Polarization effects in coherent backscattering of particles from random media

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The problem of coherent enhancement of the backscattering of particles with spin $s = 1/2$ from random media containing small-radius scatterers is solved exactly. The polarization characteristics of the angular spectrum of particles are analyzed for reflection from three- and two-dimensional disordered systems and for a medium with an Anderson disorder (periodic system of random scatterers). This analysis is carried out for the cases of magnetic and spin–orbit interactions with the scattering centers. Effects which have no analogs in coherent backscattering of light and scalar waves are predicted: these effects are the appearance of a fine structure in the angular spectrum of the reflected particles in the vicinity of the backward direction, a nonmonotonic dependence of the backscattering enhancement factor η on the angle of incidence, and discontinuous behavior of the factor η as a function of the ratio of the cross sections of the magnetic and scalar interactions when particles are reflected from systems with an Anderson disorder.

1. INTRODUCTION

Interference between the wave functions of particles moving along coincident paths in opposite directions significantly alters multiple scattering by an ensemble of randomly distributed centers and is known to be the cause of weak localization waves in a random medium. This reduces the electrical conductivity and the diffusion coefficient of electrons traveling in "dirty" metals and in semiconductors, and also alters other transport coefficients.[1–7] The wave interference effects are manifested particularly strongly in the reflection of light from turbid media,[8–13] when these effects enhance greatly the backscattering intensity in the backward direction.

Theoretical analyses have shown[2,4,5,14] that one of the important features of weak localization of waves in disordered systems is its sensitivity to the symmetry of the scattering process relative to time reversal. Those interactions which break the T-invariance, for example the spin–spin (magnetic) interaction with scatterers, can alter significantly the nature of interference in a random medium. In particular, this gives rise to a variety of physical features in the behavior of the quantum corrections to the transport coefficients of metals and semiconductors.[2–5]

It is particularly desirable to analyze the phenomena associated with the breaking of the invariance of the scattering relative to time reversal in the problem of coherent enhancement of backscattering. In this case the observed interference effects are no longer corrections to the observed transport coefficients (conductivity, diffusion coefficient, thermal conductivity, etc.), but alter radically the whole angular dependence of the backscattering intensity near the backward direction.[8–21]

The polarization effects in backscattering of light due to symmetry breaking in relation to time reversal are discussed in detail in Refs. 11, 14, and 20–25. It is shown there that the absence of the T-invariance always results in simple suppression of the peak representing coherent backscattering of waves of the appropriate polarization.

In the case of backscattering of particles with spin $s = 1/2$ the situation is very different.[26,27] In particular, in the case of the magnetic and spin–orbit interactions we can expect coherent weakening of backscattering, i.e., a dip in the backward direction may appear in the angular spectrum of the scattered intensity.

We shall give an exact analytic solution of the problem of coherent backscattering of spin $s = 1/2$ particles from three- and two-dimensional disordered ensembles of small-radius scattering centers and from a medium with an Anderson type of disorder in the form of a periodic system of random scatterers. We shall discuss in detail the characteristics of the backscattering process in the case of the magnetic (spin–spin) and spin–orbit interactions of particles with the scattering centers. We shall show that the reflection of particles from a three-dimensional system of scatterers with disordered distributions and randomly oriented spins gives rise to a "fine" structure of the angular dependence of the density of the particle flux in the backward direction, in contrast to the predictions of Refs. 26 and 27. At the center of an overall dip the angular spectrum of the backscattered particles has a local maximum. An analysis of the exact solution of the multiple scattering problem shows that this maximum is due to the difference between the effective lengths for destruction of the coherence of wave functions of particles characterized by parallel and antiparallel spin orientations. The backscattering enhancement factor reaches its maximum value for oblique incidence of particles in a medium, rather than for the normal incidence which is true of the scalar scattering processes.[14]

An investigation of the angular spectrum of the particles backscattered from two-dimensional disordered systems shows that the nature of the spectrum depends strongly on the mutual orientation of the polarization vector of the incident particles and the plane of their motion. The dip in the angular distribution in the vicinity of the backward direction appears only for the processes involving spin reversal in the scattering plane. In all other cases the backscattering is enhanced coherently.

In a system with an Anderson type of disorder the motion of a particle is influenced strongly by the Bragg diffrac-

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tion processes and the associated anomalous transmission and absorption of waves in a medium. The effects of weak localization of radiation in a periodic system of fluctuating scatterers have a number of features associated with the diffraction enhancement or weakening of the noncoherent interactions, which is a function of the orientation of the particle momentum relative to the lattice. For example, in the Bragg diffraction geometry the backscattering enhancement factor η, considered as a function of the ratio of the cross sections of the "scalar" and spin–spin interactions, can reach its maximum ($\eta = 2$) and minimum limiting values.

The effects considered below may be observed experimentally in investigations of backscattering of polarized electrons from disordered targets and also on reflection of thermal neutrons from crystals under conditions of a strong spin-incoherent nuclear interaction.

2. ANGULAR SPECTRUM OF BACKSCATTERING FROM A THREE-DIMENSIONAL DISORDERED SYSTEM

We consider the motion of a nonrelativistic spin $s = 1/2$ particle in the half-space $x > 0$ filled with randomly distributed scattering centers which have a small radius obeying $r_0 \ll \lambda$, where λ is the particle wavelength.

We begin our investigation of the polarization phenomena with an analysis of the spin–spin interaction of a particle with scatterers. The matrix representing the scattering of a particle by a center located at the point R_a is (see, for example, Ref. 28)

$$\langle \mathbf{r}|\hat{\mathcal{T}}_a|\mathbf{r}'\rangle = (2\pi/m)\,[A + B(\mathbf{s}\mathbf{S}_a)]\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(\mathbf{r}-\mathbf{R}_a), \quad (1)$$

where m is the mass of the particle, $\mathbf{S}_a$ is the spin of the scatterers, and $\hbar = 1$. The imaginary part of the scattering length A is related to the total interaction cross section σ_{tot} by the optical theorem

$$\text{Im}\,A = -\frac{p_0}{4\pi}\sigma_{tot},$$

where $\sigma_{\text{tot}} = \sigma_0 + \sigma_s + \sigma_a$; $\sigma_0 = 4\pi|A|^2$ is the "scalar" interaction cross section; $\sigma_s = \pi|B|^2 S(S+1)$ is the spin incoherent (magnetic) scattering cross section; σ_a is the cross section for the absorption of particles by the scattering center; $p_0 = 2\pi/\lambda$ is the particle momentum.

The density of a flux of backscattered radiation and its polarization characteristics can be described by averaging the one-particle density matrix over the positions of the scatterers and the directions of their spins:

$$\rho_{\alpha\beta}(\mathbf{r},\mathbf{r}') = \langle \psi_\alpha(\mathbf{r})\,\psi_\beta^*(\mathbf{r}')\rangle, \quad (2)$$

where α and β are the indices of the spin components of the wave function of the investigated particle.

If we ignore the recoil in collisions with the incident particle, we can rewrite the density matrix of Eq. (2) in the form[29,30]

$$\rho_{\alpha\gamma}(\mathbf{r}_1,\mathbf{r}_2) = \rho_{\alpha\gamma}^{(0)}(\mathbf{r}_1,\mathbf{r}_2) + \int\ldots\int d\mathbf{R}_1\,d\mathbf{R}_2\,d\mathbf{r}_1'\,d\mathbf{r}_2'\,G(\mathbf{r}_1,\mathbf{R}_1)\,G^*(\mathbf{r}_2,\mathbf{R}_2) \times \Gamma_{\alpha\beta\gamma\delta}(\mathbf{R}_1,\mathbf{r}_1';\mathbf{R}_2,\mathbf{r}_2')\,\rho_{\beta\delta}(\mathbf{r}_1',\mathbf{r}_2'), \quad (3)$$

where the repeated indices imply summation; $G(\mathbf{r},\mathbf{r}')$ is the Green's function of the scattering problem;[30,31] $\rho_{\alpha\gamma}^{(0)}(\mathbf{r},\mathbf{r}')$ is the density matrix of the particles that do not undergo incoherent interactions in the medium (the density matrix of the coherent fields):

$$\rho_{\alpha\gamma}^{(0)}(\mathbf{r},\mathbf{r}') = \rho_{\alpha\gamma}^{(0)}\,\psi_0(\mathbf{r},\mathbf{p}_0)\,\psi_0^*(\mathbf{r}',\mathbf{p}_0). \quad (4)$$

In the last expression $\rho_{\alpha\gamma}^{(0)}$ represents the polarization density matrix of the incident particles, whereas $\psi_0(\mathbf{r},\mathbf{p})$ is the wave function of the scattering problem corresponding to the boundary condition in the form of a plane wave with a momentum $\mathbf{p}$ incident on a given surface from the region $x = -\infty$ (see Refs. 30 and 31):

$$\psi_0(\mathbf{r},\mathbf{p})_{\text{inc}} = \exp(i\mathbf{p}\mathbf{r})\,|_{x\to-\infty}.$$

The tensor function $\Gamma_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ in Eq. (3) describes the evolution of the wave field in the course of multiple scattering of particles in matter. In terms of the impurity diagram technique, it is governed by a sum of connected diagrams without external lines.[29,30] Under weak localization conditions (i.e., when $p_0 l \ll 1$, where $l = (n\sigma_{\text{tot}})^{-1}$ is the mean free path and n is the number of the scatterers per unit volume) the dominant contribution to $\Gamma_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ comes from a series of ladder and fan (maximally crossed) diagrams.

A series of ladder diagrams corresponds to a sequence of independent incoherent scattering events. In the sum of such ladder diagrams we shall separate the contribution of single scattering

$$\mathcal{L}_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2') = (2\pi/m)^2\,n\sigma_{\alpha\beta\gamma\delta}\,\delta(\mathbf{r}_1-\mathbf{r}_1')\,\delta(\mathbf{r}_2-\mathbf{r}_2') \times \delta(\mathbf{r}_1-\mathbf{r}_2) + L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2'), \quad (5)$$

where the tensor $\sigma_{\alpha\beta\gamma\delta}$ is given by the expression[2,4,5]

$$\sigma_{\alpha\beta\gamma\delta} = \frac{1}{4\pi}\left[\sigma_0\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{3}\sigma_s(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})\right] \quad (6)$$

and $\boldsymbol{\sigma}_{\alpha\beta} = (\sigma^x_{\alpha\beta},\sigma^y_{\alpha\beta},\sigma^z_{\alpha\beta})$ are the Pauli matrices. Then, the function $L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ satisfies the following equation (Fig. 1)

$$L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2') = \left(\frac{2\pi}{m}\right)^4 n^2\sigma_{\alpha\mu\mu\gamma\nu}\sigma_{\mu\beta\nu\delta}\,\delta(\mathbf{r}_1-\mathbf{r}_2) \times\delta(\mathbf{r}_1'-\mathbf{r}_2')\,G(\mathbf{r}_1,\mathbf{r}_1')\,G^*(\mathbf{r}_2,\mathbf{r}_2') + \left(\frac{2\pi}{m}\right)^2 n\sigma_{\alpha\mu\gamma\nu}\,\delta(\mathbf{r}_1-\mathbf{r}_2)\iint d\mathbf{R}_1\,d\mathbf{R}_2 \times G(\mathbf{r}_1,\mathbf{R}_1)\,G^*(\mathbf{r}_2,\mathbf{R}_2)\,L_{\mu\beta\nu\delta}(\mathbf{R}_1,\mathbf{r}_1';\mathbf{R}_2,\mathbf{r}_2'). \quad (7)$$

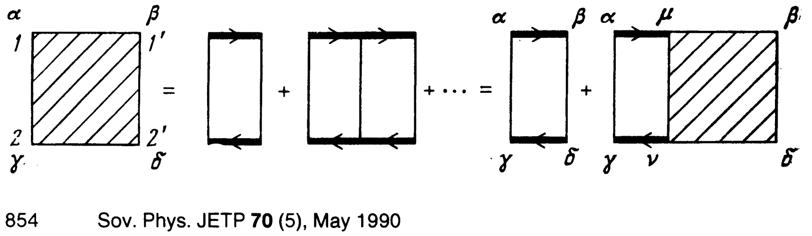

FIG. 1.

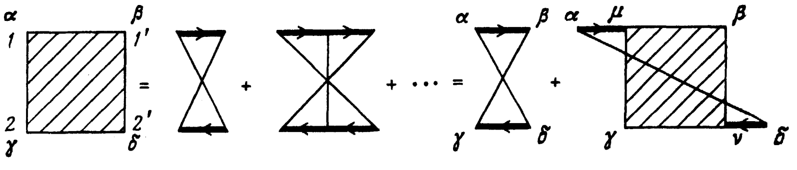

FIG. 2

Such fan diagrams describe interference between the wave functions of the particles moving in a medium along coincident paths but in opposite directions. The integral equation for the sum of such fan diagrams is (Fig. 2)

$$C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2') =\left(\frac{2\pi}{m}\right)^4 n^2\sigma_{\alpha\mu\nu\delta}\sigma_{\mu\beta\gamma\nu}\delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_2-\mathbf{r}_1')G(\mathbf{r}_1,\mathbf{r}_1') \times G^*(\mathbf{r}_2,\mathbf{r}_2')+\left(\frac{2\pi}{m}\right)^2 n\delta(\mathbf{r}_1-\mathbf{r}_2')\sigma_{\alpha\mu\nu\delta}\iint d\mathbf{R}_1\,d\mathbf{R}_2 G(\mathbf{r}_1,\mathbf{R}_1) \times G^*(\mathbf{r}_2,\mathbf{R}_2)C_{\mu\beta\gamma\nu}(\mathbf{R}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{R}_2). \quad (8)$$

In contrast to the scalar scattering case,[14,16,18,19] Eq. (8) for $C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ cannot be reduced to Eq. (7) by simple transposition of the arguments and the spin indices (because $\sigma_{\alpha\beta}\neq\sigma_{\beta\alpha}$). The physical reason for the absence of this symmetry between Eqs. (7) and (8) is the breaking of the invariance of the scattering matrix of Eq. (1), with respect to reversal of the sign of the projection of the particle spin (for the particles under consideration this is equivalent to time reversal).

The tensors $L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ and $C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ can be expanded in terms of the eigenvectors of Eqs. (7) and (8) (for details see the Appendix) and can be represented in the form

$$L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2') =\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1'-\mathbf{r}_2')\left\{\frac{\pi n}{4m^2}(\sigma_0+\sigma_s)\Pi(\sigma_0+\sigma_s;\mathbf{r}_1,\mathbf{r}_1') \times[\delta_{\alpha\beta}\delta_{\gamma\delta}+(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})] +\frac{\pi n}{4m^2}\left(\sigma_0-\frac{1}{3}\sigma_s\right)\Pi\left(\sigma_0-\frac{\sigma_s}{3};\mathbf{r}_1,\mathbf{r}_1'\right)(3\delta_{\alpha\beta}\delta_{\gamma\delta}-(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma}))\right\}, \quad (9)$$

$$C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2') =\delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_1'-\mathbf{r}_2)\left\{\frac{\pi n}{4m^2}(\sigma_0-\sigma_s)\Pi(\sigma_0-\sigma_s;\mathbf{r}_1,\mathbf{r}_1') \times[\delta_{\alpha\beta}\delta_{\gamma\delta}-(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})]+\frac{\pi n}{4m^2}\left(\sigma_0+\frac{1}{3}\sigma_s\right) \times\Pi\left(\sigma_0+\frac{1}{3}\sigma_s;\mathbf{r}_1,\mathbf{r}_1'\right)[3\delta_{\alpha\beta}\delta_{\gamma\delta}+(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})]\right\}, \quad (10)$$

where the propagator $\Pi(\sigma;\mathbf{r},\mathbf{r}')=\Pi(\sigma;|\rho-\rho'|,x,x')$ satisfies the equation

$$\Pi(\sigma;\mathbf{r},\mathbf{r}')=\frac{\pi n\sigma}{m^2}|G(\mathbf{r},\mathbf{r}')|^2 +\frac{\pi n\sigma}{m^2}\int d\mathbf{R}|G(\mathbf{r},\mathbf{R})|^2\Pi(\sigma;\mathbf{R},\mathbf{r}'). \quad (11)$$

A comparison of Eqs. (10) and (9) readily shows that the tensor $\hat{C}$ is identical with $\hat{L}$ apart from transposition of the second pair of arguments and a change in the sign in front of σ matrices. Physically, such a transformation corresponds to "reversal" of one of the two particle paths occurring in $\hat{L}$ (compare Figs. 1 and 2).

An analysis of Eq. (10) shows that the tensor factors in the first and second terms of the above expression are the operators performing projection onto states with a specific value of the total spin of two interfering particles.[32] We can therefore say that the first term in Eq. (10) represents a singlet contribution and the second a triplet contribution to the polarization density matrix of Eq. (3).

Substituting Eqs. (9) and (10) into Eq. (3), we can calculate the flux of the particle scattered along the direction defined by $\mathbf{p}_1=(p_{1x},\mathbf{p}_{1\parallel})=p_0\cos\vartheta_1, p_0\sin\vartheta_1\cos\varphi, p_0\sin\vartheta_1\sin\varphi)$:

$$J_{\alpha\gamma}(\vartheta_1,\varphi_1)=\frac{p_0^2\cos^2\vartheta_1}{(2\pi)^2\Sigma} \times\iint d\rho\,d\rho'\rho_{\alpha\gamma}(\mathbf{r},\mathbf{r}')\exp(-i\mathbf{p}_{1\parallel}\rho+i\mathbf{p}_{1\parallel}\rho')|_{x\to-\infty}, \quad (12)$$

where Σ is the surface area of the investigated medium. We then find that $J_{\alpha\gamma}(\vartheta_1,\varphi_1)$ is described by

$$J_{\alpha\gamma}(\vartheta_1,\varphi_1)=\frac{n}{\Sigma}\sigma_{\alpha\beta\gamma\delta}\rho^{(0)}_{\beta\delta}\int d\mathbf{R}|\psi_0(\mathbf{R},\mathbf{p}_0)|^2|\psi_0(\mathbf{R},-\mathbf{p}_1)|^2 +\frac{n}{\Sigma}\iint d\mathbf{R}\,d\mathbf{R}'|\psi_0(\mathbf{R},-\mathbf{p}_1)|^2|\psi_0(\mathbf{R}',\mathbf{p}_0)|^2 \times\left[\frac{\sigma_0+\sigma_s}{4\pi}\Pi(\sigma_0+\sigma_s;\mathbf{R},\mathbf{R}') \times\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}+(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})}{4}\rho^{(0)}_{\beta\delta}+\frac{\sigma_0-{}^1/_3\sigma_s}{4\pi}\Pi\left(\sigma_0-\frac{1}{3}\sigma_s;\mathbf{R},\mathbf{R}'\right) \times\frac{3\delta_{\alpha\beta}\delta_{\gamma\delta}-(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})}{4}\rho^{(0)}_{\beta\delta}\right] +\frac{n}{\Sigma}\iint d\mathbf{R}\,d\mathbf{R}'\,\psi_0(\mathbf{R},\mathbf{p}_0)\psi_0^*(\mathbf{R},-\mathbf{p}_1)\psi_0^*(\mathbf{R}',\mathbf{p}_0) \psi_0(\mathbf{R}',-\mathbf{p}_1)\left[\frac{\sigma_0-\sigma_s}{4\pi}\Pi(\sigma_0-\sigma_s;\mathbf{R},\mathbf{R}')\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}-(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})}{4}\rho^{(0)}_{\beta\delta} +\frac{\sigma_0+{}^1/_3\sigma_s}{4\pi}\Pi\left(\sigma_0+\frac{1}{3}\sigma_s;\mathbf{R},\mathbf{R}'\right)\frac{3\delta_{\alpha\beta}\delta_{\gamma\delta}-(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})}{4}\rho^{(0)}_{\beta\delta}\right]. \quad (13)$$

The first term in Eq. (13) represents the contribution of the single-scattering processes, whereas the second represents incoherent multiple scattering, and the third appears because of the interference between the wave functions of a particle in a random medium. In contrast to the scalar scattering case[14,16,18] in the presence of the spin interaction ($\sigma_s\neq 0$) the second and third terms of Eq. (13) are different along the backward direction ($\mathbf{p}_1=-\mathbf{p}_0$).

If we exclude the case of grazing incidence and the escape of particles from a medium ($\cos\vartheta_0, |\cos\vartheta_1|\gg\mu_c$, where $\mu_c\approx(n|A|)^{1/2}p_0\ll 1$ is the cosine of the critical angle), we can ignore the refraction and Fresnel reflection of waves at the boundary of the scattering medium and we can then describe $\psi_0(\mathbf{r},\mathbf{p})$ in Eq. (13) using the approximate expression

$$\psi_0(\mathbf{r},\mathbf{p})=\exp\left(i\mathbf{pr}-\frac{n\sigma_{tot}x}{2\cos\vartheta}\right),\qquad \cos\vartheta=\frac{p_x}{|\mathbf{p}|}>0,\quad (14)$$

whereas Eq. (11) with $x>0$ and $x'>0$ can be modified by substituting the Green's function for an infinite medium[31]

$$G|\mathbf{r},\mathbf{r}'|=-\frac{m}{2\pi}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\exp\left\{ip_0|\mathbf{r}-\mathbf{r}'|-\frac{n\sigma_{tot}}{2}|\mathbf{r}-\mathbf{r}'|\right\}.$$

In this case an analytic solution of Eq. (11) can be obtained explicitly (see Ref. 33). According to Ref. 33, the function

$$\Pi(\sigma;q,\xi,\xi')=2\pi\int_0^\infty\rho\,d\rho J_0(q\rho)\times\iint_0^\infty\exp(-x\xi-x'\xi')\Pi(\sigma;\rho,x,x')\,dx\,dx'\quad (15)$$

can be described by

$$\Pi(\sigma;q,\xi,\xi')=\frac{1}{\xi+\xi'}\left[H\left(\frac{n\sigma_{tot}}{\xi},\frac{\sigma}{\sigma_{tot}}\middle|\frac{q}{n\sigma_{tot}}\right)\times H\left(\frac{n\sigma_{tot}}{\xi'},\frac{\sigma}{\sigma_{tot}}\middle|\frac{q}{n\sigma_{tot}}\right)-1\right],\quad (16)$$

where

$$H(\mu,\omega|\nu)=\exp\left\{-\frac{\mu}{\pi}\int_0^\infty\frac{d\zeta}{1+\mu^2\zeta^2}\ln\left[1-\omega\frac{\operatorname{arctg}(\zeta^2+\nu^2)^{1/2}}{(\zeta^2+\nu^2)^{1/2}}\right]\right\}.\quad (17)$$

If $\nu=0$, then $H(\mu,\omega/\nu)$ is identical with the Chandrasekhar function $H(\mu,\omega)$ known from radiative transfer theory.[34,35]

The relationship (16) allows us to write down the final expression for the angular distribution of backscattered particles:

$$J_{\alpha\gamma}(\vartheta_1,\varphi_1)=\frac{1}{4\pi}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}\rho_{\beta\delta}^{(0)}\Big[(\omega_0+\omega_s)H(|\mu_1|,\omega_0+\omega_s|0)\times H(\mu_0,\omega_0+\omega_s|0)\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}+\sigma_{\alpha\beta}\sigma_{\delta\gamma}}{4}$$
$$+\left(\omega_0-\frac{1}{3}\omega_s\right)H\left(|\mu_1|,\omega_0-\frac{1}{3}\omega_s|0\right)\times H\left(\mu_0,\omega_0-\frac{1}{3}\omega_s|0\right)\frac{3\delta_{\alpha\beta}\delta_{\gamma\delta}-\sigma_{\alpha\beta}\sigma_{\delta\gamma}}{4}+(\omega_0-\omega_s)$$
$$\times\left\{\left|H\left(\tilde\mu,\omega_0-\omega_s\middle|\frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2-1\right\}\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}-\sigma_{\alpha\beta}\sigma_{\delta\gamma}}{4}$$
$$+\left(\omega_0+\frac{1}{3}\omega_s\right)\left\{\left|H\left(\tilde\mu,\omega_0+\frac{1}{3}\omega_s\middle|\frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2-1\right\}\times\frac{3\delta_{\alpha\beta}\delta_{\gamma\delta}+\sigma_{\alpha\beta}\sigma_{\delta\gamma}}{4}\Big],\quad (18)$$

where $\omega_0=\sigma_0/\sigma_{tot}$, $\omega_s=\sigma_s/\sigma_{tot}$, $\mu_i=\cos\vartheta_i$, $i=0,1$, and

$$\tilde\mu^{-1}=\frac{1}{2}(|\mu_1|^{-1}+\mu_0^{-1})+\frac{ip_0}{n\sigma}(|\mu_1|-\mu_0).\quad (19)$$

Equation (18) represents the complete solution of the problem of calculating the angular distribution of particles reflected from a disordered medium in the case of the scalar and spin–spin interactions with the scattering centers [generalization of Eq. (18) to the backscattering of particles from a disordered layer of finite thickness can be carried out using a method described in Ref. 36]. In an analysis of the interference effects in multiple spin–spin incoherent scattering we consider Eq. (18) in the special case of totally polarized incident radiation, i.e., we assume that

$$\rho_{\beta\delta}^{(0)}=\delta_{\beta+}\delta_{\delta+}={}^1/_2(\delta_{\beta\delta}+\mathbf{n}\sigma_{\beta\delta}).$$

The angular distribution of the particles reflected without a change in the polarization is

$$J_{++}(\vartheta_1,\varphi_1)=\frac{1}{4\pi}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}\Big[\frac{\omega_0+\omega_s}{2}H(|\mu_1|,\omega_0+\omega_s|0)\times H(\mu_0,\omega_0+\omega_s|0)+\frac{1}{2}\left(\omega_0-\frac{1}{3}\omega_s\right)H\left(|\mu_1|,\omega_0-\frac{1}{3}\omega_s|0\right)$$
$$\times H\left(\mu_0,\omega_0-\frac{1}{3}\omega_s|0\right)+\left(\omega_0+\frac{1}{3}\omega_s\right)\left\{\left|H\left(\tilde\mu,\omega_0+\frac{1}{3}\omega_s\middle|\frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2-1\right\}\Big],\quad (20)$$

whereas the distribution of the particles escaping from the medium with the opposite spin direction is

$$J_{--}(\vartheta_1,\varphi_1)=\frac{1}{4\pi}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}\Big[\frac{\omega_0+\omega_s}{2}H(|\mu_1|,\omega_0+\omega_s|0)H(\mu_0,\omega_0+\omega_s|0)-\frac{1}{2}\left(\omega_0-\frac{1}{3}\omega_s\right)H\left(|\mu_1|,\omega_0-\frac{1}{3}\omega_s|0\right)\times H\left(\mu_0,\omega_0-\frac{1}{3}\omega_s|0\right)$$
$$+\frac{1}{2}\left(\omega_0+\frac{1}{3}\omega_s\right)\times\left\{\left|H\left(\tilde\mu,\omega_0+\frac{1}{3}\omega_s\middle|\frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2-1\right\}-\frac{1}{2}(\omega_0-\omega_s)\left\{\left|H\left(\tilde\mu,\omega_0-\omega_s\middle|\frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2-1\right\}\Big].\quad (21)$$

The sum of Eqs. (20) and (21) gives the total intensity of the backscattered radiation:

$$J(\vartheta_1,\varphi_1)=\frac{1}{4\pi}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}\times\Big[(\omega_0+\omega_s)H(|\mu_1|,\omega_0+\omega_s|0)H(\mu_0,\omega_0+\omega_s|0)+\frac{3}{2}\left(\omega_0+\frac{1}{3}\omega_s\right)$$
$$\times\left\{\left|H\left(\tilde\mu,\omega_0+\frac{1}{3}\omega_s\middle|\frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2-1\right\}-\frac{1}{2}(\omega_0-\omega_s)\left\{\left|H\left(\tilde\mu,\omega_0-\omega_s\middle|\frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2-1\right\}\Big].\quad (22)$$

The difference between Eqs. (20) and (21) is proportional to the degree of polarization of the reflected particles.

The terms in the braces in Eq. (22) represent the interference of particles with parallel and antiparallel spins. The numerical factors in front of these terms are proportional to the degeneracy multiplicity $(2S+1)$ with respect to the "total spin" of the interfering particles (corresponding to $S=1$ and $S=0$, respectively).

The angular distributions calculated from Eqs. (20)–

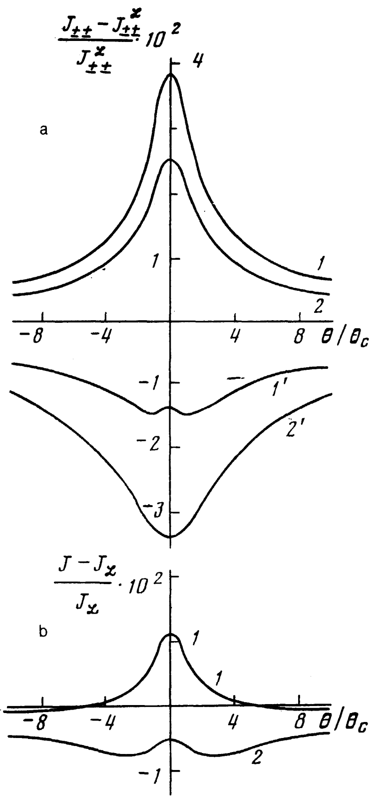

FIG. 3. Angular spectrum of reflected particles in the vicinity of the backward direction calculated for the magnetic scattering in a disordered medium: a) without (curves 1 and 2) and with (curves 1′ and 2′) spin reversal; b) total flux density. Parameters of the medium: $\theta_c = \sin^{-1}(\lambda/2\pi l) = 1°$; 1), 1′) $\omega_0 = 0.1$, $\omega_s = 0.9$; 2), 2′) $\omega_0 = 0$, $\omega_s = 1.0$. The curves are normalized to the relevant values of the incoherent flux density.

(22) are plotted in Fig. 3. The angular dependence of the flux density of the particles scattered with no change in the polarization always has a maximum in the backward direction. The angular distribution of the particle reflection accompanied by spin reversal [Eq. (21)] and the angular dependence of the intensity in the vicinity of the backward direction [Eq. (22)] are governed by the competition between the contributions of the singlet and triplet states. Since ω_s is positive, the intensity peak associated with the contribution of the triplet term is always sharper than the intensity dip caused by the singlet state. In particular, this has the effect that even in the case of pure spin–spin scattering the angular distribution of Eq. (22) has a local maximum near the $\mathbf{p}_1 = -\mathbf{p}_0$ direction.

This feature of the angular spectrum is due to the difference between the characteristic distances $l_{s=1} = 3/2(n\sigma_s)^{-1}$, and $l_{s=0} = 1/2(n\sigma_s)^{-1}$, at which the coherence of the wave functions of the particles with parallel and antiparallel spins is lost, and is the result of very many interactions between the particle and the scatterers. The distribution found in Ref. 26 using the double-scattering approximation has no intensity maximum along the $\mathbf{p}_1 = -\mathbf{p}_0$ direction.

An analytic investigation of the profile of the coherent backscattering peak is easily carried out in the case of relatively rare spin–spin interactions ($\sigma_s \ll \sigma_0$) and in the case of weak absorption ($\sigma_a \ll \sigma_0$). Using an asymptotic expression for the Chandrasekhar function $H(\mu,\omega|\nu)$ when $1-\omega \ll 1$ and $\nu \ll 1$ (Ref. 33), we find that for small angular deviations θ of the momentum $\mathbf{p}_1$ of the particles from the exactly backward direction $[\theta \ll (p_0 l)^{-1}, l = (n\sigma_{tot})^{-1}]$ the interference terms in Eq. (22) can be represented in the form

$$J^c_{S=1} = \frac{3}{16\pi}\mu_0\left[A(\mu_0) - B(\mu_0)\left(\frac{2\sigma_s+3\sigma_a}{\sigma_{tot}} + \nu^2\right)^{1/2}\right], \quad (23)$$

$$J^c_{S=0} = -\frac{1}{16\pi}\mu_0\left[A(\mu_0) - B(\mu_0)\left(\frac{6\sigma_s+3\sigma_a}{\sigma_{tot}} + \nu^2\right)^{1/2}\right], \quad (24)$$

where

$$\nu = \frac{|\mathbf{p}_{0\parallel} + \mathbf{p}_{1\parallel}|}{n\sigma_{tot}}, \quad A(\mu_0) = H^2(\mu_0, 1|0) - 1,$$

$$B(\mu_0) = 2\mu_0 H^2(\mu_0, 1|0).$$

It follows from Eqs. (23) and (24) that in the absence of absorption ($\sigma_a = 0$) the ratio of the characteristic angular widths of the triplet and singlet terms in the spectra of Eqs. (20)–(22) is $\Delta\theta_1/\Delta\theta_0 = 1/3^{1/2}$. An increase in the ratio σ_s/σ_0 gradually smooths out the coherent backscattering peak.

The behavior of the interference contribution to $J(\vartheta_1,\varphi_1)$ at high angles of deviation from the backward direction $[\theta \gg (p_0 l)^{-1}]$ also depends on the cross section ratio σ_s/σ_0. Using the asymptotic form of the function $H(\mu,\omega|\nu)$ at high values of $\nu \gg 1$ (Ref. 33), we can readily show that for

$$\sigma_s > 3^{1/2}\sigma_0(3^{1/2}+1)/(3^{1/2}-1)$$

the interference term in $J(\vartheta_1\varphi_1)$ becomes negative.

An important characteristic of the interference of waves and particles under multiple scattering conditions is the factor η representing the enhancement or weakening of backscattering, defined as the ratio of the observed flux density to the "background" density of the flux of incoherently scattered particles along the exact backward ($\mathbf{p}_1 = -\mathbf{p}_0$) direction. Using Eq. (22), we can write down η in the form

$$\eta = \eta^{sc}(\omega_0+\omega_s) - 1 + \frac{{}^3/_2(\omega_0 + {}^1/_3\omega_s)H^2(\mu_0, \omega_0 + {}^1/_3\omega_s) - {}^1/_2(\omega_0-\omega_s)H^2(\mu_0, \omega_0-\omega_s)}{(\omega_0+\omega_s)H^2(\mu_0, \omega_s)}, \quad (25)$$

where $\eta^{sc}(\omega) = 2 - H^{-2}(\mu_0,\omega)$ is the backscattering en-

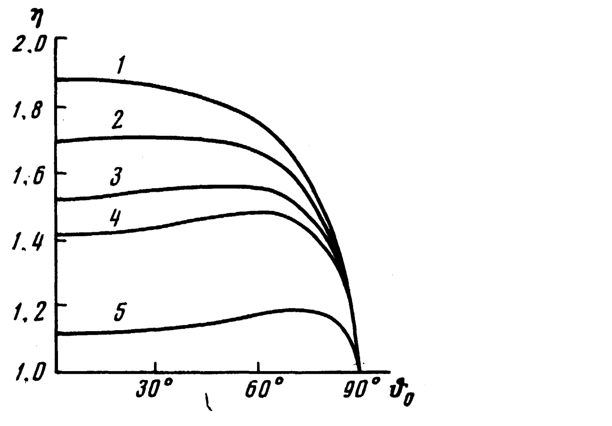

FIG. 4. Dependences of the backscattering enhancement factor on the angle of incidence of particles on the surface of a disordered medium: 1) $\omega_0 = 1.0$; $\omega_s = 0$; 2) $\omega_0 = 0.99$, $\omega_s = 0.01$; 3) $\omega_0 = 0.95$, $\omega_s = 0.05$, 4) $\omega_0 = 0.9$, $\omega_s = 0.1$; 5) $\omega_0 = 0.5$, $\omega_s = 0.5$.

hancement factor in the scalar case.[33] The dependence of the factor η on the cosine of the angle of incidence μ_0 in the case when $\sigma_s \neq 0$ is nonmonotonic (Fig. 4). The maximum value of η for $\omega_s > 0$ corresponds not to the normal incidence of radiation on a medium, as in the scalar scattering case,[33] but to some specific value $\mu_0^{max} < 1$. An increase in the ratio σ_s/σ_0 reduces $\mu_0^{\max}$. This behavior of the factor η is related to the influence of the spin–spin scattering for which the T-invariance limits the multiplicity of the collisions contributing to the interference part J^c of the angular spectrum.

In fact, in accordance with the differential of the factor η, it can be represented in the form

$$\eta = 1 + J^c [J_L (1 + J_1/J_L)]^{-1}, \tag{26}$$

where J_1 and J_L are the intensities of single and multiple ($k \geqslant 2$) incoherent backscattering. Since $(\omega_0 + \omega_s) > (\omega_0 + 1/3\omega_s)$, $(\omega_0 - \omega_s)$, the multiplicity of incoherent scattering is higher than the effective number of scattering events that destroy the coherence of the wave functions of the particles with parallel and antiparallel spins. However, a reduction in μ_0 reduces the multiplicity of the collisions resulting in incoherent backscattering and the ratio J^c/J_L rises. For $\mu_0 \ll 1$, when the main contributions to J^c and J_L come from double collisions, this ratio approaches unity. A reduction in the cosine of the angle of incidence enhances the single scattering role and the factor $(1 + J_1/J_L)^{-1}$ in Eq. (26) decreases monotonically. The maximum of η corresponds to such values of $\mu_0^{\max} < 1$ that the effective multiplicity of the collisions contributing to the interference term J^c and to the incoherent intensity J_L are quantities of the same order of magnitude. When $\mu_0 < \mu_0^{\max}$ is reduced still further, the angular distribution of the reflected particles begins to be dominated by the single scattering processes ($J_1|J_L \gg 1$) and the factor η decreases to unity.

When the particles are incident normally on a surface ($\mu_0 = 1$), the dependence $\eta = \eta(\sigma_s/\sigma_0)$ is monotonic and we have $\eta(0) = 1.8817$ and $\eta(\infty) = 0.995$.

3. POLARIZATION EFFECTS IN THE SPIN-ORBIT INTERACTION

We shall now consider to backscattering of particles from a disordered system of zero-spin centers when the spin–orbit interaction becomes important. In this case the scattering matrix considered in the momentum representation is of the form

$$\langle \mathbf{p}' | \hat{\mathcal{T}}_a | \mathbf{p} \rangle = (2\pi/m) (A + iC[\mathbf{p}', \mathbf{p}]\mathbf{s}), \tag{27}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the momenta of a particle before and after a collision.

The differential cross section for the scattering of particles by a single center is not isotropic in the spin–orbit interaction case. An allowance for the angular anisotropy of the scattering cross section greatly complicates the equations for the tensor functions $L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ and $C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$, and it makes the problem of calculation of the angular spectrum of the reflected particles very difficult. However, if we ignore the exact form of the reflection spectra, we can find those new qualitative characteristics of the angular distribution of particles which are due to the symmetry of the spin–orbit interaction under time inversion; this can be done by applying the isotropic scattering approximation. It is interesting to point out that this approximation is fully justified when collisions accompanied by spin reversal are relatively rare ($\sigma_0 \gg \sigma_{so}$, where σ_{so} is the spin–orbit scattering cross section) and the directions of particles after consecutive spin–orbit interaction events are uncorrelated.[4]

In the approximation of isotropic single scattering the equation for the function $L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ applicable to the spin–orbit interaction differs from Eq. (7) only by the replacement of σ_s with σ_{so} in the scattering tensor $\sigma_{\alpha\beta\gamma\delta}$. The equation for $C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ is obtained from Eq. (8) if instead of $\sigma_{\alpha\beta\gamma\delta}$ we substitute a tensor (see Refs. 2, 4, and 5)

$$\varkappa_{\alpha\beta\gamma\delta} = \frac{1}{4\pi}\left(\sigma_0 \delta_{\alpha\beta}\delta_{\gamma\delta} - \frac{1}{3}\sigma_{so}(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma})\right). \tag{28}$$

The difference between the tensors $\sigma_{\alpha\beta\gamma\delta}$ and $\varkappa_{\alpha\beta\gamma\delta}$ is in this case related to the time-reversal symmetry of the scattering matrix of Eq. (27):

$$\begin{aligned} \langle \mathbf{p}' | \hat{\mathcal{T}}_a{}^T | \mathbf{p} \rangle &= (2\pi/m) (A + iC[-\mathbf{p}, -\mathbf{p}'](-\mathbf{s})) \\ &= (2\pi/m) (A + iC[\mathbf{p}', \mathbf{p}]\mathbf{s}). \end{aligned}$$

The procedure of calculation of the angular distribution of backscattered particles carried out using Eq. (18) differs in no way from the procedure used above in the magnetic interaction case and it effectively reduces the modification of Eqs. (18) and (20)–(22) by replacing of σ_s with σ_{so} in the terms describing the usual incoherent scattering, and σ_s with $-\sigma_{so}$ in the interference terms. In particular, for $\mathbf{J}(\vartheta_1,\varphi_1) = J_{++}(\vartheta_1,\varphi_1) + J_{--}(\vartheta_1,\varphi_1)$, the following expression ($\omega_{so} = \sigma_{so}/\sigma_{\text{tot}}$) applies:

$$\begin{aligned} J(\vartheta_1, \varphi_1) = \frac{1}{4\pi}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0} \\ \times\Big\{ (\omega_0+\omega_{so}) H(|\mu_1|, \omega_0+\omega_{so}|0) H(\mu_0, \omega_0+\omega_{so}|0) \\ + \frac{3}{2}\left(\omega_0 - \frac{1}{3}\omega_{so}\right) \\ \times\left[\left| H\left(\tilde{\mu}, \omega_0 - \frac{1}{3}\omega_{so} \middle| \frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2 - 1\right] \\ - \frac{1}{2}(\omega_0+\omega_{so})\left[\left| H\left(\tilde{\mu}, \omega_0+\omega_{so} \middle| \frac{|\mathbf{p}_{0\parallel}+\mathbf{p}_{1\parallel}|}{n\sigma_{tot}}\right)\right|^2 - 1\right]\Big\}. \end{aligned} \tag{29}$$

The first term in Eq. (29) describes the usual angular spectrum of the incoherently scattered particles, whereas the second and third terms are the results of the triplet and singlet interference between the particles.

In the absence of absorption ($\sigma_a = 0$) if the angles of deviation from the backward direction are small [$\theta \ll (p_0 l)^{-1}$], the triplet and singlet contributions to Eq. (29) can be represented exactly as was done earlier in Eqs. (23)–(24):

$$J^c_{S=1} = \frac{3}{16\pi}\mu_0\left[A(\mu_0) - B(\mu_0)\left(\frac{4\sigma_{so}}{\sigma_{tot}} + \nu^2\right)^{1/2}\right], \tag{30}$$

$$J^c_{S=0} = -\frac{1}{16\pi}\mu_0[A(\mu_0) - B(\mu_0)\nu]. \tag{31}$$

According to Eq. (31), the same contribution to the angular spectrum of the reflected radiation has a "triangular" singularity in the backward direction, which demonstrates[20,24] the

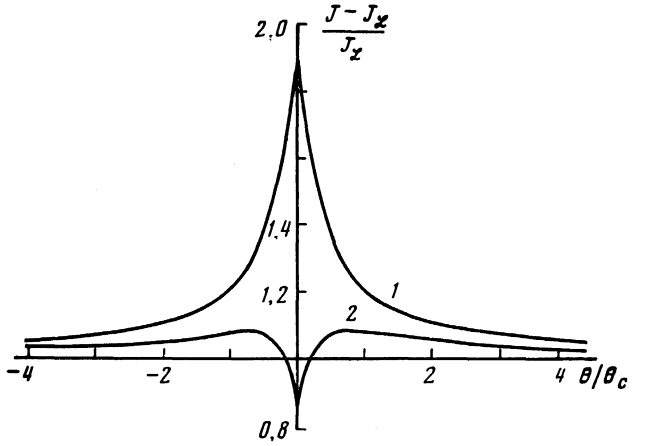

FIG. 5. Angular spectrum of the reflected particles calculated in the vicinity of the backward direction for the spin–orbit scattering in a disordered medium: 1) $\omega_0 = 1.0, \omega_{so} = 0$; 2) $\omega_0 = 0.9, \omega_{so} = 0.1$. The curves are normalized to the corresponding values of the incoherent flux density.

conservation of the coherence of the wave functions of the particles moving along oppositely directed paths.

An analysis of the above results leads us to the conclusion that the angular spectrum of the particles which are backscattered without spin reversal should exhibit a smooth maximum, whereas in the case of spin reversal there should be a dip with a triangular singularity. Such a triangular dip along the backward direction occurs also in the total distribution $J(\vartheta_1,\varphi_1)$ (Fig. 5).

These features of the angular distribution of the reflected radiation are due to the interference of the particles characterized by parallel and antiparallel spins. Since time reversal alters the scattering amplitude $\hat{\mathscr{T}}(\mathbf{p}',\mathbf{p},\mathbf{s})$ into $\hat{\mathscr{T}}(-\mathbf{p},-\mathbf{p}',-\mathbf{s})$, it follows that the T-symmetry of the amplitude of Eq. (27) conserves the coherence of the wave functions of the particles with antiparallel spins moving along coincident trajectories in opposite directions. On the other hand, the coherence of the wave functions of particles with parallel spins is destroyed over distances of the order of the mean free path relative to the spin–orbit interaction, i.e., for $l_{so} = (n\sigma_{so})^{-1}$.

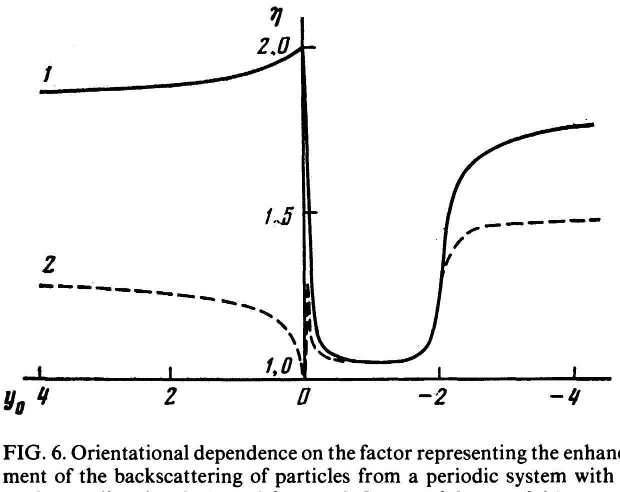

FIG. 6. Orientational dependence on the factor representing the enhancement of the backscattering of particles from a periodic system with an Anderson disorder: 1) $\omega_i = 1.0, \sigma_s = 0$; 2) $\omega_i = 0.9, \sigma_s = 0.1(\sigma_s + \sigma_i)$. Bragg geometry case, $y_0 = \varepsilon_0/2|\text{Re } V|$ (the value $y_0 = 0$ corresponds to the exact Bragg condition).

4. COHERENT BACKSCATTERING FROM A SYSTEM WITH AN ANDERSON DISORDER

Generalization of the above expressions to the scattering of particles by fluctuating periodic structures is readily made by replacing the scalar scattering cross section σ_0 in Eqs. (7)–(8) with the "isotopic" incoherent scattering cross section σ_i of Ref. 37 (the corresponding scattering channel is associated with fluctuations of the magnitude of the potential, representing the interaction of a particle with separate periodically distributed centers) and by going over from integration over the volume of the scattering medium to summation over the crystal lattice sites:[38]

$$\begin{aligned}
J_{\alpha\gamma}(\vartheta_1,\varphi_1) &= \frac{1}{\Sigma}\sigma_{\alpha\beta\gamma\delta}\rho^{(0)}_{\beta\delta}\sum_a |\Psi(\mathbf{R}_a,\mathbf{p}_0)|^2\,|\Psi(\mathbf{R}_a,-\mathbf{p}_1)|^2 \\
&+ \frac{1}{4\pi n\Sigma}\sum_{a,b}|\Psi(\mathbf{R}_a,\mathbf{p}_0)|^2\,|\Psi(\mathbf{R}_b,-\mathbf{p}_1)|^2\rho^{(0)}_{\beta\delta} \\
&\times\Big\{(\sigma_i+\sigma_s)Q(\sigma_i+\sigma_s;\mathbf{R}_a,\mathbf{R}_b) \\
&\times\frac{3\delta_{\alpha\beta}\delta_{\gamma\delta}+(\sigma_{\alpha\beta}\sigma_{\delta\gamma})}{4}+\Big(\sigma_i-\frac{1}{3}\sigma_s\Big)Q\Big(\sigma_i-\frac{1}{3}\sigma_s;\mathbf{R}_a,\mathbf{R}_b\Big) \\
&\times\frac{3\delta_{\alpha\beta}\delta_{\gamma\delta}-(\sigma_{\alpha\beta}\sigma_{\delta\gamma})}{4}\Big\} \\
&+\frac{1}{4\pi n\Sigma}\sum_{a,b}\Psi(\mathbf{R}_a,\mathbf{p}_0)\Psi^*(\mathbf{R}_a,-\mathbf{p}_1)\Psi^*(\mathbf{R}_b,\mathbf{p}_0)\cdot \\
&\times\Psi(\mathbf{R}_b,-\mathbf{p})\rho^{(0)}_{\beta\delta}\Big\{(\sigma_i-\sigma_s)Q(\sigma_i-\sigma_s;\mathbf{R}_a,\mathbf{R}_b)\,\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}-(\sigma_{\alpha\beta}\sigma_{\delta\gamma})}{4} \\
&+\Big(\sigma_i+\frac{1}{3}\sigma_s\Big)Q\Big(\sigma_i+\frac{1}{3}\sigma_s;\mathbf{R}_a,\mathbf{R}_b\Big)\frac{3\delta_{\alpha\beta}\delta_{\gamma\delta}+(\sigma_{\alpha\beta}\sigma_{\delta\gamma})}{4}\Big\}.
\end{aligned} \tag{32}$$

An equation similar to Eq. (11) for the propagator $Q(\sigma;\mathbf{R}_a\mathbf{R}_b)$ is now

$$Q(\sigma;\mathbf{R}_a,\mathbf{R}_b) = \frac{\pi n\sigma}{m^2}|G_{\rm cr}(\mathbf{R}_a,\mathbf{R}_b)|^2 + \frac{\pi n\sigma}{m^2}\sum_c |G_{\rm cr}(\mathbf{R}_a,\mathbf{R}_c)|^2 Q(\sigma;\mathbf{R}_c,\mathbf{R}_b). \tag{33}$$

The main difficulties encountered in the solution of Eq. (33) and calculation of the angular spectrum of Eq. (32) are related to the complex structure of the wave functions $\Psi(\mathbf{R},\mathbf{p})$ and of the Green's function $G_{\rm cr}(\mathbf{r},\mathbf{r}')$ describing the problem of particle diffraction by the periodic potential of a crystal.

For a weak potential we can find $\Psi(\mathbf{R},\mathbf{p})$ using the two-wave approximation of the dynamic diffraction theory while in solving Eq. (33) we can ignore the influence of the diffraction effects on multiple scattering.[37,38] The subsequent integration of Eq. (33) and calculation of the density of the flux of backscattered particles of Eq. (32) are similar to the procedures applied in Ref. 38 to scalar fields. The peak profile and the polarization features of the process of coherent backscattering are then similar to those discussed in Sec. 2.

The most interesting is the problem of how the anomalous transmission and absorption of particles in a crystal influence the backscattering enhancement factor η. We analyze the Bragg diffraction of the incident and reflected particles in which the effects of the anomalous transmission and absorption of radiation are manifested most strongly.[38] The wave function $\Psi(\mathbf{r},\mathbf{p})$ at the lattice sites is then (see, for example, Refs. 37 and 38)

$$\Psi(\mathbf{R}_a,\mathbf{p})=\exp(i\mathbf{p}\mathbf{R}_a)\left\{\frac{\varepsilon}{2V}\left[\left(1+\frac{4V}{\varepsilon}\right)^{1/2}-1\right]\right\}\cdot$$
$$\times\exp\left\{\frac{i\varepsilon x_a}{2v\cos\vartheta}\left[1-\left(1+\frac{4V}{\varepsilon}\right)^{1/2}\right]\right\},\qquad(34)$$

where $v=p/m$ is the velocity of the particles; V is the Fourier component of the periodic potential corresponding to the diffraction reflection by the reciprocal lattice vector $\mathbf{G}$, $G_x=-|\mathbf{G}|$, and the parameter ε represents the "energy" deviation from the exact Bragg condition $\varepsilon=[(\mathbf{p}+\mathbf{G})^2-\mathbf{p}^2]/2m$. The exponential nature of the dependence of the wave function on the depth of penetration of the radiation x_a into a crystal makes it possible to use Eq. (25) in calculating the backscattering factor η, provided we replace μ_0 with $\cos\vartheta_0\mathrm{Re}(1+4V/\varepsilon_0)^{1/2}$, where

$$\mathrm{Re}\left(1+\frac{4V}{\varepsilon_0}\right)^{1/2}=\left[\frac{1}{2}\left[1+\frac{2}{y_0}\right.\right.$$
$$\left.\left.+\left\{\left(1+\frac{2}{y_0}\right)^2+\frac{1}{y_0^2}\left(\frac{\mathrm{Im}\,V}{\mathrm{Re}\,V}\right)^2\right\}^{1/2}\right]\right]^{1/2},\qquad(35)$$

as $y_0=\varepsilon_0/2|\mathrm{Re}\,V|$. We can see from the above expression that in the limit $y_0\to 0$ (i.e., in the vicinity of the exact Bragg geometry) the first argument of the Chandrasekhar functions in Eq. (25) becomes infinite. The use of the familiar expression[39]

$$\lim_{\mu\to\infty}H(\mu,\omega)\,|\,0)=(1-\omega)^{-1/2}\qquad(36)$$

yields an unusual result:

$$\lim_{y_0\to 0}\eta=\begin{cases}1+\omega_i & \text{for}\quad \sigma_s=0,\\ 1 & \text{for}\quad \sigma_s>0,\end{cases}\qquad(37)$$

with $\omega_i=\sigma_i/(\sigma_i+\sigma_s+\sigma_a)$.

This discontinuous behavior of the enhancement factor η in the vicinity of the point $y_0=0$ is due to the same physical factor as the shift of the maximum of the dependence $\eta(\mu_0)$ toward grazing angles of incidence, which occurs as the ratio σ_s/σ_0 increases [see Eq. (25)]. In fact, the near-zero value of the parameter y_0 corresponds to an anomalously deep penetration of the field of the incident radiation into a crystal.[37] Therefore, in the absence of absorption, for $\omega_i=1$ and $\sigma_s=0$, the contribution to the interference term of Eq. (10) is made by the processes of the scattering of "infinitely high" multiplicity [$J_1/J_L=0$ in Eq. (26)] and, in full agreement with the familiar law of enhancement of the "multiple" part of the scattered radiation intensity,[14,16,18] the limit of Eq. (37) amounts to 2. An allowance for the T-invariant spin–spin interaction ($\sigma_s>0$) limits the multiplicity of the collisions, governing the value of J^c and suppresses absolutely the coherent backscattering process. The dependence $\eta(y_0)$ illustrating this behavior is plotted in Fig. 6. We can see from this figure that the curve corresponding to $\sigma_s>0$ lies below the curve obtained for $\sigma_s=0$, which confirms the picture of the effect discussed above.

5. BACKSCATTERING OF PARTICLES FROM TWO-DIMENSIONAL DISORDERED SYSTEMS

An important feature which distinguishes two- from three-dimensional systems is the conservation of the projection of the spin of the particles along the z axis, perpendicular to the scattering plane xy, in the magnetic and spin–orbit interactions. Consequently, when particles are reflected from a medium with a two-dimensional disorder, we find an additional dependence of the angular spectrum on the orientation of the polarization vector of the incident particles relative to the xy plane, which results in a qualitative difference between the polarization phenomena in a two-dimensional system from those discussed above in the case of a three-dimensional system.

We shall now analyze from the beginning the process of multiple scattering of particles in the magnetic interaction case. An example of a two-dimensional disordered medium with such an interaction is a system of randomly distributed vortices with random (clockwise or anticlockwise) directions of the current.

The equations for the tensor functions $L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ and $C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ are then obtained directly from Eqs. (7) and (8) if instead of the scattering tensor of Eq. (6) we substitute

$$\sigma_{\alpha\beta\gamma\delta}=\frac{p_0}{(2\pi)^2}(\sigma_0\delta_{\alpha\beta}\delta_{\gamma\delta}+\sigma_s\sigma_{\alpha\beta}{}^z\sigma_{\gamma\delta}{}^z),\qquad(38)$$

where σ_0 and σ_s are the scalar and magnetic (spin incoherent) scattering cross sections for the two-dimensional case. The cross sections σ_0 and σ_s have the dimensions of length.

Since the tensor $\sigma_{\alpha\beta\gamma\delta}$ is expressed in terms of the matrices $\delta_{\alpha\beta}$ and $\sigma^z_{\alpha\beta}$, it follows that transposition of the arguments $\mathbf{r}_2$ and $\mathbf{r}_2'$ and of the spin indices can reduce the equation for $C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ to the equation for $L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$. The solution of the latter in the two-dimensional case can be written as follows:

$$L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2',\mathbf{r}_2)$$
$$=\frac{p_0 n}{(2\pi)^2}\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1'-\mathbf{r}_2')$$
$$\times\Big[(\sigma_0+\sigma_s)\Pi(\sigma_0+\sigma_s;\mathbf{r}_1,\mathbf{r}_1')\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}+\sigma_{\alpha\beta}{}^z\sigma_{\delta\gamma}{}^z}{2}$$
$$+(\sigma_0-\sigma_s)\Pi(\sigma_0-\sigma_s;\mathbf{r}_1,\mathbf{r}_1')\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}-\sigma_{\alpha\beta}{}^z\sigma_{\delta\gamma}{}^z}{2}\Big].\qquad(39)$$

The propagator $\Pi(\sigma;\mathbf{r},\mathbf{r}')\equiv\Pi(\sigma;|y-y'|,x,x')$ satisfies the equation

$$\Pi(\sigma;\mathbf{r},\mathbf{r}')=\frac{n\sigma p_0}{m^2}|G(\mathbf{r},\mathbf{r}')|^2$$
$$+\frac{n\sigma p_0}{m^2}\int d\mathbf{r}''\,|G(\mathbf{r},\mathbf{r}'')|^2\Pi(\sigma;\mathbf{r}'',\mathbf{r}'),\qquad(40)$$

where $G(\mathbf{r},\mathbf{r}')$ is the Green's function of the appropriate two-dimensional Schrödinger equation and n is the number of scatterers per unit area. Integration in Eq. (40) is carried out over the half-plane $x''>0$.

We shall now consider the physical meaning of the various terms in Eq. (39). In the expression for the sum of the ladder diagrams $L_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ the first term describes the particle density and the second the polarization of the particles in the case of multiple incoherent scattering in a disordered medium. In the expression for the sum of the fan diagrams $C_{\alpha\beta\gamma\delta}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ the first term represents interference between the wave functions of the particles with identical signs of the spin projection on the z axis, whereas the second represents the interference of the wave functions of particles with the opposite signs, i.e., these terms give respectively the triplet and singlet interference contributions to the density matrix.

Equation (40) is the two-dimensional analog of Eq. (11). If we ignore the Fresnel reflection and refraction of particles at the boundary of the investigated medium, Eq. (40) can be solved exactly (see Ref. 33).

Substituting Eq. (39) into the expression for the density matrix of Eq. (3), we can readily calculate the angular spectrum of the backscattered particles which in the two-dimensional case is related to $\rho_{\alpha\gamma}(\mathbf{r},\mathbf{r}')$ by the expression

$$J_{\alpha\gamma}(\vartheta_1)=\frac{p_x^2}{2\pi p_0\mathcal{L}}\iint_{-\infty}^{\infty} dy\,dy'\exp\{-ip_{\parallel}(y-y')\}\rho_{\alpha\gamma}(\mathbf{r},\mathbf{r}')\,|_{x\to-\infty}$$

$$=\frac{p_x^2}{2\pi p_0\mathcal{L}}\rho_{\alpha\gamma}(x,p_{\parallel};x,p_{\parallel})\,|_{x\to-\infty}, \qquad (41)$$

where we have written $p_x=p_0\cos\vartheta_1, p_{\parallel}=p_y=p_0\sin\vartheta_1$ and $\mathcal{L}$ is the length of boundary lines separating the medium from vacuum.

The density of the flux of backscattered particles polarized along the z axis is

$$J^z(\vartheta_1)=\frac{\omega_0+\omega_s}{2\pi}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}\Big\{h(|\mu_1|,\omega_0+\omega_s|0)\times h(\mu_0,\omega_0+\omega_s|0)+\Big[\Big|h\Big(\tilde{\mu},\omega_0+\omega_s\Big|\frac{p_0}{n\sigma_{tot}}|\sin\vartheta_1+\sin\vartheta_0|\Big)\Big|^2-1\Big]\Big\}, \qquad (42)$$

where the function

$$h(\mu,\omega|v)=\exp\Big[-\frac{\mu}{\pi}\int_0^{\infty}\frac{d\zeta}{1+\mu^2\zeta^2}\ln\Big(1-\frac{\omega}{(1+v^2+\zeta^2)^{1/2}}\Big)\Big] \qquad (43)$$

is the two-dimensional analog of the "generalized" Chandrasekhar function $H(\mu,\omega|v)$ (Ref. 33). The conservation of the projection of the spin s_z in the case of multiple scattering in a two-dimensional medium makes the angular spectrum of the particles polarized along the z axis and undergoing the magnetic interaction exactly the same as the angular spectrum of the backscattering of zero-spin particles. The scattering amplitudes for the forward and backward waves with parallel spins are identical at each of the centers and the magnetic interaction does not disturb their coherence. As in the scalar case,[33] in the absence of absorption ($\sigma_a=0$) the angular spectrum of Eq. (42) has a sharp "triangular" peak in the backward direction.

Note that, in accordance with the above discussion, the only contribution to Eq. (42) is that made by the first term of Eq. (39) which describes the particle density in the sum of the ladder diagrams and represents the interference of the wave functions of particles with parallel spins in the sum of the fan diagrams.

The situation is quite different when particles polarized in the scattering plane are reflected. The projection of the spin along any axis in the xy plane is no longer an integral of motion and we can therefore have scattering with spin reversal. The wave function of a particle polarized in the xy plane can be represented as a superposition of the wave functions with $s_z=1/2$ and $s_z=-1/2$. Therefore, the interference between the wave functions of counterpropagating particles polarized in the scattering plane includes the contribution of the wave functions of particles with identical and opposite projections of the spins along the z axis. It follows that the expression for the backscattering spectrum should now contain both terms of Eq. (39).

The flux density of the reflected particles polarized in the scattering plane is

$$J^{xy}_{\pm\pm}(\vartheta_1)=\frac{1}{4\pi}\frac{|\mu_1|\mu_0}{|\mu_1|+\mu_0}\Big\{(\omega_0+\omega_s)h(|\mu_1|,\omega_0+\omega_s|0)\times h(\mu_0,\omega_0+\omega_s|0)\pm(\omega_0-\omega_s)h(|\mu_1|,\omega_0-\omega_s|0)h(\mu_0,\omega_0-\omega_s|0)+(\omega_0+\omega_s)\times\Big[\Big|h\Big(\tilde{\mu},\omega_0+\omega_s\Big|\frac{p_0}{n\sigma_{tot}}|\sin\vartheta_1+\sin\vartheta_0|\Big)\Big|^2-1\Big]\pm(\omega_0-\omega_s)\Big[\Big|h\Big(\tilde{\mu},\omega_0-\omega_s\Big|\frac{p_0}{n\sigma_{tot}}|\sin\vartheta_1+\sin\vartheta_0|\Big)\Big|^2-1\Big]\Big\}. \qquad (44)$$

Here, the plus sign in Eq. (44) corresponds to reflection without spin reversal, whereas the minus corresponds to spin reversal.

Since the magnetic interaction breaks the time-reversal symmetry, the coherence of the wave functions of the particles with antiparallel spins is destroyed over distances of the order of the mean free path under the magnetic scattering conditions, i.e., over distances $l_{so}=(n\sigma_s)^{-1}$. Consequently, the singlet contribution to Eq. (44) is not only less than the triplet contribution, but it is characterized by a smoother dependence on the angle θ representing the deviation from the backward direction.

In the absence of absorption the angular spectrum of Eq. (44) has a sharp triangular peak along the backward direction, and this is true of the backscattering accompanied by conservation of the polarization and of the processes which involve reversal of the particle spin. One should mention also that if $\sigma_0=\sigma_s$, a particle polarized in the xy plane seems to "forget" the spin direction after each scattering event: $J_{++}(\vartheta_1)=J_{--}(\vartheta_1)$.

If the polarization of the scattered particles is ignored, the expression for the angular spectrum $J^{xy}=J_{++}{}^{xy}+J_{--}{}^{xy}$ is identical with Eq. (42) obtained above.

We now analyze the polarization characteristics of the angular spectrum of the particles reflected in the presence of the spin–orbit interaction. The scattering amplitude is then[2–4]

$$\langle\mathbf{p}'|\hat{\mathcal{T}}_a|\mathbf{p}\rangle=A+iC[\mathbf{p}'\mathbf{p}]_z\hat{s}_z. \qquad (45)$$

In contrast to the case of the magnetic interaction discussed above, the amplitude of Eq. (45) is invariant under time reversal. Consequently, in the case of multiple scattering in a medium we can expect conservation of the coherence of the wave functions of the particles moving along coincident opposite paths and characterized by opposite signs of the projection of the spin along the z axis. In the case of the wave functions of particles with the same spin projections along the z axis the coherence is destroyed over distances on the order of the mean free path in the spin–orbit interaction case $l_{so} = (n\sigma_{so})^{-1}$.

In the approximation which ignores the angular anisotropy of the spin–orbit scattering[4] the expressions for the sums of the ladder and fan diagrams have the same matrix structure, but they cannot be converted from one to the other simply by transposition of the coordinates and the spin indices:

$$L_{\alpha\beta\gamma\delta}(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_2, \mathbf{r}_2') = \frac{p_0 n}{(2\pi)^2}\Big[(\sigma_0+\sigma_{so})\Pi(\sigma_0+\sigma_{so}; \mathbf{r}_1, \mathbf{r}_1')\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}+\sigma_{\alpha\beta}{}^z\sigma_{\delta\gamma}{}^z}{2} + (\sigma_0-\sigma_{so})\Pi(\sigma_0-\sigma_{so}; \mathbf{r}_1, \mathbf{r}_1')\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}-\sigma_{\alpha\beta}{}^z\sigma_{\delta\gamma}{}^z}{2}\Big] \times\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1'-\mathbf{r}_2'), \quad (46)$$

$$C_{\alpha\beta\gamma\delta}(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_2, \mathbf{r}_2') = \frac{p_0 n}{(2\pi)^2}\Big[(\sigma_0-\sigma_{so})\Pi(\sigma_0-\sigma_{so}; \mathbf{r}_1, \mathbf{r}_1')\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}+\sigma_{\alpha\beta}{}^z\sigma_{\delta\gamma}{}^z}{2} + (\sigma_0+\sigma_{so})\Pi(\sigma_0+\sigma_{so}; \mathbf{r}_1, \mathbf{r}_1')\frac{\delta_{\alpha\beta}\delta_{\gamma\delta}-\sigma_{\alpha\beta}{}^z\sigma_{\delta\gamma}{}^z}{2}\Big] \times\delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_1'-\mathbf{r}_2), \quad (47)$$

where σ_0 and σ_{so} are the cross sections of the scalar and spin–orbit interactions, whereas the propagator $\Pi(\sigma;\mathbf{r},\mathbf{r}')$ satisfies Eq. (40).

Using Eqs. (46) and (47), we readily obtain expressions for the angular distributions of backscattered particles with different polarizations in the initial and final states. The relationships for J^z and $J^{xy}_{\pm\pm}$ are similar to Eqs. (42) and (44) and can be obtained from Eqs. (42) and (44) by replacing σ_s with σ_{so} in the terms describing the distribution of incoherently scattered particles, and by replacing σ_s with $-\sigma_{so}$ in the interference terms of the angular spectrum.

The conservation of the spin projection s_z has the effect that the contribution to the angular spectrum J^z is made only by the interference of the wave functions of particles with identical projections of the spin along the z axis, the coherence of which is lost over distances of the order of l_{so}. Therefore, the peak in the angular distribution of the particles polarized along the z axis is smoothed out.[27]

On the other hand, in view of the nonconservation of the spin projection in the plane of motion, the angular spectrum $J^{xy}_{\pm\pm}$ includes also a term describing the interference of the particles with opposite signs of the spin projection along the z axis and, in contrast to the magnetic scattering case, the singlet contribution to the density of the flux of the reflected particles is much greater than the triplet contribution. Consequently, in the absence of absorption ($\sigma_a = 0$) the angular distribution of $J^{xy}_{\pm\pm}$ has a sharp triangular peak and $J^{xy}_{\mp\pm}$ has a dip of the same triangular shape.

6. CONCLUSIONS

We have obtained the exact solution of the problem of coherent backscattering of particles with $s = 1/2$ from random media containing small-radius scatterers. We classified the polarization effects observed on the reflection of particles from three- and two-dimensional spatially disordered media and from a periodic system with an Anderson disorder in the two cases of the spin–spin and spin–orbit interactions with the scatterers.

We predict effects that have no analogs in the coherent backscattering of light and scalar waves, such as the appearance of fine structure in the angular spectrum of particles in the vicinity of the backward direction, nonmonotonic dependence of the backscattering enhancement factor η on the angle of incidence, and discontinuous behavior of this factor η when particles are reflected by systems with an Anderson disorder, considered as a function of the ratio of the spin–spin and scalar interaction cross sections.

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APPENDIX 1

The eigenvectors of the collision operator in the equation for $L_{\alpha\beta\gamma\delta}$ can be found by introducing tensors

$$E_{\alpha\beta\gamma\delta}=\delta_{\alpha\beta}\delta_{\gamma\delta}, \qquad \Sigma_{\alpha\beta\gamma\delta}=(\boldsymbol{\sigma}_{\alpha\beta}\boldsymbol{\sigma}_{\delta\gamma}) \quad (A1)$$

and defining their multiplication rule as follows:

$$(\hat{\Sigma}\hat{\Sigma})_{\alpha\beta\gamma\delta}=\Sigma_{\alpha\nu\gamma\mu}\Sigma_{\nu\beta\mu\delta}. \quad (A2)$$

It readily follows from Eqs. (A1) and (A2) that

$$\hat{E}\hat{E}=\hat{E}, \quad \hat{E}\hat{\Sigma}=\hat{\Sigma}\hat{E}=\hat{\Sigma}, \quad \hat{\Sigma}\hat{\Sigma}=3\hat{E}+2\hat{\Sigma}.$$

It is clear from Eq. (7) that the selection of the eigenvectors of the collision operator reduces to a solution of the problem of the type

$$(\sigma_0\hat{E}+{}^1/_3\sigma_s\hat{\Sigma})(\hat{E}+\beta\hat{\Sigma})=g(\hat{E}+\beta\hat{\Sigma}). \quad (A4)$$

The use in Eq. (A4) of the rules of Eq. (A3) gives the following quadratic equation for β:

$$\beta^2-{}^2/_3\beta-{}^1/_3=0.$$

which yields the eigenvalues and the normalized eigenvectors of the problem of interest to us,

$$g_1=\sigma+\sigma_s, \quad {}^1/_4(\hat{E}+\hat{\Sigma}), \qquad g_2=\sigma_0-{}^1/_3\sigma_s, \quad {}^3/_4(\hat{E}-{}^1/_3\hat{\Sigma}). \quad (A5)$$

Similarly, we can calculate the eigenvectors of the collision operator of the equation for $C_{\alpha\beta\gamma\delta}$ by defining the product rule

$$(\hat{\Sigma}\hat{\Sigma})_{\alpha\beta\gamma\delta}=\Sigma_{\alpha\nu\mu\delta}\Sigma_{\nu\beta\gamma\mu}, \quad (A6)$$

which leads to

$$\hat{E}\hat{E}=\hat{E}, \quad \hat{E}\hat{\Sigma}=\hat{\Sigma}\hat{E}=\hat{\Sigma}, \quad \hat{\Sigma}\hat{\Sigma}=3\hat{E}-2\hat{\Sigma}. \quad (A7)$$

The eigenvalues and the normalized eigenvectors of the spin part of the collision integral of the equation for $C_{\alpha\beta\gamma\delta}$ deduced using the rules of Eqs. (A6) and (A7) are

$$g_1=\sigma_0-\sigma_s,\quad {}^1/_4(\hat{E}-\hat{\Sigma}),$$
$$g_2=\sigma_0+{}^1/_3\sigma_s,\quad {}^3/_4(\hat{E}+{}^1/_3\hat{\Sigma}). \tag{A8}$$

Substitution of Eqs. (A5) and (A8) into Eqs. (7) and (8) reduces this expression to the form given by Eqs. (9) and (10).

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