaining two terms described density bursts that depart from the beam-localization region at the speed of sound in opposite directions along the z direction. The matter conservation law

$$\int_{-\infty}^{\infty} d\eta \rho(\eta, \kappa, \theta) = 0$$

is satisfied at each instant of time and for any point κ .

Equation (4) is analogous to the expressions obtained in Refs. 15 and 16 dealing with plasma-density perturbations produced by a specified two-dimensional laser beam, but the influence of these perturbations were not taken into account.

The perturbation alters the acoustic field, and for a sufficiently short time interval it can be represented by $U = U_0 + U_1$. In the linear approximation we obtain for U_1 from Eq. (1) the expression

$$4i \frac{\partial U_1}{\partial \kappa} + \frac{\partial^2 U_1}{\partial \eta^2} = \alpha \rho U_0. \quad (5)$$

Equation (5) has a solution satisfying the boundary condition $U_1(\eta, \kappa = 0, \theta) = 0$ and given by

$$U_1(\eta, \kappa, \theta) = \frac{\alpha}{4i} \int_0^{\kappa} d\kappa' \int_{-\infty}^{\infty} d\eta' \rho(\eta', \kappa', \theta) U_0(\eta', \kappa') \times G(\eta - \eta', \kappa - \kappa'), \quad (6)$$

where the expression for the function G was given above.

We consider below the absolute value of the dimensionless electric-field amplitude

$$A = [(\text{Re } U_0 + \text{Re } U_1)^2 + (\text{Im } U_0 + \text{Im } U_1)^2]^{1/2}. \quad (7)$$

Equations (3), (4), and (6) provide an analytic solution of the problem of the initial stage of the dynamics of a planar beam. No similar solution can be obtained for an axial beam.

The use of perturbation theory is valid only if A differs little from $A_0 = [(\text{Re } U_0)^2 + (\text{Im } U_0)^2]^{1/2}$. Failure to meet this condition is the cause of the limited time interval during which our treatment is valid.

2. DYNAMICS OF GAUSSIAN BEAM

By way of example we consider the most typical case of a Gaussian beam with a planar front, when $f(\eta) = \exp(-\eta^2)$. From Eq. (3) we obtain²

$$U_0(\eta, \kappa) = (1 + \kappa^2)^{-1/2} \exp[-\eta^2/(1 + \kappa^2)] \times \exp\left[-i\left(\frac{1}{2} \text{arctg } \kappa - \frac{\kappa\eta^2}{1 + \kappa^2}\right)\right]. \quad (8)$$

Figure 1 shows lines of constant values of $A_0 = (1 + \kappa^2)^{-1/4} \cdot \exp[-\eta^2/(1 + \kappa^2)]$ with an interval 0.04 separating two neighboring lines.

According to (4) and (8), the density perturbation produced by the wave beam is given by

$$\rho(\eta, \kappa, \theta) = -\frac{\beta}{2(1 + \kappa^2)^{1/2}} \left\{ 2 \exp\left(-\frac{2\eta^2}{1 + \kappa^2}\right) - \exp\left[-\frac{2(\eta + \theta)^2}{1 + \kappa^2}\right] - \exp\left[-\frac{2(\eta - \theta)^2}{1 + \kappa^2}\right] \right\}. \quad (9)$$

To analyze the establishment of a stationary state, it is expedient to rewrite (9) in the form

$$\rho(\eta, \kappa, \theta) = -\frac{\beta}{(1 + \kappa^2)^{1/2}} \exp\left(-\frac{2\eta^2}{1 + \kappa^2}\right) \times \left[1 - \exp\left(-\frac{2\theta^2}{1 + \kappa^2}\right) \cdot \text{ch}\left(\frac{4\eta\theta}{1 + \kappa^2}\right) \right]. \quad (9')$$

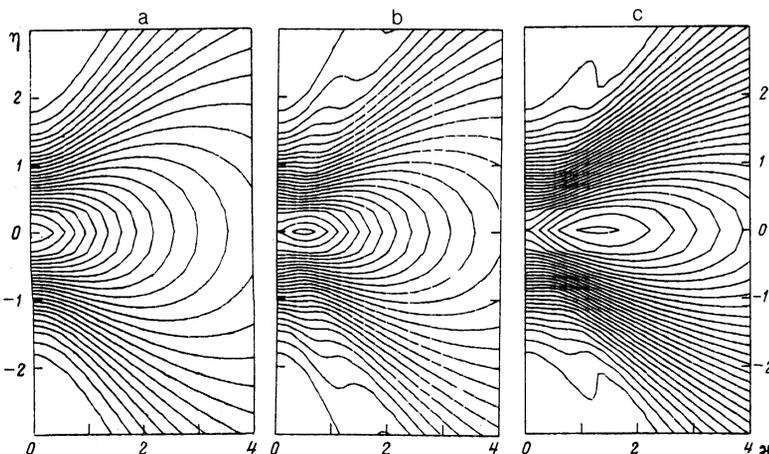


FIG. 1. Lines of constant amplitude A in the plane of variables η and κ for the instants of time $\theta = 0$ (a), 1 (b), and 2 (c). Average intensity $3.3 \cdot 10^{12}$ W/cm², interval between neighboring lines $\Delta A = 0.04$.

The density dip on the beam axis (at $\theta < \theta_c = [(1 + \kappa^2)/2]^{1/2}$) increases as the square of the time, corresponding to neglect of the thermal motion in Eq. (2). It is precisely in this approximation that the beam dynamics was investigated in Ref. 10. At $\theta < \theta_c$ the thermal pressure becomes substantial and a stationary state sets in. Taking θ_c to be time required to reach the stationary value, we can conclude that it is reached faster the closer the point x is to the boundary of the medium.

At each instant of time there are two lines $\eta_0(x, \theta)$ symmetric about the beam axis, on which the density perturbations are equal to zero. According to (9') the equation for these lines is

$$\eta_0 = \frac{1 + \kappa^2}{4\theta} \text{Arch}[\exp(2\theta^2/(1 + \kappa^2))].$$

At $\theta < \theta_c$ the position of these lines is independent of time ($\eta_0 \approx \pm \frac{1}{2}(1 + \kappa^2)^{1/2}$) and at $\theta > \theta_c$ this dependence is close to linear ($\eta_2 \approx \pm (\theta/2 - (1 + \kappa^2)(\ln 2)/2\theta)$). An analogous variation is observed for the location of the maxima of the unperturbed density $\eta_m(x, \theta)$, a location determined according to (9') by the equation

$$\eta_m \left[\text{ch} \left(\frac{4\eta_m \theta}{1 + \kappa^2} \right) - \exp \left(\frac{2\theta^2}{1 + \kappa^2} \right) \right] = \theta \text{sh} \left(\frac{4\eta_m \theta}{1 + \kappa^2} \right).$$

At $\theta < \theta_c$ the location of the maxima depends little on the time ($\eta_m \approx \pm (3^{1/2}/2)(1 + \kappa^2)^{1/2}$, while at $\theta > \theta_c$ the maxima move away at the speed of sound ($\eta_m \approx \pm \theta$) from the beam axis.

The beam-field change due to the density perturbation takes according to (6), (8), and (9') the form

$$U_1(\eta, \kappa, \theta) = \frac{\alpha\beta}{4} e^{i\pi/4} \int_0^\kappa \frac{dq}{(PQ)^{1/2}} \exp[\eta^2(3i+q)/Q] \times \left\{ 1 - \cos \left(\frac{4\eta\theta}{Q} \right) \cdot \exp \left[2i\theta^2 \frac{1+i\kappa}{PQ} \right] \right\}, \quad (10)$$

where $P = 1 + iq$ and $Q = 3(\kappa - q) - i(1 + \kappa q)$. It is impossible to evaluate the integral in (10) analytically. We confine ourselves therefore to a short time interval ($\theta < \theta_c$) and consider the field perturbation on the beam axis ($\eta = 0$). From (10) we get

$$U_1(\eta=0, \kappa, \theta < \theta_c) = \frac{\alpha\beta\theta^2}{8\sqrt{2}} \frac{(1-i)}{(1+\kappa^2)^{1/2}} \left\{ \frac{1+i\kappa}{(\kappa-i)^{1/2}} - \frac{(1-i\kappa)^2}{[(1+\kappa^2)(3\kappa-i)]^{1/2}} \right\}. \quad (11)$$

Within the context of the perturbation theory employed, Eq. (7) should be considered in an approximation linear in U_1 , when

$$A \approx A_0 + \frac{1}{A_0} (\text{Re } U_0 \text{Re } U_1 + \text{Im } U_0 \text{Im } U_1).$$

Substituting here expressions (8) for $\eta = 0$ and (11) we obtain the beam amplitude on the axis. Calculating the derivative $\partial A / \partial \kappa$ and equating it to zero, we obtain an equation for the position of the extremum of the amplitude:

$$(1 + \kappa_0^2)^{1/2} = \frac{\alpha\beta}{16} \theta^2 \Phi(\kappa_0^2), \quad (12)$$

where $\Phi(\kappa^2)$ is a rather complicated algebraic function that

decreases monotonically with increase of κ^2 and satisfies at $\kappa^2 < 1$ the approximate expression $\Phi(\kappa^2) \approx 76 - 600\kappa^2$.

Equation (12) has solutions only for $\theta > \theta_0 = 4/19\alpha\beta$, the solution for θ_0 corresponding to $\kappa_0 = 0$. With increase of θ the value of κ_0 increases like

$$\kappa_0 = \{ [19\alpha\beta\theta^2/2 - 2]/(1 + 75\alpha\beta\theta^2) \}^{1/2}. \quad (13)$$

As shown by numerical integration of Eq. (10), the results of which are given below, the extremum of the amplitude on the beam axis is a maximum. It can therefore be concluded that as the medium is forced out of the beam region an intensity maximum is produced on the beam axis. This maximum sets in at a time interval θ_0 after the start of the self-action on the boundary of the medium, and moves away from the boundary to the interior in the course of time.

3. NUMERICAL EXAMPLES

To illustrate the results of the preceding section, we have calculated the perturbations of the density ρ of the medium and the amplitude A of the field according to Eqs. (7)–(10) for a number of specific parameters of the medium and of the incident radiation.

The calculations were made for a Gaussian beam of width $z_0 = 30 \mu\text{m}$ and frequency $\omega = 2 \cdot 10^{15} \text{ s}^{-1}$, propagating in a hydrogen plasma with electron temperature 100 eV and a ratio of the electron density to critical $5 \cdot 10^{-2}$. For these parameters, the dimensionless variables η, κ , and θ are connected respectively with the transverse and longitudinal coordinates z and x and with the time t by the relations $\eta = 3.3 \cdot 10^2 z$ (cm), $\kappa = 3.3x$ (cm), $\theta = 3.3t$ (ns).

Figures 1 and 2 show the lines of constant amplitude A calculated from Eqs. (7), (8), and (10) and the constant-density lines calculated from Eq. (9), for a number of instants of time at an average incident-radiation energy-flux density $3.3 \cdot 10^{12} \text{ W/cm}^2$. It can be seen that as the density dip produced by the beam deepens and the forced-out plasma goes out of the beam region, and a maximum of radiation intensity sets in and moves away towards the boundary. Figure 3 shows the time variation of the coordinate κ_0 of this maximum and of the amplitude A_m at the maximum for a number of incident-radiation intensities. At an intensity $3.3 \cdot 10^{12} \text{ W/cm}^2$ ($\alpha\beta = 4.2$) the quantities κ_0 and A_m reach maxima at close instants of time, after which they decrease somewhat and assume stationary values. With decrease of the incident-radiation intensity this effect of motion of the amplitude from the boundary into the interior of the plasma and back becomes ever more pronounced. Finally, at an intensity $1.2 \cdot 10^{12} \text{ W/cm}^2$ ($\alpha\beta = 1.46$) the amplitude maximum produced on the boundary and moving away from it again returns to the boundary and vanishes. In the steady state the beam amplitude has no maximum outside the boundary. No maximum is produced at all at $\alpha\beta = 1.42$.

Figure 3 shows the amplitude-maximum coordinate calculated from Eq. (13) for $\alpha\beta = 4.2$. It can be seen that the instant of the onset of the maximum, obtained from Eq. (13), agrees well with the results of the numerical calculation.

As seen from Fig. 3, at an incident radiation intensity $3.3 \cdot 10^{12} \text{ W/cm}^2$ the field amplitude in the self-action process differs little from A_0 , which justifies in fact our approach for a sufficiently long time. With increase of the incident-radiation intensity the time interval during which our ap-

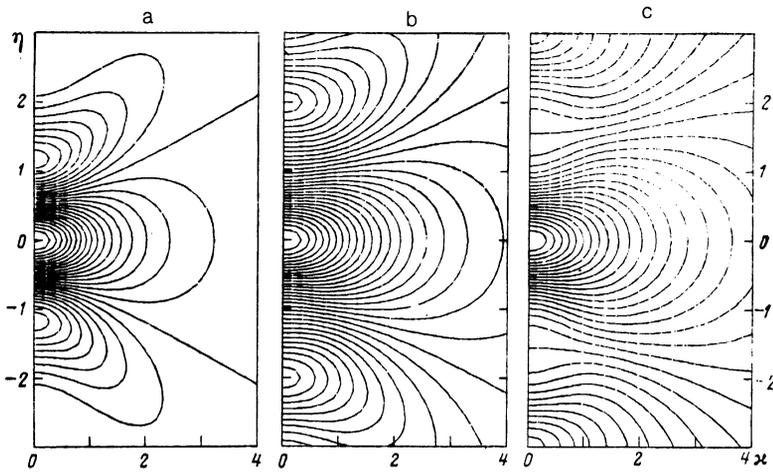


FIG. 2. Constant plasma-density ρ lines in the plane of the variables η and x for the instants of time $\theta = 1$ (a), 2 (b) and 3 (c). Average intensity $3.3 \cdot 10^{12}$ W/cm²; interval between neighboring lines $\Delta\rho = 2 \cdot 10^{-5}$.

proach is valid becomes narrower. The main features of the beam self-action dynamics are nevertheless preserved. Figure 4 shows the change of the field amplitude in the beam at the instant $\theta = 0.06$ at an intensity $6.6 \cdot 10^{14}$ W/cm². For longer times, the difference between A_0 and A becomes large and our treatment is unsuitable.

As seen from Fig. 4, refraction of the radiation by the sound waves in the interior of the medium makes the transverse intensity distribution in the beam nonmonotonic. This effect becomes clearly manifest also in numerical computations¹⁷ of the propagation of a narrow beam against the background of a broader beam in a plasma.

Our calculations for a plasma are applicable also to a liquid. Using the characteristic values of the quantities in the equations¹⁸ ($\rho_0 = 1$ g/cm³, $\rho_0(\partial\epsilon'/\partial\rho)_s = 1.5$, $(\partial p/\partial\rho)_s^{1/2} = 10^5$ cm/s, $z_0 = 30$ μ m), we find that the val-

ue $\alpha\beta = 4.2$ corresponds to an average incident radiation intensity $4 \cdot 10^9$ W/cm².

4. DISCUSSION OF RESULTS

The results can be made clear by simple qualitative considerations. The material is forced out most rapidly near the boundary, where the beam width is a minimum and the striction forces, determined by the derivative of the intensity with respect to the transverse coordinate, are largest. In addition, near the boundary the beam divergence is minimal on ac-

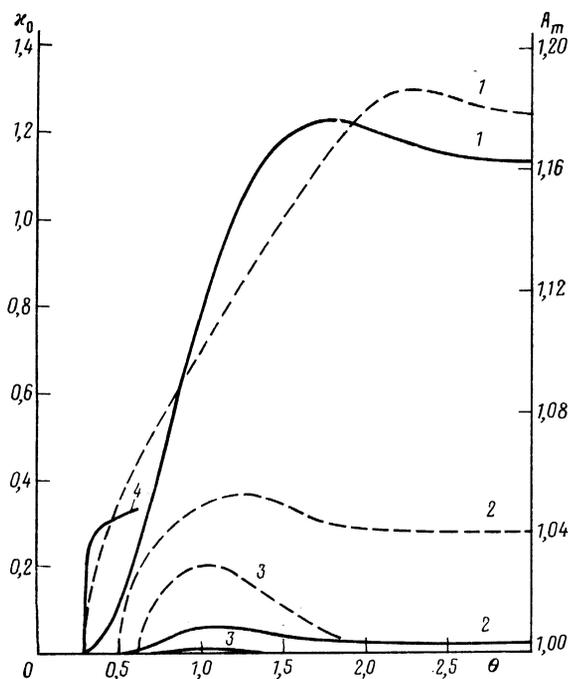


FIG. 3. The coordinates x_0 (solid lines) and maximum amplitudes A_m (dashed) vs the time θ for different values of the average incident-beam intensity (1— $3.3 \cdot 10^{12}$; 2— $6 \cdot 10^{11}$; 3— $4.6 \cdot 10^{11}$ W/cm²). Curve 4 was obtained for x_0 from Eq. (13) for an intensity $3.3 \cdot 10^{12}$ W/cm².

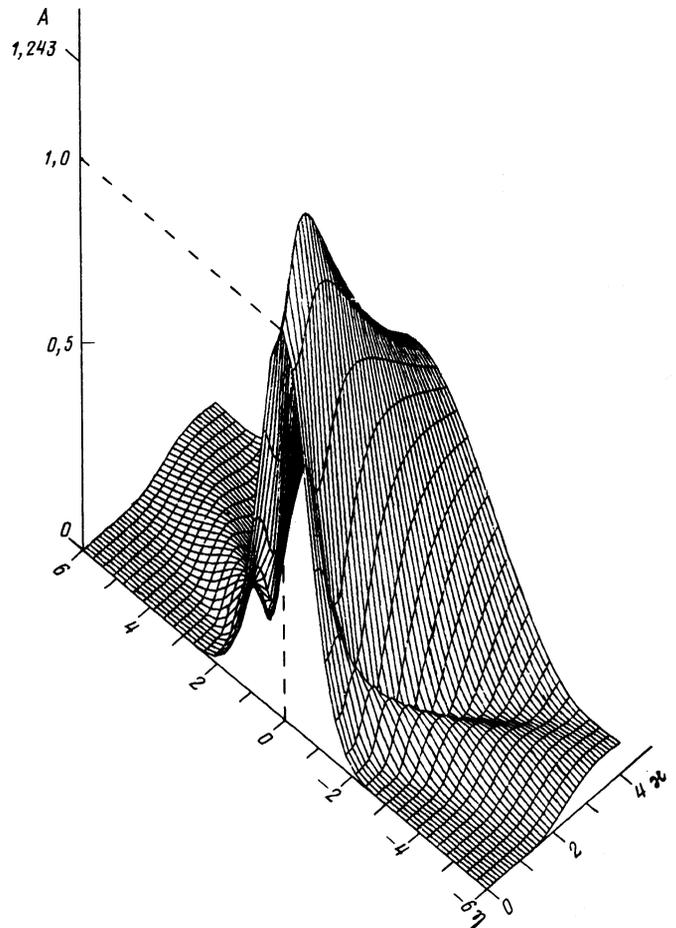


FIG. 4. Spatial distribution of field amplitude A in a beam for the instant of time $\theta = 0.06$ at an average intensity $6.6 \cdot 10^{14}$ W/cm².

count of diffraction, and the wavefront is closest to planar. It is therefore just near the boundary that the nonlinear refraction compensates most rapidly for the diffractive divergence, and near the beam axis there is produced a focusing region in which the wavefront is concave. With time, as the density dip deepens and broadens, this focusing region expands both radially and longitudinally. The field amplitude at the maximum is then increased, and the maximum itself shifts into the interior of the medium.

Near the boundary ($\kappa < 1$), at the time $\theta \sim 1$ the density perturbations of the matter forced out by the beam are located on the edges of the central dip (at $\eta \approx \pm 1$), and the nonlinear refraction determined by the value of the density drop is a maximum. Its effect on the beam focusing is then strongest. It weakens subsequently with increase of the distance between the density bursts and the beam. The largest focusing effect is therefore produced in the transient regime, and for beams with a relatively low intensity a field maximum exists in fact only in the transient regime.

Note that in our analysis one cannot rigorously speak of a stationary state. The picture of the density and field perturbations ceases to change by the instant θ only in the interval $\kappa (\lesssim (2\theta^2 - 1)^{1/2})$, and continues to change at large values of κ . In the perturbation theory used by us it is impossible to track the evolution of stationary self-action far from the boundary. In that region the zeroth-order approximation field is made weak enough by diffraction, and establishment of a stationary state entails an appreciable field variation.

It can be assumed that after establishment of the first (closest to the boundary) intensity maximum formation of a second intensity maximum farther from the boundary sets in, followed by a third, etc. The basis for this assumption is the fact that after establishment of the first maximum the field behind it is close to the field present near the boundary in the self-action process (cf. the fields at $\theta = 0$ and $\theta = 2$ for $\kappa > 1$ in Fig. 1). This can result in the periodic sequence of maxima predicted by the stationary theory for gap wave beams.²

It must be emphasized that the onset of an intensity maximum near the boundary and its motion into the interior of the medium is not connected with a special choice of boundary conditions, and in particular with assumption of a planar wavefront on the boundary. We have carried out computations also for a Gaussian beam whose phase has on the boundary a parabolic variation $\psi(0) = -\eta^2\delta$, where $\delta = kz_0^2/2R$, and R is the wavefront-curvature radius. At $\delta < 0$ (diverging beam) the local intensity maximum produced at a certain distance from the boundary increases with time and shifts into the interior of the medium. At low wavefront curvature (for $\alpha\beta = 4.2$ at $|\delta| \leq 0.5$), starting with a definite instant of time, the intensity at the maximum begins to exceed the boundary intensity. With increase of the front

curvature, the intensity in the local maximum remains all the time lower than the limiting intensity on the beam axis. At $\delta > 0$ (focused beam) the intensity maximum initially produced at the point $\kappa_0 = \delta/(1/\delta^2)$ and determined by the wavefront curvature on the boundary increases with time and shifts to the interior of the medium. The conclusion that the intensity maximum moves in the course of self-action away from the boundary of the medium is quite general and remains valid for both focused and defocused beams.

Our analysis pertains to a pulse that has with time an abrupt leading front and constant intensity. For our conclusions to be valid for a real pulse, the intensity growth time must be short and the pulse duration must be long compared with the characteristic time of the considered transient processes. For the example considered above, namely self-action of a wave beam of intensity 3.3×10^{12} W/cm² in a plasma, the time of establishment of a stationary state for the first maximum is approximately 1 ns.

In conclusion, we emphasize once more that in the case of a planar wavefront the considered onset of a field maximum near the boundary of the medium is connected with allowance for the diffractive broadening of the beam at the initial instant of time and, as a consequence, with the inhomogeneity of the density dip in the longitudinal direction. If diffraction is neglected, the density dip is homogeneous, and nonlinear refraction sets in first of all far from the boundary.

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