Multiphoton production of e^+e^- pairs in a plasma by transverse electromagnetic waves

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On the basis of the solution of the Dirac equation in lowest order of perturbation theory in the small parameter eA/ω we find the probability of multiphoton production of electron-positron pairs in a plasma by transverse electromagnetic waves. This process is possible in a plasma of ordinary densities, in contrast to the single-photon pair production, which is only possible in a superdense plasma and whose probability is almost nil due to the Pauli principle. We obtain the probability of multiphoton e^+e^- pair production in an arbitrary direction in vacuum by a periodic electric field (previously known results correspond to the case when the particles are produced in the direction perpendicular to the field).

As was shown in Ref. 1, single-photon production of e^+e^- pairs in a plasma is possible at densities of its electron component $N_e/V \equiv \rho > 3\cdot 10^{34}$ cm^{-3}. In such a superdense plasma (in the core of neutron stars) the electron gas is strongly degenerate and the Pauli principle further limits the region in which the reaction $\gamma \to e^+ + e^-$ can take place, reducing its probability to practically zero.[1] However, in a plasma of ordinary densities the multiphoton pair production process by the laser radiation field is possible. The condition for this process (reaction threshold)

$$N\hbar\omega \geqslant \frac{2mc^2}{[1-n^2(\omega)]^{1/2}} \tag{1}$$

can be realistically satisfied in a plasma with a refractive index $n(\omega) < 1$ thanks to the large number of photons ($N \gg 1$) in contrast to the single-photon pair-production case ($N = 1$).

Pair production by the laser radiation field belongs to those processes whose experimental observation requires such high intensities that the energy of interaction of the electron with the field over a wavelength becomes comparable to the electron rest mass: $\xi \equiv eA_0/mc^2 \sim 1$ (e is the charge, m is the mass of the electron, and A_0 is the amplitude of the vector potential field). Although such fields have not been achieved as yet, the study of the multiphoton pair-production channel in a plasma is nevertheless very important, particularly when one takes into account that it is the only way in which this process can take place in a plasma (it is understood that we are speaking of the plasma as a single macroscopic medium, and not about individual particles in the plasma on which "bremssthralung pair production" is possible).

Let there propagate in a plasma with the dispersion law[1)]

$$n^2(\omega) = 1 - 4\pi\rho e^2/m\omega^2$$

a plane transverse linearly polarized electromagnetic wave with frequency ω and vector potential

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_0 \sin(\omega t - \mathbf{k}\mathbf{r}), \quad |\mathbf{k}| = n(\omega)\omega/c. \tag{2}$$

It is convenient to solve the problem in the center-of-mass frame of the produced pair (C-frame), in which the wave vector of the photon is $\mathbf{k}' = 0$ [the index of refraction of the plasma in this frame is $n'(\omega' = 0)$]. The velocity of the C-frame with respect to the laboratory is $v = cn$. The traveling electromagnetic wave is transformed in that frame into a varying electric field (the magnetic field is $H' = 0$) with a vector potential

$$\mathbf{A}'(t') = \frac{\mathbf{A}_0}{2i}[\exp(i\omega' t') - \exp(-i\omega' t')],$$

$$\omega' = \omega[1-n^2(\omega)]^{1/2}. \tag{3}$$

It is easily noted that with (3) taken into account the reaction threshold condition (1) is obtained from the laws of conservation of energy $N\hbar\omega' = E'_- + E'_+$ and momentum $N\hbar\mathbf{k}' = 0 = \mathbf{p}'_- + \mathbf{p}'_+$ in the C-frame (E'_-, $\mathbf{p}'_-$, E'_+, $\mathbf{p}'_+$ are the energy and momentum of the electron and positron respectively in the C-frame).

To solve the problem of N-photon production of an e^+e^- pair in the given radiation field (2) we shall make use of the Dirac model (all vacuum negative-energy states are filled with electrons and the interaction of the external field proceeds only with this vacuum; on the other hand the interaction with the plasma electrons reduces to a refraction of the wave only).

The Dirac equation in the field (3) has the form (we set here $\hbar = c = 1$)

$$i\frac{\partial \Psi}{\partial t} = [\boldsymbol{\alpha}(\hat{\mathbf{p}}' - e\mathbf{A}'(t')) + \beta m]\Psi, \tag{4}$$

where

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

are the Dirac matrices, with $\boldsymbol{\sigma}$ the Pauli matrices.

Since in the C-frame the interaction Hamiltonian does not depend on the space coordinates, the solution of (4) can be represented in the form of a linear combination of free solutions of the Dirac equation with amplitudes $a_i(t')$ depending only on time:

$$\Psi_{\mathbf{p}'}(\mathbf{r}', t') = \sum_{i=1}^{4} a_i(t')\Psi_i^{(0)}(\mathbf{r}'\ t'). \tag{5}$$

Here

$$\Psi_{1,2}^{(0)}(\mathbf{r}',t')=\left(\frac{E'+m}{2E'}\right)^{1/2}\begin{pmatrix}\varphi_{1,2}\\ \dfrac{\boldsymbol{\sigma}\mathbf{p}'}{E'+m}\varphi_{1,2}\end{pmatrix}$$

$$\times\exp[i(\mathbf{p}'\mathbf{r}'-E't')],$$

$$\Psi_{3,4}^{(0)}(\mathbf{r}',t')=\left(\frac{E'+m}{2E'}\right)^{1/2} \tag{6}$$

$$\times\begin{pmatrix}\dfrac{-\boldsymbol{\sigma}\mathbf{p}'}{E'+m}\chi_{3,4}\\ \chi_{3,4}\end{pmatrix}\exp[i(\mathbf{p}'\mathbf{r}'+E't')],$$

where

$$E'=(p'^2+m^2)^{1/2},\quad \varphi_1=\chi_3=\begin{pmatrix}1\\0\end{pmatrix},\quad \varphi_2=\chi_4=\begin{pmatrix}0\\1\end{pmatrix}. \tag{7}$$

The solution (4) in the form (5) corresponds to an expansion of the wave function in a complete set of orthonormal functions of the electrons (positrons) with specified momentum [with energies $E'=\pm(p'^2+m^2)^{1/2}$ and spin projections $\pm 1/2$]. The latter are normalized to one particle per unit volume.

According to the assumed model only the Dirac vacuum is present prior to the turning on of the field, i.e.

$$|a_3(-\infty)|^2=|a_4(-\infty)|^2=1,\quad |a_1(-\infty)|^2=|a_2(-\infty)|^2=0$$

(the field is turned on adiabatically at $t=-\infty$). From the condition of conservation of the norm we have

$$\sum_{i=1}^{4}|a_i(t')|^2=2,$$

which expresses the equality of the number of created electrons and positrons, whose creation probability is respectively $|a_{1,2}(t')|^2$ and $1-|a_{3,4}(t')|^2$.

Substituting (5) into (4), multiplying by the Hermitian conjugate functions $\Psi_i^{(0)}+(\mathbf{r}',t')$ and taking into account orthogonality of the eigenfunctions (6) and (7), we obtain a system of differential equations for the unknown functions $a_i(t')$. Since in the C-frame there is symmetry with respect to the direction $\mathbf{A}_0$ (the y axis) we can take, without loss of generality, the vector $\mathbf{p}'$ to lie in the $x'y'$ plane ($p_z'=0$). Further, having introduced, to simplify the notation, the new symbols

$$a_1(t')\equiv b_1(t'),\quad a_4(t')\equiv b_4(t')\left(1-\frac{p_y'^2}{E'^2}\right)^{-1/2}$$
$$\times\left[\frac{p_x'p_y'}{E'(E'+m)}+i\left(1-\frac{p_y'^2}{E'(E'+m)}\right)\right],$$

we obtain for the amplitudes $b_1(t')$ and $b_4(t')$ ($|b_4(t')|=|a_4(t')|$ the following system of equations:

$$\frac{db_1(t')}{dt'}=i\frac{ep_y'A_y'(t')}{E'}b_1(t')$$
$$+ieA_y'(t')\left(1-\frac{p_y'^2}{E'^2}\right)^{1/2}b_4(t')\exp(2iE't'), \tag{8}$$

$$\frac{db_4(t')}{dt'}=-i\frac{ep_y'A_y'(t')}{E'}b_4(t')$$
$$+ieA_y'(t')\left(1-\frac{p_y'^2}{E'^2}\right)^{1/2}b_1(t')\exp(-2iE't').$$

A similar system of equations is also obtained for the amplitudes $a_2(t')$ and $a_3(t')$.

An exact analytic solution of this problem is not possible, since the system of equations (8) reduces to the Mathieu equation, just like in the case of multiphoton pair production in vacuum by a periodic electric field, which was studied by various methods in Refs. 3–11. Therefore we shall look for a solution of (8) by the method of successive approximations in the small parameter $\eta=eA_0/\omega\ll 1$, confining ourselves to lowest order perturbation theory. For the N-photon process the first nonoscillatory term (not vanishing after the wave has been turned off) to lowest order in η in $b_1(t')$ appears only due to the term $i(A_0/2)\exp(-i\omega't')$ [the contribution of the first term in the field (3) $(A_0/2i)\exp(i\omega't')$ gives rise to corrections of higher order in η].

The system of equations (8) with the initial conditions $b_1^{(0)}=0$, $b_4^{(0)}=1$ ($b_i^{(0)}$ are the Dirac electron probability amplitudes in zeroth order in the field) taken into account admits of the following recursion relations between the amplitudes of the N-photons transitions

$$b_1^{(n+1)}=i\left(\frac{eA_0}{2}\right)\frac{p_y'/E'}{2E'-(n+1)\omega'}b_1^{(n)}\exp(-i\omega't')$$
$$+i\left(\frac{eA_0}{2}\right)\frac{[1-(p_y'/E')^2]^{1/2}}{2E'-(n+1)\omega'}b_4^{(n)}\exp[i(2E'-\omega')t'],$$

$$b_4^{(n+1)}=i\left(\frac{eA_0}{2}\right)\frac{p_y'/E'}{(n+1)\omega'}b_4^{(n)}\exp(-i\omega't')-i\left(\frac{eA_0}{2}\right) \tag{9}$$
$$\times\frac{[1-(p_y'/E')^2]^{1/2}}{(n+1)\omega'}b_1^{(n)}\exp[-i(2E'+\omega')t'].$$

To obtain the probability amplitude for the production of electrons and positrons after the wave has been turned off we introduce a small detuning of the resonance $2E'=N\omega'+\gamma(\gamma<\omega')$ and look for $b_1^{(n)}$ and $b_4^{(n)}$ in the following form:

$$b_1^{(n)}=\left(i\frac{eA_0}{2\omega'}\right)^nC_1^{(n)}\exp[i(2E'-n\omega')t'],$$
$$b_4^{(n)}=\left(i\frac{eA_0}{2\omega'}\right)^nC_4^{(n)}\exp(-in\omega't'). \tag{10}$$

We then get for $C_1^{(n)}$ and $C_4^{(n)}$ from (9)

$$C_1^{(n+1)}=\frac{\alpha C_1^{(n)}}{N_0-(n+1)}+\frac{(1-\alpha^2)^{1/2}}{N_0-(n+1)}C_4^{(n)},$$
$$C_4^{(n+1)}=\frac{\alpha C_4^{(n)}}{n+1}-\frac{(1-\alpha^2)^{1/2}}{n+1}C_1^{(n)}, \tag{11}$$

where $\alpha=Py'/E'$, $N_0=2E'/\omega'=N+\gamma/\omega'$.

From the system of equations (11) we obtain the following recursion relation:

$$C_1^{(n+2)}=\frac{\alpha N_0C_1^{(n+1)}}{(n+1)[N_0-(n+2)]}-\frac{C_1^{(n)}}{(n+1)[N_0-(n+2)]}. \tag{12}$$

To determine $C_1^{(n)}$ we make use of the initial conditions $C_1^{(0)}=0$ and $C_4^{(0)}=1$. With their help we obtain from the first of the equations (11)

$$C_1^{(1)}=(1-\alpha^2)^{1/2}/(N_0-1).$$

Further, given $C_1^{(0)}$ and $C_1^{(1)}$, we determine $C_1^{(n)}$ from (12):

$$C_1^{(n)}=\begin{cases}\sum\limits_{j=0}^{(n-1)/2} C_{1,2j+1}^{(n)} & \text{for } n \text{ odd},\\ \sum\limits_{j=1}^{n/2} C_{1,2j}^{(n)} & \text{for } n \text{ even},\end{cases} \tag{13}$$

where

$$C_{1,k}^{(n)}=C_1^{(1)}\frac{(-1)^{(n-k)/2}(\alpha N_0)^{k-1}}{(n-1)!\,(N_0-2)(N_0-3)\dots(N_0-n)} \tag{14}$$

$$\times \sum_{\substack{0\leqslant s_1\leqslant s_2\leqslant\dots\\ \dots\leqslant s_{(n-k)/2}\leqslant k-1}} \prod_{l=0}^{(n-k)/2} B_{l,s_l},$$

$B_{0,s_0}=1$, B_{l,s_l}

$=(2l+s_l-1)(N_0-2l-s_l)$ for $l=1, 2,\dots (n-k)/2$.

$B_{0,s_0}=1$, $B_{l,sl}=(2l+s_l-1)(N_0-2l-s_l)$ for $l=1$, 2, ..., $(n-k)/2$.

The production probability of the e^+e^- pair, summed over the spin states, is determined by the quantity

$$|a_1(t')|^2+|a_2(t')|^2=2|a_1(t')|^2,$$

or the quantity $2|b_1(t')|^2$ [since $|a_1(t')|^2=|b_1(t')|^2$], which is given by Eqs. (10) and (14).

The differential probability of the N-photon process per unit time and phase-space volume $d^3\mathbf{p}'/(2\pi)^3$ (the normalization volume $V=1$) in the center-of-mass frame of the produced particles is given by

$$dw_N{}^C=\frac{dW_N{}^C(t')}{t'}=2\lim_{t'\to\infty}\frac{|b_1(t')|^2}{t'}\frac{d^3\mathbf{p}'}{(2\pi)^3}. \tag{15}$$

Substituting into (15) the expression (10) with (14) taken into account and making use of the definition of the δ-function in the form

$$\lim_{t'\to\infty}\frac{\sin^2\gamma t'}{\pi t'\gamma^2}=\delta(\gamma)=\delta(2E'-N\omega'),$$

we obtain

$$dw_N{}^C=\frac{\omega'^2(1-p_y'^2/E'^2)}{2\pi^2[(N-1)!]^4}\left(\frac{eA_0}{2\omega'}\right)^{2N}\Bigg[\sum_k(-1)^{(N-k)/2}$$

$$\times\left(\frac{p_y'N}{E'}\right)^{k-1}\sum_{\substack{0\leqslant s_1\leqslant s_2\leqslant\dots\\ \dots\leqslant s_{(N-k)/2}\leqslant k-1}}\prod_{l=0}^{(N-k)/2}B_{l,s_l}\Bigg]^2\delta(2E'-N\omega')\,d^3\mathbf{p}', \tag{16}$$

where the summation over k is carried out depending on the parity of N according to (13), and N_0 was replaced by N thanks to the presence of the δ-function.

Taking into account the azimuthal symmetry of the process in the C-frame and integrating (16) over energy and the azimuth angle, we obtain the angular distribution of the N-photon electron (positron) production probability:

$$dw_N{}^C=\frac{1}{8\pi}\frac{N^2\omega'^4}{[(N-1)!]^4}\left(\frac{eA_0}{2\omega'}\right)^{2N}\left(\sin^2\theta'+\frac{4m^2}{N^2\omega'^2}\cos^2\theta'\right)$$

$$\times\left(1-\frac{4m^2}{N^2\omega'^2}\right)^{1/2}\Bigg\{\sum_k(-1)^{(N-k)/2}\left[N\left(1-\frac{4m^2}{N^2\omega'^2}\right)^{1/2}\right.$$

$$\left.\times\cos\theta'\right]^{k-1}\sum_{\substack{0\leqslant s_1\leqslant s_2\leqslant\dots\\ \dots\leqslant s_{(N-k)/2}\leqslant k-1}}\prod_{l=0}^{(N-k)/2}B_{l,s_l}\Bigg\}^2\sin\theta'\,d\theta', \tag{17}$$

where θ' is the angle between the direction of the momentum of the produced electron (positron) and the electric field intensity vector.

By integrating (17) over θ' we obtain the total probability of the N-photon process w_N, which is a relativistic invariant:

$$w_N{}^C=w_N=\int_{-1}^{1}F(\cos\theta')\,d\cos\theta', \tag{18}$$

where $F(\cos\theta')$ is the expression preceding $\sin\theta' d\theta'$ in (17). Consequently (18) also gives the total probability of N-photon production of e^+e^- pairs by a transverse electromagnetic wave in a plasma in the laboratory (L) frame of reference.

Because it is unwieldy we do not give here the final expression for w_N (the integration over θ' is elementary amounting to integration of a power function over $\cos\theta'$.

As regards the angular distribution of the probability of N-photon pair production in the L-frame, it can be obtained from the expression dw_N^C for the differential probability in the C-frame by a Lorentz transformation. Here the quantity multiplying $d^3\mathbf{p}'$ in (16) transforms like the time component of the current density 4-vector of the electrons in the Dirac vacuum ($E'<0$). One must here take into account that the momentum of a real electron coincides with the momentum of the vacuum electron $\mathbf{p}'$, while the momentum of a positron equals $-\mathbf{p}'$ and the vacuum phase-space volume element $d^3\mathbf{p}'/(2\pi)^3$ (in unit volume $V=1$) goes over correspondingly into the volume element in momentum space of electrons and positrons. Further, transforming the quantities in (13) with allowance for the passage from the C-frame to the L-frame, we obtain for the differential probability of N-photon pair production per unit time in the L-frame

$$dw_N=\frac{N\omega^3(1-n^2)^2}{8\pi^2E[(N-1)!]^4}\left[1-\frac{4p_y{}^2}{N^2\omega^2(1-n^2)}\right]$$

$$\times\left[\frac{eA_0}{2\omega(1-n^2)^{1/2}}\right]^{2N}\Bigg\{\sum_k(-1)^{(N-k)/2}\left[\frac{2p_y}{\omega(1-n^2)^{1/2}}\right]^{k-1}$$

$$\times\sum_{\substack{0\leqslant s_1\leqslant s_2\leqslant\dots\\ \dots\leqslant s_{(N-k)/2}\leqslant k-1}}\prod_{l=0}^{(N-k)/2}B_{l,s_l}\Bigg\}^2\delta\left(E-np_x-\frac{N\omega(1-n^2)}{2}\right)d^3\mathbf{p}, \tag{19}$$

where E and $\mathbf{p}$ are the energy and momentum of the produced electron or positron. Integrating (19) over the electron (positron) energy we obtain the angular distribution of the probability of the N-photon production of electrons (positrons) per solid angle element $do=\sin\theta d\theta d\varphi$ (the azimuthal asymmetry of the probability in the L-frame is due to the linear polarization of the wave, in the case of circular polarization the probability distribution has azimuthal symmetry):

$$
dw_N=\sum_{\nu=1}^{2}\frac{N\omega^3(1-n^2)^2p_\nu^2\sin\theta}{8\pi^2[(N-1)!]^4(p_\nu-nE_\nu\cos\theta)}\Bigg[1-\frac{4p_\nu^2\sin^2\theta\cos^2\varphi}{N^2\omega^2(1-n^2)}\Bigg]\Bigg[\frac{eA_0}{2\omega(1-n^2)^{1/2}}\Bigg]^{2N}\Bigg\{\sum_k(-1)^{(N-k)/2}\times\Bigg[\frac{2p_\nu\sin\theta\cos\varphi}{\omega(1-n^2)^{1/2}}\Bigg]^{k-1}\times\sum_{\substack{0\leqslant s_1\leqslant s_2\leqslant\ldots\\ \ldots\leqslant s_{(N-k)/2}\leqslant k-1}}\prod_{l=0}^{(N-k)/2}B_{l,s_l}\Bigg\}^2 d\theta\, d\varphi, \tag{20}
$$

where

$$
p_{1,2}=\frac{Nn\omega(1-n^2)\cos\theta\pm[N^2\omega^2(1-n^2)^2-4m^2(1-n^2\cos^2\theta)]^{1/2}}{2(1-n^2\cos^2\theta)} \tag{21}
$$

$$
E_{1,2}=\frac{N\omega(1-n^2)\pm n\cos\theta[N^2\omega^2(1-n^2)^2-4m^2(1-n^2\cos^2\theta)]^{1/2}}{2(1-n^2\cos^2\theta)}
$$

The angle φ varies from 0 to 2π, while θ (the angle between the vectors $\mathbf{p}$ and $\mathbf{k}$) varies from 0 to $\theta_{\max}$, which is determined from the energy and momentum conservation laws (21). Further, depending on the value of the plasma refractive index $n(\omega)$, the electron (positron) production at the given angle θ is possible for a particular momentum or for one of two different in magnitude momenta. For values

$$
n(\omega)\leqslant\left(1-\frac{2m}{N\omega}\right)^{1/2}
$$

[in this case the threshold condition (1) for the process is certainly satisfied] we should take in (21) only the upper sign, corresponding to the fact that in the probability (2) only $\nu=1$ (p_1) remains and $Q_{\max}=\pi$, i.e., particles are produced in all directions for the given angle θ with definite momentum. In the opposite case we must also take into account the reaction threshold condition in the region of values of the index of refraction

$$
\left(1-\frac{2m}{N\omega}\right)^{1/2}<n(\omega)<\left(1-\frac{4m^2}{N^2\omega^2}\right)^{1/2}
$$

an electron (positron) is produced in a given direction with one of two different values of momentum p_1 and p_2 in a cone, opened forward, whose opening angle is

$$
\theta_{max}=\arcsin\left\{\frac{1}{n}\left[(1-n^2)\left[\frac{N^2\omega^2(1-n^2)}{4m^2}-1\right]\right]^{1/2}\right\}.
$$

In the case $N=1$, i.e., for single-photon pair production [naturally for frequencies satisfying the reaction threshold condition (1) for $N=1$], we obtain from (17) and (18) the familiar expression for the probability of the process $\gamma\to e^++e^-$ in a plasma.[1]

Equations (16)–(18) for the probabilities of multiphoton pair production by a transverse wave in a plasma in the C-frame also describe the process of pair production in vacuum by a uniform periodic electric field [see (3)]. As was mentioned above, this process has been studied by various methods in a number of papers, however the multiphoton probabilities were obtained only for the special case when the particles are produced in a direction perpendicular to the electric field ($\theta=90°$). Equations (16)–(18) describe the multiphoton process for arbitrary direction of particle production in an approximation in which the parameter eA_0/ω' is small, while in the special case $\theta=90°$ Eq. (16) coincides with the result of Ref. 10 (in that case pair production is only possible for absorption of an odd number $N=2k+1$) of quanta.

[1)]As is well known, the plasma refractive index depends on the field of a strong electromagnetic wave also on the intensity of the wave, which becomes significant for $\xi\gtrsim 1$. Since we consider fields with $\xi\ll 1$ the plasma dispersion law remains linear.

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