"Bags" and "strings" in macroscopic electrodynamics

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The ability of ordered electrodynamic media of the ferroelectric type to localize lines of force of the electromagnetic field of external charges in the form of compact configurations of the "string" and "bag" type is discussed. This leads to the appearance of forces (of confinement or anticonfinement) that are independent of distance and couple two unlike charges inside a medium of this type, and also to opacity of the medium to fast charged particles.

1. INTRODUCTION

Material media in a low-symmetry state with a nonzero order parameter (OP) have a number of distinct properties that have been widely illuminated in the literature. Less well known is the ability of such ordered media (OM) to localize the lines of force of a field of external sources in the form of three-dimensional compact configurations ("bags") or quasi-one-dimensional compact configurations ("strings").

For a long time, this property of OM has been discussed only in connection with vortex filaments in superconductors and superfluids. Interest in it has grown sharply as a result of progress in the theory of the strong interaction, which has pushed color-field and quark-field configurations of the string and bag type to the forefront. The phenomenon of confinement, i.e., the appearance of forces of attraction that do not fall off with distance and lead to the confinement of color, is attributed to such configurations in a particular example of an ordered medium—the physical vacuum.

Localization of the lines of force of an electromagnetic field is a consequence of the "superdielectric" ($\varepsilon = 0$) and "superdiamagnetic" ($\mu = 0$) properties of the OM, where ε and μ are the static long-wavelength values of the permittivity and magnetic permeability of the OM, connecting the field inductions and intensities by the relations

$$\mathbf{D}=\varepsilon\mathbf{E}, \quad \mathbf{B}=\mu\mathbf{H}.$$

From the condition that the free energy $D^2/2\varepsilon$ or $B^2/2\mu$ be a minimum it follows that the induction lines of force are expelled from such an OM, and it is this, in combination with the condition that the flux of the induction be constant, that leads to localization of the induction lines of force.

The Meissner effect and the appearance of vortex filaments in superconductors are explained in precisely this way. It is necessary to emphasize that a quantity of a nonelectromagnetic nature—the wave function ψ of the condensate of Cooper pairs—serves as the OP of the superconductor. The superdiamagnetism of a superconductor (a medium that is not ordered in the electromagnetic sense) is a consequence of the anomalously strong spatial dispersion of the magnetic permeability

$$\mu=(1+\varkappa^2/k^2)^{-1}$$

where $\mathbf{k}$ is the wave vector and $\varkappa \propto |\psi|$ is the inverse penetration depth. In the image of the superconductor a number of models of confinement in quantum chromodynamics (QCD) have been constructed, in which either condensates of Higgs particles (or of magnetic monopoles) or auxiliary scalar fields ensuring superdiamagnetism have been introduced (see Ref. 1). Essentially the same mechanism gives rise to the vortex filaments in a rotating superfluid, in which localization of lines of the angular velocity $\mathbf{\Omega} = \frac{1}{2}\mathrm{curl}\,\mathbf{v}$ occurs ($\mathbf{v}$ is the velocity of the liquid). This follows from the correspondence

$$\mathbf{B}\to\mathbf{\Omega},\ \mathbf{H}\to\mathbf{m},\ \mu\to 1/J$$

between magnetic and rotational quantities, where $\mathbf{m} = J\mathbf{\Omega}$ is the angular momentum of the liquid and J is its moment of inertia. Superdiamagnetism corresponds to the condition $1/J = 0$, which means simply that the liquid, with a given angular momentum, cannot be dragged by the rotation of the vessel, i.e., that the angular-velocity lines are expelled from the volume of the liquid (the analog of the Meissner effect).

Another mechanism that leads to superdielectricity or superdiamagnetism and localization was pointed out in general form by one of the authors[2] and is the basis of confinement models of another type (see the review by Adler and Piran[3]). We have in mind ordered media with an electromagnetic order parameter $\mathbf{E}(\mathbf{H})$ that arises spontaneously in the absence of an induction $\mathbf{D}(\mathbf{B})$; this corresponds to zero permittivity ε (permeability μ). It is important to stress, however, that the effectiveness of the mechanism in QCD cases doubt on its applicability in electrodynamics. This is connected with the fact that, on the level of interest to us, the electromagnetic and color fields have opposite properties: The former are characterized by screening ("zero charge"), and the latter are characterized by antiscreening (asymptotic freedom).[4] This is manifested in the fact that the known stable electrodynamic OM possess a spatially uniform order parameter $\mathbf{D}$ (a ferroelectric) or $\mathbf{B}$ (a ferromagnet), but not $\mathbf{E}$ or $\mathbf{H}$.

The general analysis carried out below shows that stable electrodynamic OM with a uniform order parameter $\mathbf{E}$ or $\mathbf{H}$ do not, indeed, exist.[1)] When created artificially they become inevitably rearranged, going over into stable ferroelectric or ferromagnetic states. However, there exist conditions under which such a transition turns out to be difficult, and the states of interest to us are rather long-lived. It is then that compact field configurations and forces that are independent of distance arise in a macroscopic medium.

More surprising is the fact that, as noted the authors,[7] similar phenomena can also arise in an equilibrium OM—in a ferroelectric or ferromagnet, in which the permittivity or permeability is not only not zero but, on the contrary, is large or even infinite. The corresponding mechanism of localiza-

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tion of the field and of the appearance of forces that are independent of distance differs substantially from that considered above.

The questions touched upon in this section constitute the content of the present article. For simplicity, we confine ourselves to only homogeneous, nongyrotropic media with a uniform (in the absence of external sources) order parameter of the electric type. The corresponding generalization does not give rise to difficulties and leads to qualitatively similar conclusions. In the article we use Heaviside units and the velocity of light is taken equal to unity.

2. GENERAL RELATIONS

Static fields in the medium are described by the Maxwell equations

$$\text{а) } \operatorname{rot}\mathbf{H}=\mathbf{j}, \quad \text{б) } \operatorname{div}\mathbf{D}=\rho, \qquad \text{в) } \operatorname{rot}\mathbf{E}=0, \quad \text{г) } \operatorname{div}\mathbf{B}=0, \tag{2.1}$$

where ρ and $\mathbf{j}$ are the densities of the external charges and currents, and by the constitutive equations relating $\mathbf{D}$ (or the polarization $\mathbf{P}=\mathbf{D}-\mathbf{E}$ of the medium) to $\mathbf{E}$, and $\mathbf{H}$ (or the magnetization $\mathbf{M}=\mathbf{B}-\mathbf{H}$ of the medium) to $\mathbf{B}$. In linear media these relations have the form of a proportional dependence between the corresponding Fourier components, from which, according to (2.1 c,d),

$$\mathbf{P}_t=0, \quad \mathbf{M}_l=0,$$

in which the subscripts l and t, here and below, correspond to the longitudinal (potential) and transverse (solenoidal) components of the vector. For a geometry that eliminates depolarization nd demagnetization effects, $\mathbf{D}$ and $\mathbf{H}$ coincide with the quantities $\mathbf{E}_0$ and $\mathbf{B}_0$—the fields of the same external sources in vacuo, as follows from the equalities

$$\mathbf{D}=\mathbf{E}_0+\mathbf{P}_t, \quad \mathbf{H}=\mathbf{B}_0-\mathbf{M}_l. \tag{2.2}$$

In nonlinear media the quantities $\mathbf{P}_t$ and $\mathbf{M}_l$ are nonzero, and the corresponding terms (2.2) can radically change the configuration of the $\mathbf{D}$ and $\mathbf{H}$ lines of force, making it, in particular, compact. These terms have opposite (in the sense of longitudinality and transversality) properties in respect of the first terms in the expressions (2.2). This makes it possible to simulate effects due to the nonlinearity of the medium by introducing into the right-hand side of Eq. (2.1d) a source of longitudinal magnetic field—the magnetic-monopole charge density $\tilde{\rho}$, and by introducing into the right-hand side of Eq. (2.1c) (with the opposite sign) a source of transverse electric field—the monopole-current density $\tilde{\mathbf{j}}$:

$$\tilde{\rho}=-\operatorname{div}\mathbf{M}, \quad \tilde{\mathbf{j}}=-\operatorname{rot}\mathbf{P}.$$

The above pertains, in particular, to nonlinear media undergoing spontaneous ordering with the appearance of an electromagnetic OP (polarization or magnetization) that can be simulated by a monopole solenoid or a monopole capacitor, respectively. Below, we shall convince ourselves that such a simulation is useful.

Going over to the analysis of the energetics of an OM with a uniform electric order parameter $\mathbf{P}$, we start with forced ordering that is induced by an external (but independent of the state of the medium) action and vanishes together with the latter. There are two types of such actions, for which the trigger is provided by either the induction $\mathbf{D}$ or by the intensity $\mathbf{E}$. It is also possible to realize these actions in two ways. The first consists in placing a sample of the medium inside a capacitor whose plates are charged to a given charge density or, respectively, are shorted across a battery with a given potential difference (see Refs. 5 and 8). The second way corresponds to choosing the sample of the medium in the form of a thin but macroscopic plate, oriented arbitrarily in an external field $\mathbf{E}_0$. The boundary conditions

$$E_{0n}=D_n, \quad \mathbf{E}_{0\tau}=\mathbf{E}_\tau \tag{2.3}$$

(the subscripts n and τ denote the directions perpendicular to and along the plate) show that a plate oriented perpendicular to the lines of force of the external field is acted upon by the induction $\mathbf{D}$, while one oriented parallel to the lines of force is acted upon by the intensity $\mathbf{E}$. The second way is preferable in the discussion of energetic questions, in that it makes allowance for such factors as the work done by the battery unnecessary.

The change of the free energy of the medium (per unit volume) under the influence of an external field $\mathbf{E}_0$ is determined by the expression for the energy of a dipole in an external field and has the form $\delta F=-\mathbf{P}\delta\mathbf{E}_0$ (unlike the standard expression $\mathbf{E}\delta\mathbf{D}$, here the energy $\mathbf{E}_0\delta\mathbf{E}_0$ of the field in vacua is absent). The conditions (2.3) lead to the expression

$$\delta F=-P_n\delta D_n-\mathbf{P}_\tau\delta\mathbf{E}_\tau, \tag{2.4}$$

which corresponds directly to what was said above about the fields acting on a plate.

Spontaneous ordering, which is what we shall be discussing below, arises in a medium free from external influences. It can also have a dual character, depending on the conditions in which the ordering occurs: A spontaneously arising order parameter $\mathbf{P}$ can be manifested either as an induction $\mathbf{D}$ or as a field intensity $\mathbf{E}$. Thus, for ordering inside a capacitor with uncharged or completely absent plates ($\mathbf{D}=0$) the quantity $\mathbf{E}$ serves as the order parameter, while if the plates are shorted or grounded ($\mathbf{E}=0$) the order parameter coincides with $\mathbf{D}$. But in the case of a plate with $\mathbf{E}_0=0$, when, according to (2.3) $D_n=0$ and $\mathbf{E}_\tau=0$, two types of ordering are possible: with order parameter $\mathbf{D}_\tau$, parallel to the plate, and with order parameter E_n, perpendicular to the plate. For brevity, states of the OM with order parameters $\mathbf{D}$ and $\mathbf{E}$ will be called hereafter *D*- and *E*-phases.

Spontaneous ordering is a consequence of instability of the medium—the growth of small polarization fluctuations. The corresponding information is contained in the effective potential (Landau functional[8]) $V(\mathbf{P})$—a quantity that is defined for all (including nonequilibrium) values of the OP and has a minimum at the point of equilibrium, where it coincides with the free energy F. The latter leads to the effective potential upon a Legendre transformation from the argument $\mathbf{E}_0$ to the OP itself. According to (2.4),

$$\delta V(\mathbf{P})=D_n\delta P_n+\mathbf{E}_\tau\delta\mathbf{P}_\tau. \tag{2.5}$$

The effective potential has the form of the sum of the quantity $V_0(\mathbf{P})$, specified by its power expansion in the spirit of Landau theory, and the quantity $V_1=p_n^2/2$, which is the energy of the dipoles in their own (depolarizing) field;

also,

$$\delta V_0 = \mathbf{E}\delta\mathbf{P} \tag{2.6}$$

This relation determines (in implicit form) the constitutive equation of the medium and the permittivity tensor

$$\varepsilon_{ik} = \delta D_i/\delta E_k = \delta_{ik} + \theta_{ik}^{-1}, \qquad \theta_{ik} = \delta^2 V_0/\delta P_i \delta P_k. \tag{2.7}$$

The conditions for stability of the medium require first of all that the derivatives of V with respect to P_n and $\mathbf{P}_\tau$ be equal to zero:

$$P_n + \delta V_0/\delta P_n = 0, \quad \delta V_0/\delta \mathbf{P}_\tau = 0, \tag{2.8}$$

and, in addition, that the tensor $\delta^2 V_0/\delta P_i \delta P_k$ be positive-definite:

$$1/\varepsilon_n \leqslant 1, \qquad \varepsilon_\tau \geqslant 1 \tag{2.9}$$

[see (2.5) and (2.7)], where ε_n and ε_τ ($\tau = \tau_1, \tau_2$) are the principal values of the tensor ε_{ik}, one of the principal axes of which is assumed to be perpendicular to the plate.

3. AN ISOTROPIC AND UNIAXIAL MEDIUM

In applying the general relations of the preceding section, we shall start from the case of an isotropic medium, for which the potential V_0 depends on $P_n^2 + P_\tau^2$ (a derivative with respect to this argument is indicated by a prime). The equations (2.8), which take the form

$$P_n(1+2V_0') = 0, \quad \mathbf{P}_\tau V_0' = 0,$$

have, for a medium capable of spontaneous ordering, two nontrivial solutions:

$$P_n = 0, \qquad V_0'(P_\tau^2) = 0$$

(the D-phase), and

$$\mathbf{P}_\tau = 0, \qquad V_0'(P_n^2) = -1/2$$

(the E-phase), The principal values of the tensor ε_{ik} [see (2.7)] have the form

$$\varepsilon_{n,\tau} = 1 + (2V_0' + 4P_{n,\tau}^2 V_0'')^{-1},$$

whence, in the D-phase (the τ_1 axis is the direction of spontaneous polarization),

$$\varepsilon_n = \varepsilon_{\tau_2} \to \infty, \qquad \varepsilon_{\tau_1} = 1 + (4P_\tau^2 V_0'')^{-1}, \tag{3.1}$$

and, in the E-phase,

$$1/\varepsilon_n = 1 - (4P_n^2 V_0'')^{-1}, \qquad \varepsilon_\tau = 0. \tag{3.2}$$

For $V_0'' > 0$ it follows from (3.1) and (3.2) that the D-phase is stable (both the criteria (2.9) are fulfilled) and the E-phase is unstable (the second of these criteria is violated). This instability is manifested in a transition to the D-phase state by a rotation of the vector $\mathbf{P}$ parallel to the plate, and is associated with the inequality $V_1 > 0$ (Sec. 2), i.e., with the extra work (against the depolarizing field) that is required for creation of the E-phase. The local instability of the E-phase (the corresponding extremal point of the effective potential is of the saddle-point type) means that this phase has a short lifetime.

In a uniaxial medium with its easy-polarization axis perpendicular to the plate, because rotation of the vector $\mathbf{P}$ is not favored it would appear that stabilization of the E-phase is possible. The effective potential of such a medium has a different dependence on P_n^2 and P_τ^2 and can be taken, for simplicity, in the form (the dots denote higher-order and gradient terms)

$$V_0 = -\alpha P_n^2/2 - \beta P_\tau^2/2 + \gamma(P_n^2 + P_\tau^2)^2/4 + \ldots, \tag{3.3}$$

the inequality $\alpha > \beta$ means that the axis of easy polarization is perpendicular to the plate, and the opposite inequality means this axis lies parallel to the plate. In fact, a trivial generalization of the relations given above for an isotropic medium shows that for sufficient anisotropy, namely, for

$$\alpha > \beta + 1, \tag{3.4}$$

both the stability criteria (2.9) are fulfilled in relation to the E-phase.

This stability, however, has only a local character, and there exists a more stable (corresponding to a deeper minimum of the effective potential) state of the medium, corresponding to a multi-domain structure with domains in the form of thin plates oriented perpendicular to the original plate (one of these domains is shown shaded in Fig. 1). Inside a domain the medium is in the D-phase state (the easy axis, along which the vector $\mathbf{P}$ is oriented, lies in the plane of the plate-domain), and there is no depolarizing field. That such a situation is favored also follows from the violation, in the E-phase, of the corresponding stability criterion [more stringent than (2.9)]

$$\varepsilon_n \geqslant 1, \tag{3.5}$$

which can be obtained from (2.9) by a simple relabeling of indices: $n_1, \tau_1, \tau_2 \to \tau_1, \tau_2, n$. At the same time, when the condition (3.4) is fulfilled the E-phase state is metastable, being separated from the multi-domain state by an energy barrier, and so lives relatively long (this is the justification for analyzing it further[2)]).

When the condition (3.4) is violated the E-phase is locally unstable against transition to the D-phase. If at the same time the easy-axis case is realized ($\beta < \alpha < \beta + 1$), then, because the criterion (3.5) is violated, the multi-domain D-phase state of the type considered above will be the most stable. In the easy-plane case ($\beta > \alpha$), on the other hand, the single-domain D-phase state will be absolutely stable.

Up to now we have not taken into account the gradient terms in (3.3), which correspond to the energy associated with the nonuniformity of the polarization distribution. These terms determine the characteristics of the domain

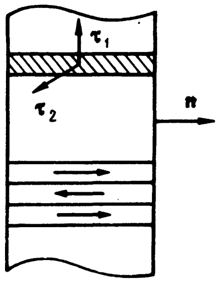

FIG. 1.

wall and the shape and size of the domains. The gradient terms raise the energy of the multi-domain phase in proportion to the surface-to-volume ratio of the domains, while not affecting the criteria for local stability of the medium.

We note that the above qualitative features of the process of electric ordering are possessed not only by a plate but also by a sample of arbitrary shape placed inside a capacitor with uncharged plates (giving the E-phase) or shorted plates (giving the D-phase) (see Sec. 2). Thus, the E-phase instability, associated with the additional work needed to create the electric field outside the medium, will be manifested in a transition to the D-phase with the appearance of a nonuniform (in particular, multi-domain) structure. As regards the ordering inside a capacitor with shorted plates, it leads to the appearance of a stable single-domain D-phase if the easy axis or easy plane is orthogonal to the plates.

To complete this section, we give the constitutive equations of the OM near extremal points of the effective potential (for small external fields):

$$D_a=P_a+\varepsilon_a E_a \quad (D\text{-phase}),$$
$$E_a=-P_a+D_a/\varepsilon_a \quad (E\text{-phase}),$$

where P is the spontaneous polarization and a is the index of the principal axes of the tensor ε_{ik}. In an isotropic medium, for the D- and E-phases, respectively we have [see (3.1) and (3.2)]

$$\mathbf{D}=P\mathbf{E}/E+\varepsilon_\tau\mathbf{E}, \quad \mathbf{E}=-P\mathbf{D}/D+\mathbf{D}/\varepsilon_n. \tag{3.6}$$

In a uniaxial medium with an easy axis ($a=1$), in the D-phase we have

$$D_1=PE_1/|E_1|+\varepsilon_1 E_1, \qquad D_{2,3}=\varepsilon_{2,3}E_{2,3} \tag{3.7}$$

and in the E-phase we have

$$E_1=-PD_1/|D_1|+D_1/\varepsilon_1, \quad E_{2,3}=D_{2,3}/\varepsilon_{2,3} \tag{3.8}$$

with analogous expressions in a medium with an easy plane.

4. CHARGES INSIDE AN ORDERED MEDIUM (THE E-PHASE)

In this section and Sec. 5 we consider the configuration of the lines of force and the law of interaction of test (heavy) charges placed in a medium with an electromagnetic OP. Specifically, we shall be concerned with a dipole with charges $\pm Q$ and length r, in the direction of the unit vector $\mathbf{v}$:

$$\rho(\mathbf{x})=Q[\delta(\mathbf{x})-\delta(\mathbf{x}-r\mathbf{v})]. \tag{4.1}$$

We begin by discussing the OM in the E-phase state. The case of an isotropic medium is not realistic because of the instability noted above. However, a medium with a constitutive equation

$$\mathbf{E}=P\mathbf{D}/D+\mathbf{D}/\varepsilon_n, \tag{4.2}$$

close to the second relation (3.6) is an electrodynamic model of the ordered QCD vacuum; the changed sign in (4.2) reflects the property of asymptotic freedom (see Sec. 1). This model has been considered by one of the authors,[2] by Adler and Piran,[3] by Lehmann and Wu,[9] and by the other author.[3)] Its properties are as follows.

In the simplest case $\varepsilon_n\to\infty$ the exact solution of Eqs. (2.1b,c) and (4.1), (4.2) has the form

$$\mathbf{E}=\mathbf{v}PQ/|Q|, \quad \mathbf{D}=Q\mathbf{v}\int_0^r dt\,\delta(\mathbf{x}-t\mathbf{v}). \tag{4.3}$$

It can be seen that the induction lines of force are localized in the form of an infinitely thin string connecting the charges. The law of interaction of the charges is given by the r-dependent part of the free energy $\int dx \int \mathbf{E}d\,\mathbf{D}$ of the medium:

$$U(r)=P|Q|r. \tag{4.4}$$

The onset of confinement can be explained by the proportionality of the energy of a uniform string to its length, by the one-dimensional Coulomb law, or simply by the action of constant forces $Q\,\mathbf{E}=\pm|Q|P\mathbf{v}$ pulling the charges together (the direction of the field $\mathbf{E}$ is fixed by the vector $\mathbf{v}$).

For finite ε_n the induction lines of force are localized in the form of a bag containing the charges inside it, and are stretched out into a string of finite thickness upon increase of the distance between the charges. In fact, the length of a force line of $\mathbf{D}$ has an upper limit. This can be seen by writing (2.1c) in the form

$$\int_{C_1} d\mathbf{l}\,\mathbf{E}=\int_{C_2} d\mathbf{l}\,\mathbf{E},$$

where the integrals are taken along lines of force, of which one makes a "straight" connection between the charges while the other has an arbitrary length L. When (4.2) is taken into account, this equality acquires a term PL that grows without limit with L and violates the equality. Outside the volume of the bag, $D=0$ and $E=P$. The surface of the bag is a singular element of the system of equations (2.1b,c), (4.1), (4.2): Going over to the potential $\mathbf{E}=-\nabla\varphi$ leads to the coefficients a_{ik} in the leading derivatives $a_{ik}\nabla_i\nabla_k$, and the determinant $\det a=1-P^2/(\nabla\varphi)^2$ of these coefficients vanishes exactly on the boundary of the bag. The singular character of the boundary is clear from the pattern (drawn in Fig. 2) of the $\mathbf{E}$ lines, which inside the bag coincide with the $\mathbf{D}$ lines and outside the bag form a family of straight lines [their curvature $|\mathrm{curl}(\mathbf{E}/E)|=0$, by virtue of (2.1c) and the fact that $E=P$], with the bag boundary as their envelope.

The pattern of the localization of the induction lines is especially lucid in the language of magnetic monopoles (Sec. 2). On the usual longitudinal induction field of the charges is

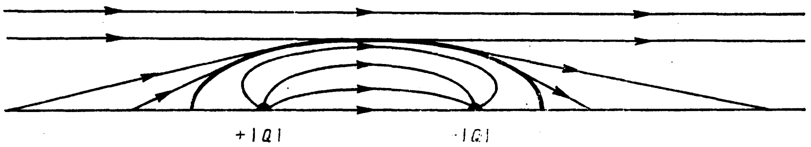

FIG. 2.

imposed the transverse field of a monopole solenoid; near the axis of the dipole these fields reinforce each other and far from the axis of the dipole they cancel each other [see (2.2)].

In the above system of equations for the fields of the dipole in the medium it is possible to change to the dimensionless variables $\mathbf{x}/r$, $\mathbf{E}/P$, and $\mathbf{D}r^2/|Q|$ and to the parameter $\lambda = P\varepsilon_{\parallel} r^2/|Q|$. This gives the following energy of interaction of the charges:

$$U(r)=P|Q|\chi(\lambda)r. \tag{4.5}$$

Here, $\chi \to -\lambda^{-1}$ as $\lambda \to 0$ (Coulomb's law $-Q^2/\varepsilon_{\parallel} r$ acts for $P=0$), and $\chi \to 1$ as $\lambda \to \infty$ [for $\varepsilon_{\parallel} \to \infty$ the formula (4.4) is valid]. Therefore, with increase of r, and, correspondingly, of λ, the interaction changes from the Coulomb law to a linear law corresponding to the transformation of the bag into a string with a transverse size determined by the parameter $(|Q|/P\varepsilon_{\parallel})^{1/2}$.

A similar picture arises in the case of the relatively stable E-phase of a uniaxial electromagnetic medium if the direction of the vector $\mathbf{v}$ in (4.1) is along the axis of easy polarization. The constitutive equation of such a medium is taken in the form (3.8), which also has meaning outside the range of its applicability (in large fields). Calculation of the free energy [see (4.4)] leads in this case as well to a linear law of interaction of the charges. However, because of the different signs, what arises in the right-hand sides of Eqs. (3.8) and (4.2) is anticonfinement—a distance-independent repulsion of unlike charges (at large distances between them).

The situation with the localization of the induction lines of force will be different from that described above. Complete localization in a uniaxial medium does not occur: The above analysis of the length of a line of force gives, when applied to (3.8), an upper bound not on this length itself but only on its projection onto the easy axis. Therefore, we should speak rather of a "pancake" of finite thickness in the direction of this axis.

Nevertheless, here too we can speak of a bag and a string (albeit with diffuse boundaries), since the induction falls off rapidly in the directions perpendicular to the pancake axis (more rapidly than any finite power of the distance). The point is that the character of this decrease is determined by the nonzero multipole moments of the system of charges and currents [according to (2.2), when referring to the induction, we should speak of the sum of the electric moments of the charges and the magnetic moments of the monopole currents simulating the nonlinearity of the medium]. The very fact of the localization along the pancake axis shows that these sums are equal to zero for all degrees of multipolarity, and it is this which implies the absence of a power decrease in a direction perpendicular to the axis.

In conclusion, we note that the formation of string-like configurations is also possible in a disordered nonuniform medium (e.g., inside a semiconducting cylinder with a permittivity greater than that of the surrounding medium[10]).

5. CHARGES INSIDE AN ORDERED MEDIUM (THE *D*-PHASE)

Localization of lines of force and confinement also arise in an equilibrium OM with an order parameter $\mathbf{D}$ (a ferroelectric). To be dealing with a single-domain structure, we place a sample of the medium between shorted or grounded plates of a capacitor, which become charged upon ordering of the medium ($\mathbf{E}=0$, $\mathbf{D}=$ const; see Sec. 2). It is assumed that the dipole (4.1) of the external charges is normal to the plates and opposite to the dipole of the plates themselves (Fig. 3).

We start from the case of an isotropic D-phase. Comparing its constitutive equation [the first equation (3.6)] with (4.2), we may expect localization of the lines of force not of the induction, as in Sec. 4, but of the intensity of the field. In fact, the force lines of $\mathbf{E}$ (generated by the external charges) do occupy a finite region of space: Gauss's theorem [see (2.1b)] and the inequality $D \geqslant P$, which follows from (3.6), place an upper bound equal to $|Q|/P$ on the area of the equipotential surface. For an isolated charge Eqs. (2.1b,c) and (3.6) give

$$\mathbf{D}=Q\mathbf{r}/4\pi r^3, \quad \mathbf{E}=(1-r^2/r_0^2)\mathbf{D}/\varepsilon_\tau, \tag{5.1}$$

and the field $\mathbf{E}$ does indeed vanish on a sphere of radius $r_0 = (|Q|/4\pi P)^{1/2}$. Thus, the charges inside a nonconducting (!) equilibrium OM experience complete screening in it.

Despite this, confinement arises in such a medium as well. To display its mechanism it is useful to begin from the exactly solvable planar problem of two charges in a two-dimensional OM. The analog of (5.1) in polar coordinates, viz.,

$$D_\rho=Q/2\pi\rho, \quad E_\rho=(1-\rho/\rho_0)D_\rho/\varepsilon_\tau, \quad D_\varphi=E_\varphi=0, \tag{5.2}$$

shows the $\mathbf{E}=0$ outside a circle of radius $\rho_0 = |Q|/2\pi P$ about each charge, where the induction is determined by the equations

$$\operatorname{div}\mathbf{D}=0, \quad D=P. \tag{5.3}$$

The general solution of (5.3) has the form [$F(x)$ is an arbitrary function]

$$D_\rho=P\cos(\Phi-\varphi), \quad D_\varphi=P\sin(\Phi-\varphi), \\ \Phi=F[\rho\cos(\Phi-\varphi)/\cos\Phi]. \tag{5.4}$$

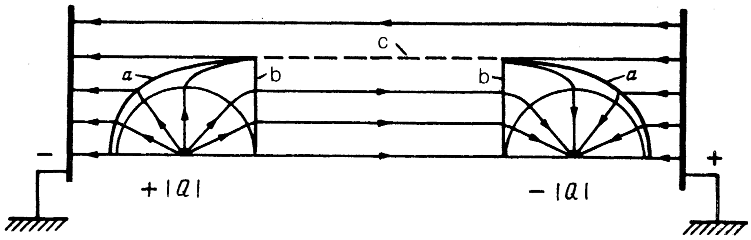

FIG. 3.

Not for any $F \neq 0$ does it go over into the solution

$$D_\rho = P\cos\varphi, \quad D_\varphi = -P\sin\varphi, \tag{5.5}$$

corresponding to the plane-parallel case and to the absence of external charges, with increase of the distance from the dipole in a transverse direction ($\rho \to \infty$, $\varphi = \pi/2$). Therefore, lines of discontinuity arise on which the external solution (5.5) matches (with satisfaction of the boundary condition that the normal components of the induction be continuous) with the internal solution in the region containing the dipole.

The internal solution also has a piecewise character. corresponding to the continuation of the solution (5.2) into the region with $\mathbf{E} = 0$ is the choice $F(x) = \arccos(\rho_0/x)$:

$$D_\rho = P\rho_0/\rho, \quad D_\varphi = \pm P(1-\rho_0^2/\rho^2)^{1/2} \tag{5.6}$$

(we are considering only one of the charges, which is taken to be at the coordinate origin). The boundary conditions for the matching of (5.6) with (5.5) and with the solution that differs in sign from (5.5) determine the lines of discontinuity a and b (Fig. 3). The line of discontinuity c separates opposite parallel induction fields. Its appearance, as the boundary of a 180-degree domain, is due to Gauss's theorem: The flux of the induction between the charges should be smaller (by an amount $|Q|$) than the flux around the plates. It is the lines c that lead to confinement. Owing to the gradient terms (Sec. 3), whose contribution to the energy has the form

$$\varkappa \int d\mathbf{x}\,(\mathrm{rot}\,\mathbf{D})^2,$$

the domain wall has a finite thickness $(\varkappa/\alpha)^{1/2}$ [see (3.3)] and a line tension $\sigma \sim (\varkappa\alpha)^{1/2}p^2$. Correspondingly, the energy of interaction of the charges is

$$U(r) = \sigma r, \quad r \gg \rho_0 \tag{5.7}$$

The generalization of the solution obtained to the real three-dimensional case corresponds to going over from the planar picture of Fig. 3 to the corresponding picture that is axially symmetric about the axis of the dipole.[7] However, in the three-dimensional case there is also another solution in which the indicated symmetry is broken, corresponding to induction lines of force in the form of helical lines wound onto the axis of the dipole in the gap between the charges (a rotating domain wall prolate because of the absence of anisotropy). In both solutions the quantity curl**D** in this gap is constant over the length of the gap, and the energy of the gradient terms is proportional to the distance between the charges, implying the appearance of confinement.

Breaking of the axial symmetry is absent (because deviation of a line of force from the direction of the easy axis is not favored) in a uniaxial OM with the easy axis normal to the plates, to the analysis of which we now turn. The constitutive equation of such a medium is given by Eq. (3.7), which leads to the inequality $|D_1| \geqslant P$. Therefore, Gauss's theorem for the equipotential surface places an upper bound $|Q|/P$ not on the actual area of this surface but only on its projection onto the plane of "hard" polarization. This leads to localization of the field **E** within a cylinder of radius r_0 with its axis along the axis of the dipole, implying, simultaneously, that all the multipole electric moments of the total (external and induced) charge density are equal to zero. From this it follows that the field **E** is localized (to within terms that fall off more rapidly than any finite power of the distance) in the direction of the axis of the dipole (see the end of Sec. 4).

If the distance between the charges is sufficiently large, then, in by far the greater part of the gap between them, E is small, $|D_1| \cong P$, and $D_{2,3} \cong 0$. Therefore, as in the case of an isotropic medium (see above), the necessary deficit in the flux of induction in this gap is a consequence of a reversal of the direction of some of the lines of force, i.e., of the appearance of an axially symmetric intermediate layer (in the limit, a domain wall) in which the direction of the induction is reversed. It is the surface tension of this layer, which arises as a consequence of the gradient terms, that leads to the confinement.[4)]

6. CONCLUSION

The account given above substantiates the statement made in the Introduction that the localization of lines of force with the formation of compact configurations of the string and bag type and the appearance of distance-independent forces (confinement or anticonfinement) can be regarded as a general property of ordered media with an electromagnetic order parameter. Depending on the conditions under which the ordering of the medium occurs, the basis of the mechanism of the phenomena enumerated is either the vanishing of the permittivity (magnetic permeability), leading to expulsion of the induction lines of force, or the appearance of a nonzero lower bound on the induction, impeding the unlimited divergence of its lines of force.

The clearest manifestation of this property of an equilibrium ordered medium (a ferroelectric) is its "opacity" to an isolated external charge—even one possessing appreciable energy. A particle with energy E cannot penetrate a distance greater than E/σ [σ is the string tension; see (5.7)] into an ordered medium from the side of the capacitor plate of the same charge. But if the particle is penetrating from the side of the capacitor plate of the opposite charge, it must surmount an energy threshold σL, where L is the macroscopic size of the sample of the medium. A quantity of the same order also characterizes the threshold for lateral penetration by a particle. Having penetrated the medium, the particle is pushed toward the plate of like charge with a force that is independent of the distance separating them. These effects are the result of the formation of a string-like configuration linking the charge with the plate of like charge (see Fig. 3, in which the right-hand plate must be moved into the gap between the charges).

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[1)] Dolgov, Losyakov, and one of the authors[5] have shown previously, that in an equilibrium disordered medium the instability threshold for uniform small fluctuations of th fields **E** and **H** (the growth of which could generate the corresponding order parameter) is not reached. It was not clear, however, whether discontinuous ordering with the appearance of a finite value of the OP is possible. Incidentally, even earlier, Linde and one of the authors[6] showed that the existence of an ordered medium in its lowest energy state and with a spontaneous uniform field **E** is impossible: An external charge or a pair created by this field itself would inevitably be accelerated by the field, taking energy away from the medium in contradiction with the original assumption.

[2)] Another way in which the E-phase can decompose is by relatively slow overflow of charges onto the plate from outside or on account of the conductivity of the medium.

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[4)] Because of the incomplete localization of the field in the direction of the easy axis, the distance-independent force of interaction between the charges is supplemented by a term that decreases exponentially with distance.

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