Diffraction of optical radiation by metallic bodies

V. S. Buldyrev, M. A. Lyalinov, V. N. Smirnov, G. A. Strokovskiĭ, and É. E. Fradkin

A. A. Zhdanov State University, Leningrad

(Submitted 18 April 1989, resubmitted 3 October 1989)
Zh. Eksp. Teor. Fiz. **97**, 733–744 (March 1990)

Theoretical and experimental investigations were made of the diffraction of optical radiation by metal bodies in the penumbra region. Asymptotic expressions for the wave field in the diffraction problems were obtained by generalization of the Leontovich boundary conditions which is valid when $\eta = 1/ka \ll 1$ and $\xi = kd \lesssim 1$ ($k = 2\pi/\lambda$ is the wave number, λ is the wavelength of the incident radiation, d is the thickness of the skin layer in the absorbing body, and a is its characteristic size). An experimental study of the scattered radiation in the bright and dark parts of the penumbra was made by illumination, with a Gaussian optical beam ($\lambda = 0.63\,\mu$m), of one "edge" or the whole cross section of cylindrical metal filaments with a diameter $2a = 15$–1500 μm. A comparison of the experimental and theoretical results showed that the agreement was good for the calculations based on the asymptotic expressions derived in the present paper.

The interest in the diffraction of optical radiation by metallic bodies has recently increased for the following three reasons:

1) the developments in laser technology, which make it possible to observe the diffraction of optical radiation by convex bodies using compact laboratory apparatus;

2) the feasibility of checking experimentally under laboratory conditions the predictions of the hf theory of diffraction;

3) the use of the optical radiation diffraction in monitoring various technological processes.

Classification of the possible cases of the scattering of the incident field by absorbing convex bodies characterized by a complex refractive index $N = N' + iN''$ can be made on the basis of two parameters: $\zeta = k_0 d$ and $\eta = 1/k_0 a$, where $k_0 = 2\pi/\lambda_0$ is the wave number, λ_0 is the wavelength of the incident radiation outside the reflecting body, $d = \lambda_0/2\pi N''$ is the thickness of the skin layer or the depth of penetration of the field inside the absorbing body, and a is the characteristic size of this body.

Up to now the theory of diffraction of electromagnetic waves by conducting bodies has been concerned mainly with the scattering of radiowaves by ideal or good conductors. In such cases we have $\zeta = 0$ or $\xi \ll 1$, $\eta \lesssim 1$ or $\eta \ll 1$. If the inequality $\zeta \ll 1$ is satisfied to a high degree of precision on the surface of a conducting body, the Leontovich boundary conditions are obeyed.[1]

In the optical range of frequencies the electrical conductivity σ is approximately 100 times less than the static value and for typical metals it is of the order of 10^{15} s^{-1}. Since in the optical range the angular frequency of the oscillations of light ω_0 is of the order of 10^{15} s^{-1}, it follows that

$$N'' \sim (2\pi\sigma/\omega_0)^{1/2} \lesssim 1$$

and the condition $\zeta = 1/N'' \ll 1$ is replaced by[1)] $\zeta \lesssim 1$. The inequality $\eta \ll 1$ is satisfied even by bodies of small macroscopic dimensions.

We can thus see that the theory of diffraction of optical radiation by metallic bodies should be developed and be valid for low values of the parameter η in the range $\eta \ll 1$, and moderately low values of the parameter ζ in the range $\zeta \lesssim 1$, i.e., it should apply to refractive indices in which the imaginary part N'' is close to or slightly greater than unity. The first question that arises in the development of such a theory is whether the Leontovich impedance conditions remain valid when $\zeta \lesssim 1$ and $\eta \ll 1$ and whether we can exclude from consideration the interior of an absorbing body by some relationships that apply to an external field, are satisfied on the surface of the absorbing body, and replace the Leontovich conditions.

We shall provide the basis of an asymptotic theory of diffraction of optical radiation by convex metallic bodies, describe experiments involving the scattering of laser beams by metal filaments, and compare the theoretical predictions with the experimental data. We shall show that such an experiment demonstrates that the proposed theory, which utilizes not the usual but the generalized Leontovich conditions, describes much better the experimental dependences particularly in the case of TE-polarized light.

1. ASYMPTOTIC BOUNDARY CONDITIONS AND DIFFRACTION EQUATIONS VALID IN THE PENUMBRA OF A WAVE FIELD SCATTERED BY A METALLIC BODY

The approximate boundary conditions on the surfaces of strongly absorbing bodies (good conductors) can be derived by asymptotic expansion of the field in the surface layer of an absorbing body and subsequent matching by the boundary conditions to the external field. In 1940, Rytov obtained an asymptotic expansion of the field for a surface layer in powers of the skin layer thickness d (Ref. 3). Essentially this was an expansion in terms of a dimensionless parameter $\xi = d/a \ll 1$ on the assumption that $\eta \sim 1$. In 1973 Kravchenko proposed an asymptotic expansion of the field in the surface layer in powers of the quantity $\zeta^{1/q} \ll 1$ (q is a natural number), i.e., in fractional powers of the parameter ζ (Ref. 4). In this case it was assumed that $\eta \ll 1$ and that the parameters η and ζ are related by $\eta \propto \zeta^{p/q}$ for some natural number p. The main terms of the Rytov and Kravchenko expansions (in the three cases $p < q$, $p = q$, and $p > q$) lead to the Leontovich conditions[1] on the surface S of an absorbing body. In all these cases the correction terms are different and they allow us to determine the influence of the curvature of the surface S and of the gradients of the parameters of the absorbing medium on the validity of the Leontovich conditions for the relevant values of the parameters ζ and η.

The correction terms to the Leontovich conditions in

 0038-5646/90/030409-07$03.00

the cases when $\zeta \ll 1$ and $\eta \sim 1$ were also investigated by Panych.[5] The case when $\zeta \lesssim 1$ and $\eta \ll 1$, i.e., the situation of main interest in the scattering of optical radiation by metals, was considered also in Refs. 6 and 7 for a homogeneous absorbing medium bounded by a plane and a cylindrical surface. In the case of an arbitrary convex surface and an inhomogeneous medium the boundary conditions corresponding to $\zeta \lesssim 1$ and $\eta \ll 1$ were obtained in Ref. 8.

We can easily see that both when $\zeta \ll 1$ and $0 < \eta \lesssim 1$, as well as when $0 < \zeta \lesssim 1$ and $\eta \ll 1$, the product $\xi = \eta\zeta = d/a$ is a small quantity: $\xi \ll 1$. We shall obtain an asymptotic expansion of the field in the surface layer of an absorbing body in powers of $\xi \ll 1$ on the assumption that $N'' \gtrsim 1$. Such an expansion is important not only on its own merits (for example, it can be used to calculate the quantity of heat released in an absorbing body allowing for the curvature of the surface and the refractive index gradients), but it can be used to obtain the asymptotic boundary condition satisfied by the external field on the surface S. Obviously, this expansion should contain the Rytov and Kravchenko expansion, which should be derivable from it subject to additional assumptions about the values of the parameters η and ζ. The asymptotic boundary conditions make it possible to calculate the corrections to the Fresnel reflection expressions which are governed by the curvature of the surface S and by the gradient of the refractive index N.

All our calculations will be made for two spatial variables. Generalization to the case of vector wave fields and three spatial variables does not require any new fundamental concepts.

We shall consider a two-dimensional region Ω with a smooth convex boundary S on a plane $\mathbf{R}^2$; we shall use $N(\mathbf{r}) = N'(\mathbf{r}) + iN''(\mathbf{r})$ to describe the complex refractive index of an inhomogeneous absorbing medium filling the region Ω. We shall assume that the wave field is

$$U = \begin{cases} U_0 = U_0^i + U_0^r, & \mathbf{r} \in \mathbf{R}^2/\Omega, \\ U_1, & \mathbf{r} \in \Omega, \end{cases}$$

where U_0^i and U_0^r are the fields incident on and reflected by Ω, and that this field satisfies the Helmholtz equation with a coefficient which is discontinuous on the surface S:

$$\Delta U + k_0^2 \chi(\mathbf{r}) U = 0, \quad \chi(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in \mathbf{R}^2/\Omega, \\ N^2(\mathbf{r}), & \mathbf{r} \in \Omega, \end{cases} \tag{1}$$

and we shall postulate that the following contact conditions are satisfied at the interface S between the media:

$$U_0|_S = U_1|_S, \quad \left.\frac{\partial U_0}{\partial n}\right|_S = \frac{1}{\varkappa} \left.\frac{\partial U_1}{\partial n}\right|_S . \tag{2}$$

Here, ∂/∂_n represents differentiation along the normal to S ($n > 0$ outside Ω) and $\varkappa$ is a function defined on S. Moreover, the reflected field U_0^r should satisfy the principle of emission of radiation. In the case of the Maxwell equations the planar diffraction problem can be reduced to a scalar formulation of Eqs. (1) and (2) by introduction of the Hertz potentials. For TM-polarized light (with the vector $\mathbf{H}$ in the $\mathbf{R}^2$ plane) we have $\varkappa = 1$, whereas for TE-polarized light we find that $\varkappa = N^2(\mathbf{r})|_S$.

We shall first obtain an asymptotic expansion of wave field in the boundary layer Ω adjoining the illuminated part of the interface S. We shall adopt dimensionless variables $\mathbf{x} = \mathbf{r}/a$ (we shall assume that the characteristic size a of the region Ω does not exceed the radius of curvature of the boundary S) and the reduced refractive index

$$\mathcal{N}(\mathbf{x}) = \zeta N(a\mathbf{x}),$$

where we now have

$$\zeta = [\min(N''(\mathbf{r})|_{\mathbf{r}\in S})]^{-1}.$$

We shall assume that $\mathcal{N}(\mathbf{x})$ and the derivatives of $\mathcal{N}(\mathbf{x})$ are of the order of unity. The latter assumption means that over distances of the order of the size of the region Ω the refractive index $N(\mathbf{r})$ does not vary more than severalfold, i.e., that the medium filling the region Ω is continuously inhomogeneous. Since $N''(\mathbf{r})$ is a function of $\mathbf{r}$, the depth of penetration of the field into Ω is different at different points of the boundary S. We shall use the symbol d to denote the maximum value of this depth ($d = k_0^{-1}\zeta$) and still assume that the parameter $\xi = d/a$ is small, so that it satisfies $\xi \ll 1$, i.e., that the dimensions of the region Ω are much greater than the maximum depth of penetration of the field inside Ω. Therefore, if $|N(\mathbf{r})| \gtrsim 1.2$, we can ignore reflections[2)] of the field refracted into the region Ω. Subject to this assumption, we shall seek the wave field in Ω in the form of an expansion in powers of ξ with a single eikonal describing the wave traveling from the boundary into the region Ω:[3)]

$$U_1 = \exp[i\xi^{-1}\tau(\mathbf{x})]u(\mathbf{x}, \xi), \quad u(\mathbf{x}, \xi) = \sum_{j=0}^{\infty} u_j(\mathbf{x})(-i\xi)^j. \tag{3}$$

In the vicinity of the boundary S we shall introduce two dimensionless coordinates s and n: we shall use n to denote the dimensionless (divided by a) distance along the normal from a given point to the boundary S, and we shall denote by s the dimensionless length of arc of S measured from a certain point on S to the base of the normal. We shall assume that the convex boundary S is smooth. This assumption implies that the dimensionless (divided by a) radius of curvature $\rho(s)$ has derivatives which are comparable with this radius. The coordinates (s,n) of the convex boundary S are regular in the range $n > -\rho(s)$ and they can be used to calculate the refracted field U_1 in a layer adjoining the boundary S and of thickness much less than the depth of penetration d; we shall not consider the field U_1 outside this layer because it is then a negligible exponentially small quantity.

Substituting the expansion of Eq. (3) into Eq. (1), rewriting the latter in dimensionless variables, and equating to zero the coefficients in front of various powers of ξ, we obtain the eikonal equation for the function $\tau(\mathbf{x})$:

$$(\nabla\tau(\mathbf{x}))^2 = \mathcal{N}^2(\mathbf{x}), \tag{4}$$

and the transfer equations for the coefficients $u_j(\mathbf{x})$. The solution of the eikonal and transfer equations will be obtained in terms of the coordinates (s,n) as an expansion in powers of n:

$$\tau(\mathbf{x}) = \sum_{m=0}^{\infty} \tau_m(s) n^m, \qquad u_j(\mathbf{x}) = \sum_{m=0}^{\infty} u_{j,m}(s) n^m. \tag{5}$$

Substituting the expansion $\tau(\mathbf{x})$ into the eikonal equation (4), expanding its right-hand side in powers of n

$$\mathcal{N}^2(\mathbf{x})=\sum_{m=0}^{\infty}\mathcal{N}_m^2(s)n^m,$$

and equating the coefficients in front of different powers of n on the left- and right-hand sides, we obtain a recurrent system of algebraic equations from which we can determine consistently the coefficients $\tau_m(s)$, where $m\geqslant 1$. The coefficient $\tau_0(s)$ cannot be deduced from the eikonal equation: it can be found from the contact conditions of Eq. (2) if the incident field U_0^i is specified. The coefficients $\tau_1(s)$ and $\tau_2(s)$ in the first two equations of the recurrent system are

$$\tau_1(s)=-[\mathcal{N}_0^2-\tau_0'^2(s)]^{1/2},$$
$$\tau_2(s)=4^{-1}\tau_1^{-1}(s)[\mathcal{N}_1^2-2\tau_0'(s)\tau_1'(s)+2\tau_1'^2(s)\rho^{-1}(s)]. \qquad (6)$$

The root branch is fixed by the condition $\mathrm{Im}[\mathcal{N}_0^2-\tau_0'^2(s)]^{1/2}>0$, and the minus sign is selected by the need to ensure that the field is attenuated as it penetrates the region Ω, i.e., as $-n$ increases.

If the wavelength of the incident radiation is much less than the characteristic size of the region Ω ($\eta\ll 1$), the incident and reflected fields can be described by the following ray expansions:

$$U_0^i=\exp[i\eta^{-1}\tau^i(\mathbf{x})]\sum_{j=0}^{\infty}u_j^n(\mathbf{x})(-i\eta)^j,$$
$$U_0^r=\exp[i\eta^{-1}\tau^r(\mathbf{x})]\sum_{j=0}^{\infty}u_j^0(\mathbf{x})(-i\eta)^j. \qquad (7)$$

It then follows from the condition (2) that

$$\eta^{-1}\tau^i(\mathbf{x})|_{x\in S}=\eta^{-1}\tau^r(\mathbf{x})|_{x\in S}=\xi^{-1}\tau_0(s).$$

Hence, we obtain

$$\tau_0(s)=\xi\eta^{-1}\tau^i(\mathbf{x})|_{x\in S}=\zeta\tau^i(\mathbf{x})|_{x\in S}$$

and

$$\tau_0'(s)=\zeta\left(\nabla\tau^i(\mathbf{x})|_{x\in S},\frac{d\mathbf{x}}{ds}\right)=\zeta\sin\theta,$$

where θ is the angle of incidence. Therefore, in this case we obtain

$$\tau_1(s)=-[\mathcal{N}_0^2(s)-\zeta^2\sin^2\theta(s)]^{1/2}.$$

If the wavelength of the incident radiation is comparable with the characteristic dimensions of the region Ω ($\eta\sim 1$), it follows that $\tau_0(s)\sim\xi$ and in the first approximation we have to assume that $\tau_0(s)=0$. We then find that $\tau_1(s)=-\mathcal{N}_0(s)$. The coefficients $u_{j,m}(s)$ in the expansion $u_j(\mathbf{x})$ are defined similarly. The coefficients $u_{j,0}$, where $j\geqslant 0$, are found from the contact conditions given by Eq. (2).

The expansion (3) for the wave field U_1 in the boundary layer allows us to exclude the field inside the region Ω from the contact conditions given by Eq. (2) and thus obtain an asymptotic form for the boundary condition which must be satisfied by the external field U_0:

$$\left.\frac{\partial U_0}{\partial n}\right|_S+i\xi^{-1}\varkappa^{-1}[\mathcal{N}_0^2(s)-\tau_0'^2(s)]^{1/2}U_0|_S$$
$$=\varkappa^{-1}\exp[i\xi^{-1}\tau_0(s)]$$
$$\times\sum_{l=0}^{\infty}B_l\left(s,\frac{d}{ds}\right)\{\exp[-i\xi^{-1}\tau_0(s)]U_0|_S\}(-i\xi)^l. \qquad (8)$$

In Eq. (8) the expression

$$B_l\left(s,\frac{d}{ds}\right)=\sum_{q=0}^{4l+1}B_{l,q}(s)\left(\frac{d}{ds}\right)^q$$

represents polynomials in powers of d/ds with coefficients dependent on s. In particular, the zeroth-order polynomial is

$$B_0\left(s,\frac{d}{ds}\right)=-\frac{1}{2\tau_1(s)}\left[2\tau_2(s)+\tau_1(s)\rho^{-1}(s)+\tau_0''(s)+2\tau_0'(s)\frac{d}{ds}\right].$$

The left-hand side of Eq. (8) yields the leading terms of the boundary condition, whereas the right-hand side provides the correction terms that introduce the radii of curvature of the boundary S and of the front of the incident wave, the derivatives of these quantities, and the derivatives of the refractive index into the boundary conditions. Moreover, higher-order tangential derivatives appear in the approximate boundary conditions if, for example, we replace an elastic plate between liquid-filled half-spaces with the approximate contact boundary conditions of Ref. 9.

When an hf wave ($\eta\ll 1$) is incident on a metallic body, we find that $\tau_0''(s)$ may be described by the following expressions:

$$\tau_0''(s)=\zeta\{-\rho^{-1}(s)\pm R^{-1}(s)[1\pm R^{-1}(s)]^{-1}\cos\theta\}\cos\theta,$$

where $R(s)$ is the dimensionless radius of curvature of the front of the incident wave [the plus sign of $R^{-1}(s)$ corresponds to the reflection of a diverging wave and the minus sign corresponds to the reflection of a converging wave]. Apart from the correction terms, we find that an incident hf wave obeys the boundary condition

$$\left.\frac{\partial U_0}{\partial n}\right|_S+i\frac{1}{\eta\varkappa}[N^2(\mathbf{x})|_S-\sin^2\theta(s)]^{1/2}U_0|_S=0. \qquad (9)$$

The condition (9) satisfied on the illuminated part of the boundary is called in Ref. 7 (where it is derived from the exact solution of the problem of diffraction by a circular homogeneous cylinder) the generalized Leontovich condition. If $|N|\gg 1$, we can simplify this generalized condition by dropping the term $\sin^2\theta$ within the radicand which then yields the ordinary Leontovich condition.

If the wavelength of this incident wave is comparable with the dimensions of the reflecting body ($\eta\sim 1$), the boundary condition of Eq. (8) together with the first correction can be written in the form

$$\left.\frac{\partial U_0}{\partial n}\right|_S+\frac{i}{\eta\varkappa}N(\mathbf{x})|_S U_0=\frac{1}{2\varkappa}\left(\frac{1}{\rho(s)}+\left.\frac{\partial\ln N(\mathbf{x})}{\partial n}\right|_S\right)U_0\Big|_S$$

which is identical with the condition derived in Ref. 3.

If in the boundary condition of Eq. (8) we replace

$U_0 = U_0^i + U_0^r$ with the ray expansions of Eq. (7) and equate terms of the same order of magnitude, we obtain the Fresnel reflection expression for the principal term of the expansion of the amplitude u_0^r and the reflection expressions for the higher terms u_j^r, where $j \geqslant 1$.

The asymptotic boundary condition in the vicinity of the points of tangential contact C between the incident beam and the boundary S, i.e., in the penumbra, can be derived similarly. Using reduced variables

$$\nu = \eta^{-2/3}[2/\rho(C)]^{1/3} n, \quad \sigma = \eta^{-1/3}[2\rho^2(C)]^{-1/3} s$$

(when the length of an arc s is measured from the point C and we have $s > 0$ in the shadow zone) this asymptotic condition can be written as an expansion in terms of fractional powers of the parameter $\xi = d_c/a$ (d_c is the depth of penetration at the point C):

$$\frac{\partial U_0}{\partial \nu}\Big|_S + \frac{i}{\varkappa}\left[\frac{\rho(C)}{2\eta}\right]^{1/3} [N^2(C) - 1]^{1/2} U_0|_S$$

$$= \frac{\exp(i\eta^{-1}s)}{\varkappa} \sum_{l=1}^{\infty} \hat{B}_l\left(\sigma, \frac{d}{d\sigma}\right)(\exp(-i\eta^{-1}s) U_0|_S)(-i\xi)^{l/3}. \tag{10}$$

Here, $\hat{B}_l(\sigma, d/d\sigma)$ are polynomials which are functions of σ and $d/d\sigma$. In particular, the first-order polynomial is

$$\hat{B}_1 = -e^{i\pi/6}[4\rho(C)]^{-1/6}[N''(C)]^{1/2}[N^2(C)-1]^{-1/4} d/d\sigma.$$

Equating the left-hand side of Eq. (10) to zero, we obtain the following expression for the boundary condition in the penumbra which is valid in the principal (leading) approximation:[4)]

$$\frac{\partial U_0}{\partial \nu}\Big|_S + \frac{i}{\varkappa}\left[\frac{\rho(C)}{2\eta}\right]^{1/3} [N^2(C) - 1]^{1/2} U_0|_S = 0. \tag{11}$$

It is identical with the condition (9) if we substitute $\theta = \pi/2$ in Eq. (9).

The asymptotic boundary condition for a three-dimensional vector problem with the Maxwell equations on the surface of a conducting body when $\eta \ll 1$ and $0 < \xi \lesssim 1$ can be obtained by similar methods. On the illuminated part of the surface we find that in the principal (leading) approximation this condition is

$$\begin{pmatrix} H^\alpha \\ H^\beta \end{pmatrix} = \begin{pmatrix} 0 & \varepsilon'(\varepsilon'\mu - \sin^2\theta)^{-1/2} \\ -\mu^{-1}(\varepsilon'\mu - \sin^2\theta)^{1/2} & 0 \end{pmatrix} \begin{pmatrix} E^\alpha \\ E^\beta \end{pmatrix}, \tag{12}$$

where α and β are the orthogonal coordinates on the surface S; $\theta = \theta(\alpha, \beta)$ is the angle of incidence of the beam and $\varepsilon' = \varepsilon + 4i\pi\sigma/\omega_0$ (ε and μ are the permittivity and the magnetic permeability, respectively). The condition (12) represents the generalized Leontovich condition for an electromagnetic field. Higher terms of the boundary condition (12) contain matrix differential operators along directions tangential to S. The first correction describes diffusion of the polarization on reflection, since the matrix operator of the first correction contains nonzero diagonal elements. The derivation of the boundary condition for the Maxwell equations in the penumbra is fully analogous to the procedure adopted in the two-dimensional scalar problem, although it is somewhat more cumbersome.

Applying the generalized Leontovich boundary condition (11) in the penumbra, we obtain an expression describing the scattered wave field in the bright part of the penumbra at some distance behind a convex absorbing cylinder. We shall give this expression without derivation:[11]

$$U_0^r = U^i(C)\exp\left\{i\eta^{-1}\tau^i(C) + i\eta^{-1}(|\mathbf{x} - \mathbf{y}(C)| + s_0) + \frac{3\pi i}{4}\right\}$$

$$\times \{2[\pi J_0(\mathbf{x})]^{1/2}\}^{-1}\left[\frac{2\eta}{\rho(C)}\right]^{1/6} [F(x) + 2i\Phi_v(\mathbf{x}) - \Phi_w(\mathbf{x})]. \tag{13}$$

Here, $\mathbf{x}$ and $\mathbf{y}(C)$ are the dimensionless radius vectors of the point of observation and of the point C; J_0 is the geometric divergence described in terms of "evolvent" coordinates (grazing rays); s_0 is the dimensionless length of an arc corresponding to the point of detachment of a grazing ray[5)] (in the bright part of the penumbra we have $s_0 \leqslant 0$); σ_0 is the reduced length of the arc up to the point of detachment; $F(\mathbf{x})$ is the Fresnel integral described by

$$F(\mathbf{x}) = \int_{\infty \cdot e^{4\pi i/3}}^{0} \exp\{i\sigma_0\zeta + i\psi\zeta^2\}\, d\zeta,$$

$$\psi = \frac{1}{4m}\left[\frac{\rho(C)}{|\mathbf{x} - \mathbf{y}(C)|} + \frac{\rho(C)}{R(C)}\right], \tag{14}$$

$$m = \eta^{-1/3}\left[\frac{\rho(C)}{2}\right]^{1/3};$$

$R(C)$ and $\rho(C)$ are the dimensionless radii of curvature of the phase front of a diverging cylindrical wave and of the investigated cylinder at the point C; in terms of the Fock integrals $\Phi_v(\mathbf{x})$ and $\Phi_w(\mathbf{x})$ (Ref. 10, p. 133) the impedance can be described by

$$Q = im\varkappa^{-1}[N^2(C) - 1]^{1/2}. \tag{15}$$

In the most interesting (from the point of view of optical experiments) case of the far-field zone, $|\mathbf{x}| \gg \rho(C)$, and a near-planar phase front, $R(C) \gg \rho(C)$, the condition $\psi \ll 1$ is obeyed and we have $F(x) \approx -i/\sigma_0$, so that Eq. (13) becomes invalid at low angles of tilt φ of a grazing angle relative to the tangential angle [for typical values of the parameters used in our experiments such a replacement of $F(x)$ in Eq. (13) gives rise to an error smaller than 10% if $\varphi \gtrsim 1°$].

In the dark part of the penumbra it is preferable to consider the complete field $U_0 = U_0^i + U_0^r$, where U_0 is described (in the range $\sigma_0 \geqslant 0$) by Eq. (13) if the integration contour in Eq. (14) is replaced with $[0, \infty \cdot e^{i\pi/3}]$ and the sign of the integrand is reversed. An expression for the scattered field in the bright part of the penumbra, characterized by a constant angular distribution, is obtained in Refs. 11 and 12 and in this expression the Fresnel integral is replaced with the Kirchhoff integral. (An asymptote of the scattering diagram of a plane wave incident on absorbing cylinder is also obtained in Ref. 12.)

The asymptote of the angular distribution of the scattered radiation in the penumbra can be described by a sum of two terms representing waves scattered in the vicinity of the points of contact C and C' of the extreme rays representing the boundary of the shadow zone. The term U_0^r corresponding to the point C can be calculated from Eq. (13) allowing for the fact that $\sigma_0 < 0$, whereas at the point C' we have to assume that $\sigma_0' < 0$ in the expression of U_0.

Calculations of the behavior of the scattered radiation in the penumbra and of the angular distribution of this radiation, carried out for the case when the generalized Leontovich condition applies at the boundary of a body, are described in Sec. 2 in connection with an analysis of the experimental results on the diffraction of a laser beam by metal filaments.

2. EXPERIMENTAL INVESTIGATION OF THE DIFFRACTION OF LIGHT BY METAL FILAMENTS AND COMPARISON OF THE EXPERIMENTAL AND CALCULATED RESULTS

The published experimental investigations of the diffraction of electromagnetic waves by metal cylinders have been carried out mainly in the microwave range, i.e., in the lf diffraction range ($\eta \sim 1$) and in the near-field zone ($|\mathbf{x}| = |\mathbf{r}|/a \gtrsim 1$). There have been very few studies in the optical range ($\eta \ll 1$) and in the far-field zone ($|\mathbf{x}| \gg 1$). In the published experimental studies of the optical range the stress has been on determination of the integral scattering coefficient and the local structure of the scattered field in the penumbra region has been practically ignored. We shall concentrate our attention on the scattered light in the penumbra because it is of greatest interest in a comparison with our calculations based on asymptotic expressions.

We used an He–Ne laser emitting at $\lambda_0 = 0.63$ μm ($k_0 \approx 10 \mu\text{m}^{-1}$). Our experiments were carried out on metal filaments with a diameter $2a = 15$–1500 μm so that the dimensionless parameter $\eta^{-1} = k_0 a$ was within the range $\eta^{-1} = 75$–2500. A cylindrical lens formed a Gaussian light beam with an elliptic cross section and the phase front of the beam was nearly planar at the point of incidence of the beam on the filament, so that in the interpretation of the experimental results we could use the asymptotic theory of the diffraction of a plane wave. Two linear polarizations of the beam incident on a filament at right-angles to its axis were selected: TM and TE. A photodetector with a narrow entry slit was used to measure the intensity $I(\varphi)$ of the scattered field as a function of the scattering angle φ in the far-field zone at a distance $r = 55$ mm from the axis of the cylindrical filament. The intensity of the field on the beam axis was I_0 in the absence of the scattering filament. The polarization of the scattered radiation was investigated with an analyzer in the form of a film Polaroid.

Two quite distinct series of experiments were carried out deliberately.

We investigated the diffracted radiation separately in the bright and dark regions of the penumbra when only one "edge" of a cylindrical filament was illuminated with a "narrow" focused beam (Fig. 1). A "waist" of the caustic surface of the incident field formed at the point of incidence of the beam on the filament. The minor semiaxis $l = 28$ μm of the elliptical cross section of the beam, perpendicular to the cylinder axis, satisfied the inequalities $a > l \gg \lambda$. This method allowed us to eliminate in practice the diffraction effects at the other edge of the filament and to ensure that the size of the geometric shadow was considerable (III in Fig. 1). The illuminated region (II), including the bright part of the penumbra, was located on the other side of the transmitted beam (I).

A study of the diffraction of the radiation in the penumbra was also made in the second series of experiments when

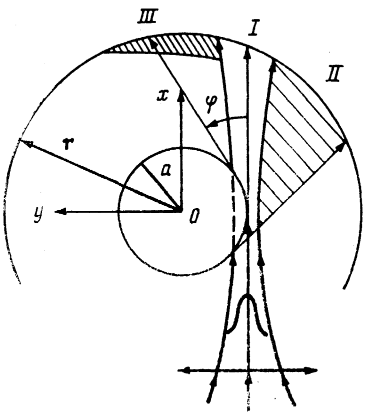

FIG. 1.

the whole cross section of a cylindrical filament was illuminated with a "wide" beam with $l = 320$ μm ($l > a \gg \lambda$) and the radius of curvature R of the phase front of the beam satisfied the condition $R \gg a$ (Fig. 2). In this case the shadow zone was located within the transmitted beam region (I) and the angular size of region I was $\leqslant 1°$. On both sides of it there were two symmetric illuminated regions including the bright part of the penumbra. This method allowed us to reveal the diffraction effects associated with the scattering from both edges of the filament. (The details of our experiments and a study of oblique incidence of a beam on a filament were reported elsewhere.[13-16])

An investigation of the polarization of the scattered light in these experiments showed that in the case of the normally incident beam with a linear TM (TE) polarization, the scattered field had the same TM (TE) polarization. If the polarization in the incident light was intermediate between TM and TE, the polarization of the scattered wave was elliptic and the parameters of the ellipse depended on the angle of observation φ. In the case of illumination of the filament at grazing angles within the range $\alpha \leqslant 30°$ relative to the filament axis, the vectors describing the scattered radiation in the penumbra retained in practice the orientation parallel to that in the incident beam. This was in agreement with the boundary condition (12) and associated with the absence of the diagonal elements in the matrix describing the

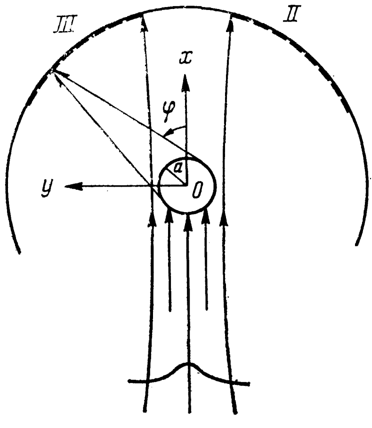

FIG. 2.

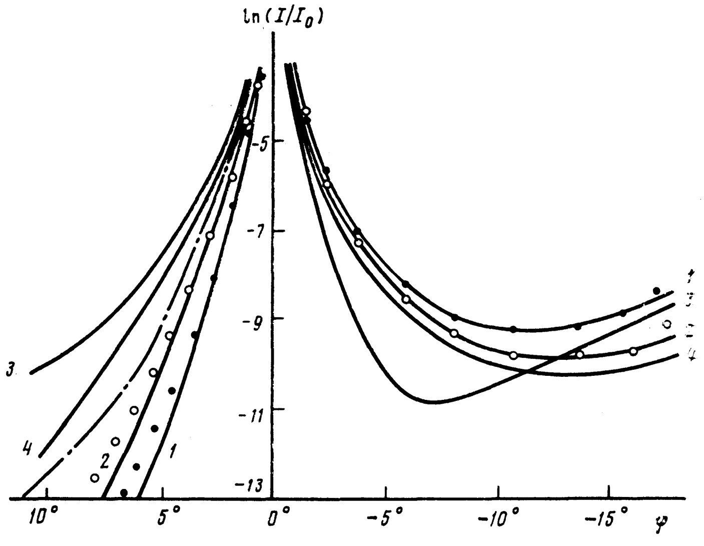

FIG. 3.

principal approximation for the boundary condition.

In the first series of experiments (Fig. 1) we found that illumination of one edge of filaments $2a = 50$–$500\ \mu m$ in diameter with a narrow beam gave rise to a "tail" of the scattered radiation intensity in the dark penumbra region (III) and this tail decreased rapidly in intensity to a background illumination level as the angle of observation was increased. On the other side of the transmitted beam in the bright part of the penumbra (II) the intensity of the diffraction field (which will be called the satellite) also fell rapidly on increase in φ and merged with the growing field due to the geometric reflection.

Figure 3 gives the angular dependences of the intensity of the scattered light obtained for a constantan filament $2a = 100\ \mu m$ in diameter ($\eta^{-1} = 500$). The black dots in Fig. 3 represent the TM polarization and the open circles correspond to the TE polarization of the incident beam. This figure includes also continuous curves representing the results of our calculations of the satellite intensity in the bright part of the penumbra ($\varphi < 0$), carried out for both polarizations on the basis of Eq. (13), where the Fresnel integral was replaced by the expression $-i/\sigma_0$. In calculation of the theoretical curves the value of Q was assumed to be ∞ for the TM polarization (curve 1), whereas for the TE polarization (curve 2) it was described by Eq. (15) in accordance with the generalized Leontovich condition (11). It is clear from Fig. 3 that curve 3 representing the intensity of the scattered fields calculated using the model of a perfectly conducting cylinder is completely unsuitable for the description of the experimental dependence in the case of the TE polarization ($Q = 0$). Curve 4 corresponding to the conventional Leontovich condition for the TE polarization, $Q = imN(C)\varkappa^{-1}$, describes — on a logarithmic scale — the experimental data on the average 10% poorer than curve 2 calculated using our condition (11). It follows from a numerical analysis that the expressions such as Eq. (13) go over continuously to the Fresnel reflection expressions and, beginning from $|\varphi| \gtrsim 20°$, the scattered field can be described with satisfactory precision by the geometric-optics approximation.

Similar results follow from a comparative analysis of the tails in the dark part of the penumbra ($\varphi > 0$). Curves 3 and 4 represent the field calculated for the TE polarization using the model of a perfectly conducting metal and the conventional Leontovich condition, respectively; the calculations were carried out by reducing Eq. (13) for $\varphi > 0$ to a series of residues by analogy with the procedure adopted in Ref. 13. Curve 2 was obtained for the total field U_0^{ζ} by calculation based on Eq. (13) specifying Q in accordance with Eq.

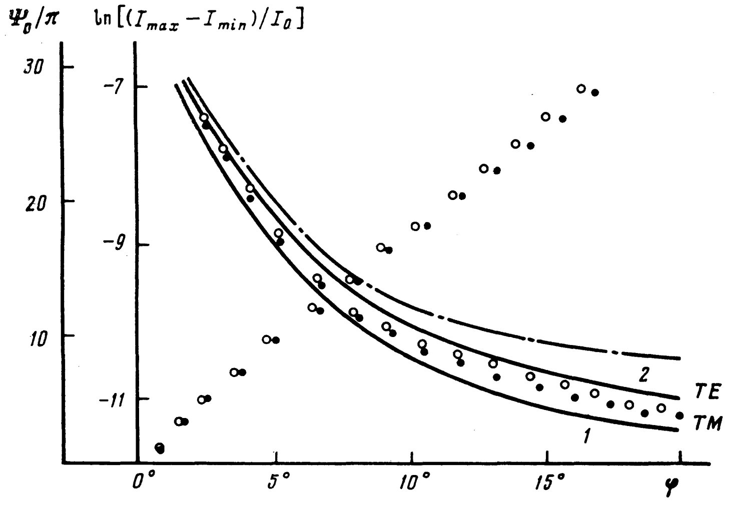

FIG. 4.

(15). We can see that in the case of the TM polarization the results of the calculation based on the model of a perfectly reflecting cylinder ($Q=\infty$) and utilizing Eq. (13) are in satisfactory agreement with the experimental data (curve 1). For comparison, Fig. 3 gives (chain curve) the calculated dependence of the intensity of the scattered field formed by diffraction of a plane wave by a half-plane with a sharp edge.[13]

In the second series of experiments (Fig. 2) we illuminated the whole cross section of filaments with diameters $2a = 15$–$50\ \mu m$ with a wide beam and observed high-contrast oscillations of the scattered light intensity in the penumbra on both sides of the transmitted beam; these results were analogous to the diffraction by a strip and the amplitude of these oscillations decreased rapidly on increase in the observation angle. An investigation of the process of formation of these oscillations was made by scanning a filament across the beam. It was found that the oscillations in the penumbra were the result of spatial interference of the waves from the diffraction tail due to one edge of the filament and the satellite due to the opposite edge.

The black dots and open circles in Fig. 4 represent the angular dependences of the peak-to-peak amplitudes of the oscillations in the bright part of the penumbra (II in Fig. 2) obtained for a Manganin filament $2a = 30\ \mu m$ in diameter ($\eta^{-1} = -150$), representing the results obtained respectively for the TM and TE polarizations of the incident field. This figure includes also the dependences of the phase shift (advance) $\Psi_0(\varphi)/\pi$ of the oscillations representing a practically equidistant distribution of extrema of the intensity on the scale of the angle φ. The amplitudes of the oscillations calculated for a Manganin cylinder with $N = 4.1 + 2.6i$ are represented in Fig. 4 by continuous curves (1 for the TM polarization and 2 for the TE polarization). It is clear from this figure that the results of calculations of the angular distribution of the scattering of a plane wave by an opaque cylinder[12,18] (see also Ref. 17) are in good quantitative agreement with the experimental dependences. The chain curve in Fig. 4 shows also a fall of the peak-to-peak oscillation amplitudes calculated using the model of diffraction by an opaque strip of width $2a$ (Ref. 13).

Experimental results similar to those presented in Figs. 3 and 4 were obtained by us for a wide range of diameters and materials of the filaments and they differed only in respect of the slopes of the angular distributions.

These experimental and theoretical studies of the diffraction of optical radiation by conducting cylinders characterized by large values of the parameter k_0a revealed a good qualitative (and in most cases also a quantitative) agreement between the experimental results and those found by calculation employing the asymptotic expressions derived above. This allowed us to draw the following conclusions.

1. The proposed experimental method is suitable for the investigation of the diffraction of optical radiation by convex metallic bodies in the penumbra where the diffraction (wave) effects are dominant.

2. In the penumbra the TE-polarized light exhibits, in contrast to the TM polarization, a strong dependence of the angular distribution of the intensity on the complex refractive index. Therefore, the behavior of the TE-polarized light in the penumbra is best used in determination of the complex refractive indices of metals which can be obtained by comparing the calculated and experimental data. For example, such a comparison shows that the complex refractive index of Manganin is $N = 4.1 + 2.6i$ at the wavelength $\lambda_0 = 0.63\ \mu m$.

3. The models of perfectly and strongly conducting conductors (corresponding to the ideal boundary conditions and the conventional Leontovich conditions) are insufficient for a satisfactory description of the diffraction of optical radiation with the TE polarization by metallic bodies, particularly in the penumbra. A satisfactory agreement between the experimental results and the calculated curves are obtained for the TE polarization only if the scattering properties of such metallic bodies are described by the generalized Leontovich boundary condition.

[1)] For example, at the frequency of $\nu = 5.5\times 10^{14}\ s^{-1}$ the complex refractive index of copper is $0.62 + 1.59i$ (Ref. 2); the thickness of the skin layer is then $d \approx 0.4\times 10^{-5}$ cm and we also have $\zeta \approx 0.6$.

[2)] If $N'' \ll N'$, the attenuation of the refracted field inside Ω is proportional to $\exp(-\xi^{-1} l/a)$, where l is the length of the segment of the refracted ray traversed by the wave. In the case of a homogeneous cylinder of radius a the minimum length of the refracted ray (corresponding to the tangential ray) is $2a(N'^2 - 1)^{1/2}/N'$ and the attenuation of the field along this ray is by a factor of at least $\exp(-\xi^{-1})$ if $N' \gtrsim 1.15$. If $N'' \propto N'$, such ray constructions are invalid and the attenuation of the field at a depth a [see Eqs. (3), (5), and (6)] is proportional to $\exp(-\xi^{-1}\,\mathrm{Im}\tau_1) \ll \exp(-\xi^{-1})$.

[3)] The factors $(-i)^j$ are introduced into the expansion (3) for convenience in calculations.

[4)] It is pointed out by Fock[10] that if the expression for the wave field in the penumbra obtained on the basis of the conventional Leontovich condition is modified by replacing N^2 with $(N^2-1)^{1/2}$, then its accuracy increases. Such a replacement corresponds to a change from the conventional Leontovich condition to the boundary condition of Eq. (11) (see also Ref. 6).

[5)] The penumbra is identified by the inequality $|s_0| \lesssim 2m^{-1}$.

[1] M. A. Leontovich, Investigations of Radiowave Propagation (ed. by B. A. Vvedenskiĭ) [in Russian], Part II, Izd. AN SSSR, Moscow (1948), p. 5.

[2] M. Born and E. Wolf, *Principles of Optics*, *4th ed.*, Pergamon Press, Oxford (1970).

[3] S. M. Rytov, Zh. Eksp. Teor. Fiz. **10**, 180 (1940).

[4] A. G. Kravchenko, Radiotekh. Elektron. **18**, 49 (1973).

[5] O. I. Panych, Dokl. Akad. Nauk SSSR **70**, 589 (1950).

[6] E. L. Feĭnberg, *Propagation of Radio Waves along the Surface of the Earth* [in Russian], Nauka, Moscow (1961).

[7] P. Langlois and A. Boivin, Can. J. Phys. **61**, 332 (1983).

[8] V. S. Buldyrev and M. A. Lyalinov, Vestn. Leningr. Univ. Mat. Mekh. Astron. No. 2, 10 (1987).

[9] D. P. Kuzov, *Problems of Radiowave Diffraction and Propagation* [in Russian], Vol. 6, Leningrad State University (1978), p. 150.

[10] V. A. Fok, *Problems of Radiowave Diffraction and Propagation* [in Russian], Sovet-skoe Radio, Moscow (1970).

[11] M. A. Lyalinov, Vestn. Leningr. Univ. Mat. Mekh. Astron. No. 1, 24 (1988).

[12] M. A. Lyalinov, Author's Abstract of Thesis for Candidate's Degree [in Russian], Leningrad State University (1988).

[13] V. S. Buldyrev, V. N. Smirnov, G. A. Strokovskiĭ, and É. E. Fradkin, Opt. Spektrosk. **59**, 1255 (1985) [Opt. Spectrosc. (USSR) **59**, 754 (1985)].

[14] V. N. Smirnov and G. A. Strokovskiĭ, Vestn. Leningr. Univ. Mat. Mekh. Astron. No. 4, 27 (1987).

[15] V. N. Smirnov and G. A. Strokovskiĭ, Opt. Spektrosk. **60**, 1053 (1986) [Opt. Spectrosc. (USSR) **60**, 652 (1986)].

[16] V. N. Smirnov and G. A. Strokovskiĭ, Opt. Spektrosk. **62**, 614 (1987) [Opt. Spectrosc. (USSR) **62**, 366 (1987)].

[17] V. I. Ivanov, Zh. Vychisl. Mat. Mat. Fiz. **15**, 126 (1975).

[18] V. N. Smirnov, Author's Abstract of Thesis for Candidate's Degree [in Russian], Leningrad State University (1987).

Translated by A. Tybulewicz