Magnetic flux penetration into layered superconductors

A. I. Buzdin and A. Yu. Simonov

M. V. Lomonosov State University, Moscow
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We show that penetration of an oblique field $H = H_{c1}$ into a layered anisotropic superconductor should take place not in the form of individual vortex lines, but rather in the form of a chain of vortices. We find the equilibrium periods of these chains for various orientations of the field and degrees of anisotropy. We investigate how a vortex lattice forms, along with the specific features of the magnetization curves for an oblique vortex lattice, and show that in an oblique field slightly greater than H_{c1}, the function $M(H)$ is nonmonotonic: in addition to a maximum corresponding to the lower critical field, it has an additional maximum connected with the degree of anisotropy and the inclination angle of the external field.

Layered high-temperature superconductors (HTSC) possess strongly anisotropic laminar electronic structures. A consequence of this is that their superconducting properties are also strongly anisotropic. The distinctive features of a system of inclined Abrikosov vortices in layered superconductors derive from the fact that screening currents flow predominantly in the planes of the layers (for the HTSC, these planes are the Cu–O planes), not in planes perpendicular to the vortex axis as is the case for normal isotropic superconductors. As a result, for anisotropic superconductors the physics of vortex penetration and formation of vortex lattices for arbitrary orientations of the external magnetic field differs strongly from that of isotropic type-II superconductors.

This paper is devoted to an investigation of the magnetic properties of anisotropic superconductors in a field inclined to the anisotropy axis **c**. In Section 1 we consider the specific features of the penetration of inclined vortices in such systems. We show that, in contrast to isotropic superconductors, the vortex penetration takes place in the form of vortex chains that lie in planes formed by the anisotropy axis **c** and the field **H**. This circumstance has been noted in a previous short communication.[1] In Sec. 2 we will discuss the formation of a vortex lattice in fields $H > H_{c1}$ and its possible distortions at oblique orientations. In Sec. 3 we present magnetization curves of an anisotropic type-II superconductor for various orientations of the external field with respect to the anisotropy axis, and discuss how these curves differ from magnetization curves for anisotropic superconductors.

1. PENETRATION OF VORTICES AND LOWER CRITICAL FIELD H_{c1}

The properties of anisotropic type-II superconductors will be investigated here in the London limit. Following Refs. 2 and 3, we write the expression for the electromagnetic part of the free energy of the superconductor in the form

$$F = \frac{1}{8\pi}\iiint [\mathbf{h}^2 + \lambda^2(\operatorname{rot}\mathbf{h})\hat{\mu}(\operatorname{rot}\mathbf{h})]\,dV, \qquad (1)$$

where $\lambda = \lambda_b$ is the London penetration depth of the magnetic field along the crystal symmetry direction **b**; $\hat{\mu}$ is the reduced effective mass tensor, whose principal values for superconductors with tetragonal symmetry are $\mu_a = M_a/M_b = \varepsilon_1 + 1$, $\mu_b = 1$, and $\mu_c = M_c/M_b = \varepsilon + 1$; M_i is the effective mass of an electron along the i axis ($i = a,b,c$); and **h** is the local magnetic field, which satisfies the London equation:[2,3]

$$\mathbf{h} + \lambda^2 \operatorname{rot}[\hat{\mu}\operatorname{rot}\mathbf{h}] = \mathbf{l}\Phi_0 \sum_i \delta(\mathbf{r}-\mathbf{r}_i). \qquad (2)$$

Here **l** is a unit vector along the vortex axis (i.e., the axis of the induction **B**); and Φ_0 is the flux quantum. As usual, we have neglected the structure of the vortex core in accordance with our assumption that $\lambda \gg \xi$. In Eqs. (1) and (2) we also assume that the vortex cores do not intersect, i.e., that the distance between them satisfies $d \gg \xi$. This latter condition is violated only when we are close to H_{c2}.

We will limit ourselves, in general, to the case of laminar anisotropies ($M_a = M_b < M_c$). In analyzing the field of a vortex it is convenient to use an orthogonal system of coordinates with its z axis along the vortex axis **l**, a y axis which coincides with the crystal symmetry axis **b**, and a new x axis which lies in the plane **cl**. In this new system, the components of the effective mass tensor appear as follows:

$$\hat{\mu} = \begin{vmatrix} 1+\varepsilon\sin^2\theta & 0 & \varepsilon\sin\theta\cos\theta \\ 0 & 1 & 0 \\ \varepsilon\sin\theta\cos\theta & 0 & 1+\varepsilon\cos^2\theta \end{vmatrix}, \qquad (3)$$

where θ is the angle between the anisotropy axis and the vortex axis.

Going from Eqs. (1) and (2) to a Fourier representation with respect to the basis vectors of the vortex lattice:

$$\mathbf{h}(\mathbf{r}) = \frac{1}{S}\sum_{\mathbf{k}} \mathbf{h}_{\mathbf{k}} e^{i\mathbf{k}\mathbf{r}}, \quad \mathbf{h}_{\mathbf{k}} = \int \mathbf{h}(\mathbf{r}) e^{-i\mathbf{k}\mathbf{r}}\, d^2\mathbf{r},$$

where S is the area of a unit cell containing one flux quantum Φ_0, we can rewrite the expression for the free energy per unit volume in the form[2,3]

$$F = \frac{\Phi_0}{8\pi S^2}\sum_{\mathbf{k}} (h_{\mathbf{k}})_z, \qquad (4)$$

where

$$(h_{\mathbf{k}})_z = \frac{\Phi_0}{\lambda^2}\frac{\lambda^{-2} + (q^2+Q^2)(1+\varepsilon v_z^2)}{(\lambda^{-2}+q^2+Q^2)(\lambda^{-2}+(1+\varepsilon)q^2+(1+\varepsilon v_z^2)Q^2]} \qquad (5)$$

and $k_x = Q$, $k_y = q$, $v_z = \cos\theta$. The dependence of the vortex energy on its orientation is given by the expression[3]

$$E_v^0 = \frac{\Phi_0^2}{(4\pi\lambda)^2}\left(\cos^2\theta + \frac{m_\parallel}{m_\perp}\sin^2\theta\right)^{1/2} \ln\varkappa(\theta), \qquad (6)$$

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where the ratio of effective masses is $m_\perp/m_\parallel = \varepsilon + 1$, and $\varkappa(\theta)$ is an angle-dependent Ginzburg–Landau parameter.

For the case of an inclined vortex the plane **cl** is singled out: as we will show below, the configuration which possesses minimum energy is not that of a solitary vortex but rather that of a vortex which is one component of a vortex chain lying in the plane **cl**. The energy of a vortex which is a component of such a chain with period a can be written in the form

$$E_v = E_v^0 + \frac{\Phi_0}{8\pi}\left(\frac{1}{a}\sum_{Q=\frac{2\pi n}{a}}\int\frac{dq}{2\pi}\,\mathbf{lh}_{qQ} - \iint\frac{dq\,dQ}{(2\pi)^2}\,\mathbf{lh}_{qQ}\right)$$
$$= E_v^0 + \frac{\Phi_0^2}{16\pi\lambda^2}\int\frac{dq}{2\pi}\left(\frac{\operatorname{cth}(\tilde a Q_1/2)-1}{Q_1} - F(q)\right.$$
$$\left.\times\left[\frac{\operatorname{cth}(\tilde a Q_1/2)-1}{Q_1} - \frac{\operatorname{cth}(\tilde a Q_2/2)-1}{Q_2}\right]\right), \quad (7)$$

where $\tilde a = a/\lambda$ and

$$Q_1 = (q^2+1)^{1/2}, \quad Q_2 = \left(\frac{(1+\varepsilon)q^2+1}{1+\varepsilon v_z^2}\right)^{1/2},$$
$$F(q) = \frac{q^2}{q^2 - \operatorname{ctg}^2\theta}. \quad (8)$$

In Eq. (7) we have separated out the interaction energy of the vortices in explicit form. By subtracting off the integral with respect to Q, we eliminate the logarithmic divergence for large values of **k**. It is clear from Eqs. (7) and (8) that for $\tilde a \gg 1$ the primary contribution to the integral (7) comes from the small-q region. In this case for $\varepsilon \gg 1$ we have $Q_2 \ll Q_1$ and the interaction energy of the vortices at large spacings is determined primarily by the last term of (7):

$$E_v \approx E_v^0 + \frac{\Phi_0^2}{16\pi\lambda^2}\int\frac{dq}{2\pi}F(q)\,\frac{\operatorname{cth}(\tilde a Q_2/2)-1}{Q_2}.$$

The expression under the integral sign is negative for small q (i.e., the interaction energy of the vortices is negative) and vanishes in the limit $\tilde a \to \infty$. On the other hand, it is obvious that for $\tilde a \ll 1$ the vortices should repel and their energy of interaction should be positive. Therefore, the minimum interaction energy of the vortices should occur at finite $\tilde a$. This conclusion agrees with the conclusions of Ref. 4 concerning reversal of the magnetic field of an inclined vortex at large distances.

Minimizing Eqs. (7) and (8) numerically with respect to $\tilde a$, we can find the equilibrium energy and equilibrium period $\tilde a_{\min}$ of the vortex chain. The results of this calculation for the case $\varepsilon = m_\perp/m_\parallel - 1 = 25$ (corresponding to $YBa_2Cu_3O_{7-\delta}$, see Ref. 5) are presented in Fig. 1.

It is clear from the results we have obtained that the energy of a vortex decreases most strongly due to interaction in the region of angles $\theta \approx 60°$; however, this decrease is relatively small, since Eq. (6) for E_v^0 contains a large logarithmic factor $\ln\varkappa$ which is not present in the difference $E_v(\theta) - E_v^0(\theta)$. Nevertheless, this implies that in fields inclined at an angle φ to the **c** axis, the lower critical field H_{c1} is somewhat decreased in comparison to the value calculated in Ref. 3:

$$H_{c1}(\varphi) = \frac{\Phi_0}{4\pi\lambda^2}(1+\varepsilon\sin^2\varphi)^{-1/2}\ln\varkappa. \quad (9a)$$

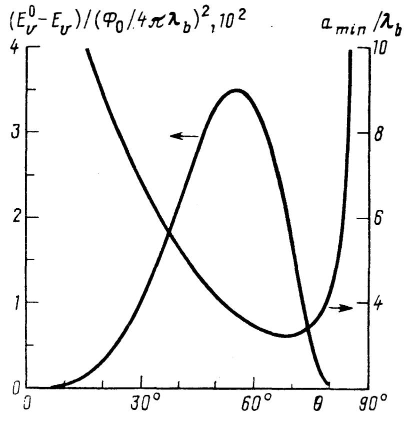

FIG. 1. Angular dependence of the vortex interaction energy in a chain and equilibrium period of such a chain for the case $m_\perp/m_\parallel = 1+\varepsilon = 26$.

Since the vortex penetrates into the superconductor almost parallel to the layers even when the angle φ between the anisotropy axis and the external field is small:[3]

$$\operatorname{tg}\theta = (\varepsilon+1)\operatorname{tg}\varphi, \quad (9b)$$

the largest deviation of the lower critical field from the function (9a) takes place for small angles ($\varphi \lesssim \arctan[1/(\varepsilon+1)]$; for Y–Ba–Cu–O, $\varphi \lesssim 10°$). The results of our calculations are shown in Fig. 2. For comparison we show the behavior obtained according to the formulas (9) as dashed curves.

For a sample having the form of an ellipsoid of rotation with an axis coinciding with the anisotropy axis **c** and a demagnetization factor along this axis n, the angles of inclination of the vortex θ and the external field φ in a field equal to the first critical field

$$H_{c1}(\varphi) = \frac{\Phi_0}{4\pi\lambda^2}\,\frac{(1-n^2)\ln\varkappa}{[4(1-n)^2(1+\varepsilon)\sin^2\varphi + (1+n)^2\cos^2\varphi]^{1/2}}, \quad (10a)$$

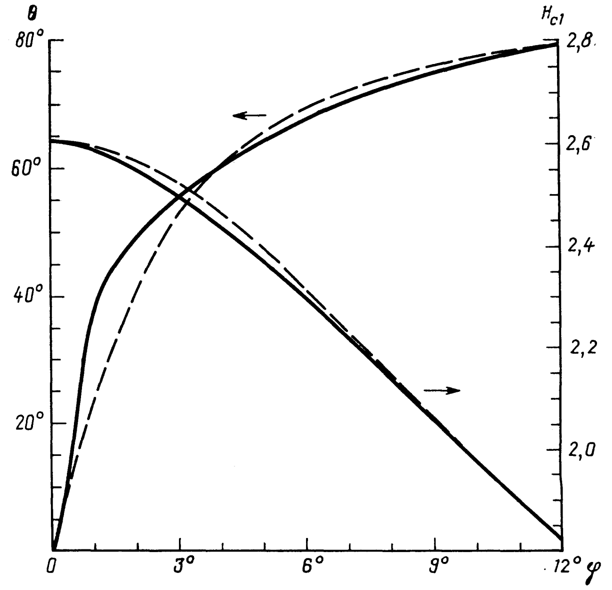

FIG. 2. Angular dependence of the lower critical field and angle of penetration of vortices into a superconductor, taking into account the penetration of vortex chains. The dashed curves indicate the dependences calculated in Ref. 3.

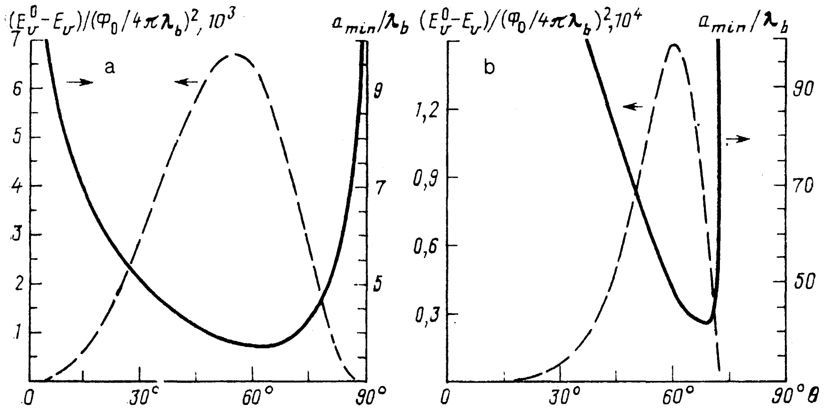

FIG. 3. Angular dependence of the vortex interaction energy in a chain and the equilibrium period of such a chain for $\varepsilon = 2499$, $\varepsilon_1 = 224$ in cases where the chain lies (a) in the plane **ac** and (b) in the plane **bc**.

are related by the expression

$$\operatorname{tg}\theta = 2\frac{1-n}{1+n}(1+\varepsilon)\operatorname{tg}\varphi. \tag{10b}$$

As a result, for angles φ corresponding to $\theta \approx 60°$, at a field $H = H_{c1}$ we should expect the appearance of vortex chains that are spaced very far apart with a period $a \sim 2\lambda$ [see Fig. 1; for $YBa_2Cu_3O_{7-\delta}$ we find $\lambda_a \approx \lambda_b \approx 300$–$500$ Å (see Ref. 6)].

The necessary condition for the presence of an attractive force between vortices in a uniaxial semiconductor is the fulfillment of the inequality $\varepsilon > 0$ (so that this effect will never be observed in quasione-dimensional superconductors). Note that we are considering only straight-line oblique vortices here, i.e., we will not treat other possible configurations, e.g., in which modulation of the inclination angle of a vortex is present.

In concluding this section, let us touch briefly on the situation in a biaxial superconductor. As in the case of a uniaxial superconductor investigated above, this system can also support chains of vortices in a field $H = H_{c1}$ lying in the plane formed by the **c** axis (i.e., the axis along which the effective mass is a maximum) and the vortex axis. This situation corresponds, e.g., to organic superconductors. In the orthogonal system of coordinates x, y, z, with the z axis along the vortex axis **l**, the y axis coinciding with the crystal symmetry axis **b**, and a new x axis lying in the plane **cl** (we will consider a case where the vortex lies in one of the principal planes), the effective mass tensor can be written as

$$\hat{\mu} = \begin{vmatrix} 1+\varepsilon_1\cos^2\theta+\varepsilon\sin^2\theta & 0 & (\varepsilon-\varepsilon_1)\sin\theta\cos\theta \\ 0 & 1 & 0 \\ (\varepsilon-\varepsilon_1)\sin\theta\cos\theta & 0 & 1+\varepsilon_1\sin^2\theta+\varepsilon\cos^2\theta \end{vmatrix}. \tag{11}$$

The interaction energy of the vortices in the chains can once again be written in the form (7); however, Q_1 and Q_2 are now roots of the biquadratic polynomial

$$z(Q) = \mu_{yy}\mu_{zz}Q^4 + [\mu_{zz}+\mu_{yy}+q^2(\mu_{zz}\mu_{yy} + \mu_a\mu_c)]Q^2 + [1+q^2(\mu_a+\mu_c)+q^4\mu_a\mu_c], \tag{12}$$

$$F(q) = [1+\mu_{zz}(q^2-Q_2^2)]/\mu_{zz}(Q_1^2-Q_2^2). \tag{13}$$

In Fig. 3 we present the results of calculations of the equilibrium value of the vortex interaction energy for a chain and the period of this chain for a case corresponding to the organic superconductor $(TMTSF)_2ClO_4$, for which measurement of the anisotropy of the upper critical field H_{c2} implies $H_{c2}^a : H_{c2}^b : H_{c2}^c = 15:1:50$ (Ref. 7). It is noteworthy that in this case the plane in which the gain in energy due to attraction of vortices in the chain is largest is the plane of maximum anisotropy **ac**, while the maximum in the angular dependence of the energy with increasing ε_1 shifts towards the region of small angles θ.

2. FORMATION OF A VORTEX LATTICE

In fields somewhat larger than H_{c1}, systems of parallel chains appear: to first approximation the period of these chains $a_{\min}$ does not change, but the spacing between chains L is determined by the force of their mutual repulsion. The contribution to the free energy due to the interaction of vortex chains is

$$E_{int} = \frac{\Phi_0}{8\pi S^2}\left(\sum_{q,Q}\mathbf{lh}_{qQ} - \frac{S}{4\pi^2}\iint \mathbf{lh}_{qQ}\,dQ\,dq\right). \tag{14}$$

If we first carry out the summation with respect to q in Eq. (14), by using the Poisson formula we obtain the increase in energy (for a single chain) due to the interaction between chains:

$$\tilde{E}_v = E_v + \frac{\Phi_0^2}{8\pi a\lambda}\frac{\nu_x^2}{(1+\varepsilon)^{1/4}}\exp\left[-\frac{L}{\lambda(1+\varepsilon)^{1/4}}\right]. \tag{15}$$

This expression is valid in the asymptotic limit $L \gg a^2/\lambda, \lambda$.

Note that for simplicity we have assumed a square lattice in deriving (15); however, since the difference between triangular and rectangular lattices is vanishingly small in this limit by virtue of the fact that the lattice is very widely spaced along the crystal symmetry axis **b** ($L \gg a$), Eq. (15) also gives a correct description of the situation for a triangular lattice, a configuration which is realized in uniaxial superconductors as well.

Using (15), it is easy to show[4] that the magnetic induction B in fields which slightly exceed H_{c1} is

$$B = \frac{\Phi_0}{a\lambda(1+\varepsilon)^{1/4}} \Big/ \ln\left[\frac{\Phi_0\nu_x^2}{2a\lambda(1+\varepsilon)(H-H_{c1})}\right]. \tag{16}$$

Note that for isotropic superconductors $B \propto \ln^{-2}[(H-H_{c1})]$ (see Ref. 8).

In higher fields, where the lattice becomes rather dense ($S \sim \lambda^2$), Eq. (15) cannot be used; in this case, in order to

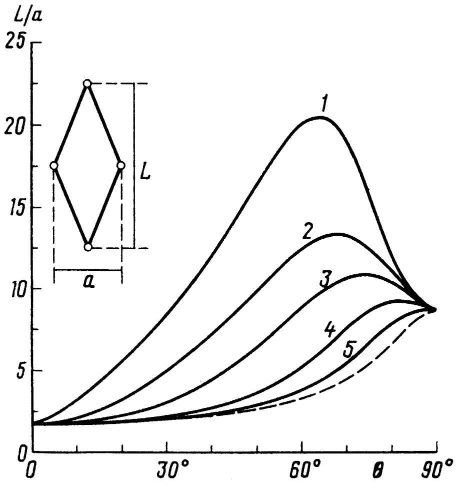

FIG. 4. Dependence of the ratio of parameters of the unit cell a/L of a vortex lattice on the angle θ for various values of the area of a unit cell S: *1*) $S=120\lambda_b^2$, *2*) $60\lambda_b^2$; *3*) $30\lambda_b^2$; *4*) $10\lambda_b^2$; *5*) $5\lambda_b^2$. The dashed curves denote the function (17). In the inset we show a unit cell of a triangular vortex lattice.

reveal the structure of the vortex lattice, it is necessary to minimize the free energy (14) numerically with respect to a for a fixed vortex direction and density of vortices (i.e., the area of an elementary cell of the vortex lattice S) for various types of vortex lattice.

For a square lattice, the summation in (14) is taken over the reciprocal lattice vectors

$$Q_\square=2\pi k/a,\quad q_\square=2\pi n/L,\quad k, n=0, \pm1, \pm2, \ldots,$$

while for the case of a triangular lattice (see the inset in Fig. 4), the reciprocal lattice vectors can easily be found:

$$Q_\triangle=2\pi(2k-n)/a,\quad q_\triangle=2\pi n/L,\quad k, n=0, \pm1, \pm2, \ldots$$

Figure 4 shows the results of these calculations.[1)] We note that, in all cases investigated, the triangular lattice was found to be preferred; however, for vortices oriented at an angle $\theta\approx 60°$ to the anisotropy axis and $S>\lambda^2$ the difference in energy between a triangular and a square lattice is $|(E_\square - E_\triangle)/E_\triangle|\approx 10^{-6}$; recall that for isotropic superconductors this parameter is almost four orders of magnitude larger.[4] From the data shown in the figure it is clear that in our case the angular dependence of the ratio of the two vortex lattice parameters is quite nonmonotonic. From this we see that in the low-field region $H\gtrsim H_{c1}$, the presence of chains can significantly distort the vortex lattice.

With increasing field these distortions disappear, and for a dense lattice ($S<\lambda^2$) we are led to previously known results[10] which do not depend on the value of the external field:

$$\frac{L}{a}=\left[\frac{3(1+\varepsilon)}{1+\varepsilon\cos^2\theta}\right]^{1/2} \qquad (17)$$

(see the dashed curves in Fig. 4).

Note that the triangular lattice is apparently preferred for all uniaxial superconductors. It is interesting to note that for biaxial superconductors a result was obtained in Ref. 11 suggesting a transition to a rectangular lattice at $\theta\approx 60°$ for certain values of the effective mass ratio. However, it is not yet clear whether or not this transition is a general property of all biaxial superconductors or appears only for certain values of ε_1. This question is important for HTSC that possess small anisotropy in the basal plane and requires a more detailed investigation.[2)]

3. MAGNETIZATION CURVES OF ANISOTROPIC SUPERCONDUCTORS

The field dependence of the magnetic moment was previously discussed in Refs. 12–14 for anisotropic superconductors; however, the behavior of the magnetization for fields close to H_{c1} was not analyzed there. The distinctive features associated with penetration of oblique vortices discussed above for anisotropic superconductors in the immediate vicinity of the lower critical field cannot be reflected in the magnetization curves.

Let us consider the Gibbs free energy of such a system

$$G=\frac{\Phi_0^2}{(4\pi\lambda)^2 S}\left(\cos^2\theta+\frac{m_\parallel}{m_\perp}\sin^2\theta\right)^{1/2}\ln\varkappa(\theta)+\frac{\Phi_0}{8\pi S^2}\left(\sum_{q,Q}\mathrm{lh}_{qQ}-\frac{S}{4\pi^2}\iint\mathrm{lh}_{qQ}\,dQ\,dq\right)-\frac{\Phi_0}{4\pi S}H\cos(\theta-\varphi), \qquad (18)$$

where the first term is a sum of the energies of individual vortices, the second term is the vortex-vortex interaction energy, and the third term is the interaction energy of the vortices with the external magnetic field. Note that the sum over Q in (18) is easily performed and the tedious calculation of the double sum can be avoided:

$$E_{int}=\frac{\Phi_0}{8\pi S^2}\Bigg\{\sum_{q=\frac{2\pi n\lambda}{L}}\frac{\tilde a}{4}\Bigg[\frac{\mathrm{sh}(\tilde aQ_2/2)}{Q_2[\mathrm{ch}(\tilde aQ_2/2)-(-1)^n]}-\frac{\cos^2\theta}{q^2\sin^2\theta-\cos^2\theta}\left(\frac{\mathrm{sh}(\tilde aQ_1/2)}{Q_1[\mathrm{ch}(\tilde aQ_1/2)-(-1)^n]}-\frac{\mathrm{sh}(\tilde aQ_2/2)}{Q_2[\mathrm{ch}(\tilde aQ_2/2)-(-1)^n]}\right)\Bigg]-\frac{S}{4\pi\lambda^2}\Bigg[\left(\frac{1+\varepsilon\cos^2\theta}{1+\varepsilon}\right)^{1/2}\times\ln\left|\frac{[q_{max}^2+1/(1+\varepsilon)]^{1/2}+q_{max}}{[q_{max}^2+1/(1+\varepsilon)]^{1/2}-q_{max}}\right|-\cos\theta\left(\ln\left|\frac{(q_{max}^2+1)^{1/2}+q_{max}}{(q_{max}^2+1)^{1/2}-q_{max}}\right|-\ln\left|\frac{\cos\theta[q_{max}^2(1+\varepsilon)+1]^{1/2}-q_{max}(1+\varepsilon\cos^2\theta)^{1/2}}{\cos\theta[q_{max}^2(1+\varepsilon)+1]^{1/2}+q_{max}(1+\varepsilon\cos^2\theta)^{1/2}}\right|\right)\Bigg]\Bigg\},$$

where Q_1 and Q_2 are defined by Eqs. (8) and $q_{\max}=2\pi\lambda_{\max}/L$ is the maximum value included in the sum over reciprocal lattice vectors ($n=0, \pm1, \pm2,\ldots, \pm n_{\max}$).

In order to find the relation that connects the induction **B** (or, which is the same thing, the area of the unit cell S and the inclination angle of the vortex axis to the anisotropy axis θ) and the external field **H**, we should seek a minimum of (18) with respect to the variables S, θ, and a/L in a given field **H**. It is not possible to carry out this program analytically; therefore, we have performed numerical calculations for various external field inclination angles φ and the anisotropy constant $\varepsilon=25$. Based on the results presented in Sec-

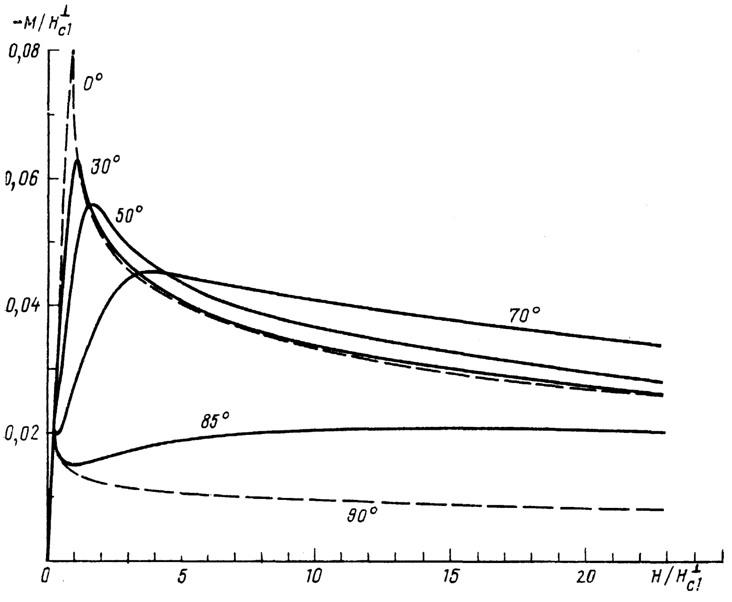

FIG. 5. Magnetization curves for various angles φ of inclination of the external field to the anisotropy axis for the case $m_\perp/m_\parallel = 1+\varepsilon = 26$.

tion 2, we have assumed the vortex lattice to be triangular in this case, with $a < L$.

It is clear from the data shown in Figs. 5 and 6 that penetration of the magnetic field into an anisotropic superconductor possesses a number of important peculiarities connected with the absence of screening of the field component parallel to the layers. This latter fact implies that starting with the field $H = H_{c1}(\varphi)$ only the component of the magnetic moment $M_\parallel$ parallel to the layers begins to decrease (Fig. 6b), while the other component of the magnetic moment $M_\perp$ (Fig. 6a) continues to grow until a certain value of the external field $H = H_M > H_{c1}$ is reached. The quantity H_M depends on the inclination angle of the external field, and also the degree of anisotropy. From this we see that the field dependence of the magnetic moment within the region of fields $H_{c1} < H < H_M$ is quite nonmonotonic: at the lower critical field the curve $M(H)$ has the logarithmic singularity implied by Eq. (16), followed by an almost linear increase associated with the increase of $M_\perp(H)$ until $H = H_M$. For $H \gg H_M$ the magnetic moment decreases logarithmically:[12]

$$-4\pi M = \frac{\Phi_0}{8\pi\lambda^2} \ln\left(\frac{\beta H_{c2}}{H}\right) \frac{1+\varepsilon(1+\varepsilon)\cos^2\theta}{1+\varepsilon\cos^2\theta}, \tag{19}$$

where β is a constant of order unity.

In connection with the shape of the magnetization curves, we should also mention that when the angle φ of inclination of the field to the anisotropy axis is small, the singularity in the magnetization at the field is only weakly expressed, while the peak at $H = H_M$ is very similar to the characteristic logarithmic singularity of the magnetization curves of isotropic superconductors at $H = H_{c1}$. From this we see that for small φ (e.g., for imprecise orientation of the field) the field H_M can be mistaken for the lower critical field, i.e., determination of $H_{c1}^{\perp}$ from a magnetization curve can lead to very serious errors.

When the external field is inclined at large angles to the anisotropy axis, the maxima in the magnetization for H_{c1} and H_M are widely spaced, and we can be confident that for this particular range of angles conditions for observing similar singularities in the curves $M(H)$ are optimal.

An important feature of the penetration of magnetic flux into anisotropic superconductors is that by increasing the external field we change not only the absolute value of the magnetic moment **M**, as in the isotropic case, but also its direction. As the external field increases, the vortex lattice, which at first (for $H \approx H_{c1}$) is directed almost parallel to the layers, aligns itself with the direction of the external field **H** (see Fig. 6). If the external field is directed at an angle $\varphi < 60°$, then as the field increases the vortex lattice lies within the range of angles discussed in Sec. 1, i.e., $\theta \approx 60°$. In this case characteristic features appear that are connected with the fact that as the system attempts to lower its energy, the vortices are "confined" within this region of angles. This effect results in a significantly nonmonotonic field dependence of the lattice parameter a (i.e., the period of the lattice in the plane **ac**) for small fields $H \approx H_{c1}$ when the angles of inclination of the external field to the anisotropy axis (see Fig. 7) are small. This unusual behavior is perhaps detectable by "decoration" methods, magneto-optic methods or neutron diffraction.

Note that for real samples the external magnetic field $\mathscr{H}$, which does not coincide with the internal Maxwell field **H**, is connected with it for a sample in the form of an ellipsoid of rotation by the expressions

$$H_\perp = \frac{1}{1-n}(\mathscr{H}_\perp - nB_\perp),$$
$$H_\parallel = \frac{2}{1+n}\left(\mathscr{H}_\parallel - \frac{1-n}{2}B_\parallel\right), \tag{20}$$

where we have used the fact that for an ellipsoid of rotation with two equal axis ($a_x = a_y \neq a_z$) the demagnetization factors $n_x = n_y$ along x and y are related to the magnetization factor $n_z = n$ along the z axis by $n_x = (1-n)/2$. For this case all the features discussed above will be observed for directions of the induction close to 60°.

Note that magnetization curves similar to the ones discussed here were observed in the experiments of Ref. 15, and that the field dependence of the inclination angle of the vortex lattice to the anisotropy axis, which we obtain in this

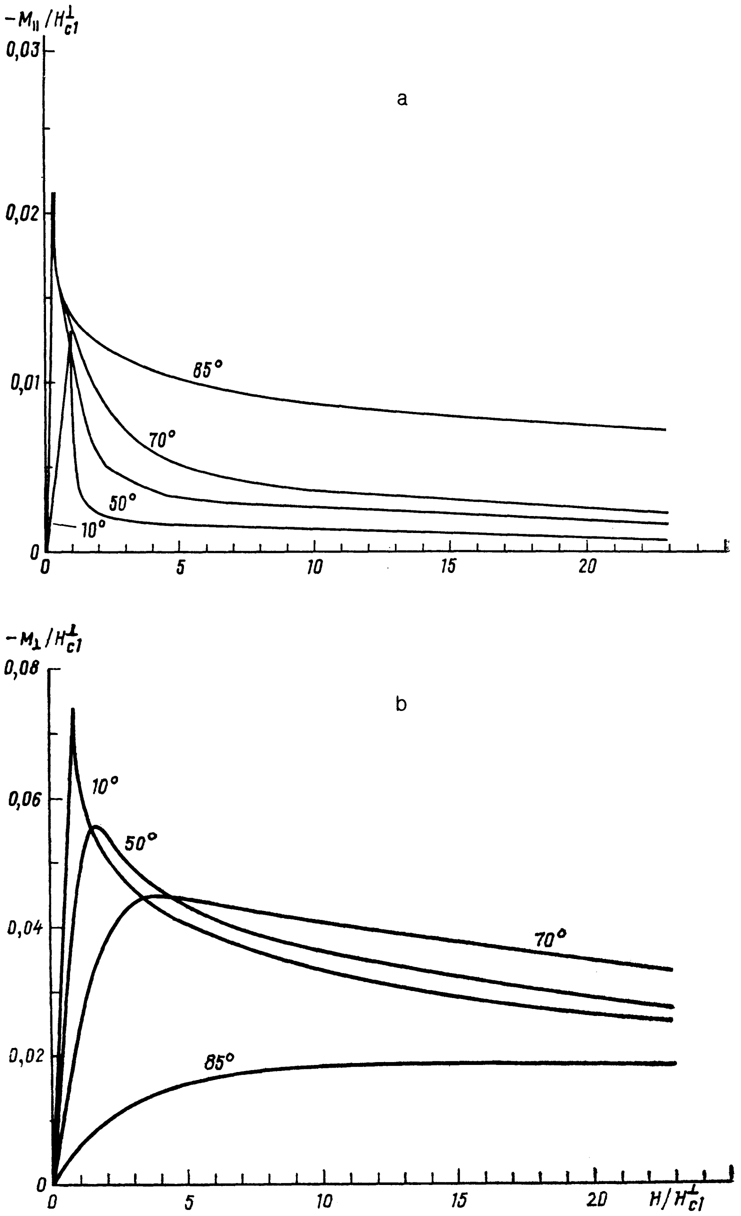

FIG. 6. Field dependence of the components of the magnetization vector $\mathbf{M} = (M_{\parallel}, M_{\perp})$ for various angles of inclination φ of the external field to the anisotropy axis for the case $m_{\perp}/m_{\parallel} = 1 + \varepsilon = 26$.

paper, is in qualitative agreement with the experimentally observed dependence in Refs. 15 and 16. The difference is apparently connected with irreversible effects which were not taken into account in our calculations.

In anisotropic superconductors, the noncollinearity of the external field and the magnetic moment gives rise to the appearance of a mechanical twisting moment

$$\mathbf{T} = [\mathbf{MH}], \qquad (21)$$

this was investigated previously in Refs. 5, 14 and 16 in the region of fields $H_{c1} \ll H \ll H_{c2}$. In this interval of fields the magnetic moment depends on the magnetic field logarithmically [see Eq. (19)], while the twisting moment (per unit volume in this calculation) is almost linear in the field and its dependence on the angle φ takes on a universal character:[14]

$$T = \frac{\Phi_0}{(8\pi\lambda^2)^2} H \ln\left(\frac{(\varepsilon+1)^{1/2}\beta H_{c2}^{\parallel}}{H(1+\varepsilon\cos^2\varphi)^{1/2}}\right)\frac{\varepsilon \sin 2\varphi}{(1+\varepsilon\cos^2\varphi)^{1/2}}. \qquad (22)$$

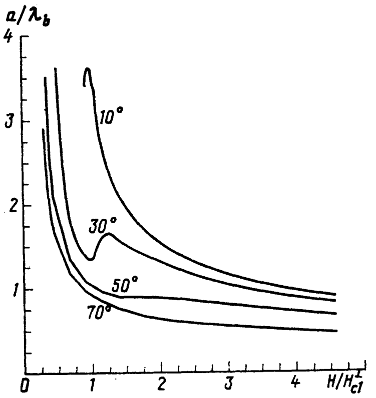

FIG. 7. Field dependence of the period a of the vortex lattice in the plane **ac** for various angles φ of inclination of the external field to the anisotropy axis for the case $m_{\perp}/m_{\parallel} = 1 + \varepsilon = 26$.

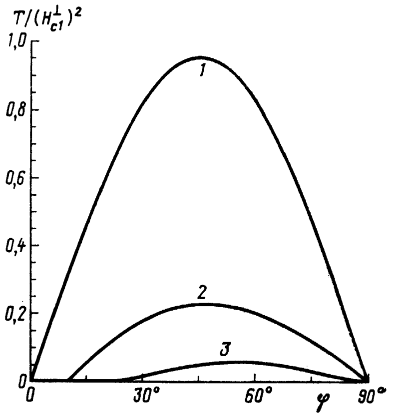

FIG. 8. Angular dependence of the mechanical twisting moment for various values of the external field: *1*) $H = 2H_{c1}^{\perp}$; *2*) $0.9H_{c1}^{\perp}$; *3*) $0.5H_{c1}^{\perp}$.

Our results show that in the neighborhood of the lower critical field both the angular and the field dependences of the twisting moment depart significantly from Eq. (22); in particular, although the point where the function $T(\varphi)$ is a maximum is not at all universal, it is a function only of the anisotropy of the superconductor, while the way in which the magnitude of the twisting moment increases as a function of field differs significantly from linear (see Fig. 8). These differences are related to the rotation of the magnetic moment vector toward the **c** axis and the nonmonotonic character of the magnetic moment discussed above within this range of fields. From this we see that the twisting moment is given by Eq. (22) only in the range of fields $H_M \ll H \ll H_{c2}$, and the anisotropy constant can be obtained correctly from magnetomechanical measurements only in this interval of fields

4. CONCLUSIONS

In this paper we have shown that the penetration of inclined Abrikosov vortices into an anisotropic superconductor at $H = H_{c1}$ takes place not in the form of individual vortex lines but rather in the form of vortex chains with period $a \sim \lambda$. Our calculations show that attraction between the vortices within a certain range of angles is a general property of all anisotropic type-II superconductors with $\varepsilon > 0$ (including biaxial ones) and the features discussed above, i.e., magnetic flux penetration and creation of vortex lattices, should be observed in these as well. Note that the small anisotropy in the **ab** plane which is observed for $YBa_2Cu_3O_{7-\delta}$ ($\varepsilon_1 = 0.2$, Ref. 17) does not alter the results presented here significantly. Magnetic flux penetration in the form of vortex chains rather than in the form of individual vortices can perhaps be observed by magnetomechanical experiments.

The presence of vortex chains significantly distorts the form of a unit cell of the vortex lattice in small fields. The presence of these distortions may perhaps be observable using magneto-optical methods or by decoration methods.

The magnetization curves presented in Section 3 differ significantly from analogous curves for isotropic superconductors in the nonmonotonic shape of the function $M(H)$ for fields that are somewhat larger than the lower critical field, and a number of the features of magnetic flux penetration in layered superconductors—in particular, the subsequent growth of the magnetic moment and the nonmonotonicity of the period of the vortex lattice—may perhaps be experimentally measurable.

In connection with this, there is also interest in studying the twisting moment for fields close to H_{c1}, as well as comparing data obtained from magnetic and magnetomechanical measurements of single crystal samples of the layered superconductors.

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[1)] We have recently been made aware of similar results derived by the authors of Ref. 18.

[2)] We should also mention Ref. 19, which contains additional results of calculations of the energies of various oblique lattices.

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