# Electromagnetic radiation emitted by ultrarelativistic particles scattered in an excited medium

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The interaction of relativistic particles with a gaseous or a condensed medium, where a high density of nondegenerate quantum states is generated, is accompanied by coherent conversion of atomic or molecular excitations into hard electromagnetic radiation.

In an analysis of hard radiation processes that accompany the passage of relativistic particles through condensed media it is usual to consider a medium as a source of an external static field in which a particle is scattered, loses momentum, and emits bremsstrahlung photons. Strictly speaking, in a complete description of radiative effects associated with the interaction of a fast particle with a many-body quantum system we need to allow for the exchange not only of the momentum but also of the energy with the system, accompanied by quantum transitions between the states of the system itself. The case of a high density of excited states is particularly interesting. A relativistic particle crossing a medium of this kind in a time shorter than the decay time of excited states is scattered in an electromagnetic field created by the currents representing transitions in the medium. In addition to the usual bremsstrahlung radiation, which is due to the Coulomb collisions, there is a further contribution resulting from the Compton scattering of the electromagnetic field in the medium by the relativistic particle. The Compton scattering cross section is small and it is customary to ignore the contribution of such scattering to hard electromagnetic radiation.

Coherent radiation emitted by a charge moving uniformly in an excited substance was first considered by Ryazanov (for details see his review paper[1] and the literature given there). The processes resulting in coherent radiation discussed in Ref. 1 are not due to acceleration of a charged particle but represent generalization of the Vavilov–Cherenkov radiation to the case of an excited substance. Nevertheless, in a number of cases when, for example, excited states of particles in a system are nondegenerate or are preconditioned in a special manner the value of the transverse acceleration of a relativistic particle (averaged over the particle positions in a system) is not zero and the emitted radiation includes a coherent contribution (as a result of coherent conversion of excitations) of spectral density with a maximum in the hard part of the spectrum and proportional to the square of the particle concentration in the system. This last feature ensures that the output of the hard radiation is significant and comparable with the incoherent bremsstrahlung background.

## 1. PRINCIPAL EQUATIONS

The classical equations of motion of a relativistic particle in an electromagnetic field are (here and later we shall assume that $\hbar = c = 1$)

$$\frac{d\mathbf{p}}{\partial t} = -e\frac{\partial \mathbf{A}}{\partial t} - e\nabla\varphi + e[\mathbf{v},\mathrm{curl}\mathbf{A}], \quad (1)$$

where $\mathbf{p} = m\mathbf{v}/1(-\mathbf{v}^2)^{1/2}$ is the momentum of a relativistic particle.

The retarded potentials of the external field are governed by the densities of the current and charge, which appear as a result of transitions between the states of a nonrelativistic quantum system in which the scattering takes place:

$$\mathbf{A}(\mathbf{r},t) = \sum_\alpha \int d\mathbf{r}'\mathbf{j}_\alpha(\mathbf{r}'-\mathbf{r}_\alpha)\exp\{-i\Omega_\alpha t + i\Omega_\alpha|\mathbf{r}-\mathbf{r}'|\}/|\mathbf{r}-\mathbf{r}'|, \quad (2)$$

$$\varphi(\mathbf{r},t) = \sum_\alpha \int d\mathbf{r}'\rho_\alpha(\mathbf{r}'-\mathbf{r}_\alpha)\exp\{-i\Omega_\alpha t + i\Omega_\alpha|\mathbf{r}-\mathbf{r}'|\}/|\mathbf{r}-\mathbf{r}'|, \quad (3)$$

$$\mathbf{j}_\alpha(\mathbf{r}'-\mathbf{r}_\alpha) = \frac{ie_\alpha}{2M_\alpha}\{\psi_i^\alpha(\mathbf{r}'-\mathbf{r}_\alpha)\nabla\psi_f^{\alpha *}(\mathbf{r}'-\mathbf{r}_\alpha) - \psi_f^{\alpha *}(\mathbf{r}'-\mathbf{r}_\alpha)\nabla\psi_i^\alpha(\mathbf{r}'-\mathbf{r}_\alpha)\}, \quad (4)$$

$$\rho_\alpha(\mathbf{r}'-\mathbf{r}_\alpha) = e_\alpha\psi_f^{\alpha *}(\mathbf{r}'-\mathbf{r}_\alpha)\psi_i^\alpha(\mathbf{r}'-\mathbf{r}_\alpha), \quad (5)$$

where $e_\alpha$ and $M_\alpha$ are the charge and mass of the $\alpha$th particle in the system; $\psi_i^\alpha$ and $\psi_f^\alpha$ are the wave functions of the initial and final states of this particle; $\Omega_\alpha = \Omega_\alpha^i - \Omega_\alpha^f$ is the transition energy.

The solution of the equations of motion (1) in the time-dependent fields of Eqs. (2) and (3) can be represented by a series in powers of a small parameter $1/E$, where $E$ is the energy of the incident particle (see, for example, Ref. 2). The transverse component of the acceleration of a particle can then be written in the form

$$\dot{\mathbf{v}}_\perp(t) = -\frac{e}{E}\left\{\frac{\partial \mathbf{A}_\perp}{\partial t} + \nabla_\perp\varphi - [\mathbf{v},\mathrm{curl}\mathbf{A}]_\perp\right\} + O\left(\frac{1}{E^2}\right). \quad (6)$$

The right-hand side of Eq. (6) contains the radius vector of unperturbed rectilinear motion $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, where $\mathbf{r}_0$ and $\mathbf{v}$ are the initial coordinate and the velocity of a particle.

The spectral density of the dipole radiation emitted in the frequency interval from $\omega$ to $\omega + d\omega$ is governed by the Fourier components of the transverse acceleration of Eq. (6) and is given by

$$\frac{dE}{d\omega} = \frac{e^2\omega}{2\pi}\int_\nu^\infty \frac{dp}{p^2}\left[1 - 2\frac{\nu}{p}\left(1 - \frac{\nu}{p}\right)\right]|\mathbf{W}(p)|^2, \quad (7)$$

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where

$$\nu=m^2\omega/2E^2,\quad \mathbf{W}(p)=\int dt\dot{\mathbf{v}}_\perp(t)e^{ipt}.$$

Equation (7) is invalid if $\omega\approx E$. However, within the limits of one coherence length the scattering angle of a particle is small compared with the characteristic angle of emission of radiation $m/E$, so that the expression for the spectral density, allowing for the recoil effect in the quasiclassical approximation, can be written in the form[3]:

$$\frac{dE}{d\omega}=\frac{e^2}{4\pi}\frac{E^2-E'^2}{EE'}\omega\int_{\nu'}^{\infty}\frac{dp}{p^2}\left[1-2\frac{\nu'}{p}\left(1-\frac{\nu'}{p}\right)\times\left(1-\frac{\omega^2}{E^2+E'^2}\right)\right]|\mathbf{W}(p)|^2, \tag{8}$$

where

$$E'=E-\omega,\qquad \nu'=m^2\omega/2EE'.$$

It follows from Eqs. (2)–(6) that the Fourier component of the transverse acceleration is (for simplicity it is assumed that $r_0=0$ and that the initial velocity $v$ is parallel to the $z$ axis):

$$\mathbf{W}(p)=\frac{2e}{E}\sum_\alpha\int d\mathbf{r}\exp\left[-\frac{i}{v}(\Omega_\alpha-p)z\right]\left\{aK_1(ar_\perp)\times\frac{\mathbf{r}_\perp}{r_\perp}[\rho_\alpha(\mathbf{r}-\mathbf{r}_\alpha)+j_\alpha''(\mathbf{r}-\mathbf{r}_\alpha)]-ipK_0(ar_\perp)\mathbf{j}_\alpha^\perp(\mathbf{r}-\mathbf{r}_\alpha)\right\}, \tag{9}$$

where

$$a^2=2\Omega_\alpha(\Omega_\alpha-p)+p^2,\qquad r_\perp=(x^2+y^2)^{1/2},$$

and $K_{0,1}(ar_\perp)$ are modified Bessel functions.

The relationships (7)–(9) are valid for $|A|\ll E$ and $\varphi\ll E$, and they provide a complete solution of the problem of the dipole radiation emitted by a relativistic particle scattered by a nonrelativistic quantum system with given current and charge densities due to quantum transitions between different states of this system if the absorption of the generated photons by a medium is ignored. This last assumption is justified in a description of generation of hard x-ray and gamma radiation in a medium of linear dimensions that do not exceed the characteristic absorption depth.

## 2. DIPOLE COHERENT CONVERSION IN A SYSTEM OF TWO-LEVEL ATOMS WHEN $\omega\ll E$

We shall consider the radiation generated as a result of the scattering of an ultrarelativistic particle by a system of identical two-level (i.e., $\Omega_\alpha=\Omega$) atoms some of which are in an excited state. In an analysis of the radiation generated as a result of scattering in a many-body quantum system we must average the spectral density over the coordinates of particles in the system. We can then distinguish between the coherent and incoherent contributions to the spectral density:

$$\frac{dE^{coh}}{d\omega}=\frac{e^2\omega}{2\pi}\int_\nu^\infty\frac{dp}{p^2}\left[1-2\frac{\nu}{p}\left(1-\frac{\nu}{p}\right)\right]|\langle\mathbf{W}(p)\rangle|^2, \tag{10}$$

$$\frac{dE^{incoh}}{d\omega}=\frac{e^2\omega}{2\pi}\int_\nu^\infty\frac{dp}{p^2}\left[1-2\frac{\nu}{p}\left(1-\frac{\nu}{p}\right)\right]\times\{\langle|\mathbf{W}(p)|^2\rangle-|\langle\mathbf{W}(p)\rangle|^2\}. \tag{11}$$

After averaging over the particle positions in a volume $V$, we find from Eq. (9) that

$$\langle\mathbf{W}(p)\rangle=-i\frac{8\pi^2ep}{Ea^2}\frac{1}{V}\sum_\alpha \mathbf{j}_\perp^\alpha\delta(\Omega-p), \tag{12}$$

where

$$\mathbf{j}_\perp^\alpha=-ie\Omega\mathbf{r}_\perp^\alpha$$

and $e\mathbf{r}_\perp^\alpha$ is the transverse component of the dipole moment of the $\alpha$th atom.

The summation in Eq. (12) is carried out over excited atoms, since for scattering accompanied by a transition from the ground to the excited state and for scattering without a change in the state we have $\Omega\leqslant0$ and, consequently, integration of the $\delta$ function in Eq. (10) causes the spectral density of the coherent radiation to vanish.[1)] Such processes contribute only to the incoherent radiation described by Eq. (11).

If we assume that the initial and final states of an atom are nondegenerate, we find that

$$\langle\mathbf{W}(p)\rangle=-\frac{8\pi^2e^2pn^*\Omega}{Ea^2}\langle\mathbf{r}_\perp\rangle\delta(\Omega-p),$$

where $n^*$ is the number of excited atoms per unit volume and

$$\langle\mathbf{r}_\perp\rangle=\int d\mathbf{r}\psi_f^*\mathbf{r}_\perp\psi_i.$$

In the case of multiple ($n$-fold) degeneracy of the initial and final states, we must average also over the orientations of the transverse component of the dipole moment, i.e.,

$$\langle\mathbf{r}_\perp\rangle=\frac{1}{N}\sum_{i,f}\int d\mathbf{r}\psi_f^*\mathbf{r}_\perp\psi_i,$$

where the summation is over the degenerate wave functions of the initial and final states in accordance with the selection rules for dipole transitions, and $N$ is the number of such allowed transitions. If this average value is zero, no coherent radiation described by Eq. (10) is produced since the average transverse acceleration is then also zero. If, however, the initial state of the system is preconditioned in a special manner so that the average dipole moment differs from zero (this can be done, for example, by pumping with high-power polarized radiation or by applying a magnetic field that lifts the degeneracy), the spectral density of coherent radiation per unit length is

$$\frac{dI^{coh}}{d\omega}=32\pi^2\frac{e^6n^{*2}|\langle\mathbf{r}_\perp\rangle|^2}{m^2\Omega}\times\frac{\omega}{\omega_m}\left[1-\frac{2\omega}{\omega_m}\left(1-\frac{\omega}{\omega_m}\right)\right]\theta(\omega_m-\omega), \tag{13}$$

where $\omega_m=2E^2\Omega/m^2$, and $\theta(x)$ is a unit Heaviside function. The upper edge of the spectrum is defined according to Eq. (13), by the inequality

$\omega \leqslant 2E^2\Omega/m^2$.

The spectral density of Eq. (13) has a maximum in the hard part of the spectrum at $\omega = \omega_m$ and it is proportional to the square of the concentration of atoms in the investigated medium. This last circumstance is an important difference which distinguishes this effect from the conventional Compton scattering of the radiation field in a medium on a relativistic beam. In the case of relativistic electrons of $E = 1$-GeV energy and $\Omega = 1$ eV we find that $\omega_m = 8$ MeV, whereas for protons of energy $E = 10$ TeV the corresponding value is $\omega_m = 0.2$ GeV. In this approximation the maximum coherent radiation intensity is independent of the particle energy and is given by

$$\left.\frac{dI^{coh}}{d\omega}\right|_{max} = 32\pi^2 \frac{e^6 n^{*2} |\langle \mathbf{r}_\perp \rangle|^2}{m^2\Omega}. \quad (14)$$

The spectral density of the incoherent radiation described by Eq. (11) and obtained as a result of scattering in a field of nuclei carrying a charge $Ze$ is described by the following expression in the spectral range $\omega \ll E$ (see, for example, Ref. 4)

$$\frac{dI^{incoh}}{d\omega} = \frac{16}{3}\frac{Z^2e^6n}{m^2}\left[\ln\frac{2E^2}{m\omega} - \frac{1}{2}\right],$$

where $n$ is the concentration of atoms in the investigated system.

The ratio of the yield of the coherent radiation in the region of the maximum to the incoherent contribution is

$$\eta = 6\pi^2 \frac{n^{*2}|\langle \mathbf{r}_\perp \rangle|^2}{nZ^2\Omega}\left[\ln\frac{m}{\Omega} - \frac{1}{2}\right]^{-1}. \quad (15)$$

If we substitute $n \approx n^* \approx 10^{22}$ cm$^{-3}$, $|\langle \mathbf{r}_\perp \rangle| \approx 10^{-8}$ cm, $\Omega \approx 1$ eV, and $Z = 4$ into Eq. (15), we find that $\eta \approx 5$, i.e., the coherent radiation of Eq. (13) may exceed the incoherent background in the region of the spectral density maximum if the optimal parameters of the nonequilibrium medium are selected.

The total energy losses due to the emission of the coherent radiation,

$$\Delta E^{coh} = \int_0^{\omega_m} \frac{dI^{coh}}{d\omega} d\omega = \frac{64\pi^2}{3} \frac{e^6 n^{*2} |\langle \mathbf{r}_\perp \rangle|^2 E^2}{m^4}$$

are proportional to the square of the particle energy and can exceed the total losses to incoherent bremsstrahlung radiation at high energies because such losses are directly proportional to the energy:

$$\Delta E^{incoh} = \frac{4Z^2e^6En}{m^2}\left[\ln\frac{2E}{m} - \frac{1}{3}\right].$$

In the case of the parameters of the medium given above, this becomes true in the range $E > 100$ GeV as demonstrated by the above expressions.

Figure 1 gives the spectral density of the bremsstrahlung radiation emitted by relativistic electrons in a nonequilibrium hydrogen plasma, calculated for different degrees of a population inversion of a $2P$–$1S$ resonant transition ($\Omega = 10.15$ eV). The calculations were carried out using Eqs. (16) and (21) (the latter is given below). It was assumed that excited atoms are in a state with a given value of the projection of the orbital momentum ($L = 1$, $M = 1$ or $L = 1$, $M = -1$). Such a state is realized, for example, when a system is pumped by a circularly polarized wave or as a result of Zeeman splitting. In the case of an equiprobable population of the states with different projections of the orbital momentum and the same transition energy the average value of the dipole moment and, consequently, the spectral density of the coherent radiation would be zero.

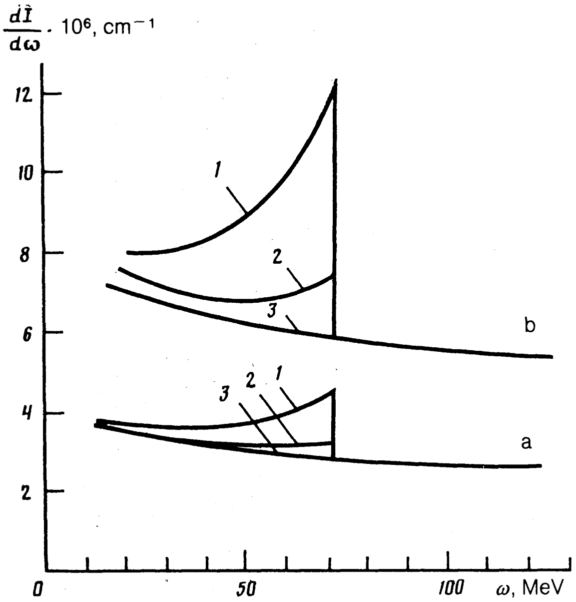


FIG. 1. Spectral density of the radiation emitted by relativistic electrons ($E = 1$ GeV) in a hydrogen plasma with atom concentrations $5\times 10^{19}$ cm$^{-3}$ (a) and $10^{20}$ cm$^{-3}$ (b), calculated for different degrees of population of an excited $2P$ state: 1) 100%; 2) 50%; 3) 0%.

It is clear from these calculated results that the coherent conversion process in a strongly excited hydrogen plasma can make a considerable contribution to the spectral density of the emitted hard radiation.

### 3. INFLUENCE OF THE RECOIL EFFECT AT $\omega \approx E$

Using Eq. (12), we obtained from Eq. (8) for the coherent radiation intensity an expression that reduces in the limit $\omega \ll E$ to the earlier result given by Eq. (13):

$$\frac{dI^{coh}}{d\omega} = 16\pi^2 \frac{E^2+E'^2}{EE'} \frac{e^6n^{*2}|\langle \mathbf{r}_\perp \rangle|^2}{m^2\Omega} \frac{E'}{E}\frac{\omega}{\omega_m'}\left[1 - \frac{2\omega}{\omega_m'} \times\left(1-\frac{\omega}{\omega_m'}\right)\left(1-\frac{\omega^2}{E^2+E'^2}\right)\right]\theta(\omega_m'-\omega), \quad (16)$$

where

$$\omega_m' = 2EE'\Omega/m^2.$$

According to Eq. (16), the upper edge of the spectrum is defined by the inequality

$$\omega \leqslant \frac{2E^2}{m^2}\Omega\frac{m^2}{m^2+2E\Omega}. \quad (17)$$

It is clear from Eq. (16) that the maximum intensity of the radiation generated as a result of coherent conversion of atomic excitations in the case when $\omega \approx E$ is reached, as in the $\omega \ll E$ case, at the edge of the spectrum. In particular, as

$E\to\infty$ we can easily show that the spectral density maximum is independent of the particle energy and is given by

$$\left.\frac{dI^{coh}}{d\omega}\right|_{max}=16\pi^2\frac{e^6n^{*2}|\langle \mathbf{r}_\perp\rangle|^2}{m^2\Omega}, \qquad (18)$$

i.e., it is half the value predicted by Eq. (13). According to Eq. (17), the edge of the spectrum obtained in the limit $E\to\infty$ is given by the expression

$$\omega_{max}=E-m^2/2\Omega. \qquad (19)$$

The coherence length $l$, corresponding to the emission of photons of energy given by Eq. (19), is

$$1/l=m^2\omega_{max}/2E(E-\omega_{max})=\Omega.$$

The mean-square multiple-scattering angle in the distance $l$ can be described with logarithmic precision by the expression

$$\langle\theta^2\rangle\approx 8\pi Z^2e^4nl/E^2. \qquad (20)$$

For a medium having the parameters $n\approx n^*\approx 10^{22}$ cm$^{-3}$, $Z=4$, and $\Omega=1$ eV it follows from Eq. (20) that the scattering angle within the coherence length $l$ is several orders of magnitude smaller than the characteristic radiation angle $m/E$. This means that Eq. (8) and, consequently Eqs. (16) and (18), remains valid also in the case of the coherent conversion process occurring at high energies when $\omega\lesssim E$.

The qualitative behavior of the spectral density (16) as a function of the electron energy is shown in Fig. 2. The tail of the spectral distribution observed at ultrahigh particle energies is characterized by a spectral density which—in contrast to the bremsstrahlung—does not fall but approaches a constant value given by Eq. (18). For comparison, we shall write down the yield of the incoherent bremsstrahlung radiation in the spectral range $\omega\approx E$ in the form [3]

$$\frac{dI^{incoh}}{d\omega}=\frac{4Z^2e^6n}{m^2}\frac{E-\omega}{E}\left[\frac{E}{E-\omega}+\frac{E-\omega}{E}-\frac{2}{3}\right]\times\left[\ln\frac{2E(E-\omega)}{m\omega}-\frac{1}{2}\right]. \qquad (21)$$

At $\omega=\omega_{max}$ it follows from Eq. (21) that

$$\frac{dI^{incoh}}{d\omega}=4\frac{Z^2e^6n}{m^2}\left[\ln\frac{m}{\Omega}-\frac{1}{2}\right].$$

The ratio of the intensities (18) and (21) differs from that given by Eq. (15) and amounts to

$$\eta=4\pi^2\frac{n^{*2}}{nZ^2\Omega}|\langle \mathbf{n}_\perp\rangle|^2\left[\ln\frac{m}{\Omega}-\frac{1}{2}\right]^{-1}.$$

## 4. CONCLUSIONS

Our analysis shows that the interaction of relativistic particles with a gaseous or a condensed medium characterized by a high density of nondegenerate excited quantum states is accompanied by coherent conversion of atomic or molecular excitations into hard electromagnetic radiation. The intensity of such radiation is proportional to the square of the concentration of excited atoms and has a maximum at the edge of the spectrum. The coherent conversion yield at the maximum can exceed the incoherent bremsstrahlung yield when the parameters of such a nonequilibrium medium are selected in an optimal manner.

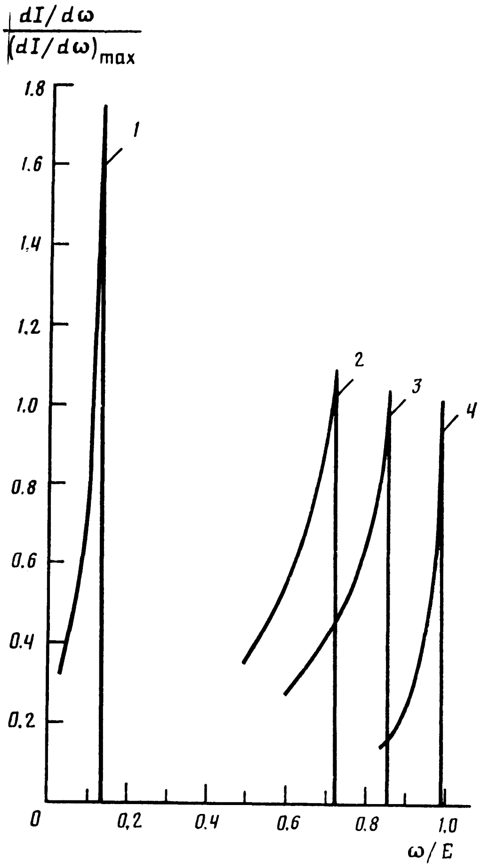


FIG. 2. Qualitative nature of the spectral density in coherent conversion involving relativistic electrons of energies 4.5 GeV (1), 70 GeV (2), 150 GeV (3), and 1 TeV (4) when $\Omega=4.5$ eV (corresponding to the maximum of the photoabsorption cross section of germanium).

[1)]The presence of the $\delta$ function in Eq. (12) means that the momentum $p$ transferred in the longitudinal direction has a fixed value $\Omega>0$ in the case of the coherent radiation described by Eq. (10). This circumstance, firstly, leads to a dependence of the photon energy on the emission angle $\omega=\Omega/(1-v\cos\theta)$ and, secondly, creates the right-hand edge of the spectral density of the output radiation [see Eq. (13) below].


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