Nonlinear theory of magnetoacoustic oscillations and of acoustic cyclotron resonance in metals

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(Submitted 14 July 1989; resubmitted 18 September 1989)
Zh. Eksp. Teor. Fiz. **97**, 343–358 (January 1990)

The damping coefficient of a longitudinal acoustic wave propagating in a metal in an oblique or perpendicular magnetic field is calculated. The nonlinear dynamics of electrons at cyclotron resonance is investigated with action and angle as the variables. It is established that electrons with a complex spectrum can have in an oblique or perpendicular magnetic field nonlinear resonances, for which the Hamiltonians are given. A connection is established between the parameters of the effective Hamiltonians and the geometry of the Fermi surface. The distribution functions of the resonance electrons are obtained and the variation of the line waveforms of magnetoacoustic and cyclotron oscillations of the absorption coefficient with increase of the sound amplitude is analyzed.

1. INTRODUCTION

Acoustic cyclotron resonance and magnetoacoustic oscillations of the absorption coefficient of sound in metals placed in a magnetic field **B** parallel to the z axis are known to be due to sound-wave interaction with resonance-electron groups. The relation between the component $\bar{v}_z$ of the average velocity of such electrons along the magnetic field and their cyclotron frequency ω_c is given by

$$k_{\parallel}\bar{v}_z-\omega-l\omega_c=0,\ l=0,\ \pm1,\ \pm2,\dots \qquad (1)$$

where ω is the wave frequency and $k_{\parallel}$ is the component of the wave vector **k** along the z axis. If the wave propagation is perpendicular to the direction of the magnetic field **B**, relation (1) goes over into the cyclotron-resonance condition

$$\omega=l\omega_c. \qquad (2)$$

The resonance conditions (1) and (2) were obtained without allowance for the influence of the wave on the trajectories of the resonance electrons. To develop a nonlinear theory of sound damping in metals we must solve the equations of motion of carriers having a complicated dispersion law $\varepsilon(\mathbf{p})$ in a magnetic field and in a wave field of finite amplitude.

We obtain in the present paper solutions of such equations of motion in the vicinities of the resonances defined by conditions (1) and (2). We use for this purpose a resonant perturbation theory[1] that makes it possible to solve this problem by introducing slow and fast phases at the resonance (and also the actions canonically conjugate to them) and by averaging over the fast phase. Calculation has shown that two different types of motion can be realized in the vicinity of the resonance (1) in an oblique field. If the resonance cross section of the Fermi surface p_z = const does not lie on a section corresponding to an extremal value of the derivative $\partial S/\partial p_z$, where S the area of the intersection then, as will be shown below, the effective Hamiltonian takes the form of that of a mathematical pendulum. The dynamics of electrons described by this Hamiltonian is quite well known. If, however, the resonant cross section is at an extremum of $\partial S/\partial p_z$, the effective Hamiltonian assumes the non-standard form (18), and the dynamics of the particles at such a resonance has a distinctive character.

Analysis of the dynamics of resonance electrons has made it possible to solve the kinetic equation near the resonances and to calculate the damping coefficient in the strong-nonlinearity regime, i.e., under conditions when the characteristic frequency of the particle oscillations at the resonance $\tilde{\omega}$ is much higher than the electron-collision frequency $\nu\equiv\tau_p^{-1}$. The nonlinearity parameter is thus the quantity $a=(\tilde{\omega}\tau_p)^{-1}$. It follows from the calculations that in an oblique field, just as in the linear theory,[2] there appear two types of magnetoacoustic oscillations connected, respectively, with the boundary points and with the extremum of $\partial S/\partial p_z$. By virtue of the different dynamics of the electrons, however, the dependence of the absorption coefficient on the amplitude of the longitudinal sound wave is different in these two cases [see Eqs. (40) and (44) below].

In a perpendicular magnetic field and in the field of a longitudinal sound wave the dynamics of electrons with a complicated spectrum, when the cyclotron frequency depends on S and p_z, differs substantially from the dynamics of electrons with a quadratic dispersion law, when the cyclotron frequency is constant. In the former case the effective Hamiltonian with the action and angle as variables has, after averaging over the fast phases, the standard form of the Hamiltonian of a mathematical pendulum [see (25)], with the reciprocal mass in this Hamiltonian proportional to $\partial\omega_c/\partial S$. For a quadratic electron spectrum, when $\partial\omega_c/\partial S\equiv0$ (in the terminology of Ref. 1 such systems are called degenerate) the nonlinearity in the equations of motion is of a different nature and is manifested at sound intensities apparently not yet attained in experiment.

The calculation, in Secs. 3 and 4, of the distribution function and of the absorption coefficient at cyclotron resonance has shown that, just as in the linear theory,[3] oscillations are produced in the sound-absorption coefficient by electrons having a cyclotron frequency that is extremal with respect to p_z, but the amplitude of the oscillations is smaller in the nonlinear regime by a factor $a^{3/2}$.

The plan of the paper is the following. In Sec. 2 we use the action and angle as variables for electrons having a complicated spectrum and located in a magnetic field. A resonance perturbation theory is used to construct effective Hamiltonians for resonances of various types in oblique and

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perpendicular magnetic fields. In Sec. 3 we solve the kinetic equation and obtain the distribution function in the region of isolated resonances. The last section is devoted to calculation of the sound-absorption coefficient and to a discussion of the results.

2. NONLINEAR DYNAMICS OF AN ELECTRON IN THE FIELD OF A SOUND WAVE

To describe the dynamics of an electron in the field of a sound wave and in a magnetic field $\mathbf{B}$ we introduce first the action and the angle as the variables of the unperturbed system. Choosing beforehand the gauge of the vector potential $\mathbf{A}$ in the form $\mathbf{A} = (0, Bx, 0)$, we write for the initial unperturbed Hamiltonian

$$H_0=\varepsilon(p_x,\ x-x_0,\ p_z), \tag{3}$$

where $x_0 = cp_y/eB = \text{const}$ is the coordinate of the center of the Larmor orbit. We change from the variables p_x, x, p_z, z to new canonical variables $I, \vartheta, \tilde{p}_z, Z$. The action variable I, defined in accordance with the usual rules

$$I=\frac{1}{2\pi}\oint p_x(\varepsilon, p_z, x)\,dx=\frac{c}{2\pi eB}S,$$

is proportional to the area of the intersection S of the Fermi surface $\varepsilon(\mathbf{p}) = \varepsilon_F$ with the plane $p_z = \text{const}$.

The change to the new canonical variables is made using a generating function F_2 that depends on the new momenta and on the old coordinates.[4] If this function is chosen in the form

$$F_2(I, \tilde{p}_z, x-x_0, z)=z\tilde{p}_z+\int p_x(I, \tilde{p}_z, x-x_0)\,dx,$$

the connection between the old and new coordinates and momenta is established by the following expressions:

$$\begin{aligned} &p_z=\tilde{p}_z, \qquad Z=z+\int\frac{\partial p_x}{\partial\tilde{p}_z}dx,\\ &p_x=p_x(I, \tilde{p}_z, x-x_0), \qquad \vartheta=\int\frac{\partial p_x}{\partial I}dx. \end{aligned} \tag{4}$$

Since the momenta p_z and $\tilde{p}_z$ coincide, we shall no longer distinguish between them. The equations of motion in the new variables take the simplest form:

$$\begin{aligned} &\dot{p}_z=0, \qquad \dot{Z}=\frac{\partial H_0(I, p_z)}{\partial p_z}=\bar{v}_z(I, p_z)=\text{const},\\ &\dot{I}=0, \qquad \dot{\vartheta}=\frac{\partial H_0(I, p_z)}{\partial I}=\omega_c(I, p_z)=\text{const}. \end{aligned} \tag{5}$$

The difference between the new variable $Z=\bar{v}_z t$ and the coordinate z is due to the nonuniform motion of the particle along the magnetic field.

The contribution to the Hamiltonian (3) from the interaction between the conduction electrons and the longitudinal sound wave is

$$H_1=\Lambda_{ik}(\mathbf{p})u_{ik}=-ku_0\Lambda(\mathbf{p})\cos(k_\parallel z+k_\perp x-\omega t), \tag{6}$$

where $\Lambda_{ik}(\mathbf{p})$ is the strain-potential tensor, u_{ik} is the strain tensor, u_0 is the amplitude of the displacement of the lattice atoms,

$$\Lambda(\mathbf{p})=\Lambda_{xx}\sin^2\varphi+\Lambda_{xz}\sin 2\varphi+\Lambda_{zz}\cos^2\varphi,$$

and φ is the angle between the vector $\mathbf{k}$ and the z axis. The total Hamiltonian, with action and angle as variables, is now written according to (4) in the form

$$\begin{aligned} H=\varepsilon(I, p_z)-ku_0\Lambda(\vartheta)\cos(k_\parallel Z-k_\parallel\Delta z(\vartheta)+k_\perp x_0\\ +k_\perp\Delta x(\vartheta)-\omega t). \end{aligned} \tag{7}$$

We transform to a reference frame that moves along the magnetic field with velocity $\omega/k_\parallel$. We introduce simultaneously a new phase $\psi = k_\parallel Z + k_\perp x_0 - \omega t$ and its canonically conjugate momentum $P = p_z/k_\parallel$, using the generating function

$$F_2=(k_\parallel Z+k_\perp x_0-\omega t)P+\vartheta I.$$

A Fourier expansion, periodic in the angle ϑ, of the perturbation in (7) yields

$$H=\varepsilon(I, P)-\omega P-\sum_{m=-\infty}^{+\infty}V_m(I, P)\cos(\psi-m\vartheta+\varphi_m), \tag{8}$$

where

$$V_m(I, P)=\frac{ku_0}{2\pi}\left|\int_{-\pi}^{\pi}d\vartheta\Lambda(\vartheta)\exp[i(\mathbf{kr}-\omega t+m\vartheta-\psi)]\right| \tag{9}$$

are the Fourier expansion coefficients and φ_m are the phases.

Let us examine in greater detail the dynamics of the particles in the vicinity of an individual lth resonance. The resonance condition (1) follows from the constancy of the phase $\psi - l\vartheta + \varphi_l \approx \text{const}$. In the spirit of resonant perturbation theory,[1] we introduce a "slow" phase $\alpha = \psi - l\vartheta + \varphi_l$ and a phase $\beta = \vartheta$. The connection between the old and new canonical variables is obtained with the aid of the generating function

$$F_2=(\psi-l\vartheta+\varphi_l)P_\alpha+\vartheta P_\beta$$

and is given by the expressions

$$\alpha=\psi-l\vartheta+\varphi_l,\ \beta=\vartheta,\ P_\alpha=P,\ I=P_\beta-lP_\alpha. \tag{10}$$

Changing in the Hamiltonian (8) to resonance variables and averaging over the fast phase, we obtain the effective Hamiltonian that describes the particle motion in the vicinity of the lth resonance:

$$H=\varepsilon(P_\alpha,\ P_\beta-lP_\alpha)-\omega P_\alpha-V_l\cos\alpha. \tag{11}$$

A rigorous corroboration of the procedure of averaging the Hamiltonian over the fast phase is contained in the Ref. 1 (see Ch. 2). Since the Hamiltonian (11) is independent of the variable β, the momentum P_β canonically conjugate to this variable remains constant near the resonance. Expanding next the Hamiltonian

$$H_0=\varepsilon(P_\alpha,\ P_\beta-lP_\alpha)-\omega P_\alpha$$

in a series in $P_\alpha - P_l$ up third order inclusive [P_l is the value of the momentum P_α corresponding to condition (1)], we get

$$H=\varepsilon(P_l, P_\beta-lP_l)-\omega P_l+\left(\frac{\partial H_0}{\partial P_\alpha}\right)_{P_\beta,P_l}(P_\alpha-P_l)$$

$$+\frac{1}{2}\left(\frac{\partial^2 H_0}{\partial P_\alpha^2}\right)_{P_\beta,P_l}(P_\alpha-P_l)^2$$

$$+\frac{1}{3!}\left(\frac{\partial^3 H_0}{\partial P_\alpha^3}\right)_{P_\beta,P_l}(P_\alpha-P_l)^3-V_l\cos\alpha, \qquad (12)$$

where $(\partial^n H_0/\partial P_\alpha^n)_{P_\beta P_l}$ denotes that the derivative is taken at a constant value of P_β at the point $P_\alpha = P_l$. The term $\varepsilon(P_l, P_\beta - lP_l) - \omega P_l$ in (12) is a constant that plays no role in the analysis of the system dynamics. The first derivative takes the form

$$\left(\frac{\partial H_0}{\partial P_\alpha}\right)_{P_\beta}=\left(\frac{\partial H_0}{\partial P_\alpha}\right)_I+\left(\frac{\partial I}{\partial P_\alpha}\right)_{P_\beta}\left(\frac{\partial H_0}{\partial I}\right)_{P_\alpha}$$

$$=k_\parallel\bar{v}_z-\omega-l\omega_c=0$$

at resonance. Obviously, the second and third derivatives can be represented in the form

$$\left(\frac{\partial^2 H_0}{\partial P_\alpha^2}\right)_{P_\beta}=\left[\frac{\partial}{\partial P_\alpha}(k_\parallel\bar{v}_z-l\omega_c)\right]_{P_\beta},$$

$$\left(\frac{\partial^3 H_0}{\partial P_\alpha^3}\right)_{P_\beta}=\left[\frac{\partial^2}{\partial P_\alpha^2}(k_\parallel\bar{v}_z-l\omega_c)\right]_{P_\beta}. \qquad (13)$$

Calculation of the partial derivative

$$\left[\frac{\partial}{\partial P_\alpha}(k_\parallel\bar{v}_z-l\omega_c)\right]_{P_\beta,P_l}$$

at constant P_β can be reduced to calculation of the derivative taken at the value ε = const (neglecting the low frequency ω of the wave). Obviously, any function $F(P_\alpha, \varepsilon(P_\alpha, P_\beta))$ satisfies the relation

$$\left(\frac{\partial F}{\partial P_\alpha}\right)_{P_\beta}=\left(\frac{\partial F}{\partial P_\alpha}\right)_\varepsilon+\left(\frac{\partial F}{\partial \varepsilon}\right)_{P_\alpha}\left(\frac{\partial \varepsilon}{\partial P_\alpha}\right)_{P_\beta},$$

and since the first derivative $(\partial\varepsilon/\partial P_\alpha)_{P_\beta}$ is equal to zero at resonance, we have

$$\left(\frac{\partial F}{\partial P_\alpha}\right)_{P_\beta,P_l}=\left(\frac{\partial F}{\partial P_\alpha}\right)_{\varepsilon,P_l}.$$

Using the well known relation

$$m_c\bar{v}_z=-\frac{1}{2\pi}\left(\frac{\partial S}{\partial p_z}\right)_\varepsilon,$$

we easily obtain

$$\frac{\partial}{\partial P_\alpha}(k_\parallel\bar{v}_z-l\omega_c)_{\varepsilon,P_l}=-\frac{1}{2\pi m_c}\left(\frac{\partial^2 S}{\partial P_\alpha^2}\right)_{\varepsilon,P_l}. \qquad (14)$$

For the second derivative taken at the point $P_\alpha = P_l$ we have

$$\left(\frac{\partial^2 F}{\partial P_\alpha^2}\right)_{P_\beta,P_l}=\left(\frac{\partial^2 F}{\partial P_\alpha^2}\right)_{\varepsilon,P_l}+\left(\frac{\partial F}{\partial \varepsilon}\right)_{P_\alpha}\frac{\partial}{\partial P_\alpha}(k_\parallel\bar{v}_z-l\omega_c)_{\varepsilon,P_l}$$

and, if the lth resonance lands on a section where the derivative $\partial S/\partial P_\alpha$ is extremal, we have in accordance with (14)

$$\left(\frac{\partial^2 F}{\partial P_\alpha^2}\right)_{P_\beta,P_l}=\left(\frac{\partial^2 F}{\partial P_\alpha^2}\right)_{\varepsilon,P_l}.$$

In this case the expression for the second derivative becomes quite simple

$$\frac{\partial^2}{\partial P_\alpha^2}(k_\parallel\bar{v}_z-l\omega_c)_{P_\beta,P_l}=-\frac{1}{2\pi m_c}\left(\frac{\partial^3 S}{\partial P_\alpha^3}\right)_{\varepsilon,P_l}. \qquad (15)$$

As a result of the foregoing calculations, the effective Hamiltonian at the lth resonance is

$$H=\frac{G}{2}(P_\alpha-P_l)^2+\frac{R}{3}(P_\alpha-P_l)^3-V_l\cos\alpha, \qquad (16)$$

where G and $2R$ stand for

$$G=\frac{\partial}{\partial P_\alpha}(k_\parallel\bar{v}_z-l\omega_c)_{\varepsilon,P_l}, \quad 2R=\frac{\partial^2}{\partial P_\alpha^2}(k_\parallel\bar{v}_z-l\omega_c)_{\varepsilon,P_l}.$$

If the resonance does not coincide with the extremum of the function $\partial S/\partial P_\alpha$, the coefficient G differs from zero and we can neglect in the Hamiltonian (16) the term of third order in the deviation from resonance. The Hamiltonian (16) goes over then into the well-studied Hamiltonian of a mathematical pendulum

$$H=\frac{G}{2}(P_\alpha-P_l)^2-V_l\cos\alpha \qquad (17)$$

the role of the mass is assumed by

$$G^{-1}=\left|\frac{1}{2\pi m_c}\left(\frac{\partial^2 S}{\partial P_\alpha^2}\right)_\varepsilon\right|^{-1},$$

and the potential-energy amplitude is $V_l(P_l)$.

If, however, the lth resonance coincides with the extremum of the function $\partial S/\partial P_\alpha$ and the coefficient G is equal to zero, we are dealing with a Hamiltonian that is cubic in $P_\alpha - P_l$

$$H=\frac{R}{3}(P_\alpha-P_l)^3-V_l\cos\alpha. \qquad (18)$$

This last case is also of great interest for an analysis of nonlinear phenomena in metals placed in a magnetic field, since the many perculiarities of the linear susceptibilities of a metal are determined just by the electron located on the Fermi surface near the section corresponding to an extremum of $\partial S/\partial P_\alpha$.

Let us examine the phase trajectories of the Hamiltonian (18). It follows from the equations of motion

$$\dot{P}_\alpha=-V_l\sin\alpha, \quad \dot{\alpha}=R(P_\alpha-P_l)^2 \qquad (19)$$

that the equilibrium positions correspond to the points $P_\alpha = P_l$ (where $\alpha = \pi n$), and that these stationary points can be regarded as a result of coalescence of stationary points of the "center" and "saddle" type. The evolution of the phase trajectories when the lth resonance reaches the section $(\partial S/\partial P_\alpha)_{ex}$ is illustrated in Figs. 1 and 2. Figure 1 shows the solutions of Eq. (1) at fixed l and three different values of B. It can be seen that when the lth resonance reaches an extremum of $\partial S/\partial P_\alpha$ (case b) the equation $k_\parallel\bar{v}_z - \omega - l\omega_c = 0$ has a unique solution. In stronger fields (case a) there is no solution at all, while in weaker fields there exist two resonances corresponding to one and the same number l. The phase portraits corresponding to these three cases are shown in Fig. 2. The trajectories shown in Fig. 2c have been obtained with the aid of the general-form Hamiltonian (16). In a weaker magnetic field the two resonances shown here and corresponding to the same l diverge, and the phase trajectories become the trajectories of mathematical pendulums. When two resonances merge (Fig. 2b) the phase curves are determined from the equations of motion (19) and the Hamiltonian resonances corresponding to one and the same number l. The phase portraits corresponding to these three cases are shown in Fig. 2. The trajectories shown in Fig. 2c have been obtained with the aid of the general-form Hamiltonian (16). In a weaker magnetic field the two resonances shown here and corresponding to the same l diverge, and the phase trajectories become the trajectories of mathematical pendulums. When two resonances merge (Fig. 2b) the phase curves are determined from the equations of motion (19) and the Hamiltonian (18); the separatrices joining stationary points are specified by the equations

$$(P_\alpha-P_l)^3=\frac{6V_l}{R}\cos^2\frac{\alpha}{2},\qquad (P_\alpha-P_l)^3=-\frac{6V_l}{R}\sin^2\frac{\alpha}{2}.$$

It follows from the equations of motion that the time of motion on the separatrices is infinite.

In stronger magnetic fields, the resonance condition is not met for any value of P_α, there are no stationary points on the phase plane, and the phase trajectories are smooth curves (Fig. 2a).

The case of wave propagation perpendicular to the magnetic field calls for a separate analysis. The Hamiltonian and the equations of motion for this case are:

$$H=\varepsilon(p_z,I)-\sum_m V_m(I,p_z)\cos(m\vartheta-\omega t+\varphi_m),\tag{20}$$

$$\dot I=-\sum_m mV_m(I,p_z)\sin(m\vartheta-\omega t+\varphi_m),\tag{21}$$

$$\dot\vartheta=\omega_c(I,p_z)-\sum_m\frac{\partial V_m}{\partial I}\cos(m\vartheta-\omega t+\varphi_m).\tag{22}$$

The Hamiltonian (20) can be obtained from the general expression (7) in which we put $k_\parallel=0$ and expand the perturbation in a Fourier series in the angle ϑ. The Fourier-expansion coefficients V_m are given by

$$V_m(I,p_z)=\frac{ku_0}{2\pi}\left|\int_{-\pi}^{\pi}d\vartheta\,\Lambda(\vartheta)\exp[i(kx(\vartheta)-m\vartheta)]\right|.$$

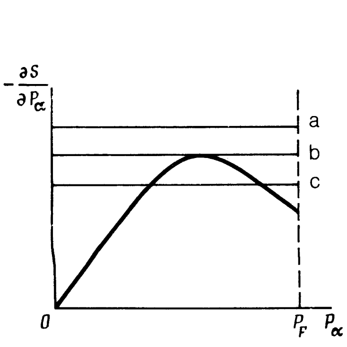

FIG. 1. Graphic solution of Eq. (1) written in the form $\partial S/\partial P_\alpha=2\pi elB/c$ for three different values of the magnetic field B (it is assumed that $\omega=0$).

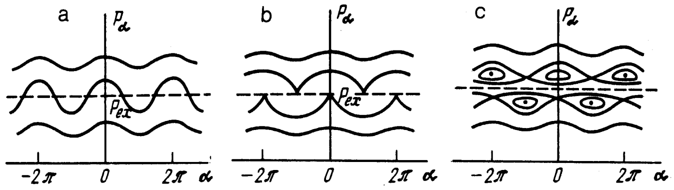

FIG. 2. Phase portrait of the dynamic system (11) in the vicinity of $P_\alpha=P_{ex}$. In case a the resonance condition (1) is not met, in cases b and c the phase trajectories were plotted with the aid of the Hamiltonians (16) and (18), respectively.

Obviously, for perpendicular propagation of the wave the momentum p_z is conserved, and enters in the Hamiltonian as a parameter. In contrast to free electrons, in metals with complicated spectra there is realized as a rule the nondegenerate case, i.e., $\partial^2\varepsilon/\partial I^2\neq 0$. Therefore, introducing the resonance phase and action, we write down the effective Hamiltonian for the lth resonance defined by the condition (2) by retaining in (21) only one resonant term with $m=l$. Introducing next the slow phase $\alpha=l\vartheta-\omega t+\varphi_l$ and neglecting in (22) small terms of order V_m we obtain the equations of motion for the lth resonance in the form

$$\dot I=-lV_l\sin\alpha,\qquad \dot\alpha=l\omega_c-\omega.\tag{23}$$

The Hamiltonian corresponding to the system (23) can be expressed as

$$H=l\varepsilon(I)-\omega I-lV_l\cos\alpha.\tag{24}$$

Expanding next (24) in terms of small deviations from the resonance action I_l determined from condition (2), we arrive again at the standard mathematical-pendulum Hamiltonian:

$$H=\frac{G}{2}l(I-I_l)^2-lV_l\cos\alpha,\tag{25}$$

where

$$G=\left(\frac{\partial^2\varepsilon}{\partial I^2}\right)_{I=I_l}\equiv\omega_c'.$$

The characteristic oscillation frequency at cyclotron resonance is correspondingly

$$\tilde\omega_l=l(GV_l)^{1/2},$$

and the resonance width is given by

$$\mathcal{J}=(V_l/G)^{1/2}.$$

We shall need the results of the present section to solve the Boltzmann kinetic equation and find the distribution functions of the resonance particles.

3. DISTRIBUTION FUNCTION OF THE RESONANCE PARTICLES

The kinetic equation, with action and angle variables, in the field of a longitudinal sound wave propagating at an angle φ to the magnetic-field direction, is of the form

$$\frac{\partial f}{\partial t}+\dot{P}\frac{\partial f}{\partial P}+\dot{I}\frac{\partial f}{\partial I}+\dot{\psi}\frac{\partial f}{\partial \psi}+\dot{\vartheta}\frac{\partial f}{\partial \theta}=\frac{f-f_0}{\tau_p}, \quad (26)$$

where $\dot{P}$, $\dot{I}$, $\dot{\psi}$, and $\dot{\vartheta}$ are defined with the aid of (8) as follows:

$$\dot{P}=-\sum_m V_m \sin(\psi-m\vartheta+\varphi_m),$$

$$\dot{I}=\sum_m mV_m \sin(\psi-m\vartheta+\varphi_m),$$

$$\dot{\psi}=k_\parallel\bar{v}_z-\omega-\sum_m \frac{\partial V_m}{\partial P}\cos(\psi-m\vartheta+\varphi_m),$$

$$\dot{\vartheta}=\omega_c-\sum_m \frac{\partial V_m}{\partial I}\cos(\psi-m\vartheta+\varphi_m).$$

The collision integral is written in the relaxation-time approximation, τ_p is the departure time, and $f_0(H+\omega P)$ is the local-equilibrium Fermi distribution function. Putting as usual $f=f_0+g$, we obtain for the g function the equation:

$$\frac{\partial g}{\partial t}+\dot{P}\frac{\partial g}{\partial P}+\dot{I}\frac{\partial g}{\partial I}+\dot{\psi}\frac{\partial g}{\partial \psi}+\dot{\vartheta}\frac{\partial g}{\partial \vartheta}+\frac{g}{\tau_p}$$
$$=\omega f_0'\sum_m V_m \sin(\psi-m\vartheta+\varphi_m). \quad (27)$$

Since we assume that the sound attenuates little over one wavelength and we seek stationary solutions in the field of a wave with specified amplitude, we put $\partial g/\partial t=0$.

To obtain Eq. (27) in the vicinity of the lth resonance, it is convenient to change over to the resonance variables P_α, α, P_β, β, and defined in accordance with (10). It is necessary next, as in Ref. 5, to expand the g function in a Fourier series with respect to the fast phase β and average with respect to it in the Boltzmann equation. After averaging, the equation for the "slow" part of the g function takes the form

$$(k_\parallel\bar{v}_z-\omega-l\omega_c)\frac{\partial g}{\partial \alpha}-V_l\sin\alpha\frac{\partial g}{\partial P_\alpha}+\frac{g}{\tau_p}=\omega V_l f_0'\sin\alpha. \quad (28)$$

As shown in Ref. 5, the fast terms and distribution functions discarded in the averaging are of higher order of smallness in the wave amplitude compared with the slow component g.[1)]

We consider first the case when the lth resonance determined by (1) does not land on an extremum of $\partial S/\partial P_\alpha$. We can then confine ourselves near resonance to terms linear in P_α in the expansion of the function

$$k_\parallel\bar{v}_z-\omega-l\omega_c\approx G(P_\alpha-P_l).$$

Introducing next the dimensionless velocity $s=(P_\alpha-P_l)/\tilde{P}$ and the dimensionless time $\tau=\tilde{\omega}_l t$, we get

$$s\frac{\partial g}{\partial \alpha}-\sin\alpha\frac{\partial g}{\partial s}+a_l g=\omega\tilde{P}_l f_0'\sin\alpha, \quad (29)$$

where $\tilde{P}_l=(V_l/G)^{1/2}$, $\tilde{\omega}_l=(GV_l)^{1/2}$, and $a_l=(\tilde{\omega}_l\tau_p)^{-1}$. A solution of Eq. (29) by the method of characteristics is given in Ref. 5 (see also Ref. 6). In our case it can be written in the form

$$g_t=\omega\tilde{P}_l f_0'[a_l\alpha-s],$$
$$g_{ut}=\omega\tilde{P}_l f_0'[a_l(\alpha-\bar{\alpha})-(s-\bar{s})], \quad (30)$$

where g_t and g_{ut} are respectively the distribution functions of the trapped and untrapped particles,

$$\bar{s}=\frac{\pi}{\varkappa\mathbf{K}(\varkappa)},\quad \bar{\alpha}=\pi\frac{F(\alpha/2,\varkappa)}{\mathbf{K}(\varkappa)},$$
$$\varkappa^{-2}=\frac{s^2}{4}+\sin^2\frac{\alpha}{2},$$

$F(\alpha/2,\varkappa)$ and $\mathbf{K}(\varkappa)$ are incomplete and complete elliptic integrals of the first kind. Trapped particles correspond to $|\varkappa|>1$ and untrapped ones to $|\varkappa|<1$.

We turn now to the case when the lth resonance coincides with an extremum of $\partial S/\partial P_\alpha$. As shown in the preceding section, the expansion of the function $k_\parallel\bar{v}_z-\omega-l\omega_c$ begins then with the quadratic terms: $k_\parallel\bar{v}_z-\omega-l\omega_c\approx R(P_\alpha-P_l)^2$. Changing again to dimensionless variables

$$s=(P_\alpha-P_l)/\tilde{P}_{ex},\quad \tau=\tilde{\omega}_{ex}t,$$

we obtain the kinetic equation

$$s^2\frac{\partial g}{\partial \alpha}-\sin\alpha\frac{\partial g}{\partial s}+a_{ex}g=\omega\tilde{P}_{ex}f_0'\sin\alpha, \quad (31)$$

where

$$\tilde{\omega}_{ex}=(RV_l^2)^{1/3},\quad \tilde{P}_{ex}=(V_l/R)^{1/3},\quad a_{ex}=(\tilde{\omega}_{ex}\tau_p)^{-1}.$$

The solution of (31) is similar to that of (29)

$$g=\omega\tilde{P}_{ex}f_0'[a_{ex}(\xi-\bar{\xi})-(s-\bar{s})], \quad (32)$$

but

$$\bar{s}=\frac{1}{T}\int_0^T s(\tau')\,d\tau'$$

(T is the period) is not expressed in terms of elliptic integrals, while the variables ξ and $\bar{\xi}$, now equal to

$$\xi=\int s(\tau)\,d\tau,\quad \bar{\xi}=\bar{s}\tau,$$

differ from the previously introduced α and $\bar{\alpha}$.

To conclude this section, we obtain the solution of the kinetic equations under conditions of cyclotron resonance in a perpendicular magnetic field.

It is easy to verify that the kinetic equation for the averaged distribution function takes, in accord with the Hamiltonian (24), the form

$$(l\omega_c-\omega)\frac{\partial g}{\partial \alpha}-lV_l\sin\alpha\frac{\partial g}{\partial I}+\frac{g}{\tau_p}=\omega V_l f_0'\sin\alpha. \quad (33)$$

Changing in (33) to the dimensionless action and dimensionless time

$$X=(I-I_l)/\mathcal{J},\quad \tau=\tilde{\omega}t,$$

where

$$\mathcal{J}=(V_l/\omega_c')^{1/2},\quad \tilde{\omega}=l(\omega_c'V_l)^{1/2},\quad \omega_c'=\left(\frac{\partial\omega_c}{\partial I}\right)_{I=I_l}$$

and solving this equation by the method of characteristics, we get

$$g=\int_{-\infty}^{\tau}\exp[a(\tau'-\tau)]\frac{dX}{d\tau'}\delta(X(\tau')-X_F)\,d\tau'. \quad (34)$$

Here, as before, $a=(\tilde{\omega}\tau_p)^{-1}$ is the nonlinearity parameter. The value of X_F is defined by the relation

$$X_F = \frac{l}{\omega J}(\varepsilon_F - \varepsilon(I_l, p_z)) \approx (I_F - I_l)/J \quad (35)$$

and specifies the distance, along the action axis I, between the resonance surface $I_l(p_z)$ and the Fermi surface for the given value of p_z, while $I_F(p_z)$ is the action on the Fermi surface.

Let us calculate the integral in (34). Recognizing that the integrand is, apart from an exponential, a periodic function of time with periods $T(\varkappa) = 2\varkappa \mathbf{K}(\varkappa)$ and $T(\varkappa) = 4\mathbf{K}(1/\varkappa)$ for untrapped and trapped particles, we can change over in (34) to integration over a single period. The integral over the period is evaluated with the aid of a delta-function and, in an approximation linear in the small parameter a, we have

$$g(\tau, \varkappa) = \begin{cases} 1-2\tau_0/T - a(1-2\tau_0/T)\tau, & 0 \leqslant \tau < \tau_0 \\ -2\tau_0/T + a(2\tau_0/T)(\tau - T/2), & \tau_0 < \tau < T-\tau_0 \\ 1-2\tau_0/T - a(1-2\tau_0/T)(\tau - T), & T-\tau_0 < \tau < T, \end{cases} \quad (36)$$

where τ_0 is the instant of time at which $X(\tau) = X_F$. From the equations of motion we obtain

$$\tau_0 = \varkappa F\left(\arcsin\left(\frac{1}{\varkappa^2} - \frac{X_F^2}{4}\right)^{1/2}, \varkappa\right)$$

for untrapped particles and

$$\tau_0 = F\left(\arccos\frac{\varkappa X_F}{2}, \varkappa^{-1}\right)$$

for trapped ones.

It must be noted that the function (36) describes, at fixed $\varkappa$, the distribution of the untrapped particles if the parameter $X_F(p_z)$ is in the range

$$\frac{4}{\varkappa^2}(1-\varkappa^2) \leqslant X_F^2 \leqslant \frac{4}{\varkappa^2}, \quad X_F > 0, \quad \varkappa > 0, \quad (37)$$

and the distribution of the trapped ones is in the range

$$0 \leqslant X_F^2 \leqslant \frac{4}{\varkappa^2}, \quad X_F \geqslant 0. \quad (38)$$

In the opposite case the distribution function vanishes. The shaded regions in Fig. 3 correspond, according to (37) and (38), to allowed values of the parameter X_F (or of the momentum p_z) for a given $\varkappa$ corresponding to untrapped or trapped electrons.

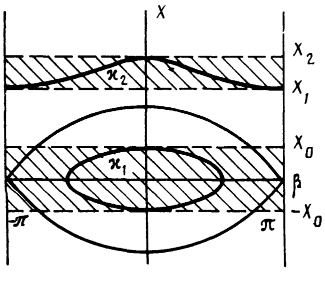

FIG. 3. Regions of the values of the parameter X_F (shaded sections), in which the g-function differs from zero at a fixed value of $\varkappa$. For $\varkappa = \varkappa_2$ the values of X_F lie in the interval $X_1 \leqslant X_F \leqslant X_2$ ("untrapped" particles); for "trapped" particles we have $-X_0 \leqslant X_F \leqslant X_0$ for $\varkappa = \varkappa_1$.

4. SOUND-ABSORPTION COEFFICIENT. DISCUSSION OF RESULTS

The obtained distribution functions yield the sound-damping coefficient in a metal with a complicated dispersion law, placed in an external uniform magnetic field. We calculate to this end the work performed by the wave on the resonance particles (the angle brackets $\langle ... \rangle$ denote averaging over the wavelength)

$$A = \left\langle n_p \frac{\partial U}{\partial t} \right\rangle,$$

and use the energy-balance equation in the wave + particle system

$$\frac{dS_0}{d\zeta} = -A = -2\Gamma S_0, \quad (39)$$

where

$$U = -\sum_m V_m \cos(\psi - m\vartheta + \varphi_m)$$

is the potential energy of the particle in the sound-wave field, $\zeta = (\mathbf{kr} - \omega t)/k$,

$$n_p = \frac{2}{(2\pi\hbar)^3}\int g\, d\mathbf{p}$$

is the nonequilibrium density of the resonant particles, Γ is the damping coefficient, and S_0 is the sound-wave average energy-flux density.

In an oblique magnetic field, the coefficient of damping by isolated resonances that do not coincide with an extremum of $\partial S/\partial P_\alpha$, is calculated with the aid of (30) and (39) just as in the case of an isotropic quadratic spectrum, i.e., by analogy with Ref. 5. The result can be written in the form

$$\Gamma = \sum_l \gamma_l 2a_l, \quad (40)$$

where the summation is over all the resonances on the Fermi surface, and γ_l is the linear damping coefficient in the lth resonance and is equal to

$$\gamma_l = \frac{4\pi^3 m_c^2(P_l)\,\omega}{(2\pi\hbar)^3 \rho c_0^2 \cos\varphi\, |\partial^2 S/\partial p_z^2|}\left(\frac{V_l}{ku_0}\right)^2; \quad (41)$$

while c_0 is the speed of sound. The difference between the nonlinear damping coefficient and the linear one is due to the appearance of an additional small factor $2a_l \ll 1$ indicative of the efficiency of electron capture by the resonances.

To analyze the dependence of the absorption coefficient (40) on the magnetic field, we examine the behavior of each of the coefficients $V_l(P_l)$ [see Eq. (9)] that determine the contribution of an individual resonance. It is known that in the linear theory the absorption is a maximum if the plane $p_z = p_l$ is tangent to the strip $\mathbf{k}\cdot\mathbf{v} = \omega$ on the Fermi surface, i.e., at the "boundary" points $p_z = p_b$. The coefficients V_l are usually determined at the boundary point itself (or far from it) by the stationary-phase method.[2,3] To describe the waveform of the oscillations, however, we must know the coefficients V_l not only at the boundary point, but also in some of its vicinity. To calculate the integral (9) we shall therefore use the following procedure. Expanding the argument of the exponential in terms of the angle ϑ in the vicinity of the value $\vartheta = w(p_z)$ ($w(p_z)$ is the solution of the equation

$$\mathbf{k}\left(\frac{d\mathbf{v}}{d\vartheta}\right)_{\vartheta = w} = 0,$$

and we assume here that $w(p_b)=0$), we get

$$\Phi_l(\vartheta)=\mathbf{kr}-\omega t-\psi+l\vartheta\approx\Phi_l(w)+\Phi_l'(w)(\vartheta-w)$$
$$+\frac{\Phi_l'''(w)}{6}(\vartheta-w)^3,$$

where it is taken into account that $\Phi_l''(w)\sim\mathbf{k}\cdot\mathbf{v}'\equiv 0$. Calculating the derivatives $\Phi_l'(w)$ and $\Phi_l'''(w)$ and substituting their values in (9), we get

$$V_l=ku_0\Lambda(w(p_l))\left(\frac{2\omega_c}{\mathbf{kv}''(w(p_l))}\right)^{1/3}\mathrm{Ai}(x_l), \qquad (42)$$

where $\mathrm{Ai}(x_l)$ are Airy functions, and x_l, equal to

$$x_l=\left(\frac{2\omega_c}{\mathbf{kv}''(w)}\right)^{1/3}\frac{\mathbf{kv}(w)-\omega}{\omega_c}, \qquad (43)$$

is taken at the resonance point $p_z=p_l$. Thus, according to (42) and (43) the Fourier component of $V_l(P_l)$ is a maximum when the plane $p_z=p_l$ is tangent to the strip $\mathbf{k}\cdot\mathbf{v}=\omega$, i.e., at the boundary point. At the tangency point the angle w is equal to zero, as is also the argument x_l of the Airy function. It is easy to verify that $\mathbf{k}\cdot\mathbf{v}(w)-\omega$, and with it x_l, becomes larger than zero for convex sections of the Fermi surface, and that the Airy function, and hence also the coefficient V_l, decreases rapidly when the resonance moves farther away from the boundary point. If $p_l<p_b$, however, the argument of x_l becomes negative when the magnetic field is decreased, and the Airy function oscillates. This behavior of the coefficients V_l explains, according to (40) and (41), the magnetoacoustic oscillations of the absorption coefficient: its next burst is produced by the resonance reaching the boundary point. The oscillation period of the absorption coefficient as a function of the magnetic field does not depend on the wave amplitude and is determined, just as in the linear theory, by the expression

$$\Delta(B^{-1})=\frac{2\pi e}{ck\cos\varphi\,|\partial S/\partial p_b|_{rp}}.$$

It follows from the foregoing that the nature of the magnetoacoustic oscillations and their form are the same in the linear and nonlinear regimes, but in the latter case, according to (40), the damping coefficient is decreased by $2a_l$ times as a result of particle capture.

Everything said above concerning the behavior of the coefficients V_l holds true under the condition $kr_L\gg 1$ (r_L is the characteristic dimension of the orbit in a plane perpendicular to the magnetic field).

As noted above, magnetoacoustic oscillations can be due also to the lth resonance reaching an extremum of the function $\partial S/\partial p_z$. At this point, on the one hand, the electron-state density is high, and on the other the dynamics of the electron at the lth resonance is not standard. It is of interest therefore to consider the magnetoacoustic oscillations connected with this extremal point.

By calculating the work A performed by the wave on the particles at the lth resonance and using the distribution functions (32) and the balance equation (39), we can obtain for the nonlinear absorption coefficient the expression

$$\Gamma_l=\Gamma_{0l}Ca_{ex}^{1/2}, \qquad (44)$$

where C is given by the integral

$$C=\frac{2^{1/2}}{\pi^2}\int_{-\infty}^{\infty}ds\int_{-\pi}^{\pi}d\alpha[\xi(s,\alpha)-\bar{\xi}(s,\alpha)]\sin\alpha.$$

The calculations yield $C=1.98$, and Γ_{0l} is the linear damping coefficient at the extremal point. Equation (44) determines thus the decrease of the amplitude of the absorption peaks with increase of the sound intensity. It must be noted that in contrast to the oscillations connected with the boundary points, where the damping decreases according to (40) like $u_0^{-1/2}$, the absorption peaks connected with the extremum $\partial S/\partial p_z$ decrease like u_0^{-1}.

In the nonlinear regime there is not only a decrease of the oscillation amplitude, but also an increase of the widths of the resonance peaks. It was noted earlier that the resonance described by the Hamiltonian (18) takes place when the resonance value of P_l occurs exactly at the point P_{ex} in the magnetic field B_l. When the magnetic field deviates from this value, the resonance splits into two, as shown in Figs. 2b and 2c. A change from one type of motion described by the Hamiltonian (18) to motion with a pendulum-type Hamiltonian (17) takes place when the separation of the resonances is of the order of $\tilde{P}_{ex}$. Obviously, it is this transition which determines the absorption width $\delta(B^{-1})$ as a function of the reciprocal magnetic field in the nonlinear regime. Using the definition of $\tilde{P}_{ex}$ and the resonance condition, we can obtain $\delta(B^{-1})$:

$$\delta(B^{-1})=\frac{\Delta(B^{-1})}{k_\parallel l_p}\frac{1}{a_{ex}}, \qquad (45)$$

where l_p is the electron mean free path. Thus, in accordance with (45), the peak width $\delta(B^{-1})$ is a_{ex}^{-1} times larger than the linear width (see, e.g., Ref. 7).

We turn now to calculate the work performed by the wave on the particles and to determine the sound-absorption coefficient for the case of perpendicular propagation. The starting point for the calculation of the work per unit time is the expression

$$A=\frac{2}{(2\pi\hbar)^3}\frac{1}{\lambda}\int dx\,d\mathbf{p}g\frac{\partial U}{\partial t}. \qquad (46)$$

Changing in (46) to integration over the new variables x_0, p_z, X, and α, and retaining in the potential energy U only the resonance term $-V_l\cos\alpha$, we obtain after integrating with respect to x_0 the work on the lth resonance:

$$A=\frac{2}{(2\pi\hbar)^3}\frac{eB\omega}{c}\int dp_z\,dX\,d\alpha JV_lg\sin\alpha.$$

Before proceeding with the calculations we must note the following. We know that the main contribution to the absorption coefficient in acoustic cyclotrom resonance is made by electrons on the section with a cyclotron frequency that is extremal with respect to p_z. It is precisely for these electrons that we calculate the work A when the resonance condition $\omega=l\omega_c^{ex}$ is met. The final result does not depend on whether the cyclotron frequency is a maximum or a minimum and, to be specific, we shall assume that the resonance lands on a section with the minimum cyclotron frequency. For these electrons the connection between p_z and the parameter X_F is, according to (35),

$$X_F(p_z)\approx\frac{X_F''(p_{ex})}{2}(p_z-p_{ex})^2, \qquad (47)$$

where the value $p_z=p_{ex}$ corresponds to an extremum of the cyclotron frequency. The expansion has no linear term, for it can be easily verified that

$$X_F' = \frac{1}{J\omega_c'}\left(\frac{\partial \omega_c}{\partial p_z}\right)_\varepsilon$$

and vanishes when the lth resonance reaches a section with extremal cyclotron frequency. The value of the second derivative X_F'' on the extremum is

$$X_F'' = \frac{1}{J\omega_c'}\left(\frac{\partial^2 \omega_c}{\partial p_z^2}\right)_\varepsilon .$$

We change now to integration over the new variables $X_F, \varkappa$, and τ. With allowance for (47) we get

$$dp_z\, dX\, d\alpha = -\frac{4}{\varkappa^3}\frac{d\varkappa\, dX_F\, d\tau}{(X_F''(p_{ex})X_F)^{1/2}} .$$

After integrating and using Eq. (34) [the limits of the integration with respect to dX_F are given by (37) and (38)] we get ultimately, for magnetic fields B ensuring satisfaction of the resonance condition $\omega = l\omega_c^{\rm ex}$,

$$A = \frac{8\omega e\tilde{J}V_l ab}{(2\pi\hbar)^3 c[X_F''(p_{ex})]^{1/2}}\left\{16\int_0^1 \frac{d\varkappa}{\varkappa^{7/2}}\left(\mathscr{E}(\varkappa) - \frac{\pi}{2}\frac{\mathscr{K}(\varkappa)}{\mathbf{K}(\varkappa)}\right) + \frac{91\cdot 2^{1/2}}{25}\left[2\mathbf{E}\left(\frac{1}{2^{1/2}}\right) - \mathbf{K}\left(\frac{1}{2^{1/2}}\right)\right]\right\}, \qquad (48)$$

where

$$\mathscr{E}(\varkappa) = \int_0^{\pi/2} dz\,(1-\varkappa^2\sin^2 z)^{1/4}, \qquad \mathscr{K}(\varkappa) = \int_0^{\pi/2} dz\,(1-\varkappa^2\sin^2 z)^{-1/4}.$$

The numerical value of the integral in the square brackets in (48) is 0.038. Using (48) and the definition (39) of the absorption coefficient of sound at the cyclotron-resonance point $\omega = l\omega_c^{\rm ex}$:

$$\Gamma = 2.11\cdot a^{3/2}\gamma_0, \qquad (49)$$

where

$$\gamma_0 = \left[2\pi^2 m_c\omega/(2\pi\hbar)^3\rho c_0^3\left(\frac{\nu}{\omega}\frac{1}{m_c}\frac{\partial^2 m_c}{\partial p_z^2}\right)^{1/2}\right]\left(\frac{V_l}{ku_0}\right)^2 \qquad (50)$$

is the linear damping coefficient. It can be shown that γ_0 in (50) agrees fully with the damping coefficient obtained by Kaner.[3]

Thus, in the linear regime, if the resonance condition $\omega = l\omega_c^{\rm ex}$ is strictly met, i.e., practically at the absorption maximum, the damping coefficient decreases in proportion to the small nonlinearity parameter raised to the 3/2 power. In the Landau nonlinear-damping theory the damping coefficient is usually linear in the parameter a. In the case considered the additional decrease of the damping is due to the relative (compared with the linear regime) decrease of the characteristic number of electrons participating in the absorption near the section $p_z = p_{\rm ex}$. The additional small factor $a^{1/2}$ appears upon integration over dp_z: whereas in the linear regime the p_z width of the interval of effective electrons is proportional to $\nu^{1/2}$, in the nonlinear regime the effective region is proportional to $\tilde{\omega}^{1/2}$. On the other hand, it is well known that if the coefficient Γ is linear in the parameter a, the width of the characteristic p_z region in the linear and nonlinear regimes is determined respectively by the frequencies ν and $\tilde{\omega}$. Note that if the acoustic-cyclotron resonance condition is met for an electron group having a cyclotron frequency $\omega_c(p_z) \neq \omega_c^{\rm ex}$, it can be shown that in this case we have

$$\Gamma_l = 2a_l\gamma_{0l}$$

for any resonance (γ_{0l} is the linear absorption coefficient).

A change in the nonlinear regime should also take place in the width (with respect to the reciprocal magnetic field) of the resonance-absorption peak. This width can be determined from the condition that the distance from the resonance surface to the Fermi surface should be of the order of $\tilde{J}$ in the action variable at a p_z value corresponding to an extremum of the cyclotron frequency. It is easy hence, using the resonance condition, to obtain an expression for the absorption-peak width in the nonlinear regime:

$$\delta(B^{-1}) = \frac{le}{c\omega}\frac{m_c'}{m_c}\tilde{J}, \qquad (51)$$

where

$$m_c' = \left(\frac{\partial m_c}{\partial I}\right)_{\varepsilon=\varepsilon_F, p_z=p_{ex}} .$$

The ratio of the peak width to the oscillation period $\Delta(B^{-1})$ in the reciprocal magnetic field is obviously $\tilde{\omega}/\omega_c^{\rm ex}$. We note in addition that according to (51) the nonresonance peaks broaden in the nonlinear regime, so that

$$\delta(B^{-1}) = \delta_L(B^{-1})/a, \qquad (52)$$

where $\delta_L(B^{-1})$ is the peak width in the linear regime.

We note in conclusion that in the present paper the resonances (1) and (2) appearing in oblique and perpendicular propagation of a sound wave were assumed to be isolated, i.e., the frequency difference ω_c between the resonances was much larger than the resonance width $\tilde{\omega}$. As shown in Ref. 5, with contemporary experimental techniques it is possible to realize the inverse situation:

$$K = \frac{2\tilde{\omega}}{\omega_c} \gg 1$$

and obtain resonance overlap on a larger part of the Fermi surface in a metal or semimetal. Under these conditions the electron dynamics becomes stochastic, and magnetoacoustic oscillations and acoustic cyclotron resonance cannot be described in the context of the theory developed here. Such a calculation will be carried out in a separate paper.

It should also be noted that nonlinear magnetoacoustic oscillations were investigated earlier by Kozub.[9] His main assumption was satisfaction of the inequality

$$b = \frac{m^* v\omega_c}{k^2\Lambda u_0} \leqslant 1, \qquad (53)$$

meaning that the Lorentz force acting on an electron is weaker than the acoustic-wave strain force. In addition, it was assumed that the angle φ between the magnetic field and the wave vector of the sound is close to $\pi/2$, and that the force acting on an electron in the magnetic-field direction is weak while the momentum p_z remain constant. In contrast to Ref. 9, we assume in the present paper an angle $\varphi \sim 1$ and a strong magnetic field

$$K \ll 1,$$

meaning that the parameter b defined by (53) satisfies in this case the inequality

$$b \gg 1.$$

For this reason, the nonlinear behavior, investigated in the present paper, of magnetoacoustic oscillations differs substantially from Kozub's results.[9] He predicted, in particular,[9] for the $\Gamma(B^{-1})$ curve new peaks which are missing from the linear theory and are connected with the existence of resonances defined by the condition

$$k_{\parallel}\bar{v}_z - \omega - \frac{l}{n}\omega_c = 0, \qquad n = 2, 3, 4, \ldots \tag{54}$$

It can be shown[5] that under conditions whose satisfaction was assumed above the absorption at half-integer resonances ($n = 2$) is proportional to the square of the amplitude of the sound wave, whereas absorption at the fundamental resonances (1) is proportional to $u_0^{-1/2}$ (see (40)]. Consequently, observation of additional absorption peaks caused even by the strongest half-integer resonances is not very likely in our case.

[1)]Equation (28) can also be obtained directly from the effective Hamiltonian (11).

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Translated by J. G. Adashko