On the possibility of stimulated generation of spin waves in magnetics with paramagnetic impurities

V. I. Ozhogin, R. M. Farzetdinova, and A. S. Mikhaĭlov

I. V. Kurchatov Institute of Atomic Energy
(Submitted 16 March 1989)
Zh. Eksp. Teor. Fiz. **97**, 863–871 (March 1990)

A method is proposed for stimulated excitation of coherent magnons in ferro- and antiferromagnetics, based on the common statistical nature of magnons in magnetics and photons in masers and lasers. The feasibility of experimental observation of the maser effect for spin waves in magnetics is discussed.

1. INTRODUCTION

In magnetically ordered crystals, low-amplitude magnetic excitations are described in the language of spin waves (SW), or magnons, which represent a gas of quasiparticles with Bose-Einstein statistics. The study of spin waves yields broad information on the properties of magnetically ordered crystals. In practice, the only method for generation of high temperature spin waves is their parametric excitation, in which pairs of spin waves with oppositely directed wave vectors are created under the action either of perpendicular or parallel pumping[1–5] in threshold fashion. However, another method of excitation of spin waves is possible—by stimulated radiation of spin waves by paramagnetic impurities in magnetics.

If paramagnetic impurities are introduced into a magnetic, the spins of which are coupled by exchange interaction with the spins of atoms of the basic lattice, then a situation arises analogous to that occurring in optical quantum generators in solids: there is in the crystal a boson field of magnons which interacts with the impurity subsystem of "active" ions, and which is capable of producing transitions between levels of the ions with emission of a single magnon.

The interaction of the impurity paramagnetic ion, which possesses strong spin-orbit coupling, with the ions of the magnetic crystal surrounding it leads to a splitting of the energy levels of the impurity. The strongest splitting for ions of the transition elements of the iron group turns out to be the splitting due to the crystal field of the lattice. In the case of an even number of electrons in the unfilled shell of an impurity ion, a crystal field of low symmetry can completely lift the degeneracy of the level.

If, however, the impurity ion contains an odd number of electrons, then, according to the theory of Kramers,[6] its energy level in the crystal field remains at least double degenerate. Lifting of the degeneracy in this case is connected with the much weaker exchange interaction of the spin of the impurity with the spin of the atoms of lattice that surround it, and also with the action of the external magnetic field.[6–8] As a result, an energy spectrum is formed in the impurity subsystem with transition frequencies between the lower levels lying in the same range as for spin waves.[9,10] If the populations of these lower levels are inverted, amplification of the spin waves is possible. At sufficiently high amplifications of the spin waves, exceeding their attenuation in the crystal, generation of spin waves is possible. For the creation of an inverted population of the two lower levels, which is necessary to obtain the maser effect, we can use optical pumping with excitation of the higher levels,[11] but in the presence of three or four nearby levels of the impurity ion, we can use microwave pumping.[12]

2. BASIC EQUATIONS

Let us consider a system consisting of two-level impurity atoms implanted in the lattice of a magnetic and interacting with the field of the spin waves. The Hamiltonian of this system has the following form:

$$H=\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} b_{\mathbf{k}}^{+} b_{\mathbf{k}} + \sum_{j} \hbar\Omega a_j^{+} a_j + \frac{1}{N}\sum_{j,\mathbf{k}} (\Psi_{j\mathbf{k}} b_{\mathbf{k}} a_j^{+} + \mathrm{H.c.}), \tag{1}$$

where $b_{\mathbf{k}}^{+}$ and $b_{\mathbf{k}}$ are the creation and annihilation operators of a magnon with frequency $\omega_{\mathbf{k}}$ and wave vector $\mathbf{k}$; the operator a_j^{+} governs the transition of the impurity ion, located at the site R_j, from the ground state to the first excited state; $\hbar\Omega$ is the energy difference of the two lower levels of the impurity; the expressions for the amplitudes of the interaction Ψ_{jk} were obtained in Ref. 13 and have the following form for a two-sublattice antiferromagnetic:

$$\Psi_{jk}=\frac{1}{2}\left(\frac{S}{8}\right)^{1/2}\left(\frac{\omega_E}{\omega_k}\right)^{1/2}(\Lambda_{xx}^{(1)}\cos\varphi_1+\Lambda_{xx}^{(2)}\cos\varphi_2 - \Lambda_{yy}^{(1)}\cos\theta_1\cos\varphi_1$$
$$+\Lambda_{yy}^{(2)}\cos\theta_2\cos\varphi_2 - i\Lambda_{yy}^{(1)}\cos\varphi_1 - i\Lambda_{yy}^{(2)}\cos\varphi_2 - i\Lambda_{xx}^{(1)}\cos\theta_1\sin\varphi_1$$
$$- i\Lambda_{xx}^{(2)}\cos\theta_2\sin\varphi_2)e^{i\mathbf{k}\mathbf{R}_j}, \tag{2}$$

where (θ_1, φ_1) and (θ_2, φ_2) are the Euler angles for the magnetizations of the two sublattices in the system of coordinates whose z axis is directed along the magnetic moment of the impurity; the tensors $\Lambda^{(1)}$ and $\Lambda^{(2)}$ describe the exchange interaction of the spins of the impurities with the spins of the atom of the basic lattice, ω_E is the exchange frequency.

In the case of a ferromagnetic, the expression for Ψ_{jk} is written in the form

$$\Psi_{jk}=\left(\frac{S}{8}\right)^{1/2}(\Lambda_{xx}-\Lambda_{yy}\cos\theta-\Lambda_{zy}\sin\theta$$
$$-i\Lambda_{xy}-i\Lambda_{yx}\cos\theta-i\Lambda_{zx}\sin\theta)e^{i\mathbf{k}\mathbf{R}_j}.$$

We note that the operators a_j^{+} and a_j, referred to a

 0038-5646/90/030483-05$03.00

single site, are connected by the Fermi permutation relation, while the operators referring to different sites commute with one another.

In addition, it is necessary to take into account the pumping that leads to the creation of an inverse population of the impurity levels. Here, it is assumed that the pumping takes place through a third, higher level, which is depleted so rapidly that its population can be neglected and we take into account the population only of the two lower levels. Therefore, we shall not include in our consideration processes associated with the formation of the inverse population of levels under the action of the given external electromagnetic coupling (this approximation is usually employed also in the theory of laser generation[14,15]). Finally, it is necessary to take into account the effect of the thermostat (thermal phonons and magnons) on the field of the spin wave interacting with it. It is known that the thermal reservoir turns out to have relaxational and fluctuational effects on the atoms of the impurity. The relaxation effect of the thermostat leads to the result that in the absence of other external actions, the average values of the operators b_k and a_j will relax to their stationary values.

The effect of the thermal reservoir is taken into account by the introduction of dissipative terms and the quantum mechanical noise of the operators in the corresponding equation of motion of the operators. The possibilities of introduction of dissipative terms and noise of the operators into the operator equation has been discussed in the literature on the theory of lasers.[14,15] In the present work we shall consider only the possibility of stimulated generation of spin waves and we shall not concern ourselves with quantum fluctuations of the emitted spin waves.

The equations for the operators a_j^+ and b_k^+, and also for the inverse difference in populations of the impurity levels $\sigma_j = 1 - 2a_j^+ a_j$, can be obtained (see Ref. 14) starting out from (1):

$$\dot{b}_{\mathbf{k}}^+ = (i\omega_{\mathbf{k}} - \Gamma_{\mathbf{k}}) b_{\mathbf{k}}^+ + i\sum_j g_{j\mathbf{k}} a_j, \qquad (3)$$

$$\dot{a}_j^+ = (i\Omega - \gamma_\perp) a_j^+ - i\sum_{\mathbf{k}} g_{j\mathbf{k}} b_{\mathbf{k}} \sigma_j, \qquad (4)$$

$$\dot{\sigma}_j = (\sigma_0 - \sigma_j)\gamma_\parallel + 2i\sum_{\mathbf{k}} g_{j\mathbf{k}} a_j^+ b_{\mathbf{k}} - 2i\sum_{\mathbf{k}} g_{j\mathbf{k}}^* a_j b_{\mathbf{k}}^+, \qquad (5)$$

$$\sigma_0 = 2P/\gamma_\parallel - d_0. \qquad (6)$$

Here $g_{jk} = \Psi_{jk}/\hbar\mathcal{N}^{1/2}$; σ_0 is the stationary value of the inverse difference in populations of the impurity levels, which were created by the action of the external pumping; d_0 is the initial equilibrium difference of populations of the impurity levels with distance $\hbar\Omega$ between the levels, at a temperature 0 equal to $d_0 = \tanh(\hbar\Omega/2\theta)$; P is the rate of pumping describing transitions from the higher levels (the specific meaning of P depends on the form of the pumping) Γ_k is the equilibrium relaxation rate of the spin waves; $\gamma_\perp$ and $\gamma_\parallel$ are the transverse and longitudinal relaxation rates of the level populations of the impurities; $\mathcal{N}$ is the total number of sites in the lattice.

The formal solution of Eq. (4) has the form

$$a_j^+ = \hat{\gamma}_\perp^{-1}\left\{-i\sum_{\mathbf{k}} g_{j\mathbf{k}} b_{\mathbf{k}}^+ \sigma_j\right\}, \qquad (7)$$

where

$$\hat{\gamma}_\perp = \gamma_\perp - i\Omega + \frac{d}{dt}.$$

Substituting (7) in (5), we find

$$\left(\frac{d}{dt} + \gamma_\parallel\right)\sigma_j = \sigma_0\gamma_\parallel - \Big(2\sum_{\mathbf{k},\mathbf{k}'} g_{j\mathbf{k}} b_{\mathbf{k}}^+ \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}}^* b_{\mathbf{k}'} + 2\sum_{\mathbf{k},\mathbf{k}'} g_{j\mathbf{k}}^* b_{\mathbf{k}} \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}} b_{\mathbf{k}'}^+\Big)\sigma_j. \qquad (8)$$

We determine σ_j from this equation:

$$\sigma_j = \sigma_0\hat{\gamma}_\parallel \Big/ \Big[\hat{\gamma}_\parallel + 2\Big(\sum_{\mathbf{k},\mathbf{k}'} g_{j\mathbf{k}} b_{\mathbf{k}}^+ \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}} b_{\mathbf{k}'} + g_{j\mathbf{k}}^* b_{\mathbf{k}} \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}'} b_{\mathbf{k}'}^+\Big)\Big], \qquad (9)$$

where

$$\hat{\gamma}_\parallel = \gamma_\parallel + \frac{d}{dt}. \qquad (10)$$

On the other hand, substituting (7) in (3), we find an equation for the operators b_k and b_k^+:

$$\left(\frac{d}{dt} - i\omega_{\mathbf{k}} + \Gamma_{\mathbf{k}}\right) b_{\mathbf{k}}^+ = \sum_{j,\mathbf{k}} g_{j\mathbf{k}}^* \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}'} b_{\mathbf{k}'}^+ \sigma_j. \qquad (11)$$

In these equations, the operators σ_j are determined by the exact expression (9), so that (11) is an equation for the SW operators. Because of the presence of the SW operators in the denominator of the expression for σ_j, these equations are essentially nonlinear and have a complicated structure.

For not too many magnons,

$$N = \sum_{\mathbf{k}} \langle b_{\mathbf{k}}^+ b_{\mathbf{k}} \rangle$$

such that the inequality

$$N/\mathcal{N} < \frac{\gamma_\perp\gamma_\parallel}{(|\Psi|/\hbar)^2}, \qquad (12)$$

is satisfied, Eq. (9) for σ_j can be written in the following form:

$$\sigma_j \approx \sigma_0 - 2\hat{\gamma}_\parallel^{-1}\sum_{\mathbf{k},\mathbf{k}_1} (g_{j\mathbf{k}} b_{\mathbf{k}}^+ \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}}^* b_{\mathbf{k}_1} + g_{j\mathbf{k}}^* b_{\mathbf{k}} \hat{\gamma}_\perp^{-1} g_{i\mathbf{k}} b_{\mathbf{k}_1}^+). \qquad (13)$$

Substituting the resultant expression for σ_j in Eq. (11) for the SW operators, we find:

$$\left(\frac{d}{dt} - i\omega_{\mathbf{k}} + \Gamma_{\mathbf{k}}\right) b_{\mathbf{k}}^+ = \sum_{j,\mathbf{k}_1} g_{j\mathbf{k}} \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}_1}^* b_{\mathbf{k}_1}^+ \Big[\sigma_0 - 2\hat{\gamma}_\parallel^{-1}\sum_{\mathbf{k}_2,\mathbf{k}_3} (g_{j\mathbf{k}_2} b_{\mathbf{k}_2}^+ \hat{\gamma}_\perp^{-1} g_{j\mathbf{k}_3}^* b_{\mathbf{k}_3} + \text{H.c.})\Big]. \qquad (14)$$

Further analysis is materially simplified if we recognize that the rates $\gamma_\parallel$ and $\gamma_\perp$ of relaxation of the populations of the levels for impurities at low temperatures amount to 10^8–10^9 s^{-1}[7] and greatly exceed the characteristic relaxation rates of SW with small wave vectors. Therefore, we can use the adiabatic approximation, i.e., we can assume that the average population of the upper level attunes itself instantaneously to the average number of magnons at this instant of time. Consequently, we can neglect the operator d/dt in comparison with $\gamma_\parallel$ and $\gamma_\perp$ in the expressions for $\hat{\gamma}_\parallel$ and $\hat{\gamma}_\perp$.

To simplify the calculations, we limit ourselves [omitting the index **k** in (14)], to consideration of single-mode generation of spin waves[1)] at exact resonance between the frequency ω of the generated spin wave and the transition frequency Ω between the lower levels of the impurity. Then (16) takes the form of a Van der Pol operator equation,[14] which has been well studied in the theory of lasers:

$$\left(\frac{d}{dt}+\Gamma-\alpha\right)b_{\mathbf{k}}^{+}+\beta b_{\mathbf{k}}^{+}b_{\mathbf{k}}b_{\mathbf{k}}^{+}=0. \qquad (15)$$

The coefficients α and β have the form

$$\alpha=c\left(\frac{|\Psi|}{\hbar}\right)^{2}\frac{\sigma_0}{\gamma_\perp}, \qquad (16)$$

$$\beta=\frac{2}{\mathcal{N}}c\left(\frac{|\Psi|}{\hbar}\right)^{4}\frac{\sigma_0}{\gamma_\parallel\gamma_\perp}, \qquad (17)$$

where c is the concentration of impurities in the sample (the ratio of the total number of impurity ions to the total number of sites in the lattice).

Introducing new variables with the help of the relation

$$B^{+}=e^{-i\omega_{\mathbf{k}}t}b_{\mathbf{k}}^{+} \qquad (18)$$

and defining the unsaturated total amplification G as

$$G=\alpha-\Gamma, \qquad (19)$$

we transform Eq. (15) to the form

$$\left(\frac{d}{dt}-G\right)B^{+}+\beta B^{+}BB^{+}=0. \qquad (20)$$

In order to make clear the dependence of G on the inversion σ_0, we set $\alpha=\Gamma\sigma_0/\sigma_c$, where σ_c is the critical value of the initial inversion at threshold, below which the generation of spin waves is absent. Then G is written in the following way:

$$G=\Gamma\left(\frac{\sigma_0}{\sigma_c}-1\right), \qquad (21)$$

where σ_c is given by the expression

$$\sigma_c=c^{-1}\gamma_\perp\Gamma(|\Psi|/\hbar)^{2}. \qquad (22)$$

We multiply Eq. (22) on the right by the operator B and join it with equation for B, that is its Hermitian conjugate and multiply on the left by B^+. We then obtain an equation for the operator of the number of magnons $\hat{N}=B^+B$:

$$\frac{1}{2}\frac{d\hat{N}}{dt}-G\hat{N}+\beta\hat{N}^{2}=0. \qquad (23)$$

Assuming that at the instant of time $t=0$, thanks to noise fluctuations, there is a number of magnons equal to $\hat{N}(t=0)=\hat{N}_0/2\beta$, the solution of Eq. (23) can be written in the following form:

$$\hat{N}=\frac{\hat{N}_0}{2\beta}\left[e^{-2Gt}\left(1-\frac{\hat{N}_0}{2G}\right)+\frac{\hat{N}_0}{2G}\right]^{-1}. \qquad (24)$$

Below threshold, when the decrement of the spin waves exceeds their increment, i.e., $G<0$ and $\sigma_0<\sigma_c$, the solution at times $t\gtrsim 1/G$ has the form

$$\hat{N}=\frac{\hat{N}_0}{2\beta}\left[\left(1-\frac{N_0}{2G}\right)\exp(-|G|t)\right]. \qquad (25)$$

Above threshold for generation of spin waves, when $G>0$, i.e., $\sigma_0>\sigma_c$, the solution (24) takes the form[2)]

$$\hat{N}=\frac{G}{\beta}\left[1+\frac{1}{2}e^{-2Gt}\left(1-\frac{2G}{\hat{N}_0}\right)\right], \qquad (26)$$

i.e., the operator of the number of magnons tends to a finite stationary value.

3. STATIONARY REGIME OF MAGNON GENERATION

We now obtain the condition for threshold generation of spin waves. We shall assume that at the instant of time $t=0$ there is a certain number of magnons N_0 in the crystal. For the generation of spin waves by inverted paramagnetic ions, it is necessary that the power absorbed by the SW system as a result of transitions of impurity ions from an excited level to the ground state exceed the power scattered by the spin waves in the lattice, i.e., that the total gain (19) satisfy the condition $G>0$. Then we obtain the threshold condition on the minimal concentration of paramagnetic ions in the crystal necessary for obtaining the maser effect

$$c_{\rm cr}=\frac{\gamma_\perp\Gamma}{\sigma_0(|\Psi|/\hbar)^{2}}. \qquad (27)$$

As estimates, we have taken the following values from Refs. 7 and 13: $\Gamma=10^6\ {\rm s}^{-1}$, $\gamma_\perp\approx 10^9\ {\rm s}^{-1}$, $|\Psi|\sim 10^{-15}$ erg and, assuming that the excited level is completely populated, i.e., $\sigma_0\approx 1$–0.1, we obtain $c_{\rm cr}\approx 10^{-8}$ for the critical concentration. It must be noted that, with account of inhomogeneous broadening, i.e., the frequency spread of the impurities $\delta\Omega$, the value of the critical concentration turns out to be somewhat higher. Actually, if the frequency spread of Ω is much greater than the relaxation rate of the impurities $\gamma_\perp$, i.e., all the impurities can be classified in separate groups, within the limits of each of which the frequency Ω differs by less than $\gamma_\perp$; in this case, the expression for the critical concentration takes the form

$$c_{\rm cr}=\frac{\delta\Omega}{\sigma_0}\frac{\Gamma}{(|\Psi|/\hbar)^{2}}. \qquad (28)$$

If the frequency spread amounts to $\delta\Omega\sim 10^{10}\ {\rm s}^{-1}$, then $c_{\rm cr}\sim 10^{-7}$.

However, it should be noted that, at high concentrations of paramagnetic impurities, the single impurity approximation becomes invalid and the effect of impurity-impurity interaction through virtual magnons leads to a collective rearrangement of the spectrum of spin waves and impurities; as calculated in Ref. 8, this concentration of impurities for $CoCO_3$ is equal to $c^*\sim 10^{-3}$. Thus, the concentration of impurities necessary for obtaining the maser effect of spin waves should lie in the following range:

$$10^{-7}\leqslant c<10^{-3}. \qquad (29)$$

In crystals of ferro- and antiferromagnetics used in experiments on parallel pumping of SW, the lines of UHF absorption of the impurities were observed with concentrations of the order of hundredths of a percent,[3] which satisfies the condition (29). Consequently, in the creation of a sufficiently high inverse difference of populations of the impurity levels, one can organize the generation of spin waves with a frequency equal to the transition frequency ω between the lower levels of the impurity ion.

Above the transition threshold, when the impurity ion, as a result of emission of a magnon, passes from the excited state to a lower one and begins to absorb the energy of the SW field, the inverse difference of populations of the levels 0 begins to decrease (the "saturation" effect). Using Eq. (13) for the case in which N magnons with frequency $\omega = \Omega$ are excited in the crystal, we can find the steady-state inverse difference of populations σ. It decreases in comparison with the initial σ_0 and turns out to be equal to

$$\sigma=\sigma_0\left(1-\frac{N}{N_c}\right), \tag{30}$$

where $N_c = \alpha/\beta$.

The steady-state value of the number $\overline{N}$, of excited magnons is determined from the stationary equation (23)

$$\overline{N}=\frac{\Gamma}{\beta}\left(\frac{\sigma_0}{\sigma_c}-1\right), \tag{31}$$

and the critical value of the initial inverse difference of populations σ_c, above which the generation of spin waves begins, is given by Eq. (22).

It must be remarked that the spin waves thus created will propagate coherently, in contrast to the spin waves generated in parallel pumping.

The minimum pumping rate P at which the critical inverse of the difference of populations of the impurity levels is achieved is determined from the stationary equation (4):

$$P_c=\sigma_c\gamma_{\parallel}/2. \tag{32}$$

The rate of pumping of the crystal is connected with the power W fed to the apparatus responsible for the pumping in the following fashion:[16]

$$P=\eta W/Vm\hbar\Omega, \tag{33}$$

where V is the volume of the crystal, m is the total number of paramagnetic impurities in the sample, η is a parameter describing the pumping efficiency.

In the steady-state regime, the power transferred by the impurity ions to the SW system, per unit volume, is equal to

$$W_{\text{rad}}=\hbar\Omega\overline{N}c(|\Psi|\hbar)^2\sigma_0/\gamma_{\perp}. \tag{34}$$

setting $c = 10^{-5}$, $\gamma_{\parallel} \sim 10^9\ \text{s}^{-1}$,[7] $\Omega \sim 10^{11}\ \text{s}^{-1}$, at the values of the other parameters given above, we get the following value for the numerical estimate of the critical inverse difference in populations: $\sigma_c \approx 10^{-4}$. The minimum pumping rate P_c necessary for attaining the critical difference in populations will amount of $P_c \approx 10^5$–$10^6\ \text{s}^{-1}$; correspondingly, the threshold power of pumping per unit volume of the crystal will be equal to $W_{\text{thres}} \approx 10^8$ erg cm^2/s, and the expected power, that has been radiated by the impurity ions into the system in the steady-state stationary regime, will be $W_{\text{rad}} \approx 10^7$–$10^6$ erg cm^3/s. The steady-state stationary value of the number of excited magnons in a unit volume will be equal to $\overline{N} \approx 10^{17}\ \text{cm}^{-3}$, and the amplification coefficient, determined by the expression (21), will be $G \approx 10^7\ \text{s}^{-1}$.

4. DISCUSSION OF THE POSSIBILITY OF EXPERIMENTAL OBSERVATION OF THE MASER EFFECT FOR SPIN WAVES

For the experimental observation of the maser effect for spin waves, there is interest in the following impurity paramagnetic ions and magnetically ordered materials. First, we can suggest the ferromagnet $Y_3Fe_5O_{12}$ with the impurity ions Cr^{3+}, which replace the octahedral ions of iron. The Cr^{3+} ion has an electron configuration $3d^7$, which corresponds to the ground level of $^4Fe_{3/2}$ of the free ion. The ground state of the Cr^{3+} ion in the octahedral field is an orbital singlet.[5] In the case of distortion of the octahedron along both the tetragonal and the trigonal axes with account of spin-orbit interaction, the degeneracy is lifted and the level is split into two Kramers doublets. Under the action of the external magnetic field and exchange interaction of ions of chromium with iron ions, a final splitting of the levels takes place. Thus, the lower energy levels of Cr^{3+} in $Y_3Fe_5O_{12}$ will be four Zeeman levels with distance between levels lying in the microwave range.

We can also suggest that antiferromagnetic rhombohedral crystal hematite α-Fe_2O_3 in its easy-plane phase at $T_M < T < T_N$, where $T_M = 262$ K is the Morin temperature, $T_N = 960$ K is the Néel temperature with the impurity ions Co^{2+}. The electronic configuration of the Co^{2+} ion, similar to the Cr^{3+} ion, in a crystal field of rhombohedral symmetry with account of the external magnetic field and exchange interaction of the Co^{2+} ions with the ions of iron will be four Zeeman levels. The transitions between the lower levels has been observed in hematite with Co^{2+} impurities of composition[18] α-$Fe_{2-x-y}Co_xSi_yO_3$ (Si is introduced for compensation of the electric charge). For the existence of the maser effect for spin waves, the concentration of Co^{2+} ions should be $c \leqslant 0.013$; above this concentration, coherent rearrangement of the spectrum begins. As established in Ref. 18, the distance between the lower levels of the Co^{2+} ion in hematite lies in the microwave range and has a strong dependence on the magnetic field. According to the calculations of the authors of Ref. 18, the value of the parameter of exchange interaction Λ for α-Fe_2O_3:Co^{2+} is equal to $\Lambda \approx 6.4\ \text{cm}^{-1}$.

There is also interest in the easy-plane antiferromagnetic crystals $MnCO_3$ and $CoCO_3$ with the impurity ions Fe^{3+} and[3] Ni^{2+} or the weakly anisotropic antiferromagnetics $KMnF_3$ and $RbMnF_3$ with Ni^{2+} impurity ions.[19] The $KMnF_3$ and $RbMnF_3$ crystals have cubic structures. At $T < 81$ K, the $KMnF_3$ crystal undergoes tetragonal distortion.

The crystals $MnCO_3$ and $CoCO_3$ have rhombohedral structure. For the Fe_2^+ ion in a trigonal rhombohedral field of the ligands CO_3^{2-}, the ground state is orbitally degenerate, while when account is taken of the spin-orbit interaction the ground state remains doubly degenerate. As is shown in Ref. 8, the exchange interaction of the spin of the impurity ion with the spins of the atoms of the matrix $CoCO_3$ and $MnCO_3$ remove this degeneracy and the value of the splitting of the lower level of the Fe^2 turns out to be equal to 2–3 cm^{-1}. This is comparable with the SW frequency corresponding to the low-frequency branch of the spectrum. The distance Ω between the lower levels can be regulated by changing the applied external magnetic field, which would allow us to control the frequency of the coherent magnons generated.

Resonance transitions in the impurity ions Fe^{2+} in the $MnCO_3$ crystals, which are accompanied by the absorption of a single spin wave, are obviously already observed in experiments on parallel pumping of spin waves.[3] In Ref. 17, the intersection of the lines of antiferromagnetic resonance with the frequency of the impurity mode has been observed in $CoCO_3$ crystals at $\Omega \approx 1.5$ cm^{-1} (Fe^{2+} ions evidently serve as the impurity).

The initial state of the Ni^{2+} ion, introduced in crystals of $KMnF_3$ and $RbMnF_3$, represents a spin triplet and an orbital singlet. The effective exchange field of interaction of the Ni^{2+} ion with the neighboring magnetic atoms of the basic lattice splits the spin triplet into three magnetic sublevels;[19] the distance between the levels in the range of the SW frequencies.

However, it should be noted that, as is seen from experiments on the parametric excitation of spin waves in antiferromagnets, the resonance lines on the impurity ions are inhomogeneously broadened, thanks to the local fluctuations of the crystalline and exchange fields at the impurity ions as a consequence of the inhomogeneities in the crystal. This effect worsens somewhat the conditions for the observation of the laser effect for spin waves. Therefore, it is desirable to prepare more nearly perfect crystals with as homogeneous as possible distribution of the paramagnetic impurities.

It should be noted again that the method of stimulated generation makes it possible to obtain coherent spin waves.

[1)] Using a special cavity, for example, made of two films of permalloy placed on two plane-parallel faces of a crystal, we can obviously separate one mode of spin waves with wave vector k_0 parallel to the normal to these faces.

[2)] Above generation threshold, when the excited spin waves are sufficiently intense, the classical and quantum descriptions coincide and the operator of the number of magnons N is simply the number of excited spin waves.

[1] Ya. A. Monosov, *Nonlinear Ferromagnetic Resonance* (in Russian) Nauka, Moscow, 1971, p. 65.

[2] V. E. Zakharov, V. S. L'vov and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974); [Sov. Phys. Usp. **17**, 896 (1975)].

[3] V. V. Kveder, B. Y, Kotyuzhanskiĭ and L. A. Prozorova, Zh. Eksp. Teor. Fiz. **62**, 2199 (1972); [Sov. Phys JETP **35**, 1050 (1972)].

[4] V. I. Ozhogin and A. Yu. Yakubovskiĭ, Zh. Eksp. Teor. Fiz. **67**, 287 (1974); [Sov. Phys. JETP **40**, 144 (1975)].

[5] A. I. Smirnov, Zh. Eksp. Teor. Fiz. **73**, 2254 (1977); [Sov. Phys. JETP **46**, 1180 (1977)].

[6] S. A. Al'tshuler and B. M. Kozyrev, *EPR Compounds of Elements of the Intermediate Groups* (in Russian) Nauka, Moscow, 1972, Ch. 3.

[7] A. P. Gurevich, *Magnetic Resonance in Ferrites and Antiferromagnets* (in Russian), Nauka, Moscow, 1972, Ch. 2.

[8] V. M. Doktev, Dissertation for doctorate in physical-mathematical sciences, Institute of theoretical physics, Academy of Sciences, Ukranian SR, Kiev, 1983.

[9] A. S. Borovik-Romanov and V. F. Mescheryakov, Pis'ma Zh. Eksp. Teor. Fiz. **8**, 425 (1968) [JETP Lett. **8**, 262 (1968)].

[10] A. S. Prohorov and E. G. Rudashevskii, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 214 (1975) [JETP Lett. **22**, 99 (1975)].

[11] J. F. Ready and D. Chen, Proc. IRE, **50**, 329 (1962).

[12] A. E. Siegmann, *Microwave Solid State Masers* McGraw, 1964.

[13] A. S. Mikhaĭlov and R. M. Farzetdinova, Zh. Eksp. Teor. Fiz. **80**, 1524 (1981); Sov. Phys. JETP **54**, 782 (1981).

[14] H. P. Haken and V. Weidlich, in *Quantum Optics and Electronics, Quantum Fluctuations of Radiation*, J. Glauber ed., Gordon and Breach, 1965.

[15] M. Lax, *Fluctuations and Coherent in Classical and Quantum Physics Phenemona*, Gordon and Breach, N.Y., 1968.

[16] V. Svelto, *Principles of Lasers*, Plenum, 1976, Ch. 3.

[17] B. S. Dumesh, V. N. Egorov and V. D. Meshcheryakov, Zh. Eksp. Teor. Phys. **61**, 320 (1971); [Sov. Phys. JETP **34**, 168 (1972)].

[18] V. N. Vasil'ev, E. N. Matveiko, *Physical Properties of Magneto-dielectrics* [in Russian], Krasnoyarsk, 1987, p. 46.

[19] V. V. Eremenko, *Introduction to Optical Spectroscopy* [in Russian], Nauk, Dumka, Kiev, 1975.

Translated by R. Beyer