Stationary propagation of streamers in electronegative gases

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(Submitted 16 April 1990)
Zh. Eksp. Teor. Fiz. **98**, 895–907 (September 1990)

A qualitative theory of streamer discharge in electronegative gases is presented. The condition is found under which electron attachment in the plasma channel appreciably affects the streamer propagation. It is shown that there exists a critical value of the external homogeneous field $\tilde{\mathscr{E}}_c$ at which stationary development of the streamer is possible. With growth of the attachment rate, the field strength $\tilde{\mathscr{E}}_c$ increases and reaches a final value E^* (the field strength at which impact ionization is compensated by attachment), at which it remains. The field, charge and electron concentration distributions along the streamer channel are found. The results are in agreement with the available experimental data and with numerical calculations.

1. INTRODUCTION

A streamer is a thin highly conductive plasma filament growing at high speed in a discharge gap. At the head of the streamer is a strong electric field in which impact ionization occurs, leading to a lengthening of the plasma filament. This field is created by a charge distributed along the streamer filament, while the newly produced sections of the filament are charged by the current flowing along the streamer channel. If the high conductivity of the channel is preserved during the entire time of streamer development, then that current exists over the entire length of the streamer filament. Under these conditions the streamer, propagating at constant velocity from one of the electrodes in the homogeneous external field (of a plane parallel gap), is a uniformly charged filament along which a current flows from the electrode and maintains a constant linear charge density along the filament, and the field at its head.

The picture changes appreciably in electronegative gases since fast attachment of the electrons to neutral atoms or molecules occurs in the streamer channel, with formation of negative ions whose mobility is negligibly small. Therefore, the streamer's conductivity vanishes at some distance from its head. So, for example, the attachment length (i.e., the length that the streamer runs until attachment occurs) for a streamer in air is several centimeters, while the distance over which the streamer is propagated can reach several meters.[1] Therefore, galvanic coupling between the electrode and the head is absent. The question arises how the streamer-head charge necessary for the creation of the strong field preceding the front is maintained under these conditions.

A discussion of this question is contained in the book by Bazelyan and Razhanskiĭ. The basic idea is that the strong field at the head is created by the polarization, in the external field, of a short conducting part of the channel (the plasma element) with a length on the order of the attachment length. This idea permits the qualitative explanation of the experimental fact that the external field $\mathscr{E}_c = \tilde{\mathscr{E}}_c$ necessary for the stationary development of the streamer is increased by the addition of electronegative components.

Another picture of streamer propagation in the strongly electronegative gas SF_6 follows from the results of numerical calculations carried out in Refs. 2–5. These results indicate that the electron concentration in the streamer channel decreases not to zero, but to some finite value with increasing distance from the head. The field E^*, reached in the channel is such that the attachment is precisely compensated by the process of impact ionization. (For a streamer in air the value of the field in the channel is less than E^*, so that attachment dominates and the electron concentration falls to zero.)

Gallimberti[3] has carried out numerical calculations for a mixture of air and SF_6. It follows from his work that upon increase in the percentage of the SF_6 the magnitude of the field $\tilde{\mathscr{E}}_c$ at which stationary propagation is possible is increased and tends to E^*.

Up to now there does not even exist a qualitative theory of streamer propagation in electronegative gases. The available theoretical investigations were based solely on numerical calculations carried out for specific experimental conditions.

In the present work we will determine the conditions of stationary streamer propagation in electronegative gases in a homogeneous external field, and we will also ascertain the character of the charge, field, and electron concentration distributions along the streamer channel.

This problem was solved by us earlier without considering attachment.[6] Expressions for all the basic streamer parameters were found with an accuracy of the numerical coefficients on the order of unity. The needed essential results of Ref. 6 consist of the following.

Stationary streamer propagation (with constant velocity V and head radius r_0) is possible only at a definite critical value of the external field $\mathscr{E} = \mathscr{E}_c$, at which the current in the channel maintains the necessary charge of the head and the field ahead of the front. If $\mathscr{E} > \mathscr{E}_c$ ($\mathscr{E} < \mathscr{E}_c$) the radius of the head and the velocity of propagation are increased (decreased) with time. We note that the existence of a critical value of the external field at which a stationary streamer is propagated was first detected experimentally by Phelps.[7] Gallimberti[8] obtained for that field an estimate coinciding with the results[7] for air, with the help of numerical calculations based on the model of Dawson and Winn.[9] It appears to us that underlying this model are the unfounded assumptions that the streamer charge is concentrated at its head, and that the conductivity of the channel is neglegibly small (see also Refs. 1, 6, 10, and 11).

Further, there exists a characteristic time t_0 of establishment of the stationary development of the streamer, determined by the values of r_0 and V and by the conductivity of the channel σ, and a corresponding characteristic length $l_0 = V/t_0$. The expressions for $\mathscr{E}_c$ and t_0 are presented in the Appendix.

The quantity t_0 is the interval in which the charge has time to spread along the growing streamer filament. At $t < t_0$ the charge distribution is close to electrostatic. At $t > t_0$ the charge does not have time to spread along the filament, and the field in the channel is not screened and is equal to the external field. The time t_0 determines the streamer lag (for example, the development of the streamer must continue in that time after the the external field is shut off). One can also state that the streamer head is effectively connected only with a section of the channel of length on the order of l_0. In particular, the field before the head of a growing filament with length $l \gg l_0$ is of just the order of magnitude which an electrostatically polarized filament of length l_0 would create.[6] The processes occuring in the channel at distances greater than l_0 from the head do not influence streamer development.

From these statements it is clear that the role of attachment depends on the ratio of the attachment time t_a and the time t_0 (or, equivalently, on the ratio of the attachment length $l_a = V/t_a$ and the length l_0.) It is evident that attachment significantly affects streamer development only for $t_a \lesssim t_0$.

The calculation of the critical value of the external field $\mathscr{E}_c$, at which stationary streamer propagation in electronegative gases is possible, will be our fundamental problem. We will show that attachment increases $\tilde{\mathscr{E}}_c$ in comparison with the critical value $\mathscr{E}_c$ in the absence of attachment. However, at $t_a \gg t_0$ the difference of $\tilde{\mathscr{E}}_c$ from $\mathscr{E}_c$ is very small in spite of the fact that, thanks to attachment, the conductivity of the channel falls exponentially with a characteristic length l_a. At $t_a \lesssim t_0$ the size of $\tilde{\mathscr{E}}_c$ grows appreciably, reaching the value E^* for sufficiently fast attachment. As a consequence, the conductivity of the channel falls to some finite value with increasing distance from the head, and then remains invariable.

Hence, either the picture of propagation presented in Ref. 1 for a streamer in air, or the picture following from the numerical calculations[2-5] for the strongly electronegative gas SF_6, can be implemented, depending on the attachment rate.

2. FORMULATION OF THE PROBLEM AND THE BASIC EQUATIONS

The problem is completely analogous to that examined in Ref. 6: a streamer filament of radius r_0 grows in a homogeneous external field $\mathscr{E}$ at a constant velocity V. Let us assume the presence of free electrons with a small concentration n_0 (created, for example, by photoionizing radiation of the streamer) that are multiplied by impact ionization in the strong field in front of the head.

The problem of streamer propagation can be divided into two parts. The first consists of the examination of the processes in the streamer head (impact ionization and Maxwellian relaxation). As a result of this examination[6,11] one may determine the field E_m before the front, the conductivity σ_m, and the field $\mathscr{E}_c$ in the channel directly behind the front, and also the connection between the speed of propagation V and the radius of the head r_0 (see the Appendix). The second question consists of determining of the charge, current, and field distributions along the streamer channel. The values of σ_m and $\mathscr{E}_c$ found by solving the first problem must be used as the boundary conditions for the equations describing the spreading of the charge and for the change of the conductivity in the channel. It is significant that the quantities σ_m and $\mathscr{E}_c$ do not depend on the external field $\mathscr{E}$, but are determined only by the character of the field dependence of the impact-ionization frequency $\beta(E)$.

The only specific feature of an electronegative gas lies in the fact that, due to attachment, the conductivity of the channel decreases away from the streamer head. Therefore, it is sufficient for us only to solve the second of the above problems, utilizing the given boundary values σ_m and $\mathscr{E}_c$.

The channel conductivity varies like[1)]

$$\partial\sigma/\partial t=\tilde{\beta}(E)\sigma,\quad \tilde{\beta}(E)=v_d(\alpha-\eta), \tag{1}$$

where v_d is the electron drift velocity, which we will take to be linearly dependent on the field for simplicity, and α and η are the coefficients of impact ionization and of attachment, respectively. With increasing field the coefficient α grows, but the coefficient η decreases, so that there exists a field E^* at which $\tilde{\beta}(E^*) = 0$. In the streamer channel we have $E < E^*$ and $\tilde{\beta}(E) < 0$. On the contrary, $E > E^*$ ahead of the streamer front, and one may, as a rule, neglect attachment.

The spreading of charge along the streamer filament is described by the equation of continuity

$$\frac{\partial\rho_l}{\partial t}+\frac{\partial}{\partial z}\left[\pi\sigma r_0^2\left(\mathscr{E}-\frac{\partial\varphi}{\partial z}\right)\right]=0, \tag{2}$$

where z is the coordinate along the direction of streamer propagation, ρ_l is the linear charge density, $\mathscr{E}$ is the external field, and φ is the potential creating the charges of the filament. For the total field E in Eq. (1) we have $E = \mathscr{E} - \partial\varphi/\partial z$. If the charge density ρ_l varies weakly at distances on the order of the filament radius r_0, then the approximation of local capacitance

$$\varphi=\rho_l\Lambda_2 \tag{3}$$

[$\Lambda_2 = \ln(a/r_0)$, and a is the characteristic length over which ρ_l varies ($a \gg r_0$)], is valid.

We will seek a self-similar solution of Eqs. (1)–(3), introducing the variable $x = Vt - z$ (the distance from the streamer head). We obtain the equations

$$V\frac{dD}{dx}=\tilde{\beta}(E)D, \tag{4}$$

$$V\frac{d\varphi}{dx}-\frac{d}{dx}D\left(\mathscr{E}+\frac{d\varphi}{dx}\right)=0, \tag{5}$$

where the quantity $D = \pi r_0^2\sigma\Lambda_2$ has the meaning of the diffusion coefficient that determines the spreading of the charge along the filament, and where $E = \mathscr{E} + d\varphi/dx$.

The boundary conditions for Eqs. (4) and (5) at $x = 0$ (i.e., at the streamer head) have the form

$$D=D_m=\pi r_0^2\sigma_m\Lambda_2, \tag{6}$$

$$V\varphi=D(\mathscr{E}+d\varphi/dx), \tag{7}$$

$$\varphi=\varphi_m. \tag{8}$$

The quantity φ_m is expressed via the field ahead of the front E_m: $\varphi_m \sim E_m r_0 \Lambda_2$. This follows from Eq. (3) and the ratio $E_m \sim \rho_l/r_0$. Condition (7) expresses the requirement of absence of total current at the streamer head.[6] This condition connects φ_m with the field in the channel directly behind the front, $\mathscr{E}_c = \mathscr{E} + (d\varphi/dx)|_{x=0}$:

$$\varphi_m = \mathscr{E}_c D_m / V. \tag{9}$$

From Eq. (5) it follows that condition (7) is valid along practically the entire channel length, and not only at $x = 0$. Thus, the problem is reduced to the solution of Eqs. (4) and (7) with the boundary conditions given in Eqs. (6) and (8) at $x = 0$.

We note that for unchanging channel conductivity, when $D = D_m$ everywhere, Eq. (7) has a solution bounded as $x \to \infty$ ($\varphi = \varphi_m$) only for $\mathscr{E} = \mathscr{E}_c$. This means that at $\sigma = \text{const}$ stationary streamer propagation is possible only at an external field equal to $\mathscr{E}_c$.

We now turn to dimensionless variables. We will measure the quantities φ, D, and x respectively in units of φ_m, D_m, and $l_0 = D_m/V$. We then obtain the equations

$$\frac{dD}{dx} = -k(E)D, \quad E = \mathscr{E} + \mathscr{E}_c \frac{d\varphi}{dx}, \tag{10}$$

$$\frac{d\varphi}{dx} = \frac{\varphi}{D} - \frac{\mathscr{E}}{\mathscr{E}_c} \tag{11}$$

with the boundary conditions $D = 1$ and $\varphi = 1$ at $x = 0$. Here,

$$k(E) = -\tilde{\beta}(E) t_0, \quad t_0 = D_m/V^2 = l_0/V. \tag{12}$$

In the region of the fields considered ($E < E^*$) the quantity k is positive.

The quantity t_0 is the characteristic time of establishment of stationary propagation (at constant conductivity σ_m) referred to in the Introduction. The expression for t_0 is given by Eq. (A5) of the Appendix, in which it is necessary to replace $\beta(E_m)$ by $\tilde{\beta}(E_m)$.

For a given $\tilde{\beta}(E)$ dependence the solutions of Eqs. (10) and (11) determine the distributions of the conductivity and of the field along the streamer channel. We will show that physically meaningful solution (bounded as $x \to \infty$) exists only at a definite value of the external field $\mathscr{E} = \tilde{\mathscr{E}}_c$.

3. MODEL WITH CONSTANT ATTACHMENT

We will examine the simplest case, when the dependence of the coefficient $\tilde{\beta}$ on the field can be neglected in the streamer channel. In such a model the quantity k in Eq. (10) is a constant parameter, equal to the ratio of the time t_0 to the attachment time $t_a = |\tilde{\beta}|^{-1}$.

In this case, $D = \exp(-kx)$ follows from Eq. (10). Using this expression we find the solution of Eq. (11) with the boundary condition $\varphi(0) = 1$:

$$\varphi(x) = \exp\left(\frac{e^{kx}-1}{k}\right)\left[1 - \frac{\mathscr{E}}{\mathscr{E}_c}\int_0^x \exp\left(\frac{1-e^{ky}}{k}\right) dy\right]. \tag{13}$$

It is easy to see that this solution remains bounded as $x \to \infty$ only if the expression in the square brackets tends to zero. Therefore, the critical value of the external field ($\mathscr{E} = \tilde{\mathscr{E}}_c$) at which stationary propagation is possible is given by the formula

$$\frac{\mathscr{E}_c}{\tilde{\mathscr{E}}_c} = \int_0^\infty dy \exp\left(\frac{1-e^{ky}}{k}\right) = \int_0^\infty dt \frac{e^{-t}}{1+kt}. \tag{14}$$

For $k \ll 1$ (weak attachment) Eq. (14) gives

$$\tilde{\mathscr{E}}_c = \mathscr{E}_c(1+k), \tag{15}$$

i.e., the critical field increases neglegibly in comparison with $\mathscr{E}_c$.

For strong attachment ($k \gg 1$)

$$\tilde{\mathscr{E}}_c = \mathscr{E}_c k/(\ln k - C), \tag{16}$$

where $C = 0.577$ is Euler's constant.

A plot of the dependence of $\tilde{\mathscr{E}}_c$ on the parameter k, calculated according to Eq. (14), is shown in Fig. (1).

From Eq. (13), applying Eq. (14), we find the following expression for the dimensionless potential:

$$\varphi(x) = \frac{\Phi(x)}{\Phi(0)}, \quad \Phi(x) = \int_0^\infty dt \frac{e^{-t}}{e^{kx}+kt}. \tag{17}$$

The distributions of the conductivity, of the charge density (which, in agreement with Eq. (3), is proportional to the potential φ), and of the field along the streamer channel are shown in Fig. 2.

Thus, in a model with constant attachment, the conductivity and charge density fall to zero with increasing distance from the head, which is consequently not connected the electrode. However, if $t_a \gg t_0$ ($k \ll 1$), streamer propagation in a homogeneous field proceeds practically as if attachment were absent. The reason, as already mentioned in the Introduction, is that processes occuring at distances from the head greater than the characteristic length $l_0 = Vt_0$ are not important for the development of the streamer. At $t_a \lesssim t_0$ ($k \gtrsim 1$) the critical field $\tilde{\mathscr{E}}_c$ increases significantly in comparison with $\mathscr{E}_c$ in the absence of attachment. This increase agrees qualitatively with the estimate made in Ref. 1. However, we note that in our opinion, the premise in Ref. 1 that the charge distribution along the streamer filament corresponds to polarization of a section of length l_a in the external field (Fig. 2).

In the model considered the field $\tilde{\mathscr{E}}_c$ increases without limit for increasing k. In very fact, for sufficiently large values $\tilde{\mathscr{E}}_c$, when the field along the channel changes strongly, the model with constant attachment becomes inapplicable, since in Eq. (10) one must take into account the $k(E)$ de-

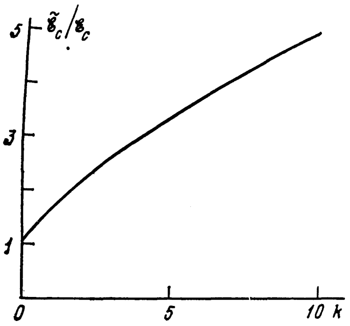

FIG. 1. Dependence of the stationary propagation field $\tilde{\mathscr{E}}_c$ on the parameter $k = t_0/t_a$ in the model with constant attachment.

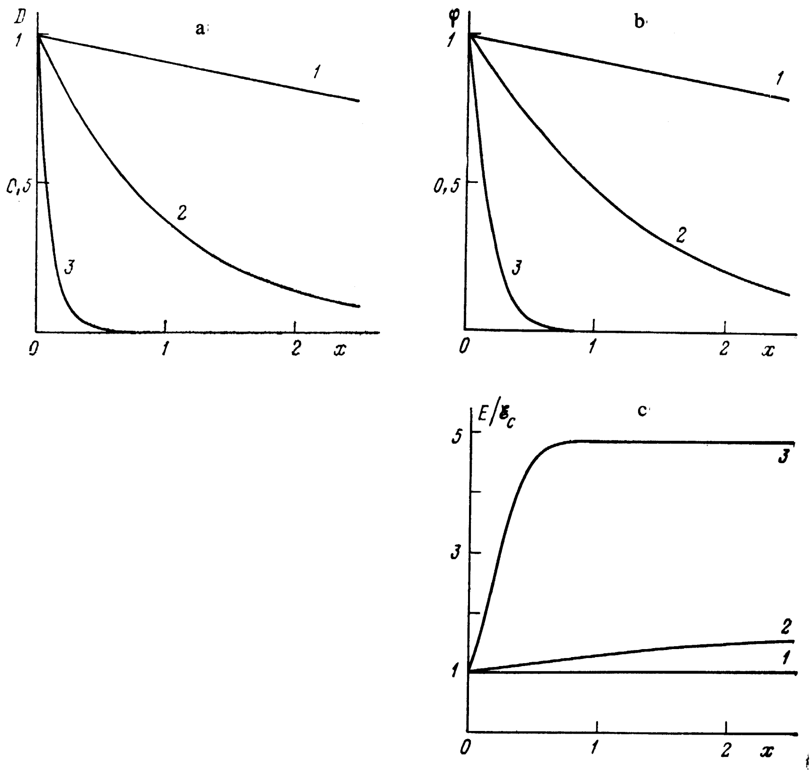

FIG. 2. Dependences on the nondimensional coordinate x of (a) the conductivity of the channel, (b) the charge density, and (c) the field, relative to their values in the streamer head. The calculations were carried out in the constant attachment model for $k = 0.1$ (1), $k = 1$ (2), and $k = 10$ (3).

pendence. Since at $E = E^*$ attachment is compensated by impact ionization, and $k(E^*) = 0$, the field $\tilde{\mathscr{E}}_c$ cannot exceed E^*.

The model with constant attachment is applicable if the relative variation of $k(E)$ is small in the region of fields from $\mathscr{E}_c$ to $\tilde{\mathscr{E}}_c$.

4. MODEL WITH A LINEAR $k(E)$ DEPENDENCE

We examine the case of a linear $k(E)$ dependence:

$$k(E) = k_0(1 - E/E^*). \tag{18}$$

Such an approximation is valid at fields E near to the value E^*, where

$$k_0 = -E^*(dk/dE)_{E=E^*}.$$

Equation (18) is model-like far from the value $E = E^*$.

We find the equations

$$\frac{d\varphi}{dx} = \frac{1}{D}\left(\varphi - \frac{\mathscr{E}}{\mathscr{E}_c}D\right), \tag{19}$$

$$\frac{dD}{dx} = k_0(\varepsilon\varphi - D), \quad \varepsilon = \frac{\mathscr{E}_c}{E^*} \tag{20}$$

from Eqs. (10), (11), and (18), with boundary conditions $\varphi(0) = D(0) = 1$. As has been shown earlier, a physically meaningful solution of Eqs. (19) and (20), bounded at $x \to \infty$, exists only at the specific value $\mathscr{E}_c = \tilde{\mathscr{E}}_c$. The form of the solution and the quantity $\tilde{\mathscr{E}}_c$ are determined by the values of the two parameters k_0 and ε (assuming that $\varepsilon < 1$). For arbitrary values of these parameters the quantity $\tilde{\mathscr{E}}_c$ can be found only by numerical integration of Eqs. (19) and (20). The dependences of the field $\tilde{\mathscr{E}}_c$ on k_0 at fixed values ε are presented in Fig. 3. With growth of k_0 the quantity $\tilde{\mathscr{E}}_c$ increases; at $k_0 = k_0^*$ it approaches the value E^* and does not change further.

Several properties of these dependences can be established analytically.

1) At $k_0 \ll 1$ Eq. (15), in which one replaces k by $k(\mathscr{E}_c) = k_0(1 - \varepsilon)$, is valid. Indeed, in this case the relative change of the quantity $k(E)$ in the interval $\mathscr{E}_c < E < \tilde{\mathscr{E}}_c$ has a value on the order of $k_0\varepsilon \ll 1$ and the model with constant attachment applies.

2) Let us establish the region of parameters k_0 and ε in which $\tilde{\mathscr{E}}_c = E^*$. Taking $\tilde{\mathscr{E}}_c = E^*$, we find from Eqs. (19) and (20) that

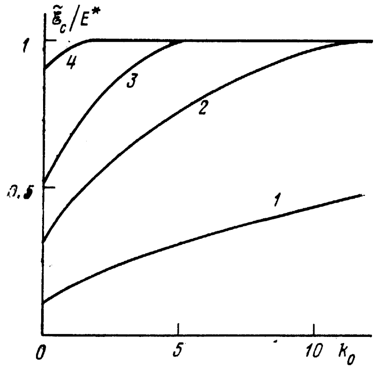

FIG. 3. The dependence of the field $\tilde{\mathscr{E}}_c$ on the parameter k_0 in the model with a linear function $k(E)$ for values of the parameter $\varepsilon = \mathscr{E}_c/E^*$, where $\varepsilon = 0.1$ (1), $\varepsilon = 0.3$ (2), $\varepsilon = 0.5$ (3), and $\varepsilon = 0.9$ (4).

$$\frac{d\varphi}{dD}=\frac{1}{k_0\varepsilon D}, \tag{21}$$

whence, subject to the boundary conditions, we obtain a connection between φ and D:

$$\varphi=1+\frac{1}{k_0\varepsilon}\ln D. \tag{22}$$

Equation (20) can then be rewritten in the form

$$dD/dx=F(D),\ F(D)=k_0\varepsilon+\ln D-k_0D, \tag{23}$$

where $D(0) = 1$. The function $F(D)$ attains its maximum value

$$F_{max}=k_0\varepsilon-1-\ln k_0$$

at $D = k_0^{-1}$. If $F_{max} < 0$, then $F(D) < 0$ everywhere; there are no bounded solutions here for D and φ. Therefore, it is necessary that the condition $F_{max} > 0$ be fulfilled; i.e.,

$$(1+\ln k_0)/k_0<\varepsilon. \tag{24}$$

With the fulfillment of this condition Eq. (23) has two stationary points D_1 and D_2 ($D_2 \geqslant D_1$) in which $F(D) = 0$, where point D_1 is unstable and point D_2 is stable (Fig. 4). As is seen from Fig. 4, for $D_1 > D(0) = 1$ the solution of Eqs. (22) and (23) leads to the point $D = 0$, $\varphi = \infty$, a result having physical meaning. If $D_1 < 1$, then as $x \to \infty$ it follows from Eqs. (22) and (23) that $D \to D_2$, $\varphi \to D_2/\varepsilon$, $E \to E^*$. It is easily seen that under the condition (24) and $\varepsilon < 1$ the requirement $D_1 < 1$ is fulfilled only for $k_0 < 1$. Here, $D_2 < 1$ as well.

The region of parameters ε, k_0 in which $\tilde{\mathscr{E}}_c = E^*$ is presented in Fig. 5.

3) One can also explain the character of the approach of $\tilde{\mathscr{E}}_c$ to E^* when k_0 tends to a threshold value k_0^* lying on the boundary of the region of Eq. (24) (the shaded region in Fig. 5):

$$\frac{\tilde{\mathscr{E}}_c}{E^*}=1-C\left(\frac{\ln k_0^*}{k_0^*}\right)^{2/3}(k_0^*-k_0)^{2/3}, \tag{25}$$

where $C = (2|b|^3)^{-1/2} \approx 0.2$ and b is the first zero of the Airy function.

The x-dependence of the quantities D, φ, and $E/\mathscr{E}_c = \varphi/D$, which determine, respectively, the conductivity, charge, and field in the channel, referred to the values directly behind the streamer front, are shown in Fig. 6.

The behavior of these quantities is significantly different in the cases $k_0 < k_0^*$ and $k_0 > k_0^*$. In the first case the conductivity and charge fall to zero, but the field in the channel tends to the value of the external field, which is equal to $\mathscr{E}_c < E^*$ for stationary propagation. In the second case (sufficiently strong attachment) the external field necessary for stationary development equals E^*. Here the conductivity and the charge tend to finite $D = D_2$ and $\varphi = D_2/\varepsilon$, and the field in the channel tends to E^*. With increasing k_0 the quantity D_2 increases from the value $1/k_0^*$ at $k_0 = k_0^*$ to the value ε at $k_0 \gg k_0^*$. The dependence of the ratio of the conductivity of the channel far from the head (σ_∞) to the conductivity at the head (σ_m) on k_0 is shown in Fig. 7. At $k_0 < k_0^*$ we have $\sigma_\infty = 0$; for $k_0 > k_0^*$ this ratio equals D_2.

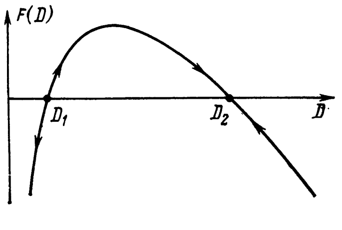

FIG. 4. Form of the function $F(D)$ under condition (24).

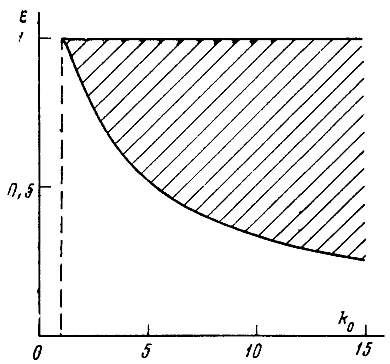

FIG. 5. Region of the values of the parameters ε and k_0 (shaded area) in which $\tilde{\mathscr{E}}_c = E^*$. The region is bounded by inequality (24) and with the conditions $k_0 > 1$ and $\varepsilon < 1$.

5. STREAMER GROWTH IN STRONG INHOMOGENEOUS FIELDS

We will now briefly discuss the case when a streamer propagates from a sharp tip. This case was examined by us earlier for the absence of attachment.[11,12] It was shown that the streamer can propagate during a time on the order of t_0, while the charge distribution along the streamer filament is close to electrostatic and the potential at the head does not differ strongly from the potential U of the tip. Here, the speed of the streamer and the radius of its head are proportional to U. It was shown that in the streamer channel there exists a definite field that creates the current ensuring the head charge necessary for stationary development. This field coincides with the quantity[2)] $\mathscr{E}_c$ introduced above. The entire length of the streamer to its stop is also proportional to the potential of the tip: $l \sim l_0 = Vt_0 \sim U/\mathscr{E}_c$, and is virtually determined by the condition that the voltage drop on the streamer length $\mathscr{E}_c l$ be on the order of U.

We now estimate the total length of the streamer l at the beginning of attachment. For this, we calculate the voltage drop on the streamer length, assuming this drop to be small in comparison with the potential of the tip. When this condition is fulfilled, the speed of propagation is practically constant and one can use the self-similar Eqs. (19) and (20), in which one sets the external field $\mathscr{E}$ equal to zero. (The field at the tip can be neglected at distances much larger than its radius of curvature.) Moreover, in the right-hand sides of these equations one can set $\varphi = 1$, which corresponds to small changes of the potential along the streamer. We recall that in Eqs. (19) and (20) the potential φ is measured in units of the head potential, which in the case considered

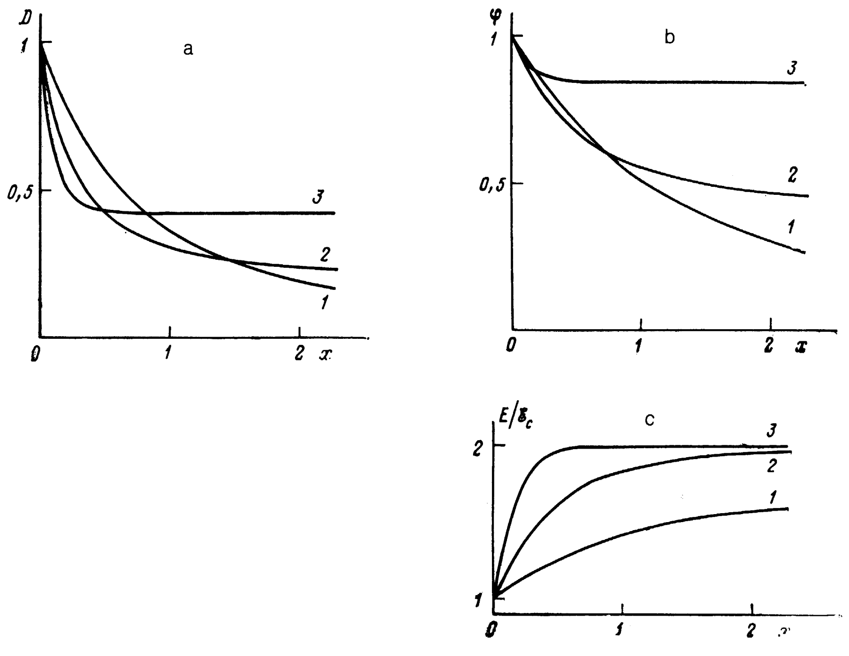

FIG. 6. The same as Fig. 2, for a model with linear $k(E)$ dependence for $\varepsilon = 0.5$, $k_0^* = 5.36$ and $k_0 = k_0^*/2$ (1), $k_0 = k_0^*$ (2), $k_0 = 2k_0^*$ (3).

differs little from the potential U of the tip. The length x is measured in the units $l_0 = U/\mathscr{E}_c$.

From Eq. (20) we find

$$D(x)=\varepsilon+(1-\varepsilon)\exp(-k_0x), \tag{26}$$

and from Eq. (19) we obtain the following expression for the voltage drop $\Delta\varphi$:

$$\Delta\varphi=\int_0^{l/l_0}\frac{dx}{D(x)}=\frac{1}{k_0\varepsilon}\ln\left[1-\varepsilon+\exp\left(k_0\frac{l}{l_0}\right)\right]. \tag{27}$$

Equation (27) is valid when $\Delta\varphi \ll 1$; however, the maximum length of the streamer l can be estimated from the condition $\Delta\varphi \sim 1$.

For weak attachment ($k_0 \ll 1$) we obtain from this the previous result $l \sim l_0 \sim U/\mathscr{E}_c$. For strong attachment ($k_0 \to \infty$) we find $l \sim l_0 \varepsilon$; i.e., $l \sim U/E^*$. This result is explained by the fact that for strong attachment the field practically coincides with the quantity E^* in the principal part of the channel.

Thus, in the extreme cases of weak and strong attachment the full length of a streamer propagating from a sharp tip satisfies the estimate

$$l \sim U/\tilde{\mathscr{E}}_c, \tag{28}$$

where $\tilde{\mathscr{E}}_c$ is the value of the external homogeneous field in which the streamer is capable of stationary propagation. Equation (28) can be considered as an interpolation in the intermediate cases.

The results obtained in this section agree with the experimental data of Gallimberti[3] and with his numerical calculations for the mixture of air and SF_6. We note, however, that for these calculations (see also Ref. 2) the radius of the streamer channel is considered constant and is specified as an independent parameter. In agreement with our assumptions,[11] for propagation from a sharp tip, the radius of the head is uniquely determined by its potential. The potential of the head decreases as the streamer develops, and its radius and speed also decrease; it is this which leads to the stopping of the streamer.

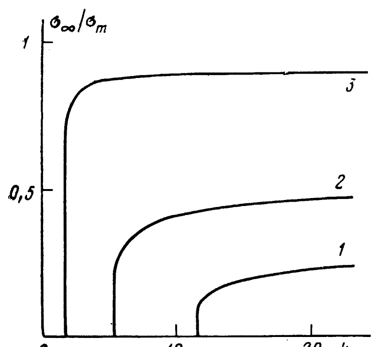

FIG. 7. Dependence of the ratio σ_∞/σ_m on k_0 at $\varepsilon = 0.3$ (1), $\varepsilon = 0.5$ (2), and $\varepsilon = 0.9$ (3).

6. COMPARISON WITH THE RESULTS OF NUMERICAL CALCULATIONS

Dhaly and Pal[4] solved numerically the problem of streamer propagation in a strong electronegative gas (SF_6) in a homogeneous field. It is significant that in this work the total system of equations describing the streamer discharge was solved correctly in contrast to the majority of similar calculations utilizing the so-called "disk method," in which the problem is artifically reduced to a single dimension.

The $\tilde{\beta}(E)$ dependence used in Ref. 4 is shown in Fig. 8. In agreement with the calculations,[4] the maximum field E_m at the head was $(2\text{–}3)E^*$, and the field $\mathscr{E}_c$ directly behind the streamer front was $(0.25\text{–}0.5)E^*$. The ratio of $\mathscr{E}_c$ and

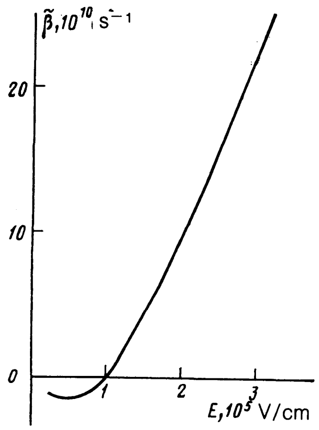

FIG. 8. The function $\tilde{\beta}(E)$ used in the numerical calculations for SF_6 in Ref. 4.

E_m agrees with the estimate (A4).

The estimate of the maximum field preceding the front ($E_m \sim E_0$) was made by us earlier[11] for the absence of attachment and with the dependence

$$\beta(E)=\beta_0\left(\frac{E}{E_0}\right)\exp\left(-\frac{E_0}{E}\right).$$

The function $\tilde{\beta}(E)$ in Fig. 8 has a completely different character. At $E > E^*$, in a sufficiently wide region of the field ($E - E^* \sim E^*$), the function $\tilde{\beta}(E)$ is nearly linear. If the quantity $\tilde{\beta}$ depends linearly on the field for $E > E^*$, then with the help of arguments analogous to those presented in Ref. 11 one can show that $E_m \sim E^*$. Indeed, if at some instant E_m is close to E^* ($E_m - E^* \ll E^*$), then the condition $\tilde{\beta} > 0$ is fulfilled only in a small region (with a dimension much smaller than the radius of the head) near the point where the field reaches the maximum value E_m. Therefore, the radius of the head will decrease, and the field preceding the front increases. In other words, if $E_m \gg E^*$ then, as shown in Ref. 13, the radius must increase, and the field E_m decreases.

Hence, for a linear function $\tilde{\beta}(E)$ in the region $E > E^*$ the maximum field E_m must exceed E^* at values on the order of E^*. This conclusion agrees with the result $E_m = (2\text{–}3)E^*$ of a numerical modeling.[4]

The maximum electron concentration in the streamer head is determined by the formula

$$N=\frac{1}{4\pi e}\int_{E^*}^{E_m}(\alpha-\eta)\,dE, \tag{29}$$

which is completely analogous to Eq. (A.2). Calculation according to this formula yields values of the concentration N two to three times smaller than obtained with numerical modeling. Hence, the results agree in order of magnitude. The difference between them is not fundamental, especially if one recognizes that in the region preceding the streamer front the concentration changes by 5–7 orders of magnitude.

An estimate of the streamer propagation speed according to Eq. (A.3), with the replacement of $\beta(E_m)$ by $\tilde{\beta}(E_m)$ and with the use of the numerical values of the parameters r_0, E_m, and Λ_1 from Ref. 4, leads to a result that agrees well with the results of numerical calculations. We note that the ratio of V to the electron drift velocity in the field E_m at the head was of the order of 10, which ensured the applicability of the theoretical estimates cited here (see footnote 1).

The characteristic time t_0 [see Eq. (A5)] was of the order of 1 nsec, and the attachment time $t_a \sim |\beta(\mathscr{E}_c)|^{-1}$ was of the order of 0.1 nsec, so that $t_0/t_a \gg 1$, and the situation corresponds to strong attachment. Here, as was shown above, the field in the streamer channel must grow from the value $\mathscr{E}_c$ directly behind the front to the value E^*, and the electron concentration falls, tending to a finite value distant from the head. Just such a behavior was demonstrated also by the numerical calculations.

APPENDIX

We present a summary of results obtained by us earlier and used in the present work, accurate to within the numerical factors of the fundamental streamer parameters.

1. The maximum field preceding the front E_m must be such that the characteristic length on which the impact ionization frequency $\beta(E)$ varies ahead of the streamer front, is on the order of the radius r_0 of the head.[11] In the absence of attachment, for the usual dependence

$$\beta(E)=\beta_0(E/E_0)\exp(-E_0/E)$$

this condition leads to the relation $E_m \sim E_0$.

2. The conductivity σ_m directly behind the front is determined by Eq. (6),

$$\sigma_m=\frac{1}{4\pi}\int_0^{E_m}\frac{\beta(E)}{E}\,dE, \tag{A.1}$$

which leads to the estimate[11,13] $\sigma_m \sim \beta(E_m)$. Equation (A.1) holds for a linear dependence of the electron drift velocity v_d on the field. The equivalent of expression (A.1) for the electron density N behind the front has the form

$$N=\frac{1}{4\pi e}\int_0^{E_m}\alpha(E)\,dE. \tag{A.2}$$

We note that expression (A.2) is valid for arbitrary functions $v_d(E)$.

3. The speed of streamer propagation V is proportional to the radius of the head:[11,13]

$$V \sim \frac{\beta(E_m)r_0}{\Lambda_1}, \qquad \Lambda_1=\ln\frac{N}{n_0}, \tag{A.3}$$

where n_0 is the electron density ahead of the front.

4. The radius of the head r_0 is determined only by the conditions of the origin of the streamer[6] for stationary propagation in a homogeneous field. For propagation from a sharp tip, the radius of the head is proportional to the potential of the tip.[11]

5. The field in the channel directly behind the front is given by the expression[6,11]

$$\mathscr{E}_c \sim \frac{V\sigma_m}{r_0}E_m \sim \frac{E_m}{\Lambda_1}. \tag{A.4}$$

6. The characteristic time of establishment of stationary development is[6,11]

$$t_0 \sim \frac{\sigma_m r_0^2}{V^2}\Lambda_2 \sim \frac{\Lambda_1^2\Lambda_2}{\beta(E_m)}, \qquad (A.5)$$

where $\Lambda_2 = \ln(a/r_0)$, and where a is the characteristic distance over which the linear charge density changes.

[1)] A drift term—$\operatorname{div}(\sigma v_d)$ is omitted in the right-hand side of Eq. (1). Neglect of these terms is justified if the streamer velocity is much greater than the drift speed v_d (see Ref. 6). We will assume that this condition is fulfilled. For $V \gg v_d$ the properties of the anode and cathode streamers are identical.[6]

[2)] In Ref. 11 a streamer in a semiconductor was considered, and saturation, typical of semiconductors, of the carrier drift speed in large fields was taken into account. Therefore, the expression for the field in the channel (denoted in Ref. 11 by E_z) differed from the value (A.4) for a streamer in gas.

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Translated by R. J. Brewer