Dynamics of a two-dimensional domain wall in a ferromagnetic film with a uniaxial anisotropy

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The (2 + 1)-dimensional Landau–Lifshitz equation is solved numerically in order to study the motion of a solitary domain wall in a ferromagnetic film with a uniaxial anisotropy subjected to a homogeneous magnetic field. The internal dynamics of a domain wall in the velocity saturation region is determined as a function of the intensity of the applied magnetic field.

INTRODUCTION

Domain walls have a considerable influence on a number of processes such as quasistatic and dynamic magnetization reversal (switching), electrodynamic losses, motion of isolated domains, etc., which occur in magnetically ordered materials (see, for example, Ref. 1). Moreover, domain walls significantly influence the spectra of magnetic excitations, resonance properties, wave propagation, and other phenomena (see, for example, Ref. 2). This accounts for the major interest in the dynamic behavior of domain walls.

Since the paper by Landau and Lifshitz[3] where the results of a calculation of domain wall velocities were presented for the first time, much work has been done on the dynamic behavior of these walls (for reviews see Refs. 4–6). The present direction of research is aimed to gain an understanding of the major role played in the dynamics of domain walls by their internal structure, which can change considerably in the process of domain wall motion.

The most general equation which can be used in a study of the domain wall dynamics is the Landau–Lifshitz equation.[3] However, because of the great complexity of this nonlinear integrodifferential equation, it is usual to consider simpler models of the domain wall structure and the associated simpler equations of motion,[4–17] which nevertheless allow for the main features of the original problem.

The simplest model which can be used to obtain an analytic solution of the Landau–Lifshitz equation is a one-dimensional rigid domain wall.[7] It is true in this case that we can describe only the steady-state behavior of the wall. However, the model is far from reality in the case of thin magnetic (and particularly magnetically uniaxial) films with an easy magnetization axis perpendicular to the film surface. In films of this kind the structure of a domain wall is at least two-dimensional, due to the existence of magnetostatic poles on the film surface. Consequently, the domain wall is twisted.[8]

The major progress in the study of the dynamics of twisted domain walls was made by Slonczewski,[9] who reduced the two-dimensional problem with a nonlocal magnetostatic interaction to a one-dimensional local problem. He obtained two relatively simple equations assuming that the distribution of the domain wall magnetization in the polar angle is the same as in a Block domain wall. An analytic investigation of the dynamic behavior of a domain wall was subsequently made[10,11] by introducing the concept of horizontal Block lines which dynamically transform the domain wall structure and represent the transition region between two domain walls with different forms of chirality.

According to the experimental data (see, for example, Ref. 12), the dependence of the domain wall velocity on the intensity of the external driving magnetic field is initially linear in many materials, followed by a saturation region. The theory put forward in Refs. 10 and 11 relates the linear–saturation transition to the formation of a horizontal Bloch line in some critical field. The two best known theoretical models of the dynamic modification of the domain wall structure in the saturation region are 1) generation and breakthrough of a single horizontal Bloch line at the film surface;[10] 2) generation and formation of clusters of horizontal Bloch lines followed by subsequent expulsion of these lines to the film, surface and their annihilation.[11] However, the values of the saturation velocity obtained on the basis of these two models do not agree even qualitatively with the experimental results (see, for example, Ref. 12).

The first numerical investigations of the Slonczewski equations were made by Hubert[13] who demonstrated that the nature of the dynamic transformations of the domain wall structure depends on the film thickness. Subsequent investigations of the Slonczewski equations by numerical methods were reported in Refs. 14–16. They demonstrated that both of the above internal dynamics mechanisms are possible. Moreover, certain new features were established: for example, it was shown in Ref. 15 that horizontal Bloch lines may form in relatively weak fields, but they cannot break through to the surface.

The strong influence of the nature of the dynamic modification of a domain wall on the average velocity demonstrates the need to analyze in greater detail the mechanism of the internal dynamics of domain walls without any model assumptions underlying the existing theories. Our aim will be therefore to investigate the domain wall dynamics numerically, retaining the rigorous micromagnetic formulation of the problem within the framework of the Landau–Lifshitz equation.

This approach had been used earlier[18–20] to study processes unrelated to the motion of a domain wall. The equation of motion of the magnetization allowing for the damping has been used as an analog for the steady-state method to obtain two-dimensional static solutions.

We shall list the main assumptions of a new numerical tracking method and give the results of an investigation of a (2 + 1)-dimensional domain-wall equation of motion

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which allows for a nonlocal demagnetizing field. We shall describe in greater detail a new mechanism of generation and breakthrough of horizontal Bloch lines which corresponds to the region of saturation of the domain wall velocity reported earlier in Ref. 21.

FORMULATION OF THE PROBLEM AND SOLUTION METHOD

Consider a ferromagnetic film with a uniaxial anisotropy (assumed to be a material with bubble domain walls). We assume that the $z(\mathbf{k})$ axis coincides with the anisotropy axis and is perpendicular to the film plane, while the $x(\mathbf{i})$ axis is perpendicular to the domain wall and lies in the plane of the film, and the $y(\mathbf{j})$ axis is directed along the domain wall in such a way that the coordinate system is right-handed. We assume that all the investigated magnetization distributions are translationally invariant along the y axis (Fig. 1).

We assume that

$$t_1=|\gamma|M_S t,\quad l=\frac{(AK)^{1/2}}{(\pi M_S^2)^{1/2}},\quad Q=\frac{K}{2\pi M_S^2},$$
$$\mathbf{v}(x,z,t)=\frac{\mathbf{M}(x,z,t)}{M_S},\quad \mathbf{h}(x,z,t)=\frac{\mathbf{H}(x,z,t)}{M_S}.$$

Here, $\mathbf{M}(x, z, t)$ is the distribution of the magnetization; M_s is the saturation magnetization; $\mathbf{H}(x, z, t)$ is the effective field; γ is the gyromagnetic ratio; l is a characteristic length; A is the exchange constant; K is the uniaxial anisotropy constant; Q is the quality factor. We write down the Landau–Lifshitz equation in its dimensionless form:

$$(1+\alpha^2)\frac{\partial \mathbf{v}}{\partial t}=[\mathbf{hv}]-\alpha[\mathbf{v}[\mathbf{vh}]],$$
$$\mathbf{h}(x,z,t)=-4\pi Q\mathbf{v}_\perp+(\pi/Q)\nabla^2\mathbf{v}+\mathbf{h}_{d_1}+\mathbf{h}_{d_2}+\mathbf{h}_0, \qquad (1)$$
$$\mathbf{h}_{d_1}(x,z,t)=-\int_\Omega\frac{(\nabla\mathbf{v})2\boldsymbol{\rho}}{\boldsymbol{\rho}^2}dx'dz'+\int_{\partial\Omega}\frac{(\mathbf{vn})2\boldsymbol{\rho}}{\boldsymbol{\rho}^2}ds',$$

where

$$\mathbf{v}=v_x\mathbf{i}+v_y\mathbf{j}+v_z\mathbf{k},\qquad |\mathbf{v}|=1,\qquad \mathbf{v}_\perp=v_x\mathbf{i}+v_y\mathbf{j},$$
$$\nabla=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial z}\right),\quad \boldsymbol{\rho}=(x-x')\mathbf{i}+(z-z')\mathbf{k},$$

$\mathbf{n}$ is the external normal to $\partial\Omega$, $\Omega=[-L_1/2, L_1/2]$ $[-L_2/2, L_2/2]$ is the region where the calculations are carried out, $\mathbf{h}_{d_2}(x, z)$ is the demagnetizing field of the neighboring domains, $\mathbf{h}_0$ is the external magnetic field, and α is the damping parameter. The boundary and initial conditions are

$$\mathbf{v}\left(\pm\frac{L_1}{2},z,t\right)=\pm\mathbf{k},\quad \frac{\partial\mathbf{v}}{\partial z}\left(x,\pm\frac{L_2}{2},t\right)=0,$$
$$\mathbf{v}(x,z,0)=\mathbf{v}_0(x,z),\quad (x,z)\in\Omega,\quad t\in[0,T]. \qquad (2)$$

We introduce a uniform network w in a region Ω (Fig. 1). We approximate the effective exchange field by a standard fourth-order difference operator and the demagnetizing field by piecewise-constant finite elements matched to the network w (Refs. 18 and 20), and the effective anisotropy field as well as the external magnetic field by standard projections on a set of network functions. The boundary conditions can be approximated by postulating additional boundary layers. Time differencing was done by a stable difference scheme (using the predictor–corrector method with monitoring of the accuracy at each step).

Since there are no phase surfaces separating the domain from its wall,[22] it is desirable to artificially introduce boundary surfaces and to identify a domain wall as a two-dimensional region for the study of its dynamics.

We call the kernel of a domain wall the subregion $\Omega_W(t)$ of the region Ω where the main rotation of the magnetization vector from one domain to another takes place. The subregion $\Omega_W(t)$ is bounded by the lines $z=\pm L_2/2$ and $v_z(x, z) = \pm 0.9$ (we recall that $|\mathbf{v}| = 1$).

Assume that $x = SL$ and $x = FL$ are lines $\Omega_W(t)$ at $t = 0$. The subregion bounded by the lines $z = \pm L_2/2$, $x = SL$, $x = FL$ will be labeled Ω_{WD}.

Application of an external magnetic drive field $\mathbf{h}_0 = -a\mathbf{k}$ ($a = \text{const} > 0$) displaces the domain wall toward a domain with its magnetization opposite to the external field. If the network is fixed, then after traveling a certain distance this wall stops at the edge of the calculation region Ω because of the boundary condition (2). The numerical tracking method involves retention of the domain wall core inside the region Ω_{WD}. After a time interval Δt a "photograph" of the domain wall is taken, i.e., the region of localization, the displacement of the center of the wall from the center of the calculation region, and the dynamic characteristics are calculated; if the line $v_z(x, z, t) = \pm 0.9$ intersects the line SL or FL, the network is shifted by several nodes in such a way as to ensure $\Omega_W(t)\subset\Omega_{WD}$. The quantities SL, FL, L_1, and Δt and the displacement are obtained by trial and error, depending on the capabilities of the computer and the characteristics of the domain wall motion. It is found that these quantities can be selected so that the error in the displacement is less than the error due to the approximation involved in the original equation.

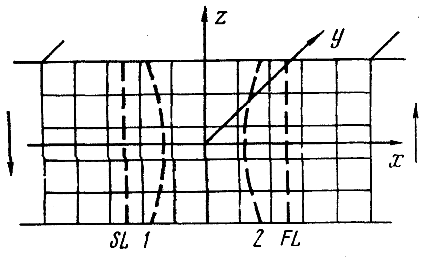

FIG. 1. Geometry of the calculation region: 1), 2) curves bounding the domain wall kernel at the initial time.

RESULTS OF CALCULATIONS

Assume that $\varphi(x, z, t)$ is the angle between the projection of the vector $\mathbf{v}(x, z, t)$ onto the xy plane and the $y(\mathbf{j})$ axis, and $\theta(x, z, t)$ is the angle between $\mathbf{v}(x, z, t)$ and the $z(\mathbf{k})$ axis. The process of mapping a two-dimensional three-component dynamic vector field is by itself a complex task and it is best solved by producing a computer-generated film on the screen of a display. We lacked facilities for such a procedure and were therefore able to obtain only certain "frames" of the dynamic process.

We consider the following characteristics: the curve representing the angle of twist of a domain wall $\psi(z, t)$ [$\psi(z, t) = \varphi(x, z, t)$ on the central line $\theta(x, z, t) = \pi/2$]; the localization region of a domain wall $\Omega_W(t)$, the velocities of three "key" points S_1, S_2, and S_3 also located on the central domain wall line at the upper surface of the film, in the central plane of the film, and the lower surface of the film, respectively.

In the case of thick films [obeying the condition $\Lambda/L_2 \ll 1$, where $\Lambda = (A/2\pi M_s^2)^{1/2}$ is the nominal width of a horizontal Bloch line] a horizontal Bloch line is usually understood to be a sharply localized region of a domain wall where a considerable change in the angle of twist $\psi(z, t)$ takes place. Horizontal Bloch lines then join static branches of the twist curves $\psi(z)$. In our case, for the sake of brevity, we use the term "horizontal Bloch line" to denote parts of the domain wall where the angle of twist varies rapidly, although there may be no clear localization ($\Lambda/L_2 \approx 0.26$) and a horizontal Bloch line does not necessarily join different static branches of $\psi_0(z)$.

We report the results of calculations for the following parameters: $Q = 4$, $L_2 = 3$ (in units of l), $\alpha = 0.2$, $\mathbf{h}_0 = -a\mathbf{k}, a \in [0, 5]$ (in units of M_S). We consider in greatest detail the case $a = 4$. The majority of the calculations were carried out for a $N_x \times N_z = 40 \times 40$ network. The initial condition was a two-dimensional distribution of the magnetization obtained by solving the static variational micromagnetic problem in the case when $h_0 = 0$. Next, in the course of integration of the equations of motion we assumed that a static external drive magnetic field was applied abruptly (in the form of a step).

If $0 < a < a_1 = 1.2$, the dependence of the domain wall velocity on the drive field is linear. The evolution of the angle $\psi(z, t)$ is then slow compared with the domain wall velocity. The $\psi(z, t)$ curve is typical of this case and very similar to the curves reported in Ref. 6 (Fig. 17.2b). This case fully deserves the term "quasisteady".

In $a_1 < a < a_2 = 2.7$ a perturbation of the twist angle created at the lower surface of the film grows quite rapidly reaching the value $\approx 2\pi$ (giving rise to a 2π horizontal Bloch line) and then the Bloch line breaks through to the upper surface of the film. The process is then repeated: a new horizontal Bloch line is created at the top, i.e., it grows, drops downward, and becomes annihilated there, and so on. The motion of the horizontal Bloch line is faster than that of the domain wall; such a line has a retarding influence on the part of the wall where it is localized. An increase of the external field intensity enhances the amplitude of the perturbations, although the average domain wall velocity still remains approximately constant. Averaging over several periods of passage of a horizontal Bloch line gives the relative domain velocity 0.44. The internal dynamics mechanism is then qualitatively similar to the mechanism described in Ref. 10.

For $a_2 < a < 5$, the average velocity remains as before, but the internal dynamics mechanism changes significantly. We describe this in greater detail by splitting the process into stages (see Figs. 2–5, which correspond to the case when $a = 4$).

1. A perturbation of the twist angle $\psi(z, t)$ is created at the lower surface of the film by an external field. After a time, a perturbation wave rises to the central plane of the film and grows at the center, whereas the edges are characterized by stable values $-\pi/2$ and $\pi/2$ which are due to the action of the demagnetizing fields of the adjacent domains. We say that a pair of horizontal Bloch lines is formed: the upper line is greater and the lower line is smaller (curves 1–5 in Fig. 2a).

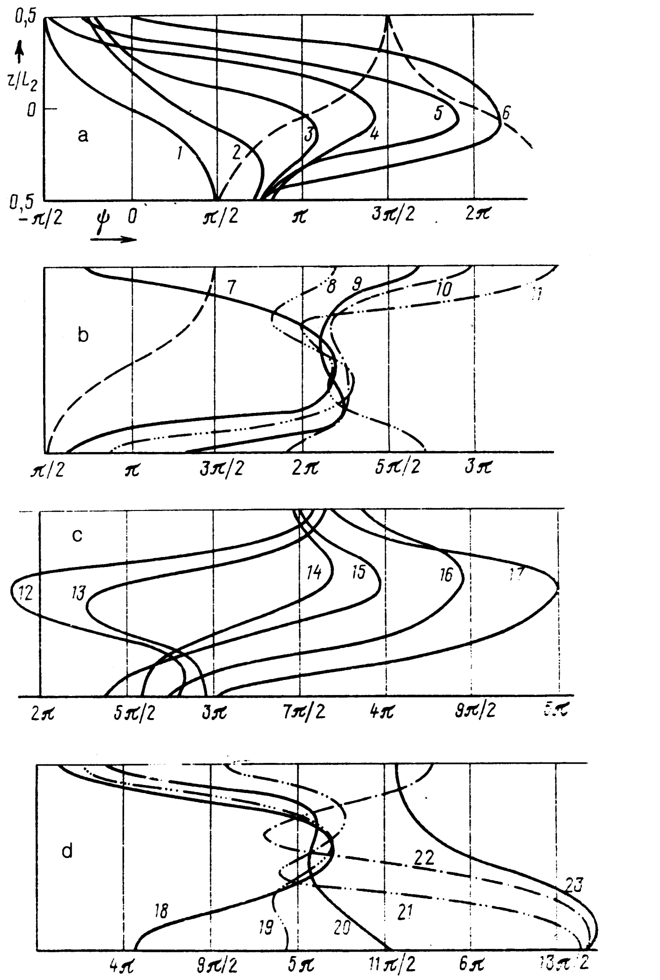

FIG. 2. Dynamics of the curve representing the twist angle $\psi(z, t)$ of a domain wall. The dashed curves are the static branches of the twist angle $\psi_0(z)$. The numbers alongside the curves correspond to the following times: 1) $t = 0$; 2) 0.274; 3) 0.52; 4) 0.767; 5) 1.014; 6) 1.359; 7) 1.507; 8) 1.754; 9) 2.002; 10) 2.076; 11) 2.248; 12) 2.371; 13) 2.617; 14) 2.893; 15) 2.992; 16) 3.484; 17) 3.932; 18) 4.077; 19) 4.250; 20) 4.475; 21) 4.843; 22) 5.114; 23) 5.361.

2. The upper horizontal Bloch line first rotates by an angle $\approx 4\pi$ and then the lower line rotates by an angle $\approx 2\pi$ (curves 6–11 in Fig. 2b). The energy of a stressed twisted state (curve 5 in Fig. 2a) is sufficiently high for the magnetization vector on the upper plane of the film to jump across the stable position $3\pi/2$.

3. The breakthrough of a horizontal Bloch line has the effect that the central part of the $\psi(z, t)$ curve lags behind the surface regions. The center of the curve then begins to move, whereas the surface vectors are in the vicinity of the stable states $5\pi/2$ and $7\pi/2$ (curves 12–14 in Fig. 2c).

The stages 1–3 are then repeated but the polarity of the domain wall is reversed: the large horizontal Bloch line is now at the bottom and the small one at the top (curves 15–17 in Fig. 2c); this is followed by breakthrough involving rotation by angles of $\approx 4\pi$ and $\approx 2\pi$, respectively (curves 18–21 in Fig. 2d); finally, the central region of the $\psi(z, t)$ curve reaches its surface and the complete cycle is repeated.

Since the motion of a domain wall begins from a static

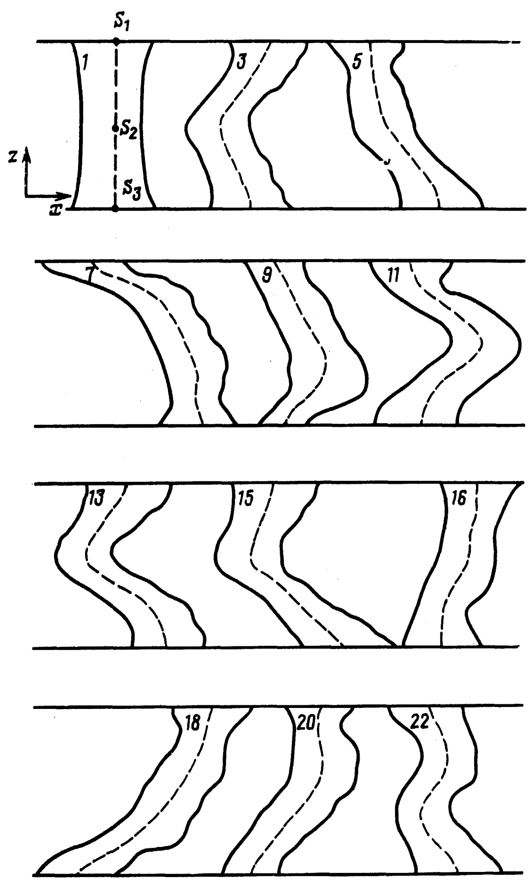

FIG. 3. Dynamics of boundaries and of the central line of the domain wall core.

state, the process is of steady-state nature and the exact periodicity is not obeyed.

The behavior of the domain wall core [region $\Omega_W(t)$] is shown in Fig. 3. The numbers of the curves in Fig. 3 are the same (and the times are the same) as in Fig. 2: the states are plotted for the same moments in time. It is quite clear that the formation and breakthrough of the horizontal Bloch line distort the domain wall structure.

Fuller information on the reciprocating nature of the motion of a domain wall is presented in Fig. 4, which gives the velocities of the three points S_1, S_2, and S_3 whose positions at the initial moment are shown in Fig. 3. The velocity was calculated from the expression $V_i = \Delta S_i/\Delta t$, where $\Delta t = 0.02$ and ΔS_i is the path traveled by the relevant point. Note that the breakthrough of a horizontal Bloch line and rapid motion of the central region of the $\psi(z, t)$ curve are accompanied by reciprocating motion of the relevant parts of the domain wall.

Figures 5a–5d give certain characteristic two-dimensional distributions of the magnetization. It is interesting to note that the most "critical" moments of the domain wall retain their rotational nature, which is due to the magnetostatic interaction.

DISCUSSION

Direct numerical integration of the Landau–Lifshitz equation revealed three internal dynamic mechanisms of a moving isolated two-dimensional domain wall. The results of calculations confirmed the hypothesis that saturation of the domain wall velocity as a function of the drive field is due to the internal transformations of the domain wall structure. There is a qualitative agreement between some effects of the two-dimensional domain wall dynamics and the results reported in Refs. 6 and 14–16, obtained by solving the Slonczewski equations. We now list them.

1. A horizontal Bloch line is created "automatically" (in a natural manner) at the film surfaces in the vicinity of the critical points where the effective field intensity is least. The formation of the horizontal Bloch line is due to the dynamic properties of the Landau–Lifshitz equation and due to the topological limit $|v| = 1$.

2. The existence of the quasisteady range $0 < a < a_1$, corresponding to initiation of deformation of the $\psi(z, t)$ curve, is localized within the film and there is no breakthrough to the surface.

3. When the first critical value a_1 is exceeded by the drag field, the horizontal Bloch line exhibits regular and irregular breakthrough to this film surface and the surface magnetization vector then rotates by an angle $\approx 2\pi$, resulting in annihilation of the horizontal Bloch line. The internal dynamics then follows the Slonczewski model.

4. In the course of motion of the domain wall the region where the horizontal Bloch line is localized moves at a lower velocity than the rest of the domain wall. The velocity decreases on increase in the angle of twist of the horizontal Bloch line.

5. When the horizontal Bloch line breaks through the surface region, it exhibits reciprocating motion, whereas the rest of the wall moves forward.

However, there are some differences. For example, when the drive field is increased still further, the horizontal Bloch line no longer rises to the surface and the central part of the line travels forward much further than the nearest static branch $\psi_0(z)$, while the edges are in potential wells created by the demagnetizing fields of the adjacent domains,

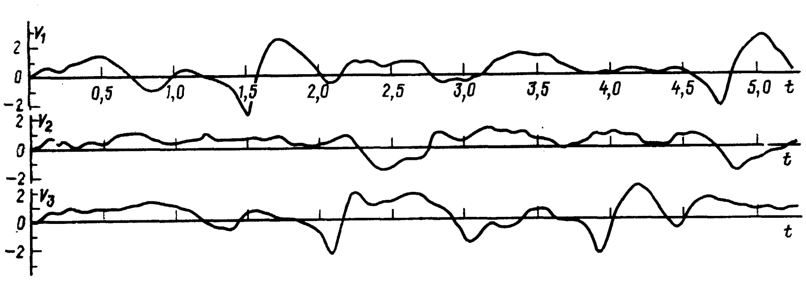

FIG. 4. Plots showing the velocities of the points S_1, S_2, and S_3.

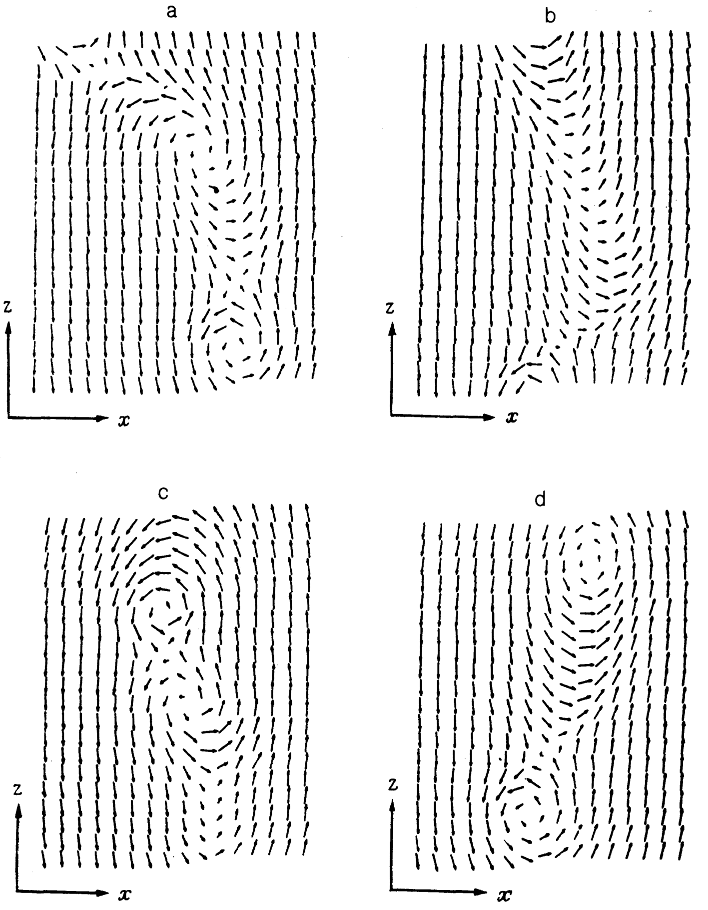

FIG. 5. Components (v_x, v_z) of the distribution of the magnetization at the following moments: a) $t = 1.507$; b) $t = 2.002$; c) $t = 2.493$; d) $t = 3.484$.

so that $\psi(\pm L_2/2, t) \approx \mp \pi/2$. Strongly twisted regions form on the surface which for brevity we call horizontal Bloch lines, although it is clear from the figures that the structure is now somewhat different. Strongly twisted regions are smoother than "classical" horizontal Bloch lines. Moreover, fast rotation (in the form of annihilation of horizontal Bloch lines) occurs not only on the film surfaces, but also in the interior where the central region catches up with the edges (Fig. 2c). A mechanical analogy can be quoted in support of this mechanism: the twist curve resembles an elastic filament with small loads at the ends moving on a surface with a relief under the influence of a distributed force.

In our opinion the smoother nature of the dynamics due to the Landau–Lifshitz equation is the result of the intrinsic magnetostatics of a domain wall which is allowed for more rigorously than in the case of the Slonczewski equation.

The results of our calculations demonstrate that there is a great variety of internal dynamic processes corresponding to the saturation region of the domain wall velocity in an external field. The saturation velocity obtained in our model (0.44 nominal units of the length l per unit dimensionless time t_1) is higher than the velocity deduced using the empirical de Leeuw expression $v = \pi/5Q$, which creates a certain margin of the strength since an allowance for dislocations, irregularities of the parameters of the material, and stabilizing fields used in real experiments reduces the value of this velocity.

An estimate of the accuracy and reliability of the numerical solution of the system of equations (1)–(2) is a difficult task and a detailed analysis of this task is impossible within the framework of the present paper. We simply stress some aspects of the solution of this nonlinear problem.

The absence of a proper analytic solution of the domain wall dynamics makes it more difficult to test the code and to estimate the precision of the calculations. We investigated separately the quality of the approximations in terms of the temporal and spatial variables by considering model motion paths obtained in the course of preliminary calculations using coarser networks.

It is essential to study the solution using nested networks at different time intervals, since we found that the nature of the solutions depends on the network parameters. For example, if the number of nodes in a network is insufficient in the regions where the magnetization varies rapidly, there may be internal breakthroughs which disappear when finer networks are used and which are in conflict with the physical meaning of the micromagnetic formation of the problem.

All the reported results were obtained using a package of routines UNIMAG.DW (in the Fortran-77 language) intended for the calculation of static and dynamic structures of two-dimensional domain walls. The time needed for the calculation of several cycles of creation and annihilation of the horizontal Bloch line on a computer capable of 10^7 operations/s was 3–4 h of central processor time.

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